

FAQs, Facts and Epacts

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“I wonder why Easter falls when it does?”

This is a Frequently Asked Question by people of the “normal” persuasion. The implied corollary is that only a severely *abnormal* person would actually try to answer what is clearly a convoluted problem. Fortunately or otherwise, the straitjacket of presumed normalcy has long since been gathering dust at the back of my closet (right behind my suits!), and the real me has emerged unencumbered from said enclosure to fearlessly tackle life’s great mysteries.

In 2000 the Easter question surfaced more frequently than usual, due to its unusually late date of April 23. As he prepared for a CBC Radio interview on the subject, my good friend Russ Sampson innocently raised a question that sent us both careening down this particular slippery slope amidst an avalanche of emails, Web sites, graphs, and raw data.

My own fascination with calendars predates even that with astronomy, but the two are inextricably intertwined. In *The Timetables of History* (Grun 1982), the first entry in the category Science, Technology, Growth reads, “[5000 to 4001 B.C.E.] The Egyptian calendar, regulated by sun and moon: 360 days, 12 months of 30 days each.” This suggests that not only is astronomy the oldest of the sciences (some would argue for physics), but that its first application was to measure the passage of time. Or, as Eisenberg (1958) states simply, “All calendars that have ever been in use are based on a study of the skies.”

The calculation of Easter is the subject of fascinating chapters in histories of science and religion. Steel (2000) declares,

“The single factor which has caused most controversy and division in the Christian religions, and which is pivotal in calendar definitions, is the calculation of the date of Easter.” Duncan (1998) makes the extraordinary claim that at the start of the Middle Ages, “the calculation of the date of Easter became the slender thread that science would hang by over the coming centuries,” in effect becoming the primary justification for continuing astronomy and mathematics. According to Gould (1997), “In Christian history, the need to reconcile solar and lunar cycles has centered on one of the most complex and persistently vexatious problems in the history of calendrics: the calculation of Easter. Books, indeed libraries, have been written on the subject, and great scholars have devoted their lives to devising rules and procedures for getting this cardinal day right... Easter became more problematic than any other calendrical day, or any other movable feast, because its definition includes both solar and lunar elements, and its date cannot be determined until we know how to reconcile all the great, and distressingly fractional, cosmic cycles. For Easter falls on the Sunday following the first full moon (the lunar component) after the vernal equinox (the solar contribution).”

Fortunately, the relatively small integers 235 and 19 comprise a good working ratio of lunar to solar cycles. Although he was likely beaten to the punch by Chinese and Babylonian astronomers, Meton of Athens is generally credited with the discovery of this relationship in the 5th century B.C.E. The

relationship is therefore known as the Metonic cycle. Its implications are that lunar phases occur on the same date at 19-year intervals. In the Gregorian calendar, which is strictly solar, the dates of lunar phases gradually creep forward through the months, resulting in a given phase occurring twice in a month, such as the so-called “blue moon,” 7 times in 19 years (McCurdy 1999). By comparison, the luni-solar Hebrew calendar deals with this by having 12 “common” years of 12 lunar months and 7 “embolismic” years of 13 months in a 19-year cycle. An intercalary month, We-Adar or 2nd Adar, is added to Years 3, 6, 8, 11, 14, 17, and 19 of the lunar cycle (Parise 1982).

The Council of Nicæa in 325 C.E. adopted the convention that Easter must occur on a Sunday. This introduced a third factor, namely that the artificial construct of the seven-day week be superimposed on the already complex interrelationship between the solar year and the lunar month. Other conventions introduced by Dionysius Exiguus in the 6th century and Aloysius Lilius and Christopher Clavius in the 16th century replaced astronomical observations with a more rigid structure. The vernal equinox was assumed to occur always on March 21, while the date of the Paschal Full Moon was limited to 29 possibilities, called epacts, derived from ecclesiastical tables. Easter was thus confined to the 35-day period from March 22 to April 25. Their methods may seem crude today, but given the mathematical and observational tools at their disposal, they were in fact remarkably sophisticated, even elegant.

