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The variation of short-period comet size and decay rate with perihelion distance

David W. Hughes*

Department of Physics and Astronomy, The University, Sheffield S3 7RH

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ABSTRACT

Out of over 200 known short-period comets, we analyse a self-consistent list of 105 comets which have accurately estimated nuclei radii. It is found that both the median size and the size distribution index of these comets vary as a function of the perihelion distance, q, of the cometary orbit. A value of $q \approx 2.7$ au divides the comets into an outer solar system group which are hardly affected by decay, and an inner solar system group which are decaying quickly. It is estimated that 10, 20 and 30 per cent of the 105 comets will have decayed away after 1000, 2000 and 3000 yr, respectively.

Key words: comets: general.

1 INTRODUCTION

Short-period comets are defined as having periods, P, of less than 20 yr, but this limit is rather flexible because only 1.4 per cent of the known short-period comets have periods in the $17.0 < P < 20.0$ yr range, and only 5.7 per cent in the $14 < P < 17.0$ yr range. Short-period comets tend to have aphelia close to the orbit of Jupiter and they have been captured into the inner solar system by the gravitational influence of that planet. For this reason they are often referred to as Jupiter-family comets. Hughes (2002) noted that known short-period comets have a median perihelion distance of about 1.84 au, and a median orbital period of about 7.2 yr. These values indicate that the nuclei of short-period comets have the potential to decay quickly and this is backed up by observations of extensive comae and tails in the near-perihelion portions of their orbits.

As solar heating is responsible for the mass loss and size reduction of the nucleus, it is to be expected that comets with small perihelion distances decay faster than those with large perihelion distances. The perihelion distance of a cometary orbit is, however, not fixed. All cometary orbits vary with time as they are perturbed to greater and lesser extents by the gravitational influence of the major planets. The observed temporal variation of q with time for three reasonably typical short-period comets, 16P/Brooks, 4P/Faye and 6P/d'Arrest, are shown in Fig. 1. It can be seen that over the previous 20 or so apparitions the perihelion distances vary by around ± 0.1 au, but do not vary drastically from a mean position. There are cases, however, where the changes can be much more impressive. Comet P/Brooks 2 approached Jupiter to within 0.00096 au in 1886. The perihelion of its former intermediate-period orbit ($P \approx 40$ yr), became the aphelion of the new orbit ($P \approx 7$ yr), the perihelion distance changing from 5.45 to 1.95 au. It is expected to stay in the Jupiter family for at least a century (see Kazimirchak-Polonskaya 1972). In what follows we are going to assume that the mean perihelion distance of typical short-period comets remains reasonably constant over time periods of the order of 30 000 yr. As we have assumed that these comets all have aphelion distances close to the orbit of Jupiter, the 'constant q' assumption is equivalent to an assumption of a constant semi-major axis. We are thus assuming that, over the short term, the change in cometary energy produced by Jovian perturbations is randomly either positive or negative.

The longer term dynamical behaviour of short-period comets was studied in detail by Levison & Duncan (1994). They found that short-period comets moved backwards and forwards from the Jupiter-family to the Halley (intermediate-period) family on average about 12 times during their dynamical lifetime (the Jupiter and Halley families were defined as having Tisserand parameters greater than and lesser than 2.0, respectively). By integrating the known short-period cometary population for 10^7 yr they found that the median dynamical lifetime was approximately 4.5×10^5 yr, the dynamical 'death' being about 15 : 1 in favour of ejection from the solar system as opposed to destruction by impact with the solar surface.

A twelfth of 4.5×10^5 yr, (37 500 yr) is still a long time in the life of a short-period comet. The typical comet will have passed perihelion over 5000 times in this interval. If the comet has suffered negligible decay during such an interval, the multiple transitions between Jupiter and Halley families will simply mix up old and new comets, neither group having decayed significantly. In these circumstances there should no correlation between nucleus size and orbital perihelion distance. If, however, short-period comets decay quickly, and the nucleus undergoes significant size change in 5000 orbits, i.e. 37 500 yr, then comets with small perihelion distances would be expected to be smaller than comets with large perihelion distances.

Hughes (2002) concluded that the known short-period comet population *did* decay quickly. It was estimated that 50 per cent of the known short-period comets will decay away completely in the next 2600 yr, a further 25 per cent disappearing in the following 2300 yr. Under these circumstances it is completely nugatory to integrate the

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*E-mail: d.hughes@Sheffield.ac.uk

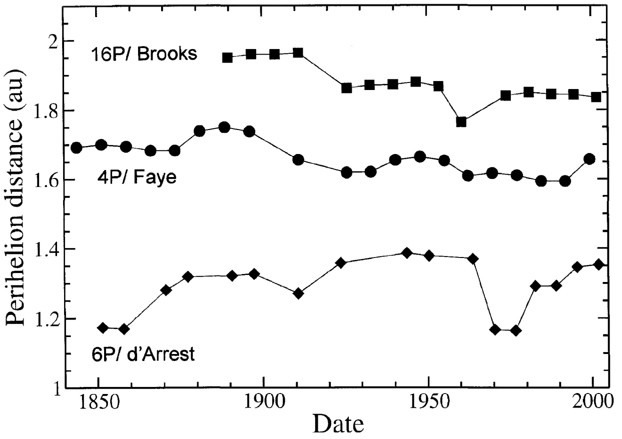

Figure 1. The observed variation in the orbital perihelion distance of three typical short-period comets as a function of date. The data has been taken from Marsden & Williams (2001).

orbital evolution of the population for 10^7 yr. In fact the 37 500 yr mentioned above, this being the average dynamical residence time in the Jupiter family, is still 14 times longer than the suggested decay lifetime of half the known population. Three 'lifetimes' have to be borne in mind. There is the 'dynamical' lifetime of approximately 450 000 yr, after which a typical short-period comet is either ejected from the solar system or perturbed into the Sun. There is the 'residence' lifetime of 37 500 yr, this being the typical time a comet spends in either the Jupiter or the Halley family before being transferred from one to the other. Then there is the 'decay' lifetime, after which a short-period comet has completely disappeared due to mass loss during perihelion passage. If the 'decay' lifetime is as short as Hughes (2002) suggests it is, then it is expected that the mean size of the cometary nuclei in the short-period comet population will increase as a function of the orbital perihelion distance, and it is this supposition that we are going to investigate in this paper.

