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Extreme recoils: impact on the detection of gravitational waves from massive black hole binaries

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ABSTRACT

Recent numerical simulations of the coalescence of highly spinning massive black hole binaries (MBHBs) suggest that the remnant can suffer a recoil velocity of the order of few thousand $\mathrm{km\,s^{-1}}$. We study here, by means of dedicated simulations of black hole build-up, how such extreme recoils could affect the cosmological coalescence rate of MBHBs, placing a robust *lower limit* for the predicted number of gravitational wave (GW) sources detectable by future space-borne missions (such as the *Laser Interferometer Space Antenna*, *LISA*). We consider two main routes for black hole formation: one where seeds are light remnants of Population III stars ($\simeq 10^2\,\mathrm{M_\odot}$), and one where seeds are much heavier ($\gtrsim 10^4\,\mathrm{M_\odot}$), formed via the direct gas collapse in primordial nuclear discs. We find that extreme recoil velocities do not compromise the efficient MBHB detection by *LISA*. If seeds are already massive and/or relatively rare, the detection rate is reduced by only $\sim$15 per cent. The number of detections drops substantially (by $\sim$60 per cent) if seeds are instead light and abundant, but in this case the number of predicted coalescences is so high that at least $\sim$10 sources in a three-year observation are guaranteed.

Key words: black hole physics – gravitational waves – cosmology: theory.

1 INTRODUCTION

Massive black hole (MBH) binaries (MBHBs) are among the primary candidate sources of gravitational waves (GWs) at mHz frequencies, the range probed by the space-based *Laser Interferometer Space Antenna* (*LISA*, Bender et al. 1998). Today, MBHs are ubiquitous in the nuclei of nearby galaxies (see, e.g. Magorrian et al. 1998). If MBHs were also common in the past, and if their host galaxies experienced multiple mergers during their lifetime, as dictated by popular cold dark matter hierarchical cosmologies, then MBHBs inevitably formed in large numbers during cosmic history. Provided MBHBs do not 'stall', their inspirals, which is driven by radiation reaction, follow the merger of galaxies and protogalactic structures at high redshifts. MBHBs coalescing in less than a Hubble time would give origin to the loudest GW signals in the Universe, and a low-frequency detector such as *LISA* will be sensitive to GWs from binaries with total masses in the range 10^3–$10^7\,\mathrm{M_\odot}$ out to $z \approx 20$ (Hughes 2002).

The formation and evolution of MBHs has been investigated recently by several groups in the framework of hierarchical clustering cosmology (e.g. Menou, Haiman & Narayanan 2001; Volonteri, Haardt & Madau 2003; Koushiappas, Bullock & Dekel 2004). The inferred *LISA* detection rate, ranging from a few to a few hundred per year, were derived in a number of papers (Jaffe & Backer 2003; Wyithe & Loeb 2003; Sesana et al. 2004, 2005; Enoki et al. 2004; Rhook & Wyithe 2005). More recently, Sesana, Volonteri & Haardt (2007) investigated the imprint of MBH formation models on the expected MBHB coalescence rate, finding that at least $\sim$10 (considering a model that marginally reproduces the observational constrains and that can be taken as a robust lower bound) sources should be safely regarded as observable by *LISA*, assuming a 3-yr lifetime mission.

GWs emitted during the final plunge of the binary carry away a net linear momentum, causing a recoil of the MBHB centre of mass in the opposite direction (Redmount & Rees 1989). This GW recoil could have interesting astrophysical effects, as many coalescence remnants can be ejected from their host galaxies and dark matter haloes (e.g. Madau et al. 2004; Merritt et al. 2004; Micic, Abel & Sigurdsson 2006). This justifies the increasing effort to obtain accurate estimates of the recoil velocity. In the case of non-spinning black holes, the latest analytical (e.g. Favata, Hughes & Holz 2004; Blanchet, Qusailah & Will 2005; Damour & Gopakumar 2006) and numerical (Baker et al. 2006; Gonzalez et al. 2007) approaches are now both converging to maximum recoil velocities v_r in the range 100–250 $\mathrm{km\,s^{-1}}$ for binaries with mass ratio $q = m_2/m_1 \sim 0.4$ (m_1 and m_2, where $m_2 < m_1$, are the masses of the two binary members). The expected values are only slightly higher if the binary is eccentric; Sopuerta, Yunes & Laguna (2007) found $v_\mathrm{r} \propto (1+e)$.