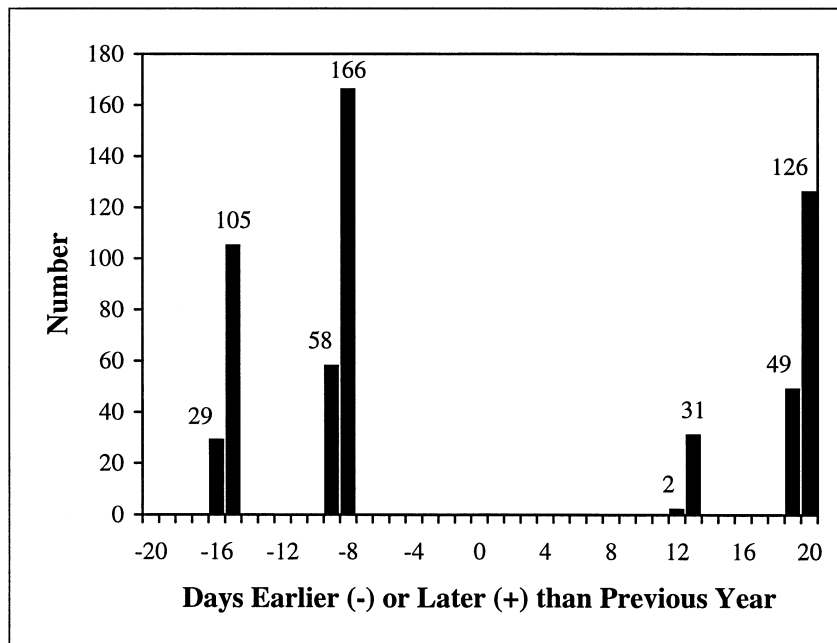


Figure 1: The intervals from each Easter to the next for the period 1583 to 2150 show the range of eight possibilities. In 12/19 (63%) of all cases, Easter moves earlier in the calendar. Note that leap-year Easters, as displayed by the shorter bars on the left of each pair, are disproportionate because leap years are smoothed out of the ecclesiastical tables but not, of course, on the calendar. One result is that the occurrence of Easter twelve days later than the previous year is exceptionally rare, occurring only in 1748 and 2076 in the period under review. (All diagrams courtesy of Russell D. Sampson.)

Since I didn't wish to write my own book, much less an entire library, on the subject, the primary subject of my research was to seek patterns in the date of Easter in the short and medium term. First, how does the date of Easter change from one year to the next? There is a range of eight possibilities, four of which apply to leap years only (www.tondering.dk/claus/calendar.html), as shown in Figure 1. Given that neither the common lunar year of 354 days nor the embolismic year of 384 days is close to the solar year of 365.24 days, one result is that Easter always differs by more than a week from one year to the next, seemingly jumping all over the chart. One night while under voluntary house arrest (I was volunteering at a Bingo), I spent an enjoyable six hours poring over about 5,000 digits of Easter dates, from calendar reform in 1583 through to 2150. (Did I mention that I'm not normal?) I tried to remove some of the inherent noise by reorganizing the data.

One fact that was immediately

apparent was the extremely regular application of the Metonic cycle. Easter moves later in the calendar exactly 7 times every 19 years, in a cycle of 3 / 3 / 2 / 3 / 3 / 3 / 2, precisely the same frequency as the intercalary insertion of We-Adar. The pattern resembles a rhythm cycle, or *tala*, used in East Indian music, where a long complex bar is subdivided into regular, smaller intervals. As the most common subdivision is three years long, I took a set of 60 Easters from 1976 to 2035 and sorted them into three columns, as follows:

1976/04/18	1977/04/10	1978/03/26
1979/04/15	1980/04/06	1981/04/19**
1982/04/11	1983/04/03	1984/04/22*
1985/04/07	1986/03/30	1987/04/19
1988/04/03	1989/03/26	1990/04/15
1991/03/31	1992/04/19**	1993/04/11
1994/04/03*	1995/04/16	1996/04/07
1997/03/30	1998/04/12	1999/04/04
2000/04/23**	2001/04/15*	2002/03/31
2003/04/20	2004/04/11	2005/03/27
2006/04/16	2007/04/08	2008/03/23
2009/04/12	2010/04/04	2011/04/24***

2012/04/08	2013/03/31	2014/04/20
2015/04/05	2016/03/27	2017/04/16
2018/04/01	2019/04/21**	2020/04/12
2021/04/04*	2022/04/17	2023/04/09
2024/03/31	2025/04/20*	2026/04/05
2027/03/28	2028/04/16	2029/04/01
2030/04/21**	2031/04/13	2032/03/28
2033/04/17	2034/04/09	2035/03/25

Each vertical column therefore displays 20 Easters at three-year intervals. Each column is not a Metonic cycle *per se*, but simulates one. Note how the bottom row compares to the top; after 57 years the calendar shift is one day, and the moon phase is identical.