2 THE RELATIONSHIP BETWEEN NUCLEUS RADIUS AND PERIHELION DISTANCE

In the last few years cometary astronomers have been allowed to use the *Hubble Space Telescope* (*HST*) and some of the largest ground-based telescopes to observe bare cometary nuclei at large heliocentric distances, at a time when they are not obscured by a surrounding coma. These observations have lead to a direct measurement of the brightness of the sunlight reflected from the nucleus. This brightness is proportional to the product of the geometric albedo of the nucleus and the mean cross-sectional area. Most researchers assume that the albedo is about 0.04 and thus derive the mean radius. The word 'mean' is used because the nuclei are expected to be potato-shaped, the longest dimension being about twice the shortest. Unfortunately the spatial resolution of *HST* at a typical comet–Earth distance is about two orders of magnitude greater than the radius of an average short-period cometary nucleus. The only Jupiter-family comet to have the size of its nucleus measured directly is 19P/Borrelly. The imaging team using the *Deep Space 1* spacecraft (see Buratti et al. 2002) found the dimensions to be 8.0 × 3.15 km au (i.e. mean radius 2.5 km). The geometric albedo was found to be 0.029 ± 0.006.

Many astronomers have tackled the difficult problem of observing comets when far from the Sun, estimating the amount of light scattered by the cometary nucleus and converting that nuclear magnitude into a value for the radius of the nucleus (see, for example, Roemer 1976; Svoreñ 1987; Meech & Newburn 1998; Lamy et al. 2000; Tancredi et al. 2000; Lincandro et al. 2000; Lowry & Fitzsimmons 2001; Boehnhardt et al. 2002; Nesluan 2003).

Tancredi et al. (2000) took great care in reducing the data, using similar colour corrections, phase coefficients and albedos. The end-product was a list of 105 short-period cometary nucleus radii, this list being divided into four quality classes dependent on whether the magnitude of the nucleus was accurate to $<\pm0.3$ mag, between ±0.3 and ±0.6 mag, between ±0.6 and ±1.0 mag, $>\pm1.0$ mag. The relationship between the error in the magnitude and the error in the radius can be estimated using equation (1) in Tancredi et al. (2000), this being

$$\log\left(\pi p_{\mathrm{v}} R_{\mathrm{N}}^2\right) = 16.85 + 0.4 \times [m_{\mathrm{O}} - H_{\mathrm{N}}], \qquad (1)$$

where p_{v} is the geometric albedo of the nucleus in the visual, R_{N} is the radius of the cometary nucleus, m_{O} is the apparent visual magnitude of the Sun (−26.77), and H_{N} is the absolute visual nuclear magnitude of the comet (this being the apparent magnitude it would have if seen at zero phase when both 1 au from the Earth and the Sun). The Tancredi et al. (2000) perihelion distance and nucleus radius data is give in both Table 1 and Fig. 2.

All cometary data sets suffer from observational selection. In the context of the present paper it is clear that large cometary nuclei are brighter than small cometary nuclei and are thus easier to detect. As the amount of gas and dust emitted by a cometary nucleus varies significantly as a function of the heliocentric distance of the comet, it is clear also that the perihelion distance of a comet plays a large role in its discoverability. Fig. 2 shows that the minimum sized short-period comet in the Tancredi et al. (2000) list increases with perihelion distance, its radius R_{min} being roughly 0.5 km for $0.25 < q < 1.0$ au, 0.6 km for $1.0 < q < 1.75$ au, 0.6 km for $1.75 < q < 2.5$ au, 0.8 km for $2.5 < q < 3.25$ au, 1.5 km for $3.25 < q < 4.0$ au and 2.1 km for $4.0 < q < 4.75$ au. This change in R_{min} explains why there is a 'data free' region in the lower right-hand corner of Fig. 2.

The minimum size, of 0.5 km, for small perihelion distance comets is also worthy of comment. A comet of this size has a nucleus magnitude of about 19.1 (using equation 1 above), and a total coma magnitude, H_{10} of about 10.4 (according to equation 13 in Hughes 2002). Both these figures suggest that smaller, fainter comets should easily be detected by today's telescopic instrumentation, if they exist. Fernändez et al. (1999) suggested that these very small cometary nuclei are not there, simply because they disintegrate very quickly into meteoroid streams. This would mean that their very short lifetime at that size mitigated against their detection. It is, however, difficult to envisage the speedy disintegration of a dirty snowball nucleus of radius 0.5 km into dust and ice-flakes occurring without a considerable sudden increase in light-scattering area, and a concomitant brightness outburst. Maybe the speed of decay when the nucleus reaches this size points strongly towards the nucleus having both an extremely fragile structure, and a low density. The later might indicate that the interior of cometary nuclei was full of very large holes. Another possibility is that $R_{\mathrm{N}} < 0.5$ km cometary nuclei speedily loose all their potentially volatile icy material and thus become physically indistinguishable from small faint asteroids. Under these circumstances we would be looking for $H_{\mathrm{N}} > 19.1$ 'asteroid-like' objects and not $H_{10} > 10.4$ small comets.

