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The multiperiodic δ Scuti star 4 Canum Venaticorum: amplitude variability

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ABSTRACT

Recent multisite campaigns of the Delta Scuti Network have revealed 34 frequencies of pulsation for the star 4 CVn. Our present knowledge of the frequencies makes it possible to reanalyse the shorter data sets in the literature, photometric observations from 1966 to 1997.

4 CVn shows strong amplitude variability with time-scales of ten years or longer, although for neighbouring years the amplitudes usually are similar. Seven of the eight dominant modes show annual variability of ~12 per cent. The variability increases to ~40 per cent over a decade. The formally derived time-scale of variation of 30 years can only be a rough estimate, since this is also the length of the available data span. The variability is compared with that of FG Vir, which shows lower amplitude variability.

The cyclic behaviour of the amplitude variations excludes an evolutionary origin. There exists some evidence that a mode at 6.12 d^{-1}, which appeared during 1996 and 1997, may have been present with small amplitudes in the 1976–1978 time period.

The pulsation mode at 7.375 d^{-1} exhibited the most rapid decrease found so far: the *V* amplitude dropped from the highest known value of 15 mmag in 1974 to 4 mmag in 1976 and 1 mmag in 1977. After that the mode has been increasing in amplitude. There exists a phase jump between 1976 and 1977, suggesting the growth of a new mode. It is interesting to note that this mode also has the strongest coupling with other modes with combination frequencies, $f_i \pm f_j$. The amplitudes of these combination frequencies are also strongly variable from year to year. We speculate that power is transferred between the modes through mode-coupling.

Key words: stars: individual: 4 CVn (AI CVn) – stars: oscillations – δ Scuti.

1 INTRODUCTION

The variability of the evolved δ Scuti star 4 CVn (HR 4715 = HD 107904 = AI CVn, F3III-IV) was discovered by Jones & Haslam (1966). Since then a number of photometric studies have become available. A list is given in Table 1, where several short or less accurate studies were omitted. The table also contains detailed information of importance to the analyses carried out later in this paper.

The puzzling and seemingly contradictory behaviour of 4 CVn mentioned in a variety of papers in the literature is caused by the amplitude variability of a number of pulsation modes with essentially constant frequencies (Breger 1990a). So far, 34 frequencies of pulsation have been discovered with the majority confirmed in independent studies.

The multifrequency structure of 4 CVn is shown in Fig. 1. It is divided into three separate regions: the main pulsation modes occur between 4.7 and 8.6 d^{-1}, while the peaks in the power spectrum outside this region (i.e. below 4 d^{-1} and above 10 d^{-1}) can be identified with frequency combinations, $f_i - f_j$ and $f_i + f_j$. It is unclear at this time whether these modes are true combinations of the pulsation modes in the main pulsation frequency region or whether they could be modes driven by resonance.

The origin of the amplitude variability in δ Scuti stars is presently not understood. 4 CVn is probably the best-studied δ Scuti star with a high degree of amplitude variability. The new data provide an excellent opportunity to examine the phenomenon of amplitude variability.

2 MULTIFREQUENCY ANALYSES OF PREVIOUS DATA

The knowledge of the frequency spectrum from the two years 1996 and 1997 makes it possible to reanalyse the available data from the previous decades in order to see which of the 34 known

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Table 1. Photometric observations used.

Year	Observer/Reference	No. nights used	Comments
1966–1970			[a]
1966	Jones & Haslam (1966)	0	Discovery paper, radial velocity only
1966	Landolt (1966)	5	
1966	Danziger & Dickens (1967)	3	
1966	Hayes & Heiser (1968)	7	
1967	Hayes & Heiser (1968)	5	
1969	Shaw (1977)	3	
1970	Fitch (1980)	5	[b]
1974–1978			
1974	Fitch (1980)	24	[b]
1976	Loumos (1980)	4	[c]
	Fitch (1980)	7	[b]
1977	Loumos (1980)	9	[c]
1977	Warman, Pena & Arrelano-Ferro (1979)	5	
1978	Loumos (1980)	6	[c]
1983–1984			
1983	Breger et al. (1990)	8	5 frequencies found
1984	Breger et al. (1990)	14	5 frequencies found
1996–1997			
1996	Breger et al. (1999)	53	34 frequencies found
1997	Breger & Hiesberger (1999)	32	19 frequencies found

Notes to Table 1:

[a] The comparison star used for the 1966–1970 data, HD 108100, is a γ Dor variable (Breger et al. 1997) with frequencies of 1.321 and 1.404 d^{-1}. These frequencies are outside the δ Scuti range (except for combination frequencies) and should not affect the analyses in a serious way. Pre-whitening these low frequencies with variable amplitudes is not practical due to the small amount of data.

[b] The unpublished extensive data by Fitch were slightly edited with poor nights removed. The amplitudes of 1974 *b* measurements were multiplied by 0.83 to simulate *V* amplitudes. In the same data set, we also pre-whitened the low frequencies of 1.321 and 1.404 d^{-1}, since they originate in the comparison star, HD 108 100.

[c] In the 1974–1978 analyses, the Loumos data were given double weight due to their high precision relative to the other data sets in this time period.