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On the other hand, recent relativistic numerical simulations of spinning black hole binaries (Herrmann et al. 2007; Schnittman & Buonanno 2007) suggest that $v_{\rm r}$ increases linearly with the black hole spin parameter a, where $0 \leqslant a \leqslant 1$, and in the case of highly spinning black holes ($a > 0.8$) the magnitude of the kick suffered by the remnant could be of the order of a few thousand $\mathrm{km\,s^{-1}}$ (Tichy & Marronetti 2007). Campanelli et al. (2007) report values of $v_{\rm r}$ as high as $\sim$4000 $\mathrm{km\,s^{-1}}$ for equal mass binaries, if both spins lie in the binary orbital plane. Such a kick is sufficient to eject the remnant not only from a dwarf galaxy, where the escape velocity is $\sim$300 $\mathrm{km\,s^{-1}}$, but also even from the centre of a giant elliptical, for which the escape velocity can reach 2000 $\mathrm{km\,s^{-1}}$.

Though it is likely that MBHs acquire high spins (e.g. Volonteri et al. 2005) during their accretion history, the impact of the resulting recoil on the MBH assembly has never been studied in details so far. If extreme recoil is indeed the rule, the ejection of a large fraction of MBHs formed through the coalescence of a binary systems can cause a significant drop in the number of expected coalescing events on the way of MBH assembly. Volonteri (2007) recently showed that current assembly models are able to reproduce the major observational constraints even if the extreme recoil prescription by Campanelli et al. (2007) is taken into account. However, high kick velocities could seriously affect the expected number counts predicted for *LISA*, as the ejection of remnants by their host haloes would avoid subsequent MBHB formation.

In this Letter we estimate a robust *lower limit* for the predicted number of *LISA* sources. We use the Monte Carlo realizations of the merger history performed by Volonteri (2007) to show that even in the worse case (for GW observations) scenario in which *during each merger* the two MBH spins are counter-aligned in the MBHB orbital plane and extreme recoil is at work, current MBH assembly models predict that at least 10 sources will be detectable by *LISA*. In practice, the lower limit of the expected number of *LISA* sources does not substantially drop with respect to models employing non-spinning MBH recoil prescriptions (e.g. Volonteri et al. 2003).

2 MODELS OF BLACK HOLE FORMATION

In the hierarchical assembly framework, MBHs form by growing as a result of mergers and accretion from seed black holes at high redshift. There are two main scenarios for MBH assembly, namely the light seed and the heavy seed models. In the light seed models, seed MBHs typically form with masses $m_{\rm seed} \sim$ few $\times\, 10^2\,\mathrm{M_\odot}$, in haloes collapsing at $z \sim 20$, and are thought to be the end-product of the first generation of stars (Madau & Rees 2001). In the heavy seed models, black hole seeds are already massive when formed (10^4–$10^5\,\mathrm{M_\odot}$) from the low angular momentum tail of gas in protogalaxies at high redshifts. The angular momentum distribution of the gas in early-forming haloes can be determined by means of cosmological *N*-body simulations (Bullock et al. 2001); haloes with low spin parameters are prone to global dynamical instabilities, leading to the formation of a massive seed black hole (Koushiappas, Bullock & Dekel 2004; Begelman, Volonteri & Rees 2006; Lodato & Natarajan 2006).

We focus here on the two specific models discussed in Volonteri (2007) that are representative of these two classes of MBH assembly scenario: the Volonteri, Haardt and Madaa (VHM) and the Begelman Volonteri and Rees law feedback (BVRlf) models. In the VHM model, representative of the light seed scenarios (see Sesana et al. 2007 for details), seed MBHs form with masses $m_{\rm seed} \sim$ few $\times$ $10^2\,\mathrm{M_\odot}$, in haloes collapsing at $z = 20$ from rare 3.5σ peaks of the primordial density field. In the BVRlf model, representative of the heavy seed scenarios (see Sesana et al. 2007 for details), black hole seeds form in haloes subject to runaway gravitational instabilities, via the so-called 'bars within bars' mechanism (Shlosman, Frank & Begelman 1989). MBH seed formation is assumed to be efficient only in metal-free haloes with virial temperatures $T_{\rm vir} \gtrsim 10^4$ K, leading to a population of massive seed black holes with $m_{\rm seed} \sim$ few $\times\, 10^4\,\mathrm{M_\odot}$.