Hughes (2000) modelled the cometary decay process and the macro-features of the surface layers of a cometary nucleus. He suggested that the complete devolatilization of a $R_{\mathrm{N}} = 0.5$ km dirty snowball nucleus was very unlikely bearing in mind the suggested

Table 1. The perihelion distances, q, and nucleus radii R_N of short-period comets, taken from Tancredi et al. (2000). The comets are divided into different accuracy classes and listed in order of perihelion distance. (Quality class 1 has an accuracy approximately equivalent to $\pm \log R_N = 0.04$; quality class 2 has $\pm \log R_N = 0.09$, quality class 3 has $\pm \log R_N = 0.16$ and quality class 4 has $\pm \log R_N = 0.24$.) ΔR is the average reduction in the nucleus radius per year, assuming that the nucleus has a mean density of 150 kg m^{-3}. The decay lifetime of the comet, τ, is give by $\tau = R_N/\Delta R$.

	q (au)	R_N (km)	ΔR (m yr^{-1})	τ (yr)
Quality Class 1				
26P/Grigg-Skjellerup	0.732	1.3	2.36	552
67P/Churyumov-Gerasimenko	1.285	2.5	0.896	2790
49P/Arend-Rigaux	1.386	3.2	0.716	4470
19P/Borrelly	1.395	3.0	0.703	4270
28P/Neujmin 1	1.529	9.1	0.526	17300
9P/Tempel 1	1.562	2.3	0.484	4750
46P/Wirtanen	1.635	0.7	0.495	1730
31P/Schwassmann-Wachmann 2	2.090	3.2	0.108	29800
47P/Ashbrook-Jackson	2.311	2.9	0.0510	56900
Quality Class 2				
21P/Giacobini-Zinner	0.931	1.0	1.647	606
6P/D'Arrest	1.163	1.5	1.112	1350
10P/Tempel 2	1.344	2.9	0.784	3700
124P/Mrkos	1.411	1.6	0.679	2360
125P/Spacewatch	1.544	0.8	0.506	1580
71P/Clark	1.560	1.3	0.487	2670
4P/Faye	1.692	2.2	0.354	6210
22P/Kopff	1.699	1.8	0.349	5160
137P/Shoemaker-Levy 2	1.844	2.9	0.229	12650
48P/Johnson	2.248	2.2	0.0668	32900
43P/Wolf-Harrington	2.428	1.8	0.0313	57600
110P/Hartley 3	2.454	1.9	0.0281	67600
36P/Whipple	2.484	2.3	0.0249	92300
65P/Gunn	3.306	4.8	5.63×10^{-4}	8.51×10^{6}
82P/Gehrels 3	3.424	2.0	3.45×10^{-4}	5.79×10^{6}
117P/Helin-Roman-Alu 1	3.707	3.5	1.14×10^{-4}	3.08×10^{7}
P/1996A1 (Jedicke)	4.055	5.0	3.25×10^{-5}	1.54×10^{8}
29P/Schwassmann-Wachmann1	5.495	13.2	4.66×10^{-7}	2.83×10^{10}
Quality Class 3				
2P/Encke	0.336	1.3	5.00	260
7P/Pons-Winnecke	0.772	1.5	2.20	681
41P/Tuttle-Giacobini-Kresák	1.140	0.7	1.154	607
P/1994A1 (Kushida)	1.367	1.2	0.746	1610
76P/West-Kohoutek-Ikemura	1.398	1.3	0.698	1860
81P/Wild 2	1.491	2.2	0.577	3810
37P/Forbes	1.528	1.0	0.528	1900
75P/Kohoutek	1.568	1.8	0.477	3770
70P/Kojima	1.631	1.2	0.409	2935
40P/Väisälä 1	1.762	1.5	0.298	5025
68P/Klemola	1.763	2.2	0.297	7395
32P/Comas-Sola	1.772	2.5	0.289	8650
60P/Tsuchinshan 2	1.796	0.8	0.267	2990
30P/Reinmuth 1	1.860	1.3	0.218	5960
88P/Howell	1.916	1.1	0.184	5990
16P/Brooks 2	1.950	1.7	0.166	10250
116P/Wild 4	1.989	3.5	0.148	23670
118P/Sheomaker-Levy 4	2.019	1.7	0.135	12624
17P/Holmes	2.141	2.0	0.0918	21780
94P/Russell 4	2.125	1.9	0.0964	19700
130P/McNaught-Hughes	2.125	1.7	0.0964	17630
89P/Russell 2	2.159	1.1	0.0869	12650
87P/Bus	2.183	1.3	0.0809	16070
61P/Shajn-Schaldach	2.234	1.1	0.0696	15810
86P/Wild 3	2.288	0.9	0.0562	16000
78P/Gehrels2	2.348	2.1	0.0435	48230
53P/Van Biesbroeck	2.414	3.8	0.0331	1.15×10^{5}
56P/Slaughter-Burnham	2.544	1.5	0.0189	7.92×10^{4}
101P/Chernykh	2.568	2.2	0.0169	1.301×10^{5}

