AN ALTERNATIVE FOR ENTROPY-ALPHA CLASSIFICATION FOR POLARIMETRIC SAR IMAGE

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ABSTRACT

In this work we discuss SAR target entropy and alpha angle relations to other scattering covariance matrix characteristics and similarity invariants. It is shown that the sum of squared elements of the coherency matrix, normalized by its trace and determinant, has many common features with target entropy parameter. The first element of the matrix is very similar to alpha angle parameter describing scattering mechanism. Possibilities to use the sum of squared elements, determinant and first element of normalized coherency matrix for classification are studied. It appears that classification schemes very similar to entropy-alpha can be established. However, classification results differ slightly from those of entropy-alpha classification as here discussed two-parameter classifications depend on three variables, although parameters are in all cases the same. As an example, NASA/JPL AIRSAR L-Band image of the San Francisco Bay was classified with both proposed schemes and original entropy-alpha classification. The size of the used image was 224 x 256 pixels. The new algorithms classified 97% and 96%, respectively, of pixels to the same classes as entropy-alpha classification. The discussed similarity invariants are straightforward to calculate and they have been used to describe covariance matrix properties in statistics. Virtually are proposed classification algorithms equivalent with entropy-alpha classification because all three use the same amount of information from covariance matrix. However, proposed parameter pairs are much easier to calculate, as they do not require the computation of eigenvalues and eigenvectors.

1 ENTROPY ALPHA CLASSIFICATION

The entropy-alpha classification scheme of polarimetric SAR image was proposed by Cloude and Pottier in [1]. The classification is based on two parameters describing polarimetric properties of response: scattering randomness (called target entropy) and scattering mechanism (called alpha angle). The parameter space formed by alpha and entropy is divided into nine categories based on the nature of scattering. Both parameters are calculated using the eigenvalues and eigenvectors of target average coherency matrix.

To clarify the following discussion, we provide here the basic definitions. Target average coherency matrix *[T]* is defined in Eq. 1 by using measurement vector $\vec{k}$, defined in Eq. 2 by the means of scattering matrix elements. The asterisk in Eq. 1 means complex conjugate transpose and the outer brackets denote ensemble average.

$$\langle [T] \rangle = \left\langle \vec{k}\vec{k}^{*} \right\rangle \tag{1}$$

$$\vec{k} = \frac{1}{\sqrt{2}} \left[S_{hh} + S_{vv}, S_{hh} - S_{vv}, S_{hv} + S_{vh} \right]^T \tag{2}$$

Target entropy H is defined in Eq. 3 by using normalized eigenvalues p_i of matrix *[T]*. Normalized eigenvalues are defined in Eq. 5. In the similar way the target average alpha angle is defined in Eq. 4. where α_i describe the position of corresponding eigenvector. The average alpha angle describes scattering mechanism.

$$H = -\sum_{i=1}^{3} p_i \log p_i \tag{3}$$

$$\overline{\alpha} = \sum_{i=1}^{3} p_i \alpha_i \tag{4}$$

$$p_i = \frac{\lambda_i}{\sum \lambda} \tag{5}$$

The eigenvalues are coherency matrix similarity invariants, because they remain constant under similarity transforms. By similarity transform the coherency matrix can be transformed into a covariance matrix in linear basis and also into circular basis. The alpha angle parameter is not similarity invariant and it is dependent on basis in which covariance matrix is represented.

2 OTHER SIMILARITY INVARIANTS FOR CLASSIFICATION

Eigenvalues are closely related to other similarity invariants of a matrix. A convenient way to study classification schemes similar to Entropy–alpha classification is to define normalized coherency matrix as in Eq.6 as proposed in [2].

$$[N] = \left\langle \overline{k}^* \overline{k} \right\rangle^{-1} \left\langle \overline{k}\overline{k}^* \right\rangle = \frac{T}{trace(T)} \tag{6}$$

Eigenvalues of such a matrix are directly the p_i values used in Eq. 3 and Eq. 4 to calculate the entropy and alpha parameter. The matrix *[N]* is also a Hermitian as matrix *[T]*. Hermitian matrices have the following similarity invariants: trace of the matrix is equal to the sum of eigenvalues as in Eq.7, sum of squared elements of the matrix is equal to the sum of squares of eigenvalues as shown in Eq.8, and matrix determinant is equal to the product of eigenvalues as in Eq.9.