The subsequent MBH evolution relies only on a few simple assumptions. Nuclear activity is triggered by halo mergers: in each major merger the more massive hole accretes gas until its mass scales with the fifth power of the circular velocity of the host halo, normalized to reproduce the observed local correlation between MBH mass and velocity dispersion ($m_{\rm BH}$–σ_* relation). MBHB coalescence is assumed to occur efficiently following halo mergers.

For both the VHM and the BVRlf models, we consider two cases that bound the effect of recoil in the assembly of MBHs and, as a consequence, *LISA* events: (i) no gravitational recoil takes place and (ii) maximal gravitational recoil is associated to every MBHB merger, using the model by Volonteri (2007), which is based on the estimates reported by Campanelli et al. (2007). For the latter we use the merger tree realizations presented in Volonteri (2007). The model consistently takes into account the cosmic evolution of the mass ratio distribution of merging binaries and of their spin parameters (see discussion in Volonteri 2007). In each single merger, the mass ratio and the MBH spin magnitudes are therefore fixed by the merger hierarchy; the spin orientations are instead chosen so as to maximize the recoil. MBH spins are assumed to lie initially in the binary orbital plane, counter-aligned one to each other. The recoil velocity is then evaluated according to equation (1) of Campanelli et al. (2007), that in this case simplifies as:

$$\boldsymbol{v}_{\rm r}(q, a_i) = A\,\frac{q^2(1-q)}{(1+q)^5}\left[1 + B\,\frac{q}{(1+q)^2}\right]\hat{e}_\parallel + K\cos(\Theta - \Theta_0)\frac{q^2}{(1+q)^5}(a_2 + q a_1)\,\hat{e}_\perp. \qquad (1)$$

Here $A = 1.2 \times 10^4\ \mathrm{km\,s^{-1}}$, $B = -0.93$, $K = 6 \times 10^4\ \mathrm{km\,s^{-1}}$, a_1 and a_2 are the magnitudes of the spin parameters of the two holes, $\hat{e}_\parallel$ is a unit vector in the binary orbital plane and $\hat{e}_\perp$ defines the direction perpendicular to the orbital plane. The component of $\boldsymbol{v}_{\rm r}$ along the $\hat{e}_\perp$ direction depends sinusoidally upon the angle Θ between the MBH spins and their initial linear momenta. To get the maximum recoil we set $\Theta = \Theta_0$ ($\simeq$0.184).

We would like to emphasize that the prescription that we have chosen for (ii), the main features of which we have just summarized, is the least favourable for GW observations and (probably) unlikely to occur in these extreme circumstances during MBH assembly (Bogdanovic, Reynolds & Miller 2007).

3 GRAVITATIONAL WAVE SIGNAL

Full discussion of the GW signal produced by an inspiralling MBHB can be found in Sesana et al. (2005), along with all the relevant references. Here we just recall that a MBH binary at (comoving) distance $r(z)$ with chirp mass $\mathcal{M} = m_1^{3/5} m_2^{3/5}/(m_1 + m_2)^{1/5}$ generates a GW signal with a characteristic strain given by (Sesana et al. 2005)

$$h_{\rm c} = \frac{1}{3^{1/2}\pi^{2/3}}\,\frac{G^{5/6}\mathcal{M}^{5/6}}{c^{3/2} r(z)}\, f_{\rm r}^{-1/6}. \qquad (2)$$

An inspiralling binary is then detected if the signal-to-noise ratio (S/N) *integrated over the observation* is larger than a given detection threshold, where the optimal S/N is given by Flanagan &

Hughes (1998)

$$S/N = \sqrt{\int d\ln f' \left[\frac{h_c(f'_r)}{h_{rms}(f')}\right]^2}. \quad (3)$$

Here, $f = f_r/(1+z)$ is the (observed) frequency emitted at time $t = 0$ of the observation, and the integral is performed over the frequency interval spanned by the shifting binary during the observational time. Finally, $h_{rms} = \sqrt{5 f S_h(f)}$ is the effective rms noise of the instrument; $S_h(f)$ is the one-sided noise spectral density, and the factor $\sqrt{5}$ takes into account the random directions and orientation of the wave; h_{rms} is obtained by adding in the instrumental noise contribution (given by e.g. the Larson's online sensitivity curve generator, http://www.srl.caltech.edu/~shane/sensitivity), and the confusion noise from unresolved galactic (Nelemans, Yungelson & Portegies-Zwart 2001) and extragalactic (Farmer & Phinney 2003) white dwarf–white dwarf (WD–WD) binaries. Notice that extreme mass-ratio inspirals (EMRI) could also contribute to the confusion noise in the mHz frequency range (Barack & Cutler 2004).

