Physical Models of Haidinger's Brush

by Maxwell B. Fairbairn (mbfairbairn@hotmail.com)

The visual effect that occurs when observing plane polarized light was first described by the mineralogist Haidinger in 1844. More than a century later Shurcliff (1955) reported that a similar effect occurs with circularly polarized light. To this day the cause of the effect is not fully understood, although it is almost certain that it involves the absorption of short wavelengths in the *macula lutea*, the "yellow spot" of the retina.

It is surprising that knowledge of the existence of the effect is not widespread, even among scientists and others working in optics and related fields. It appears that astronomers are no exception to this general rule.

Description of the Brush

Although the effect does have a medical application in the treatment of amblyopia ("wandering" or "lazy" eye), as far as the author is aware it has never been subject to rigorous clinical study for a significant number of individuals, so that knowledge of its properties is largely based on anecdotal evidence. Like the related effect of Maxwell's Spot, the image is entoptic, *i.e.* it originates in the eye and is not that of a real external object and cannot be photographed.

The effect is both weak and transient. For plane polarization the observer sees a faint elongated yellow stain pinched at the centre and variously described as an hourglass, a bow tie or one half of a Maltese cross, perpendicular to the plane of polarization, *i.e.* the plane of the electric vector. Generally more difficult to see, and parallel to the plane of polarization is a similar shape in blue, indigo or purple stretching about only half the distance of the yellow arm. The figure is centred on the fovea (that part of the retina in which our vision is most acute, and on which we fixate when scrutinizing small objects), subtending a visual angle of about 3 degrees, and fades in about 3 seconds. If the plane of polarization is slowly rotated, the figure will re-appear and rotate with the plane of polarization, remaining visible as long as the plane of polarization rotates. Most people see the yellow arm as continuous and the blue disjoint, although this may reverse as the image fades. Others claim that the continuous colour is the one closest to being vertical, and many see the yellow arm to be narrowest when it is horizontal, *i.e.* parallel to the line joining the eyes.

If the observer waits long enough for the image to fade and then averts the gaze to an unpolarized source, a negative after-image will appear briefly, with the formerly yellow arms blue and vice-versa (J.B. Tatum, personal communication 2000). The author has verified the after-image existence for himself and in others, but has never read in any literature, scientific or otherwise, any mention of it.

The effect can only be seen in light sources with a blue component (wavelengths nominally less than 500 nm) and so is invisible in, say, yellow and red sources. The effect seems to be enhanced in purely blue light, the yellowness replaced by a darker blue sensation.

For right-hand circular polarization the (upright) observer, in both left and right eye, sees the yellow brush with its long axis oriented in an upwards to the right and downwards to the left direction, an azimuth of approximately +45°. The image is fixed with respect to the retina, so that it will rotate only if the observer tilts the head. For left-handed polarization the azimuth is −45° approximately, so that the left and right figures appear to be rotated with respect to each other by ninety degrees. The orientation may differ slightly between the left and right eye.

In his original report, Shurcliff stated that the figure is as prominent for circular polarization as for plane polarization. However, he later wrote that for some observers the circular effect is significantly weaker than the plane and for others the opposite is the case. Most people can see the effect, although many need some tutoring beforehand, while others see it quite spontaneously, and can even detect the partial polarization of the blue sky, without the aid of an analyzer.

Techniques for Observing the Brush

To see the effect most vividly one needs a bright uniform source with no distracting patterns. Blue sky, or a white cloud, away from the direction in which the sky light is most polarized, or a brightly illuminated sheet of white paper or a light bench make good sources. A clear computer screen in a darkened room is also suitable. If such sources are viewed through a plane polarizing sheet, the image will appear and fade as described. If the observer waits a few seconds and then quickly rotates the polaroid by ninety degrees, an image, *more vivid than the original*, will spring into view with both blue and yellow arms more or less equally prominent — a Maltese cross. (It is the author's contention that the image so formed is not Haidinger's brush, but rather the brush reinforced by its previous after-image). Once the image has faded, then it can be refreshed to equal vigour by quickly rotating another ninety degrees in either direction.

To see the after-image, the observer needs only to wait until the figure fades and then quickly remove the polarizing sheet from the field of view.

For circular polarization, the equivalent of the above technique is to switch quickly between two circularly polarizing sheets of opposite handedness.

What's Causing It?

Although the cause of the effect is not fully understood (indeed the nature of the effect has not been fully investigated) the most plausible theories suggest that it is a manifestation of Maxwell's Spot in the case of polarized light, a model first proposed by Helmholtz in 1866 (Helmholtz 1924)[1]. Normally, we are not aware of the existence of the blue-absorbing macula (*i.e.* we do not wander around with yellow spots in the centre of our field of view) and it only makes itself apparent, and then only briefly, under special circumstances, such as switching one's gaze from a yellow field of view to a blue field.