$$trace([N]) = p_1 + p_2 + p_3 \tag{7}$$

$$\sum_i |N_i|^2 = p_1{}^2 + p_2{}^2 + p_3{}^2 \tag{8}$$

$$|[N]| = p_1 p_2 p_3 \tag{9}$$

Those invariants are easy to calculate to any matrix. By taking into account that the trace of a normalized covariance matrix is equal to one, it can be shown, that the eigenvalues are roots of a polynomial equation Eq. 10.

$$p_i^3 - p_i^2 + p_i\left(1 - \sum_i |N_i|^2\right)0.5 = |N| \tag{10}$$

All eigenvalues of *[N]* can then be calculated from matrix determinant and sum of squared elements and sum of diagonal elements. Three invariants form equivalent parameter set to eigenvalues. This implies that also entropy parameter *H* can be presented as a function of matrix determinant and sum of squared elements. This function is complicated and nonlinear. However, sum of squared elements behaves already in a way very similar to target entropy. It is easy to show that if entropy is maximal (equal to one), then sum of squared elements is minimal (equal to 0.333) and if entropy is minimal (equal to zero) then sum of squared elements is maximal (equal to 1). It can be said that the parameter describes scattering regularity rather than randomness.
Another classification parameter, alpha angle is not a similarity invariant of coherency matrix. It is dependent on current basis in which the covariance matrix is presented. By studying the definition of coherency matrix eigenvector it can be shown that the first element of normalized coherency matrix has a form similar to alpha angle definition as shown in Eq. 11.

$$N_1 = \sum_{i=1}^{3} p_i \cos^2 \alpha_i \tag{11}$$

The parameter can be interpreted as a fraction of surface scattering from all types of scattering. It can be also interpreted as a fraction of right left polarized response from total backscattered power in circular basis. As shown in [2] this parameter is linearly related for low entropy scatterers to average alpha and almost linearly related to alpha for high entropy scatterers.

3 NORMALIZED COHERENCY MATRIX BASED CLASSIFICATION SCHEME

The sum or determinant combined with the first element of the normalized coherency matrix (NCM) form classification space very similar to the well-known entropy alpha classification space. However, classes correspond to each other with some uncertainty. This is caused by the fact that function between sum of squared elements and entropy and also function relating alpha angle and N_1 both depend on two normalized eigenvalues. The relating function has so always one free parameter to cause variation.
It is possible to form similar classification spaces as shown in Fig.1 and divide those spaces to similar classes. By using the concept of maximal possible entropy (or zero anisotropy) for certain scattering mechanism it is possible to derive the boundary line of feasible region for proposed parameter spaces. Maximal possible entropy is met when two eigenvalues are equal and the third is the first element of the NCM. The equation of borderline for the sum of squared elements and N_1 is given in Eq. 12 and for the determinant and N_1 in Eq.13.

$$\sum_i |N_i|^2 = N_1^2 + \frac{1}{2}(1 - N_1)^2 . \tag{12}$$

$$|[N]| = \frac{1}{4} N_1 (1 - N_1)^2 \tag{13}$$

Threshold values for classes are derived from scatterplots and they presented in Table 1.