4 RESULTS

4.1 Coalescence rates

Fig. 1 shows the number of MBH binary coalescences per unit $\log\mathcal{M}$ per unit *observed* year, $dN/d\log\mathcal{M}\,dt$, predicted by the two models that we have considered, for both cases where recoil is neglected and extreme recoil is taken into account. Each panel shows the rates for different redshift intervals. Note that when extreme recoil in included, the rate predicted by the BVRlf model at any redshift is only marginally affected, while the VHM model is more sensitive to the GW recoil: at $z > 15$, GW kicks do not affect the coalescence rate; on the contrary, at $z < 15$, the rate drops by a factor of ~ 3 for $\mathcal{M} \gtrsim 10^3\,M_\odot$, if extreme kicks are included in the evolution. This is related to the fraction of seeds that experience multiple coalescences during the MBH assembly history. We can schematically think of the assembly history as a sequence of coalescence rounds, as also recently suggested by Schnittman (2007). After each round extreme recoil depletes a large fraction of remnants, and the relative importance of each subsequent round drops accordingly. In the VHM model, about 65 per cent of the remnants of the first round will undergo a second round of coalescences, so the second round has an important relative weight in the computation of the total rate. When extreme recoil is taken into account, a large fraction of the first round remnants is ejected from their hosting haloes. We find that the effective fraction of remnants that can experience a second coalescence drops to ~ 30 per cent. This is the reason why the number of coalescences involving light black holes ($\mathcal{M} < 10^3\,M_\odot$) does not drop at any redshift, while the number of coalescences involving more massive binaries drops by a factor ≈ 3. In the BVRlf scenario, seeds are rarer, and the fraction of first coalescence remnants that continue to the second round is around 25 per cent; switching on the extreme recoil has a significantly smaller impact on the global rate in this case. Moreover, in this model seeds are more massive and the bulk of merging events happens at lower redshift, where the hosting halo potential wells are deeper and consequently larger kicks are needed to eject the coalescence remnants. As a matter of fact, the seed abundance sets the mean number of major mergers that a seed is expected to undergo during the cosmic history, and this basically sets the ability of extreme kicks to reduce the coalescence rate.

4.2 *LISA* detection rate

We now discuss how the number of GW sources detectable by *LISA* is influenced by extreme GW recoils. To facilitate the comparison with our previous works, all the results shown here assume an observation time of 3 yr, a sharp low-frequency wall at 10^{-4} Hz in the instrumental sensitivity (see Sesana et al. 2007), and a detection threshold $S/N = 5$ (see equation 3); the confusion noise includes only galactic and extragalactic WDs and ignores a possible contribution from EMRIs (Barack & Cutler 2004). At the end of this section, we will briefly discuss the impact of the former assumptions on the number of detectable sources. Fig. 2 shows the redshift distribution of MBHBs detected by *LISA*. The effect of extreme GW recoils on the source number counts is drastically affected by the abundance

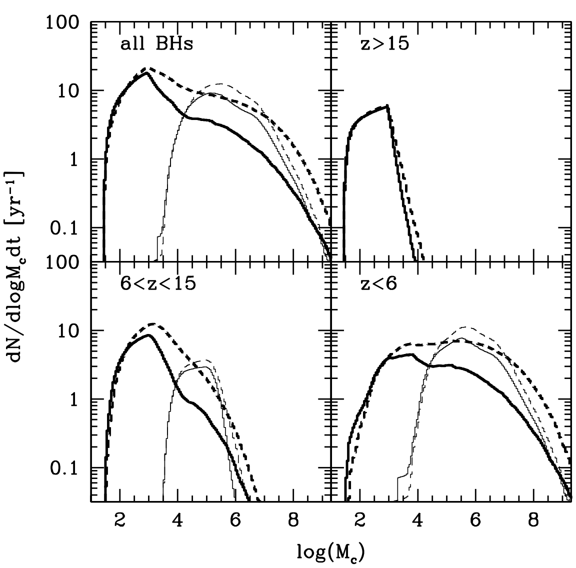

Figure 1. Number of MBHB coalescences per observed year at $z = 0$, per unit log chirp mass, in different redshift intervals. Solid lines: GW recoil neglected; dashed lines: extreme GW recoil included. Thick lines: VHM model; thin lines: BVRlf model.