Helmholtz observed that the yellow spot contains a special layer of nerve fibres, the fibres of Henle, which "run radially out from the centre of the fovea in all directions, proceeding principally parallel to the surface of the retina". He proposed that these fibres, like many organic fibres and membranes, were birefringent and behaved like uniaxial crystals, with the optical axis in the fibre's longitudinal, and that the extraordinary ray of blue is more strongly absorbed than the ordinary ray, *i.e* the fibres are dichroic. With a radial alignment of these fibres and vertical polarization, yellowness is enhanced above and below the foveola, the central point of the fovea, and blueness either side of it.

It is interesting to note that the text and accompanying colour plate simulation in Helmholtz's treatise clearly indicate the *yellow arm aligned with the plane of polarization*. Since we now define the plane of polarization in terms of the direction of the electric field vector, could it be that the plane of polarization was defined differently in 1866, some seven years before the publication of Maxwell's electromagnetic theory (1873) and when the nature of light was considered to be "transverse vibrations of the luminiferous aether"?

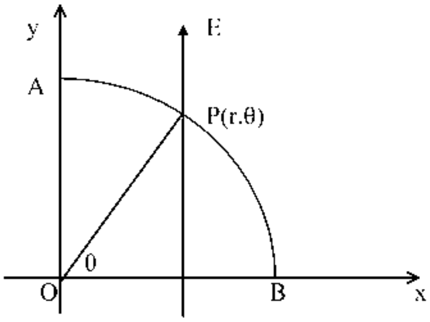
It turns out that this is not the case. In giving his explanation of the cause of the effect, Helmholtz clearly indicates that he takes the plane of polarization of the extraordinary ray to contain the optical axis. Any textbook on optics will state that the extraordinary ray is polarized this way and it can be easily verified experimentally with the following equipment: a crystal of Iceland spar, a plane polarizing sheet and a bowl of water.

The author can only conclude that when Helmholtz wrote his book, he mistakenly believed that the yellow arm aligned with the plane of polarization and, although the mistake may have been detected later, it was never corrected. Subsequent editions of the treatise were added on to, but not revised. This observation does not in any way invalidate the Helmholtz model of the cause of the effect, but simply requires that it is the ordinary ray that is more absorbed, not the extraordinary ray, as happens in dichroic crystals, some absorbing the extraordinary ray and others the ordinary ray. The author does have a theory as to why Helmholtz would make such a mistake, but it would be too lengthy a digression to elaborate on it here.

The Helmholtz Model and its Simulation

It is now known that macular absorption is caused by two organic molecules. The macular carotenoids, xanthophyll and zeaxanthin are isomers ($C_{40}H_{56}O_2$) and feature long zig-zag chains with puckered ring structures at each end (Merck Index 1996). Because of the dissymmetry of each they both exhibit circular dichroism (Joyce M. Edward 2000, private communication). By biochemical standards they are rather simple molecules and are to be found in the food we eat, *e.g.* zeaxanthin is the agent that gives corn its colour.

The macular absorption profile is well known (Wyszecki & Stiles 1967; Bone, Landrum & Cairns 1992), peaking near 458 nm with a small blended secondary peak near 485 nm. Absorption for wavelengths longer than 540 nm is negligible. It is known that the density of the macular pigment may vary by more than an order of magnitude between individuals, which may explain why some people can see the effect quite spontaneously, whereas others cannot see it at all. Being dichroic, however, is not enough to induce the effect; there must be some alignment of the long axes of the molecules, in which case they will exhibit linear dichroism. In this case we consider a tendency to align about the foveola.



As shown in the diagram, consider a plane vertically polarized beam incident on a layer of thickness τ of aligned macular pigments. The alignment may be with long axes radial, as shown by line OP, or concentric as shown by arc APB. The electric field vector E has radial and tangential components $E\cos\theta$ and $E\sin\theta$ respectively. If O is at the centre of the foveola, then τ will decrease with increasing eccentricity. At $\lambda = 458$ nm, where the macular absorption is at its maximum,

---

[1] Helmholtz's important treatise on physiological optics was first published in 1866 but not translated into English until 1924. Both Maxwell's Spot and Haidinger's Brush are discussed in detail in this work.

let κ and $\kappa+\varepsilon$ be the attenuation coefficients perpendicular and parallel to the molecular axes respectively. With $\alpha=e^{-2\kappa\tau}$ and $\beta=e^{-2\varepsilon\tau}$ and radial alignment, the absorption is

$$A_\lambda\left(r,\theta\right)=E_\lambda^2\left[1-\alpha(\cos^2\theta+\beta\sin^2\theta)\right] \quad (1)$$

For concentric alignment, it is simply a matter of swapping the trigonometrical factors. The effect of macular absorption is to subtract blueness from its input, or, alternatively, to add yellowness. We may define a yellowness quantity ΔY such that

$$\Delta Y\left(r,\theta\right)=\int_0^\infty M\left(\lambda\right)A_\lambda d\lambda \quad (2)$$

where $M(\lambda)$ is the normalized empirical macular absorption profile, a weighting function inside the integral (hence the λ is parenthesized rather than subscripted). In practice this function is available in tabulated form, and the range of integration would be 380–540 nm, the function being zero elsewhere. For random alignment, or unpolarized light, ΔY is independent of θ, and, if there is no component less than 540 nm, it is zero.