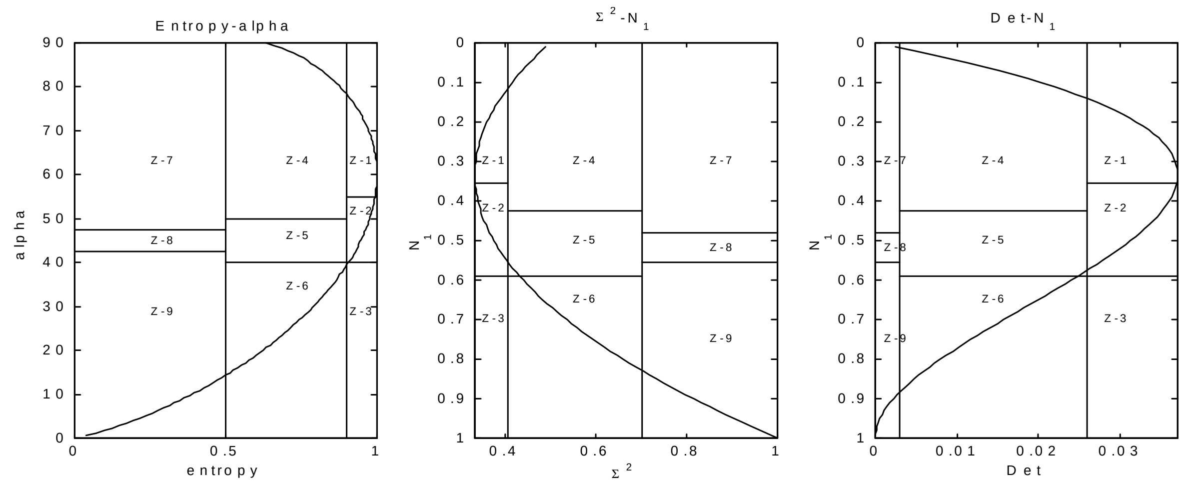

Fig 1.The entropy-alpha, $\sum |N_i|^2$ -N_1 and *Det*-N_1 parameter spaces with classification boundaries.

Zone 9: Low Entropy Surface Scatter, Zone 8: Low Entropy Dipole Scattering, Zone 7: Low Entropy Multiple Scattering Events, Zone 6: Medium Entropy Surface Scatter, Zone 5: Medium Entropy Vegetation Scattering, Zone 4: Medium Entropy Multiple Scattering, Zone 3: High Entropy Surface Scatter (not measurable), Zone 2: High Entropy Vegetation Scattering, Zone 1: High Entropy Multiple Scattering.

Table 1. Threshold values for parameters.

	Z-1	**Z-2**	**Z-3**	**Z-4**	**Z-5**	**Z-6**	**Z-7**	**Z-8**	**Z-9**
$\overline{\alpha}$	55 .. 90	40 .. 55	0 .. 40	50 .. 90	40 .. 50	0 .. 40	47.5 .. 90	42.5 .. 47.5	0 .. 47.5
N1	0 .. 0.355	0.355 .. 0.59		0 .. 0.425	0.425..0.59	0.59 .. 1	0 .. 0.48	0.48..0.555	0.555 .. 1
Entropy	0.9 .. 1			0.5 .. 0.9			0 .. 0.5		
$\sum \lvert N_i \rvert^2$	0.33 .. 0.4			0.4 .. 0.7			0.7 .. 1		
$\lvert [N] \rvert$	0.026 .. 0.037			0.003 .. 0.026			0 .. 0.003		

4 CLASSIFICATION OF EXAMPLE IMAGE

As an example, a NASA/JPL AIRSAR L-Band image for the San Francisco Bay was classified with the two proposed algorithms and with original entropy-alpha classification scheme. The size of the used image was 224 x 256 pixels. The algorithms classified 97 % and 96%, respectively, of the total number of pixels into the same classes as entropy-alpha classification. Misclassification rate of new classification schemes compared to the entropy-alpha classification for each class is presented in Table 1. Misclassification rate for most classes is less than 10 % for sum N1 classification. The sum of squared elements is notably closer to original entropy parameter than determinant.

Table 2. Percentage of pixels classified into wrong classes by NCM based classifications compared to entropy-alpha classification.

	Z-1	Z-2	Z-4	Z-5	Z-6	Z-7	Z-8	Z-9	All
Sum - N_1	10%	12%	3%	3%	5%	6%	7%	0.3%	3%
Det - N_1	12%	13%	3%	5%	7%	25%	25%	2%	4%

