The Titius-Bode Rule, Part 1:

Discovering the Asteroids

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For quite some time, I have been fascinated by the Titius-Bode rule (also simply called Bode's law). This simple arithmetic formula is an approximate fit to the spacing of the planetary orbits in the Solar system. As far as I know, there is no scientific explanation of the rule, and it is widely believed to be a coincidence. In this month's column, I will review the history of the Titius-Bode rule, including its apparent success in predicting the distance of undiscovered bodies. In the next issue, we will explore the scientific validity of the Titius-Bode rule and similar relations.

Johann Daniel Titius (1729–1796)

Whether or not you believe that the Titius-Bode rule has any scientific basis, it runs like a golden thread through the tapestry of 18th and 19th century astronomy, joining diverse efforts to discover new bodies in the solar system. The story begins with the German astronomer Titius, born in Konitz, Prussia (now Chonize, Poland). He was raised by his uncle, a naturalist who encouraged Johann's interest in science. Titius received a Master's degree from the University of Leipzig and became a professor at the University of Wittenberg, a post he held for the remainder of his life.

The lasting contribution of Titius to science is his observation in 1766 that the mean distance of the known planets from the Sun approximately follows a sequence that can be expressed in a simple formula. Titius published his findings by inserting a paragraph into his German translation of the book *Contemplation de la nature* by Swiss naturalist Charles Bonnet[1] (1720–1793). Others had noticed the sequence, but apparently only Titius saw the arithmetic relation. From Mercury out to Saturn, the formula for the mean radii of the planetary orbits (in astronomical units) is as follows:

$$D = 0.4 + 0.3\,n,$$
$$\text{where } n = \{0, 1, 2, 4, 8, 16, 32\}.$$

Note the doubling in the space between successive orbits. The degree of success of this formula is shown in Table I.

A little low here, a little high there, and one anomaly: the "missing" planet at 2.8 A.U. Not too bad! Unfortunately for Titius, no one acknowledged his curious relation until much later.

Johann Elert Bode (1747–1826)

Bode, another German astronomer, was a bit of a child prodigy. Born in Hamburg, he was the son of a teacher, and by the end of his teenage years he was already writing astronomy textbooks. He was an assistant to the mathematician Johann Heinrich Lambert[2] (1728–1777) and eventually became director of the Berlin Observatory in 1786. In 1801 he produced *Uranographia*, consisting of 20 star charts and a large catalogue of star positions.

Bode popularized the Titius formula in the second edition of one of his popular astronomy textbooks, and other astronomers began calling it Bode's law, especially after William Herschel (1738–1822) discovered Uranus at orbital distance 19.6 A.U., exactly the distance Titius and Bode would have "predicted." However, there remained the anomalous gap between Mars and Jupiter, which astronomers such as Kepler (1571–1630) had suggested was too wide, and must contain an undiscovered planet. Now the search was on!

Giuseppe Piazzi (1746–1826)

Piazzi was an Italian priest trained in philosophy, but he took up mathematics and astronomy later in life. In 1780, the independent kingdom of Naples asked Piazzi to establish observatories in Palermo and Naples. He went on a fact-finding trip to France and England, where he fell off William Herschel's observing ladder and broke his arm.

On New Year's Day, 1801, working from the observatory in Palermo, Sicily, Piazzi fortuitously discovered an 8th-magnitude object in the constellation Taurus. He was in the process of cataloguing the positions of stars using a precision instrument fabricated by Jesse Ramsden (1735–1800). By the rate of its daily motion

Table I: Planetary orbit data supporting the Titius-Bode rule

Distance from Sun in A.U	*Mercury*	*Venus*	*Earth*	*Mars*	-	*Jupiter*	*Saturn*
True (mean)	0.39	0.72	1.0	1.5	-	5.2	9.6
Titius-Bode rule	0.4	0.7	1.0	1.6	2.8	5.2	10.0

[1] Bonnet was the first to make use of the word evolution.

[2] Lambert proved π to be an irrational quantity and introduced the term *albedo* to quantify the diffuse scattering of light by bodies.

among the stars, Piazzi suspected that the object occupied an orbit between Mars and Jupiter. Eventually he lost track of his object and could not recover it in the Sun's dazzle. Piazzi made some attempts to determine an orbit, but he fell sick and turned his observations over to Bode and others.

At that time, the young German mathematician Johann Karl Friedrich Gauss (1777–1855) had just developed the method of least squares for fitting model curves to data. From the few observations that Piazzi had made, he was able to calculate an orbit for the new object. This calculation enabled the Hungarian amateur astronomer Baron Franz Xavier von Zach (1754-1832) to recover the object and confirm that its orbit indeed lay between Mars and Jupiter.