Table 1 – *continued*

	q (au)	R_{N} (km)	ΔR (m yr^{-1})	τ (yr)
134P/Kowal-Vávrová	2.609	1.4	0.0140	1.003×10^5
135P/Shoemaker-Levy 8	2.711	1.5	0.00878	1.71×10^5
P/1993X1 (Kushida-Muramatsu)	2.745	2.3	0.00755	3.05×10^5
120P/Mueller 1	2.747	0.8	0.00749	1.07×10^5
111P/Helin-Roman-Crockett	3.470	1.5	2.87×10^{-4}	5.23×10^6
P/1997C1 (Gehrels)	3.565	2.3	1.96×10^{-4}	1.17×10^7
P/1995A1 (Jedicke)	4.083	3.0	2.95×10^{-5}	1.016×10^8
99P/Kowal 1	4.660	4.8	4.65×10^{-6}	1.022×10^9
Quality Class 4				
45P/Honda-Mrkos-Pajdušáková	0.559	0.5	3.057	163
103P/Hartley 2	0.952	3.8	1.591	2390
15P/Finlay	0.998	0.9	1.480	608
73P/Schwassmann-Wachmann 3	1.011	1.0	1.444	692
24P/Schaumasse	1.226	0.8	1.008	793
57P/Du-Toit-Neujmin-Delport	1.305	1.6	0.856	1870
64P/Swift-Gehrals	1.358	1.7	0.761	2234
33P/Daniel	1.382	0.9	0.722	1246
58P/Jackson-Neujmin	1.463	0.6	0.610	983
112P/Urata-Niijima	1.449	0.7	0.628	1113
62P/Tsuchinshan 1	1.486	0.8	0.583	1370
100P/Hartley 1	1.540	1.3	0.512	2540
14P/Wolf	1.572	1.3	0.472	2750
98P/Takamizawa	1.595	2.4	0.446	5380
P/1992Q1 (Brewington)	1.601	1.5	0.439	3410
84P/Giclas	1.627	1.4	0.413	3390
138P/Shoemaker-Levy 7	1.630	0.8	0.410	1950
97P/Metcalf-Brewington	1.631	1.3	0.409	3180
51P/Harrington	1.694	1.4	0.353	3970
52P/Harrington-Abell	1.774	1.1	0.287	3830
113P/Spitaler	1.816	1.1	0.251	4390
50P/Arend	1.821	3.0	0.247	12160
44P/Reinmuth 2	1.867	1.5	0.213	7030
132P/Helin-Roman-Alu 2	1.930	0.9	0.176	5110
105P/Singer-Brewster	1.955	1.0	0.163	6120
42P/Neujmin 3	2.042	0.6	0.125	4800
131P/Mueller 2	2.083	0.8	0.110	7280
123P/West-Hartley	2.129	2.2	0.0952	23100
77P/Longmore	2.402	2.4	0.0348	69040
91P/Russell 3	2.510	1.3	0.0225	57800
90P/Gehrels 1	2.935	2.8	0.00298	9.40×10^5
P/1994J3 (Shoemaker 4)	2.944	3.3	0.00285	1.16×10^6
136P/Mueller 3	2.998	1.9	0.00221	8.59×10^5
119P/Parker-Hartley	3.026	2.5	0.00194	1.288×10^6
128P/Shoemaker-Holt 1	3.053	2.0	0.00172	1.166×10^6
P/1993K2 (Helin-Lawrence)	3.090	4.6	0.00145	3.17×10^6
P/1997V1 (Larsen)	3.293	3.6	2.87×10^{-4}	6.04×10^6
39P/Oterma	3.389	9.1	3.99×10^{-4}	2.28×10^7
74P/Smirnova-Chernykh	3.546	6.0	2.07×10^{-4}	2.83×10^7
P/1997G1 (Montani)	4.217	2.5	1.88×10^{-5}	1.32×10^8
P/1993W1 (Mueller 5)	4.250	2.1	1.69×10^{-5}	1.246×10^8

values for the thermal conductivity of cometary material. Very little heat would be transmitted through the thin dusty surface layers during the short perihelion passage period. Snow more than a few metres or so below the surface would not reach a high enough temperature for sublimation to take place. It seems that a sudden acceleration in the cometary decay process is only likely when the radius of the comet has a value similar to the size of the typical vacant holes in the cometary interior. It is thus the existence of large interior holes that most probably explains the $R_{\mathrm{min}} = 0.5$ km mentioned above. (The idea that a cometary nucleus is full of large holes has also been put forward as an explanation for cometary outbursts; see Hughes 1991.)

Observational selection does not explain the 'data free' region on the left of Fig. 2. Why are there no recorded short-period comets in the $R_{\mathrm{N}} > 1.5$ km, $q < 0.95$ au region? These are certainly big enough and close enough to both the Sun and the Earth to have been detected if they exist. We contend that these comets have all been removed by decay. The two 'data free' regions show up even more clearly if the data is plotted logarithmically, as shown in Fig. 3.

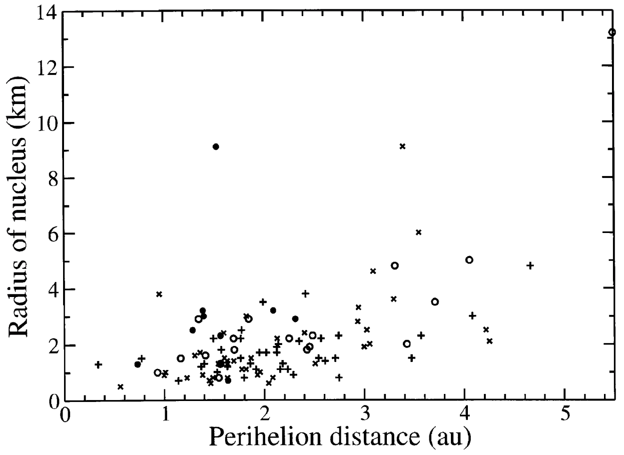

Figure 2. The radii of 105 short-period cometary nuclei are plotted as a function of their orbital perihelion distance. The data has been taken from Tancredi et al. (2000), see also Table 1. The data is divided into four sets, according to its quality. Quality class 1 (the best) is represented by filled circles, class 2 by open circles, class 3 by plus signs and class 4 (the worst) by multiplication signs.

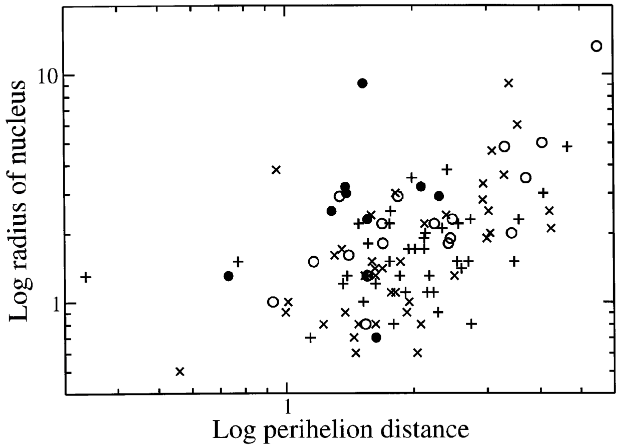

Figure 3. A plot of the logarithm of the nucleus radius as a function of the logarithm of the perihelion distance for the 105 short-period comets listed by Tancredi et al. (2000).