At three-year intervals, a regular pattern starts to emerge. The Paschal Full Moon moves ahead at extremely regular intervals of three days (occasionally two or four), as seen in Figure 2. Easter Sunday itself is shown in Figure 3; it typically moves earlier by four days every three years (three if there is no leap day in that time). About 10% of the time, however, Easter moves later by a similar shift of three or four days. This occurs when the Paschal Full Moon falls in the first instance on a Saturday, and the second on a Sunday, forcing Easter back by an entire week. These are indicated with an asterisk.

Two dates of each set of 19 contain a major shift backwards (double asterisk). These shifts occur when there is what I call a "double correction," or two embolismic years out of three. In such cases there are 38 lunations between Easters three years apart, rather than the usual 37, resulting in a major leap back in the calendar to cancel out the otherwise forward creep in the dates. All of the extreme early Easters occur just prior to a double correction, and all the extreme late ones just after, as can be seen in the following table:

1902/03/30	1905/04/23	1913/03/23	1916/04/23
1921/03/27	1924/04/20	1932/03/27	1935/04/21
1940/03/24	1943/04/25	1951/03/25	1954/04/18
1959/03/29	1962/04/22	1970/03/29	1973/04/22
1978/03/26	1981/04/19	1989/03/26	1992/04/19
1997/03/30	2000/04/23	2008/03/23	2011/04/24***
2016/03/27	2019/04/21	2027/03/28	2030/04/21
2035/03/25	2038/04/25	2046/03/25	etc.

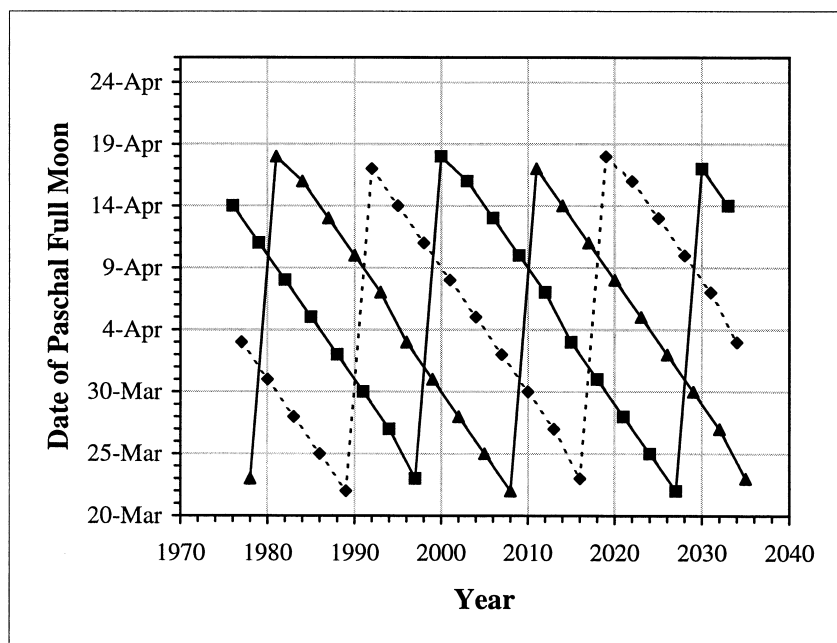


Figure 2: The Paschal Full Moon is derived from preordained ecclesiastical tables causing exceptionally regular intervals. The period 1976 to 2035 is represented here, sorted into three-year intervals to approximate three overlapping Metonic cycles of identical shape. In a given series, each successive Paschal Full Moon occurs two epacts earlier than its predecessor, which typically translates into three-day intervals. Two-day intervals occur at the very top of the graph due to the exceptional case of Epacts 24 and 25, which cause periodic bunching of three epacts. This problem was caused by the fact that the epacts are determined at New Year's during a "full" month of 30 days, but are applied for the purposes of Easter to a "hollow" month of 29 days.