For radial alignment it can be seen that ΔY is maximized along the y-axis and minimized along the x-axis, whereas the reverse is true for concentric alignment. For a partial alignment, radial or concentric, we would expect that, in the presence of plane polarized light that ΔY would be enhanced above the norm of unpolarized light in some parts and depressed in others, depending on the trigonometrical factors.

How does Equation 2 lead us to a computer-generated simulation? The answer turns out to be the essence of simplicity. Computer graphics limit us to the rendering of colour by the use of varying strengths of three broadband additive primary colours, Red, Green and Blue (RGB). By a happy coincidence, when both are normalized to the same scale, the emission profile of the standard blue primary, CIE chromaticity coordinates (0.15, 0.06), is strikingly similar to the macular absorption profile, the former peaking at 450 nm, and negligible for wavelengths greater than 540 nm. Both peak in that part of the spectrum that we would describe as Indigo.

With the plane of polarization inclined at angle ψ with respect to the x-axis and concentric alignment, the pattern $R'G'B'$ $(r,\theta+\psi)$resulting from such an impinging uniform plane polarized RGB source is thus

$$\begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}=\rho\left(r\right)\begin{bmatrix} 1 & 0 & -\xi\cos^2\theta \\ 0 & 1 & -\xi\cos^2\theta \\ 0 & 0 & 1-\xi\sin^2\theta \end{bmatrix}\begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (3)$$

where $\xi< 1$ is a measure of the difference between maximum and minimum absorption and $\rho(r)$ is the tendency of the molecules to align as a function of the distance from the centre. It should be noted that the above transformation is also valid for elliptical polarization with the major axis aligned with ψ. Programmers should note that for some RGB and ξ values the result may be out of gamut and produce spurious images, this limitation being imposed by the hardware.

The exact form of $\rho(r)$, which must fade away eventually to confine the image to just a few degrees, is of course not known, but if we assume that it is roughly proportional to the macular pigment density itself, then a simple exponential decay, e^{-r}, serves as a suitable starting point. Indeed, an acceptable likeness of the brush, in both colour and form, is produced when this is the case.

Circular Polarization

By itself, the Helmholtz model is not sufficient to explain the appearance of the image in the cases of circular polarization. Following Shurcliff's discovery, attempts were made to either modify the Helmholtz model or propose alternatives. Some, however, such as those proposing that the eye contains the equivalent of an anisotropic-dichroic crystal (Summers, Friedmann & Clements 1970) or a quarter-wave plate (Seliger & McElroy 1963), do not stand up to scrutiny and are readily dismissed.

Shurcliff & Ballard (1964) suggested that some transparent layer in front of the macula is birefringent and behaves like a retarder, but did not go into the detailed implications. For such a retarder with the slow axis very nearly horizontal, the left and right handed polarizations will be "squeezed" into left and right elliptical polarization with the major axes aligned at ±45° to the fast axis, independent of the phase difference δ, which controls the eccentricity but not the orientation, and would form fixed images on the retina as described.

The effect of the same retarder on plane polarization would also be to induce elliptical polarization, with two exceptions, when the plane of polarization is parallel to the slow and fast axes, which would then constitute two preferential directions for detecting the brush. In general, the orientation of the major axis φ of the ellipses and the angle of the plane of polarization ψ, to the slow axis, is given by

$$\tan 2\varphi = \cos\delta \tan 2\psi \quad (4)$$

where δ is the phase difference. A consequence of this is that the figure will rotate as the plane of polarization rotates, but the arms of the brushes will not line up precisely parallel and perpendicular to the plane of polarization (with exceptions at $\psi = 0, \pi/4, \pi/2, 3\pi/2$) but will be rotated slightly clockwise or anticlockwise. For the effect to be much the same strength for both plane and circular polarizations, the retarder should approximate an "eighth-wave" plate. At $\delta = \pi/4$ the maximum amount of this rotation is nearly 5 degrees, a discrepancy which, if it exists, the weakness of the effect notwithstanding, may be discernible under properly controlled clinical conditions[2].