The effects of cometary decay can be estimated by considering the observed mass loss of Comet 1P/Halley. During its close approach to the Sun in 1910 and in 1986, the mass loss was estimated to be 2.8×10^{11} kg (see Hughes 1985) and 5×10^{11} kg (see Whipple 1986), respectively. The GIOTTO observations of 1P/Halley gave the mean radius as 5.6 km (see Keller et al. 1987). Assuming that the cometary nucleus has a mean density of ρ kg m^{-3}, the mass losses given above are equivalent to the whole nucleus of 1P/Halley losing a layer from the surface of approximate depth

$$\Delta R = 1000/\rho \text{ m} \qquad (2)$$

at each perihelion passage. This idea of the whole surface of the nucleus retreating by a depth ΔR per apparition is clearly a rather too simplified picture. At any specific apparition the mass loss will only occur from a few active regions. However, these regions are not permanent features of the cometary topography. As time progresses, some seal up and others break out. Over many apparitions it is reasonable to consider the comet nucleus decreasing in size by progressively shedding ‘skins’, the process being akin to peeling off the layers of an onion.

Unfortunately the value of the mean density of a cometary nucleus is still a matter of considerable debate. (No cometary masses are accurately known!) Considering the paradigm of the cometary nucleus as a weak, low-density, fragile, friable, slightly dusty snow-ball (see Hughes 2000, for example) we shall follow Rickman et al. (1987) who obtained a density value of 150 kg m^{-3} from their study of the effect of non-gravitational parameters on near-perihelion cometary light curves. Assuming that the mean density of a cometary nucleus is $150/F$ kg m^{-3} (where, in our opinion, F is a factor close to 1.0), equation (2) thus becomes

$$\Delta R_{\mathrm{Halley}} = 6.7F \text{ m}. \qquad (3)$$

Let us now move from the specific case of 1P/Halley, to the case of a general short-period comet. Following Fernández (1984), the short-period comets are assumed to have aphelia at 5.2 au, the semi-major axis of Jupiter’s orbit. The eccentricity of the orbits therefore decrease and the orbital periods increase as the perihelion distances get larger. Fernández assumed that the cometary nuclei were spinning quickly and that the surface of the nucleus had a uniform temperature at a specific heliocentric distance. H_2O sublimation is powered by absorbed solar radiation and the mass loss per unit area of the cometary nucleus was then obtained by integrating the sublimation mass loss around the orbit. The graphical mass-loss data given in Fernández (1984) have been converted to an assumed nucleus albedo of 0.04. Values of the cometary mass-loss per unit area per unit time, ΔM, are listed in Table 2 for Jupiter-family comets of differing perihelion distances. The values of ΔM quoted for the individual comets in Table 1 have then been obtained by extrapolating between the values listed in Table 2.

The average rate of radius decrease, ΔR, of a specific cometary nucleus can then be found by dividing ΔM by the mean density of the nucleus. The values quoted in Table 1 assume that this density is 150 kg m^{-3}. Notice that rate of radius decrease quoted above for Comet Halley is comparable with the values quoted in Table 1 for short-period comets of comparable perihelion distance. Assuming that the perihelion distance of a specific short-period comet remains

Table 2. The mass loss, ΔM, per unit area, per year from the nuclei of short-period comets is given as a function of their perihelion distance, q. It has been assumed that the all the comets have aphelion distances of 5.2 au, and the eccentricity and orbital period vary accordingly. The albedo of the nucleus has been taken to be 0.04. The figures have been obtained by adapting graphical data given in Fernández (1984). If the nucleus is assumed to have a uniform density of ρ, the surface retreats at an average rate of ΔR m yr^{-1}, where $\Delta R = \Delta M/\rho$. The values quoted for ΔR assume $\rho = 150$ kg m^{-3}.

q (au)	ΔM (kg m^{-2} yr^{-1})	ΔR (m yr^{-1})
0.5	511	3.40
0.75	345	2.30
1.00	221	1.47
1.25	146	0.972
1.50	85.1	0.567
1.75	46.6	0.311
2.00	21.5	0.143
2.25	9.97	0.0665
2.50	3.51	0.0234
2.75	1.11	0.00739
3.00	0.329	0.00219

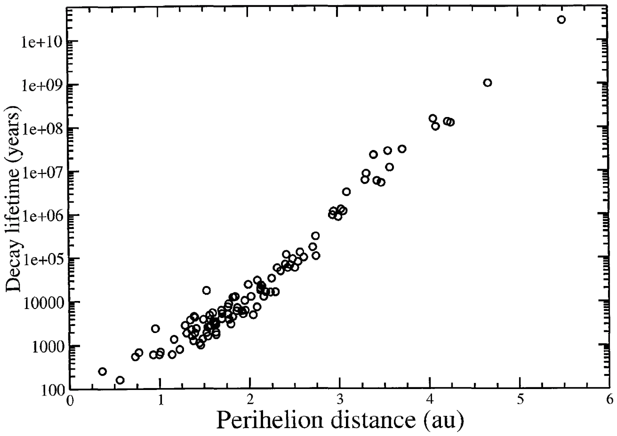

Figure 4. The decay lifetime of the short-period comets listed in Table 1 is plotted as a function of perihelion distance. These values depend on many assumptions not least of which is that the nucleus has an albedo of 0.04, and a density of 150 kg m^{-3}. It has also been assumed that the comets decay according the model put forward by Fernández (1984), and that their perihelion distances remain constant during the relevant time periods.