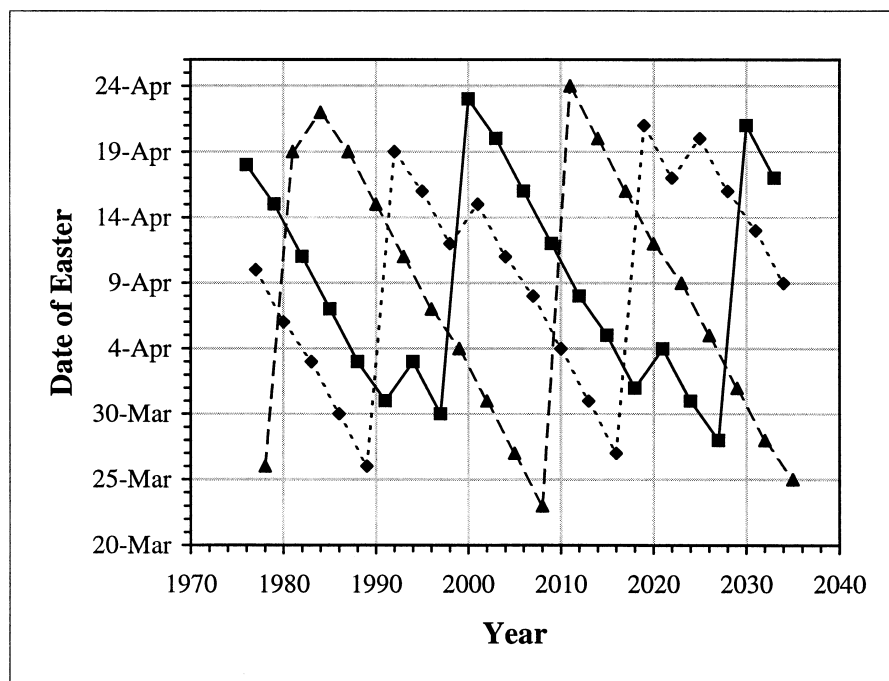


Figure 3: Dates of Easter for the same period, also sorted into three-year intervals. Generally the forward progression within a series is three to four days, depending on the fall of leap years. Occasional corrections "backwards" are due to the fact that 37 lunations are slightly longer than 156 weeks.

While a few of these dates have moderated towards the other end of the week, these four Metonic cycles carry *all* of the extreme dates of Easter in our lifetimes. Note that all the late Easters occur exactly three years after the early ones. Of particular note is the pair in 2008 and 2011, when both corrections occur simultaneously (triple asterisk). The two dates involved are separated by the maximum possible 161 weeks. Remarkably, the two most extreme dates in over 60 years will occur only three years apart.

Let's look at the medium term patterns by returning to our database of the 568 Easters beginning in 1583. In the complex Gregorian system, every few centuries the table of epacts is rotated by a day, due to a change in the "Clavian Differential" caused by the inexactitude of any of the measured periods (www.friesian.com/easter.htm). A "solar adjustment" automatically occurs in non-leap century years, while a "lunar adjustment" in the opposite direction is inserted eight times in 2,500 years to correct for the Metonic cycle. Occasionally the two cancel each other out, as in 1800 and 2100 (Richards 1998). The rotation of the epacts is graphically displayed in Figure 4 as quantum jumps from otherwise absolute linearity.

This linearity is much less in evidence in Figure 5, a nifty graph Russ produced that shows all the dates of Easters rather than their related Paschal moons. The most striking pattern is a series of diagonals, indicating groups of Easters slowly moving later in the calendar by roughly one day per twenty years. What's going on?

Let's start by looking at any narrow vertical segment of the graph representing, say, a quarter century. At a given time there are roughly eight clusters of possible dates, each separated by half-week intervals. This gives nicely with our previous finding that Easters at three-year intervals have almost invariably moved ahead or back by three or four days. There is a natural resonance reminiscent of Kirkwood gaps.