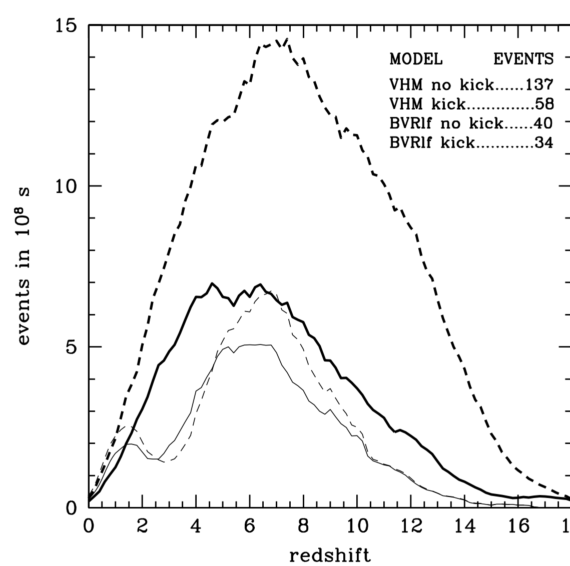

Figure 2. Redshift distribution of MBHBs resolved with $S/N > 5$ by *LISA* in a 3-yr mission. Line style as in Fig. 1. The top right-hand corner label lists the total number of expected detections.

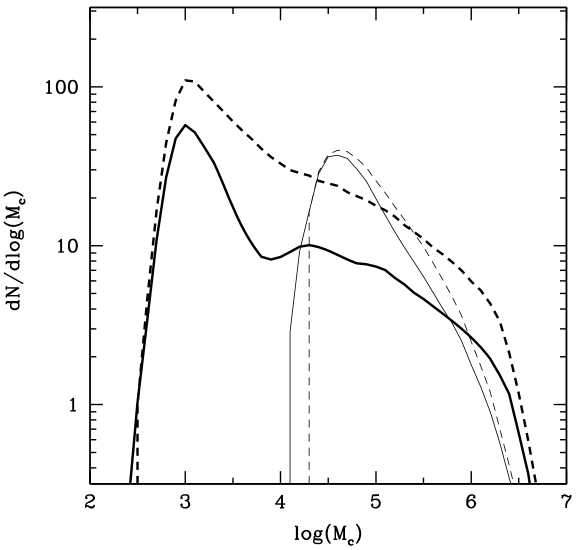

Figure 3. Chirp mass function of MBHBs resolved with S/N > 5 by *LISA* in a 3-yr mission. Line style as in Fig. 1.

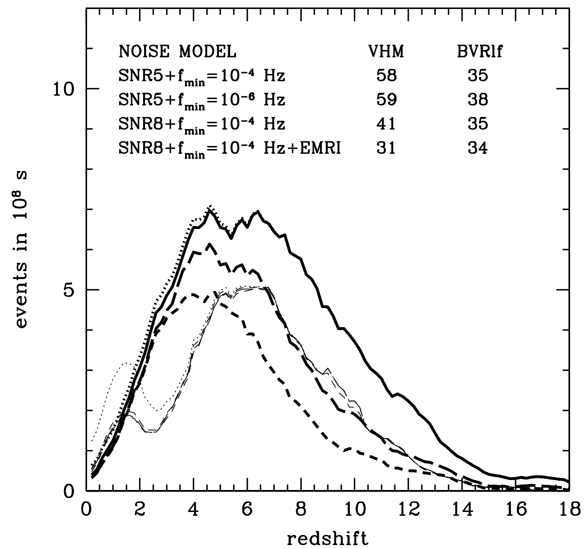

Figure 4. Impact of *LISA* sensitivity details on the number of detected sources. Solid lines: $S/R = 5$ and sensitivity cut-off at $f = 10^{-4}$ Hz; long-dashed lines: $S/R = 8$ and sensitivity cut-off at $f = 10^{-4}$ Hz; short-dashed lines: $S/R = 8$, sensitivity cut-off at $f = 10^{-4}$ Hz and EMRI confusion noise added; dotted lines: $S/R = 5$ and sensitivity cut-off at $f = 10^{-6}$ Hz. Thick lines are for the VHM model, thin lines are for the BVRlf model. The number of detected MBHBs in a 3-yr observation, under the different detection assumptions, are also listed. Extreme recoil is assumed.