reasonably constant with time, the time, τ, taken for that comet to decay away completely can be estimated by noting that $\tau = R/\Delta R$. These 'decay' lifetimes are also listed in Table 1. Fig. 4 shows the decay times of the short-period comets in Table 1 as a function of perihelion distance. Remembering the provisos (i.e the assumed density, the decay rates and the assumed constant perihelion distance), 10 per cent of the 105 comets in Table 1 decay away in the next 1000 yr. The τ values in Table 1 show that 20, 30, 40, 50, 60, 70 and 80 per cent of the 105 comets will have decayed in the next 2000, 3000, 4800, 7100, 17 000, 57 000 and 200 000 yr, respectively. Fig. 4 shows the very strong relationship between decay lifetime and perihelion distance. The 80 per cent of the comets that decay in the next 200 000 yr or so all have present perihelion distance less than about 2.75 au.

Let us return to the work of Levison & Duncan (1994). They predicted that Jovian gravitational perturbations made the typical short-period comet change orbit between the Jupiter family ($P < 20$ yr) and the intermediate family ($20 < P < 200$ yr) about every 37 500 yr. Fig. 4 shows that all the $q < 2.3 \pm 0.3$ au short-period comets listed in Table 1 decay away completely in this time period, about 68 per cent of the Table 1 comets being in this category. We would thus expect a perihelion distance of this order to be a boundary between an inner solar system population that is visibly decaying, and inside which the sizes of the cometary nuclei will vary drastically as a function of perihelion distance, and an outer solar system 'well-mixed' population in which the mean cometary size was not a noticeable function of perihelion distance, and decay was much less important. The quoted uncertainly of ±0.3 au is rather conservative. It is a function of, among other things, the assumed density of the nucleus. If this density was, say, 2×150 kg m^{-3}, the estimated lifetime of the comet would be double the values given in Table 1, and the estimated boundary at 2.3 au would move towards the Sun by about 0.2 au. A similar change in the opposite direction would occur if the density was decreased to 0.5×150 kg m^{-3}.

Imagine a system where the observational selection criteria did not change over the whole $0.0 < q < 6.0$ au range. Say this was equivalent to us being able to detect *all* the short-period comets with nuclei larger than some limiting radius R_{lim} over this perihelion range. If we then calculated the median size of the $R > R_{\text{lim}}$ comets, i.e. those unaffected by observational selection, we would expect this to be large and constant for perihelion distances greater than the 2.3-au boundary. As we moved from the boundary towards the Sun, we would expect the average size of the $R > R_{\text{lim}}$ comets to decrease. The data listed in Table 1 was sorted (independent of quality class) into six equally populated perihelion distance ranges. Fig. 2 indicates that an $R_{\text{lim}} = 1.5$ km cut-off might be a reasonable starting point. The median radii of the remaining $R \geqslant 1.5$ km comets were calculated for the comets in each group. These median radii are plotted as a function of perihelion distance in Fig. 5. This figure shows that there is a step change, of about 1.0 ± 0.1 km, in the median radius value in the 2.6 to 3.0 au perihelion distance region. There is even a hint of the median radius decreasing with decreasing perihelion distance for comets with perihelia inside this boundary. A similar analysis was carried out with an R_{lim} of 1.7 and 2.0 km. Step changes of about 1 km were again found to occur at about the same perihelion distance. We are convinced that this change is not due to observational selection. The median values shown in Fig. 5 are dependent on both the perihelion distance bin sizes and the bin start points. Progressively varying the bin start points throughout the data set indicates that the step occurs at about 2.9 ± 0.1 au.

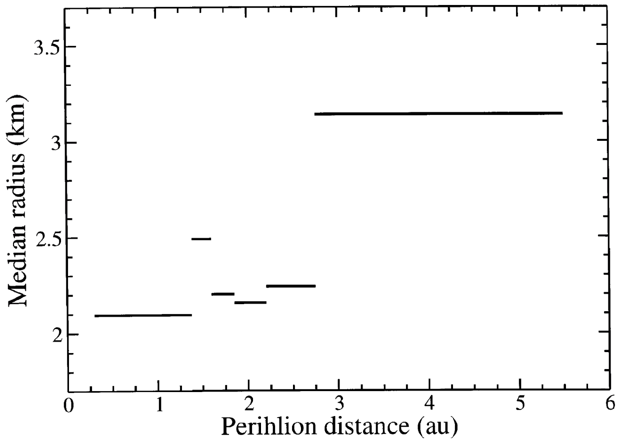

Figure 5. The $R \geqslant 1.5$ km short-period comets listed in Table 1 have been sorted into six equally populated perihelion distance groups and the median radius of the comets in each group is plotted as a function of perihelion distance.

Again, dealing with only the $R > R_{\text{lim}}$ comets, we might expect their size distribution to be constant for perihelion distances beyond the boundary, and to be a strong function of heliocentric distance for perihelion distances within the boundary. Let us assume that the number, N, of comets with radii greater than a specific value is proportion to $R^{-\gamma}$, where γ is the size distribution index. An example of this distribution is shown in Fig. 6, where the logarithm of the cumulative number of comets with radii greater than R is plotted as a function of log R, for half the comets listed in Table 1, i.e. those with $q > 1.90$ au. The linearity of the right-hand portion of the data plot in Fig. 6 is taken as justification for the choice of the $N \propto R^{-\gamma}$ relationship mentioned above. It also indicates that γ is constant for radii larger than the 'knee' radius in Fig. 6. The 'knee' occurs at a radius of around 1.6 to 1.8 km and this is taken as justification for the R_{lim} radii chosen above. The Table 1 data was again sorted (independent of quality class) into equally populated perihelion distance ranges and logarithmic plots of N verses R were produced for each group. The gradients of the right-hand portions of these plots gave the size distribution index, γ, and these indices are plotted as a function of perihelion distance in Fig. 7. It can be