Meanwhile, examine any given (non-extreme) date horizontally across the chart. Typically, Easter will recur on that date two, three, or even four times at

intervals of 11 years, followed by gaps of a half century or so before another cluster. Eleven-year recurrences are common due to two near resonances. Eleven years is the most natural fraction — along with its complement, 8 — of a Metonic cycle; in 11 years the lunar phase moves earlier in the calendar by only one or two days. Eleven years is also the most natural fraction — along with its complement, 17 — of a solar cycle of 28 years, in which a full rotation of possible calendars is completed. A given date occurs on the same weekday as 11 years previously 75% of the time; *e.g.*, April 15, 1979, April 15, 1990, and April 15, 2001 all happened to be Sundays. As the Paschal Full Moon occurred during the previous week in all three cases (April 11, 10, and 8, respectively), in all three of those years Easter fell on April 15. Next year's Easter is part of a foursome (1991, 2002, 2013, 2024) that occur on March 31. Such foursomes always end with a leap year (Meeus 1997a). However, a leap year falling earlier in a cycle results in the next Easter being pushed back by a day, for example:

1998/04/12	1988/04/03
2009/04/12	1999/04/04
2020/04/12	2010/04/04
2031/04/13	2021/04/04

Thus it appears that the fall of leap years — specifically, the one-in-four *absence* of a leap day in an 11-year cycle — is the primary cause of the gradual upward slopes of the diagonals in Figure 5. It is instructive to note that foursomes disappear and an unusual number of “singletons” occur around non-leap century years, where a leap day is removed from *all* cycles, which are thus shifted to one calendar day later. For example, Easters at 11-year intervals occur on:

1689/04/10	1700/04/11
1690/03/26	1701/03/27
1691/04/15	1702/04/16
1692/04/06	1703/04/08
1693/03/22	1704/03/23
	<i>etc.</i>

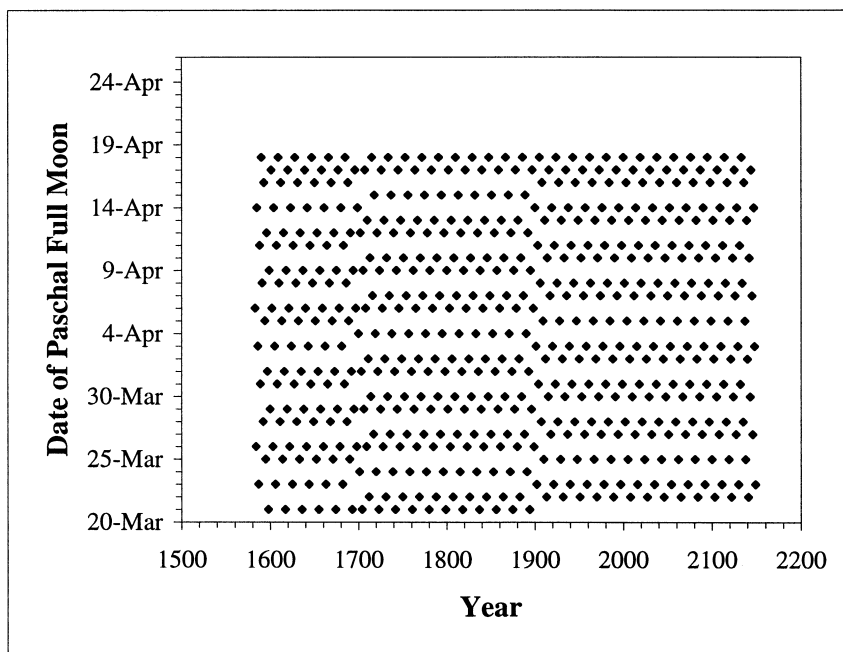


Figure 4: The distribution of Paschal Full Moons for the period 1583 to 2150 shows an extremely regular pattern. The data points on a given horizontal line are separated by intervals of 19 years. In the period shown, there are four solar corrections of one day that occur in the non-leap century years 1700, 1800, 1900, and 2100; however, lunar corrections of one day in the opposite direction in 1800 and 2100 effectively cancel out two of them. As a result, the cycle of epacts shifted, or rotated, only in 1700 and 1900, as can be readily seen. Note that in the current era, ecclesiastical full moons cannot occur on March 21; as a result, there are no March 22 Easters between 1818 and 2285.