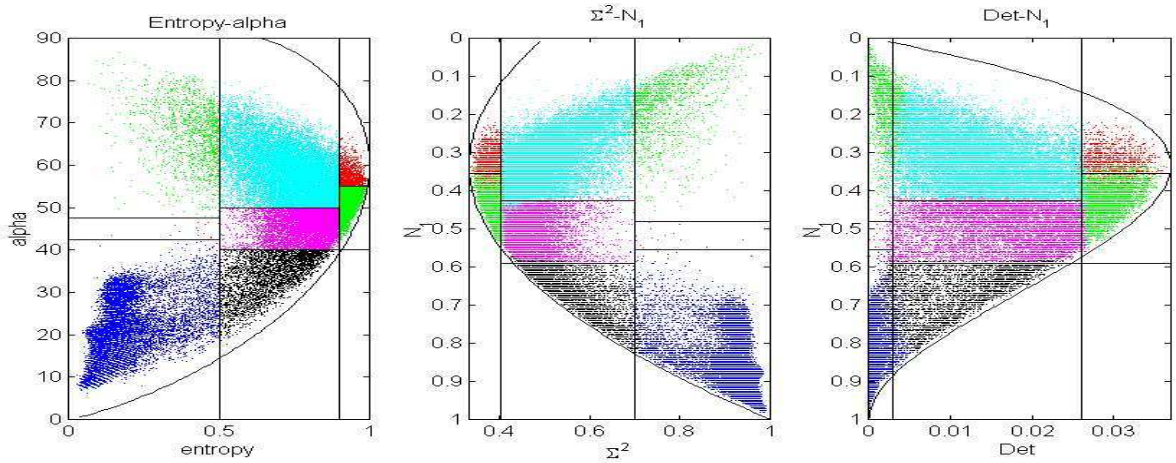

Fig 2. The scatterplots of the San Francisco Bay image in different parameter spaces. The image pixels are classified by using the entropy-alpha classification, result is showed in other two classification spaces. As it can be seen on the image, class boundaries get blurred in two other classification schemes.

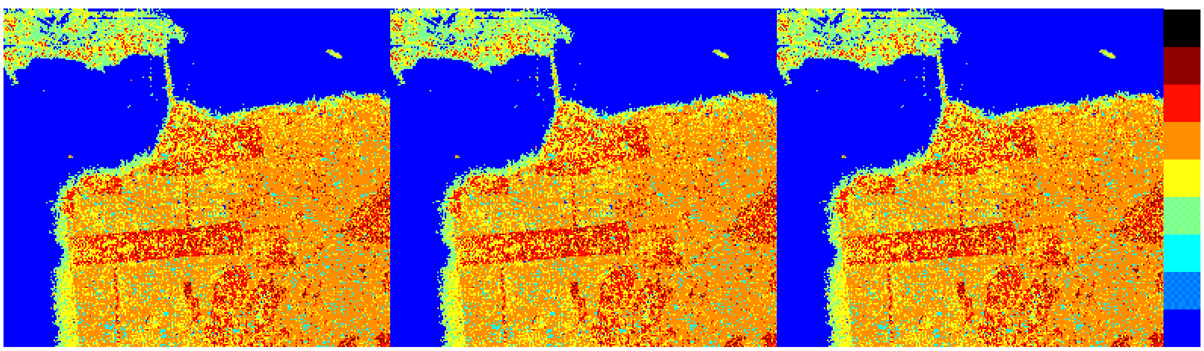

Fig 3. NASA/JPL AIRSAR L-Band image of San Francisco bay classified with three different classification schemes. From right to left, Entropy-alpha, $\sum |N_i|^2$ -N_1 and *Det*-N_1 classification

5 CONCLUSION

It has been proposed two polarimetric classification schemes similar to well known entropy-alpha classification. It has been shown theoretically and also by classifying an example image that proposed schemes similar to entropy -alpha classification. Proposed algorithms use similarity invariants and the first element of normalized coherency matrix. Proposed parameters are very simple to calculate for any matrix in almost any remote sensing software. First element of normalized coherency matrix has simple interpretation as surface scattering fraction. It is shown here that all three discussed classification parameter pairs use same amount of information from covariance matrix and therefore suite equally well for polarimetric SAR image classification.

6 REFERENCES

1. Cloude, S. R., E. Pottier, 1997: An Entropy Based Classification Scheme for Land Applications of Polarimetric SAR, IEEE Trans. Geosci. Remote Sensing, vol 35, pp. 68-78, Jan. 1997.

2. Praks, J., M. Hallikainen, 2000: A Novel Approach in Polarimetric Covariance Matrix Eigendecomposition, Digest IEEE International Symposium on Geoscience and Remote Sensing (IGARSS'2000), Honolulu, USA