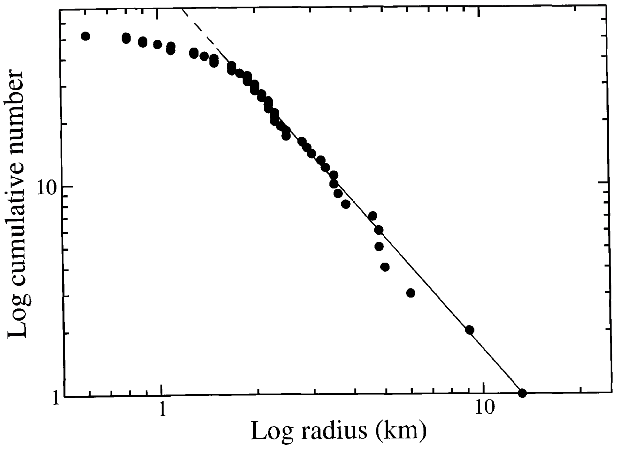

Figure 6. The logarithm of the cumulative number of short-period comets with radii greater than R is plotted as a function of log R, for those Table 1 comets with $q > 1.90$ au.

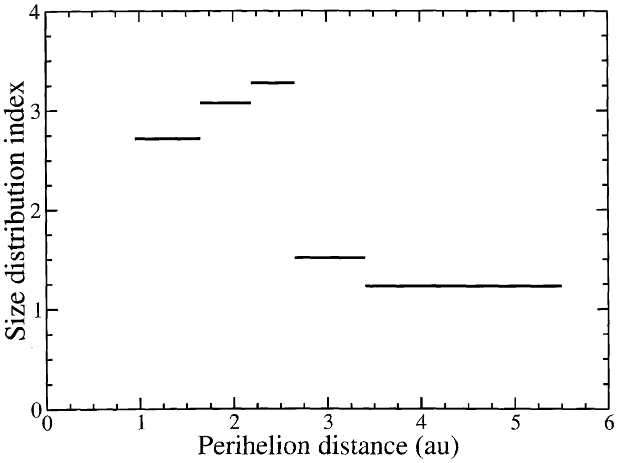

Figure 7. The size distribution index, γ, is plotted as a function of perihelion distance for five equally populated groups of comets. The group boundaries have been set by the division into five.

seen that there is a drastic change in the value of this quantity at a perihelion distance of around 2.5 au.

Unfortunately both the 'median radii' and the 'size distribution index' analyses are severely limited by the size of the available data set. The fact that only six median radii were shown in Fig. 5, and five size distribution indices in Fig. 7 is simply because there are only 105 comets in Table 1. It is, however, perfectly clear from this analysis that a boundary exists, somewhere around 2.9 au from the Sun. Greater precision will unfortunately only be possible when many more short-period comets have been found and measured.

There is a strong possibility that this boundary position is governed by the sublimation of water ice, this sublimation being the major driver of cometary mass loss. The surface temperature, T_S K, of one square metre of the cometary nucleus surface (albedo A), perpendicular to the solar radiation flux, is given approximately by

$$T_\mathrm{S} = \sqrt[4]{\frac{2.40 \times 10^{10}(1 - A)}{r^2}}. \qquad (4)$$

If we follow the majority by assuming that the albedo is about 0.04, and that water ice starts to sublimate profusely into the vacuum of space at a temperature of around 230 K, equation (4) indicates that $r = 2.90$ au separates active from inactive comets.

The conclusion presented above depends (among other things) on the calculation of the rate of mass loss from a short-period comet (see Fernández 1984, and Table 2). Many assume that Q, the number of gas molecules lost per unit time at a point on the inner solar system orbit of a comet, is proportional to $r^{-\alpha}$ where r is the heliocentric distance of the comet at that time. In the inner solar system it seems reasonable to assume that Q is proportional to the amount of solar radiation absorbed per unit time by the nucleus. Thus $\alpha = 2$. Some of the absorbed heat can, however, be re-radiated, and the fraction that is used for sublimation probably changes with heliocentric distance (i.e. the temperature of the nucleus surface). Huebner (1990) points out that 'the dependence of the production rate on the heliocentric distance seems in almost all cases to be steeper than that corresponding to r^{-2}'. This observation covered the $0.5 < r < 2.0$ au range, and Huebner suggested that the complex relationship between the water sublimation rate and the temperature, the delay produced by heat transmission through a thin surface refractory dust mantle, and the possible gradual enlargement of active regions during solar approach are possible reasons. Many examples can be quoted; for example, Keller & Lillie (1974) found an $r^{-2.3}$ relationship for the production of OH and H in Comet Bennett, and Weaver et al. (1981) found an $r^{-3.2}$ relationship for the production of H_2O in Comet Bradfield.

The work of Levison & Duncan (1994) stresses that the population of short-period comets is fluid. Jupiter's gravitational field can not only capture new comets on to $P < 20$ yr orbits but can also remove members of the short-period population by placing them on to $P > 20$ yr orbits. This capture and removal process can take place many times and is expected to take place continuously. So from the decay standpoint the known short-period cometary population contains both young, middle-aged and old comets. Unfortunately, the presently accepted paradigm of a uniform cometary interior, and the 'onion skin' model of average cometary decay, makes it impossible to distinguish old comets from young ones. We are also not sure whether other parameters, such as the fraction of the nucleus surface area that is active, are age-dependent.