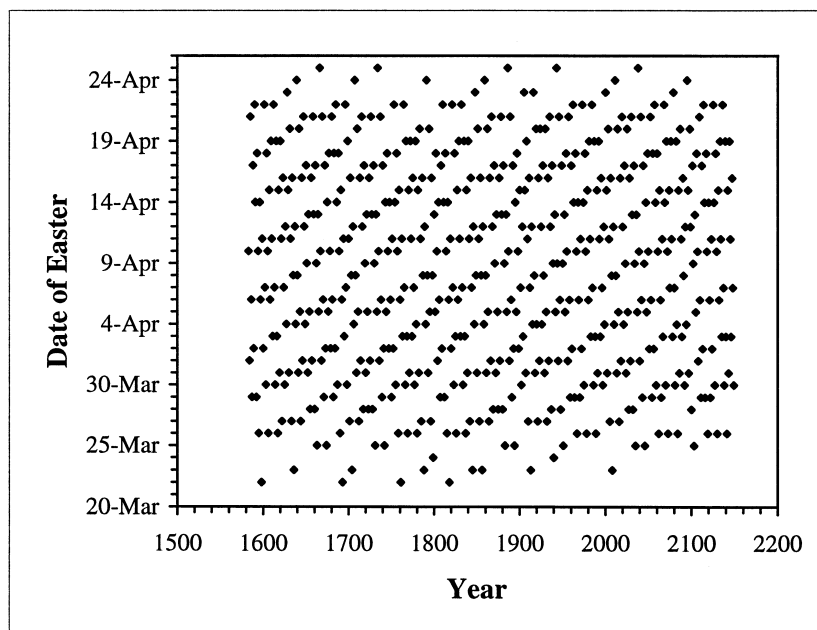


Figure 5: Dates of Easter from 1583 to 2150 graphically show the effect of the superimposition of the seven-day week. The horizontal rows of Figure 4 have largely disappeared, to be replaced by a pattern of diagonal lines. These show an apparent advance in the date of Easter by one day every ~19 years (a coincidence unrelated to the Metonic cycle). Easters on a given date frequently show a repetition of three or even four occurrences at 11-year intervals, followed by gaps of half a century or so.

Note the rare two-day lag between 1692 and 1703, because there is only *one* leap day in the 11-year interval. Similar two-day jumps account for the largest intervals in the entire period between Easters for a common (non-extreme) date. Because Easter jumps from April 8, 1792 to April 10, 1803, Easter does not occur on April 9 from 1730 until 1871; likewise a jump from March 27, 1796 to March 29, 1807 results in no March 28 Easters between 1728 and 1875. These two exceptions show up as gaps in the diagonal lines on Figure 5, which are otherwise remarkably inclusive.

On Figure 5 there are also some 30 "triplets" where three Easters occur on the same date within 11 years. The middle one is always the result of one of the corrections described in Figure 3. Curiously, all of the triplets are completely isolated in that there are no further Easters either 11 years before or after the triplet. Doublets of two Easters in five or six years are less common, but similarly isolated.

No doubt there are other patterns in the graphs that might emerge to the interested reader. I would be happy to correspond with anybody who has questions or discoveries.

There are still longer periods that can be examined. Meeus (1997b) refers to a periodicity of some 6,000 years and to the cyclical recurrence of all Easter dates over 5,700,000 years! Such immense time spans are beyond the scope of this study, so I have to conclude that they are of even less practical value.

REFERENCES

- Duncan, D. E. 1998, *Calendar: Humanity's Epic Struggle to Determine a True and Accurate Year* (Avon Books Inc.), 59
- Eisenberg, A. 1958, *The Story of the Jewish Calendar* (Abelard-Schuman Ltd.), 25
- Gould, S. J. 1997, *Questioning the Millennium: A Rationalist's Guide to a Precisely Arbitrary Countdown* (Harmony Books), 154
- Grun, B. 1982, *The Timetables of History* (Simon & Schuster Inc.), 3

- McCurdy, B. 1999, *JRASC*, 93, 68
- Meeus, J. 1997a, *Mathematical Astronomy Morsels* (Willmann-Bell Inc.), 361
- Meeus, J. 1997b, *Mathematical Astronomy Morsels* (Willmann-Bell Inc.), 354
- Parise, F. 1982, *The Book of Calendars* (Facts on File Inc.), 12
- Richards, E. G. 1998, *Mapping Time: The Calendar and its History* (Oxford University Press Inc.), 367
- Ross, K. E., www.friesian.com/easter.htm
- Steel, D. 2000, *Marking Time: The Epic Quest to Invent the Perfect Calendar* (John Wiley & Sons Inc.), 93
- Tondering, C., www.tondering.dk/claus/calendar.html

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