Concluding Remarks

In order to finalize a suitable model it is necessary to answer three questions. Is the alignment radial or concentric? Why

---

[2] The author has observed that in his own case the figure does not always align precisely parallel and perpendicular to the plane of polarization, and seems sometimes rotated slightly clockwise or anticlockwise. By itself, this observation of course does not constitute concrete scientific evidence supporting the above.

does the image fade? What is the source of the birefringent layer?

There are three candidates for the birefringent layer: the cornea, the lens and the arrangement of nerve cells and fibres in front of the macula. Whether or not the lens is involved could be established by a simple experiment. If the lens were the source, those people who have had the lens of one eye replaced by an artificial lens as treatment for cataract would be able to detect circular polarization in one eye, but not in the other.

Shurcliff & Ballard (1964) propose that the alignment of the axes of the macular caretenoids must be radial, and explain the effect as follows. "Incident linearly polarized light will then be absorbed more strongly in some parts of the pattern than in other parts and consequently some parts will fatigue more than others. When the vibration direction of the light is suddenly changed, the varying degrees of fatigue are revealed as a subjective radial pattern. Presumably no such dichroism or orientation pattern applies to longer wavelength (yellow and red) light; consequently a yellow sensation dominates in those regions where fatigue-to-blue has occurred."

The author supports the following view. It is entirely feasible that the macular caretenoids may be aligned concentrically in or among the radial fibres, rather like the concentric threads of a spider web. If this were the case and an observer suddenly switches the gaze to vertically polarized light, then blueness will be enhanced above and below the foveola, and yellowness to the left and right. In other words, in principle, Helmholtz was right. The image then fades because of fatigue and the associated development of a negative after-image, which because of the entoptic nature of the effect, remains superimposed exactly on the original. Since the effect is weak, the two images soon wash each other out (and Maxwell's Spot fades for the same reason), unless the polarization is suddenly removed, whereupon the after-image becomes apparent. This argument is also valid for purely blue light, since the L and M cone sensitivities are active in the blue region (Stockman & Sharpe 2000). If the plane of polarization is suddenly rotated by 90 degrees, or the handedness is suddenly switched, the after-image will reinforce the refreshed image.

To demonstrate the fact that the effect has as much to do with colour fatigue and after-images as it does with polarization, try the following experiment. Draw a circle, 2 to 3 cm diameter, on a white sheet of paper (alternatively, use a computer drawing program). Divide the circle into four equal sectors and uniformly fill one pair of vertically opposite sectors with indigo and the other with yellow. This makes a very poor simulation of Haidinger's brush, but that is not the aim of the exercise.

From a suitable distance, fixate upon this figure long enough to develop a negative after-image, which may then be observed on a white background by averting the gaze slightly, or on a black background by closing the eyes. At first the after-image will be a crisp complementary colour version of the original, but as it fades it will distort so that one colour will become continuous, as happens with Haidinger's brush. A criticism of the simulation is that it does not show the yellow to be continuous, but rather the arms of the figure appear to radiate from a point. However the fading after-image is an excellent likeness of the brush, *i.e.* the after-image of the simulation is a better simulation than the simulation!

Those readers who have not observed the effect are strongly urged to do so, if only for the sheer pleasure of discovering a "new" sense. Plane polarizing sheets are readily available, and even a pair of Polaroid sunglasses, though not ideal, will do the job.

ACKNOWLEDGEMENTS

The author would like to thank Jeremy B. Tatum for many illuminating discussions and for deriving Equation 4.

REFERENCES

Bone, R.A., Landrum J.T., & Cairns, A. 1992, *Vision Research*, 32, 105
Merck Index 1996, 12th ed, 10197, 10248
Seliger, H.H., & McElroy, W.D. 1963, *Light: Physical and Biological Action*, (Academic Press Inc.: New York), 303
Shurcliff, W.A. 1955, *JOSA*, 45, 399
Shurcliff, W.A., & Ballard, S.S. 1964, *Polarized Light*, (D. Van Nostrand Inc.: Princeton), 95
Stockman, A., & Sharpe, L.T. 2000, *Vision Research*, 40, 1711
Summers, D.M., Friedmann, G.B., & Clements, R.M. 1970, *JOSA*, 60, 2
von Helmholtz, H. 1964, *Treatise on Physiological Optics*, transl. ed. J.P.C. Southall, (Dover Publications, Inc.: New York), 2, 304
Wyszecki, G., & Stiles, W.S. 1967, *Color Science: Concepts and Methods, Quantitative Data and Formulae*, (1st ed.; Wiley: New York)

Max Fairbairn completed his MSc. in Astrophysics at the University of Victoria, BC, in 1972. Since then he has taught Physics and Computer Programming at various tertiary institutions. He lives near Sydney, Australia.