Piazzi, credited with the discovery, named his object Ceres after the patron goddess of Sicily. Ceres turned out to be the first of thousands of minor planets in the zone. Herschel suggested they be called "asteroids," owing to their star-like appearance, as they were dim and did not show disks like the planets. In Piazzi's lifetime, only 4 asteroids were discovered.

I hear you asking, "So, what about the Titius-Bode rule?" It turns out that Ceres has an eccentric orbit, but the semi-major axis of the ellipse is 2.77 AU, making it an excellent candidate for the missing planet in the Titius-Bode sequence, except for the fact that its diameter of about 1000 km falls short of the diameter of a full-size planet. The astronomers were not satisfied with this small fry and continued looking for the big fish.

Heinrich Wilhelm Matthäus Olbers (1758–1840)

Olbers was a German medical doctor, but a very dedicated amateur astronomer, having converted the top story of his house into an observatory. He was a keen comet hunter, discovering five in his lifetime, and he worked out a method for determining comet orbits from observations. Following the death of the last of his family in 1820, he devoted his life entirely to astronomy. He is probably best known for being the first to ask "Why is the night sky black?" His discussion of this topic has come to be known as "Olbers' paradox."

Olbers was a leader in the effort to find a planet between Mars and Jupiter. He belonged to a group of six astronomers known as "the celestial police," who were coordinating the search for the planet deemed missing by the Titius-Bode rule. The group included fellow amateurs von Zach and Johann Hieronymous Schröter (1745–1816). They intended to recruit other astronomers (including Piazzi) to the organized chase; however, their plans were somewhat foiled by Piazzi's accidental discovery. Not discouraged, Olbers carried on the hunt and soon discovered two asteroids: Pallas in 1802 and Vesta in 1807. In between, Karl Ludwig Harding (1765–1834) discovered Juno in 1804. It became evident that there were several minor planets occupying the space reserved for the elusive planet, and Olbers was the first to suggest the planetary-explosion origin for the asteroids. (It is now believed that the asteroids are remnants from the birth of the Solar system and that the asteroid belt marks the transition between the inner and outer zones of the Solar system.)

As an interesting aside, a little digging on the Internet uncovered the site `pdssbn.astro.umd.edu/nodehtml/sbdb.html` which has an interactive search engine linking asteroid numbers, official designations, and names. Asteroids 998–1002 and 1998 have names relevant to this article, as shown in Table II.

If Titius-Bode were not a scientific rule but a mutual fund, and if you had invested early, this would have been a good time to sell. The rule, based on minimal evidence, had been bolstered by the discovery of a major planet. Following that, astronomers had been encouraged by belief in the rule to search for a "missing" planet, leading to the discovery of several minor planets. Pretty good dividends! The share price was about to fall, however.

Neptune and Pluto

In my April 2001 *Reflections* column, I described the discovery of Neptune by Johann Galle (1812–1910), based on the calculations of Urbain le Verrier (1811–1877). Applying standard techniques of celestial mechanics, Le Verrier analyzed anomalies in the motion of Uranus and deduced the existence and position of a trans-Uranian planet. In 1846 Neptune was discovered to follow an orbit of radius 30.1 AU, while the Titius-Bode rule suggests 38.8 AU. Interestingly, both le Verrier and Adams used the Titius-Bode rule in their analysis, but I expect this was more along the lines of an educated guess to get the iterative calculations started, rather than a critical assumption. If you hadn't sold your Titius-Bode shares by then, you were about to lose the shirt off your back!

American astronomer Clyde William Tombaugh (1906–1997) discovered Pluto in 1930. The orbit of Pluto is odd in many respects, so it is not surprising that its semi-major axis of only 39.5 AU is a far cry from the Titius-Bode "prediction" of 77.2 AU. The Titius-Bode rule was quickly losing its appeal as a scientific tool.

Science or Numerology?

Looking back on what I have written, I think I have ended up being a little harsh on the Titius-Bode rule. There are few scientific rules that do not break down eventually when you push their application

Table II: Asteroids with connections to the Titius-Bode law

Asteroid Number	*Asteroid Name*	*Discovery Date*	*Discoverer*
998	Bodea	1923-08-06	K. Reinmuth
999	Zachia	1923-08-09	K. Reinmuth
1000	Piazzia	1923-08-12	K. Reinmuth
1001	Gaussia	1923-08-08	S. Belyavskij
1002	Olbersia	1923-08-15	V. Albitzkij
1998	Titius	1938-02-24	A. Bohrmann