It is also clear that the comets captured by Jupiter have a range of sizes, and not just one specific radius. The 'residence' time-period of 37 500 yr suggested by Levison & Duncan (1994) is of little important for comets inside the 2.7-au boundary mentioned above. Most have decayed completely before their 'residence' ends. It can be seen from Table 2 that these comets lose a considerable amount of mass each time they pass perihelion. Short-period comets typically have an aphelion distance that is approximately equal to the semi-major axis of the Jovian orbit. Kepler's Third Law then indicates that

$$P = \left(\frac{q + 5.202}{2}\right)^{3/2}.$$

Thus comets with perihelion distances of 1.0, 2.0 and 3.0 au are expected to have orbital periods of 5.5, 6.8 and 8.4 yr, respectively. The median $q < 2.7$ au comet has a $q = 1.76$ au, and a period of about 6.5 yr. This comet will have passed perihelion around 5800 times in the Levison & Duncan 'residence' period. There is no wonder that Fig. 5 shows that the $q < 2.7$ au comets have median radii that are 1.0 km smaller than the $q > 2.7$ au comets.

3 DISCUSSION AND CONCLUSIONS

In the eagerly expected book 'Comets II', Lamy et al. (2004) have reviewed the sizes, shapes, albedos and colours of cometary nuclei and have 'investigated a possible trend of nucleus size with perihelion distance'. They concluded that 'no such trend is apparent'. This might be true for $q > 2.7$ au comets, but we find that it is not the case for the $q < 2.7$ au group. Hughes (2002) stressed the transient nature of the majority of the known short-period cometary population. The Jovian induced transfers between short- and intermediate-period groups is only important for the $q > 2.7$ au group. Most of the $q < 2.7$ au comets decay so quickly that they disappear altogether in a fraction of the Levison & Duncan 'residence' time. Hughes (2002) used total coma magnitude to estimate the radii of cometary nuclei and suggested that that about 50 per cent of today's known short-period comets will decay away completely in the next 2600 yr. The present paper uses much more accurate radii for the nuclei, and a more rigorous approach to the variability of mass loss with perihelion distance. The decay of 50 per cent of the short-period cometary sample is now found to require about 7100 yr.

The present paper finds that $q = 2.7$ au is a real boundary in the short-period cometary population. This distance separates comets which have negligible water sublimation from comets that are decaying speedily. The 'outer' group contains two types of comets; ones that have always been on $q > 2.7$ au orbits and have never decayed, and ones that, at previous transfers from the intermediate population, were placed on $q < 2.7$ au orbits and were large enough to have not been completely destroyed before being returned to the intermediate population. Fernández (1984) indicated that the Jovian perturbations responsible for the transfer of intermediate period comets on to short-period orbits is such that about 37 per cent of the resultant orbits will have $0 < q < 2.7$ au, the remaining 63 per cent having $2.7 < q < 5.2$ au.

A cursory glance at a plot of nucleus radii as a function of perihelion distance (see Fig. 2) indicates that the small perihelion distance short-period comets in Table 1 are smaller than the large perihelion distance ones. Observational selection will ensure this, simply because the limiting size of a detectable comet increase drastically with increasing heliocentric distance. We endeavoured to circumvent the observational selection problem by just considering comets larger than 1.5 km. Fig. 5 shows that the median radius of the $R > 1.5$ km comets climbs gently through the $0 < q < 2.7$ au range and then jumps about 0.9 km when the $q = 2.7$ au boundary is crossed. The gradient of this 'climb' is very indistinct. The five median radii, R_{med}, on the $q < 2.7$ au side of the boundary can be tentatively fitted by the relationship

$$R_{\mathrm{med}} = (2.18 \pm 0.24) + (0.03 \pm 0.14)q.$$

The variability of mass-loss rate with perihelion distance indicated in Table 2 makes one expect a much steeper gradient than 0.03 km au^{-1}. The fact that the standard deviation of this gradient (± 0.14) is nearly five times the calculated gradient value gives you very little confidence in the 0.03 km au^{-1} value. Maybe a steeper gradient will be found in the future, when added cometary data decreases the standard deviations of the median radii given in Fig. 5. Another possible explanation for the 0.03 km au^{-1} gradient is that the variability of perihelion distance with time illustrated in Fig. 1 could mix up the inner solar system cometary population thus reducing the variability of size with perihelion distance in the $0 < q < 2.7$ au region.

The $q = 2.7$ au boundary separates short-period comets into two groups, the decaying group ($q < 2.7$ au) having a size distribution index $\gamma = 3.02 \pm 0.18$ and the 'negligable-decay' group ($q > 2.7$ au) having $\gamma = 1.37 \pm 0.14$. In the past, Hughes (2001, 2002) estimated cometary magnitude distribution indices, a, and mass distribution indices, s. Here the cumulative number, N, of comets brighter than absolute magnitude H and the cumulative number, N, of comets more massive than mass $\mathcal{M}$ is given by

$$\log_{10} N = b_1 + H \log_{10} a, \quad \text{and}$$

$$\log_{10} N = b_2 + (1 - s) \log_{10} \mathcal{M}.$$

Thus $\gamma = 5\log_{10} a$ and $s = 1 + (5/3)\log_{10} a$. Therefore $\gamma = 3.02 \pm 0.18$ corresponds to $\log_{10} a = 0.60 \pm 0.04$ and $s = 2.01 \pm 0.15$, and $\gamma = 1.37 \pm 0.14$ corresponds to $\log_{10} a = 0.27 \pm 0.03$ and $s = 1.45 \pm 0.05$.

Research in this field is seriously hindered by the scarcity of observations. Even though just over 200 Jupiter-family comets are known today, it can be seen from Fig. 2 that there are huge numbers awaiting discovery. These potential members of the family are very faint, this faintness being because the comets are too small, or too distant or both. If we could double the number of family members, we could enhance the perihelion distance resolution in Figs 5 and 7 by a factor of 2. Research is also hindered by the accuracy of observations. Table 1 lists 105 known radii divided into four quality groups. The top quality (class 1) contains only 9 per cent of the total, classes 2, 3 and 4 containing 17, 35 and 39 per cent, respectively. Not only should more comets be measured, but efforts should be made to move the measured comets into the top quality classes.

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