and nature of the seeds, along the lines discussed in the previous section. In the VHM model, the number of detectable sources drops by a factor $\sim$ 60 per cent, and the number of the potential *LISA* detections is reduced from $\approx$140, if the recoil is neglected, to $\approx$60, if extreme recoil is included. Vice versa, the detection rate predicted by the BVRlf model is only weakly affected by the extreme recoil prescription, and it drops by about 15 per cent (from 40 to 34 events in 3 yr of observation). Note that though the overall number of coalescences in the VHM model decreases only by about 25 per cent when extreme recoil is considered, the number of *LISA* detections is reduced by a much larger factor. This is because if the seeds are light, *LISA* can not detect the bulk of the first coalescences of light binaries that take place at high redshift, which are responsible for the major contribution to the coalescence rate and are not affected by the recoil. *LISA* can observe later events, involving more massive binaries, which are largely suppressed by the MBH depopulation due to extreme GW kicks. In the BVRlf model, on the other hand, seeds are more massive, and the second coalescence round is less important; in this case, the *LISA* sensitivity is sufficient to observe almost all the first coalescences, and the number of detections is only mildly reduced. As the kicks affect the merger rate starting from the second round, its signature consists in a slight decrease of the mean chirp mass of the detected binaries (see Fig. 3).

We emphasize here two aspects (i) at this present time it is not clear if *LISA* will be able to shed light on the importance of recoil in MBH assembly, even in this extreme case, as the uncertainty introduced in the number counts is at most of a factor of $\sim$3, comparable with uncertainties due to our ignorance in the MBH accretion history and in the detailed dynamics of MBHBs (see e.g. discussion in Sesana et al. 2007); (ii) on the other hand, this fact confirm that MBHBs are *LISA* safe targets – because extreme recoil effects increase with the seed abundance, we expect the drop in the detections to be more significant for those scenarios that predict a larger number of sources. In Fig. 4 we show how different assumptions on the detection threshold, the instrumental noise below 10^{-4} and the confusion noise from EMRI affect the *LISA* detection rates. If seeds are massive, the results shown in Fig. 2 are hardly affected. If seeds are light, EMRI confusion noise and a more conservative detection threshold, say S/N = 8, can halve the number of sources detected by LISA. For both scenarios, extending the *LISA* sensitivity window below 10^{-4} has also minimal effect on the number of detections.

5 DISCUSSION

Here we have considered two specific MBH assembly models, representative of two different MBH seed formation scenarios. However, our findings can be considered, at least qualitatively, valid in general. Given the size and the abundance of the seeds, our 'coalescence round' picture depends on the details of the models. For example, in the VHM model we checked that by changing the accretion prescription (see Volonteri & Rees 2006) the total number of events would change by a factor of 2 (note that the accretion prescription considered in the models described in the previous section gives the minimum number of coalescences); however, the relative weights of the different coalescence rounds do not change significantly. Thus we can safely conclude that a decrease of $\gtrsim$50 per cent in the expected *LISA* sources should be a general trend for all those models in which the MBH assembly starts from light seeds at high redshift. In this class of models the number of predicted coalescing events is so high ($\gtrsim$100 yr^{-1}) that at least a few tens of MBHBs should be guaranteed *LISA* sources. On the other hand, extreme recoils should not be an issue at all for *LISA* if the MBH seeds are massive and/or rare. We remark here that in the BVRlf model we assumed MBH seed formation to be efficient only in metal-free haloes with virial temperatures $T_{\rm vir} \gtrsim 10^4$ K, i.e. we have considered atomic hydrogen to be the only coolant. Assuming efficient molecular hydrogen gas cooling (e.g. Koushiappas et al. 2004), the number of seed MBHs increases by an order of magnitude (and because the seeds are massive, *LISA* would be able to detect the first coalescence round), and the GW kick would not be an issue at all. Relying on this result, the estimate of $\sim$10 detections in 3 yr predicted by the BVRhf model described in Sesana et al. (2007) does not change under the assumption of extreme recoils (seeds are heavy and rare), and can be considered a robust *LISA* detection lower limit. To conclude, in Sesana et al. (2007) we explored different MBH assembly scenarios to quantify the imprint of the MBH seed prescription on the *LISA* data stream. Motivated by recent studies on extreme GW recoils, we have quantified in this Letter their impact on the MBHB coalescence and on the *LISA* detection rate, confirming that the detection of at least $\sim$10 coalescing binaries in a 3-yr mission is a robust prediction even considering extreme GW recoils.

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