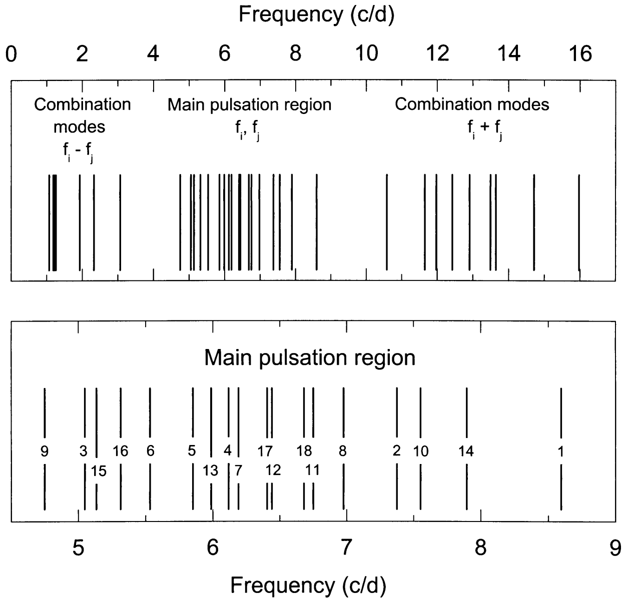

Figure 1. Pulsation structure of 4 CVn in the frequency domain. The number shown in the lower panel for each mode refers to the numbering scheme by Breger et al. (1999) and represents a sequence of decreasing photometric amplitudes. This sequence is approximate because of amplitude variability.

frequencies are also present in the older data and to examine the amplitude variability in these earlier years.

The difficulty with this type of analysis is caused by the fact that the older data sets are much shorter than the recent investigations. A statistical analysis assuming the presence of all 34 frequencies can fail because this approach requires the determination of the amplitude and phase of each frequency, i.e. $(2 \times 34 + 1) = 69$ unknowns for each data set. If a small but reasonable amount of frequency variability is included, the number of unknowns increases to 103. Such a high number of unknowns may cause an overinterpretation of the shorter data sets, e.g. the six nights of data available for the year 1978. The most common effect of the overinterpretation is to spuriously increase the amplitudes for the different modes through noise and aliasing. This problem cannot be treated with the available formal statistical tests, but its effect can be minimized by introducing minimum amplitude limits to be determined experimentally. This method of computing fits with all 34 frequencies is hereafter called the *Brute Force Method* and should be applied with great caution.

A more conservative approach is provided by the *Fourier Peak Method*. If *n* frequencies are already known to be present in a particular data set, a least-squares algorithm can fit these *n* simultaneous frequencies. Then the power spectrum of the residuals is computed from the residuals. The main peaks in this

Table 2. Amplitude variability of the main pulsation modes of 4 CVn.

	Frequency	V amplitude in mmag									
	d^{-1}	1966/7	1969/70	1974	1976	1977	1978	1983	1984	1996	1997
Limit of No Detection, ND		3	3	1.5	1.5	1	2	1	1	0.5	0.7
f_1	8.595	13	11	15	16	21	24	14	13	15	17
f_2	7.375	8	12	15	4	1	8	6	5	12	12
f_3	5.048	26	22	9	6	5	6	7	7	11	11
f_4	6.117	ND	ND	ND	3	3	4	ND	ND	9	10
f_5	5.851	17	13	13	12	11	9	10	8	10	10
f_6	5.532	11	8	9	13	12	12	4	2	6	5
f_7	6.190	3	3	3	5	4	5	6	4	6	5
f_8	6.976	6	8	6	8	9	9	6	4	5	5
f_9	4.749	ND	ND	2	ND	ND	ND	2	2	3	2
f_{10}	7.552	ND	ND	3	ND	ND	ND	ND	ND	3	4
f_{11}	6.750	ND	ND	ND	2	2	3	4	4	1	1
f_{12}	6.440	ND	ND	ND	ND	ND	ND	ND	ND	2	2
f_{13}	5.986	ND	ND	ND	ND	ND	ND	ND	ND	1	1
f_{14}	7.896	ND	ND	ND	ND	ND	ND	ND	ND	1	1
f_{15}	5.132	ND	ND	ND	ND	ND	ND	ND	ND	1	1
f_{16}	5.314	ND	ND	3	ND	ND	ND	ND	ND	1	ND
f_{17}	6.404	ND	ND	ND	ND	ND	ND	ND	ND	1	ND
f_{18}	6.680	ND	ND	ND	ND	ND	ND	ND	ND	ND	1

power spectrum are then examined for statistical significance and agreement with known frequency values.

For the different 1966–1984 data sets, between five and seven frequencies were previously found. The two methods, together with the frequency determinations from 1996 and 1997, allow us to extend the number of frequencies. The values of the frequencies f_i, can be found below in Table 2.

2.1 1983 and 1984 multisite data

The first multisite campaign of 4 CVn by the Delta Scuti Network, carried out during 1983 and 1984, revealed five pulsation frequencies. The power spectra of the residuals indicated the possible presence of additional frequencies, but their detection was not regarded as statistically significant due to inconsistent results for the years 1983 and 1984.

The Brute Force Method of fitting the 34 known modes to the combined 1983/4 data confirms the five frequencies (f_1, f_2, f_3, f_5, f_8) in the literature. Two additional frequencies (f_7 and f_{11}) have convincingly large amplitudes of 4 and 3 mmag, respectively. Furthermore, f_4 and f_6 may also be present with amplitudes of 2 mmag. All other modes have smaller amplitudes and must be regarded as undetected.

Next, the Fourier Peak Method was applied to the same data, still assuming constant amplitudes for the two years. The best fit with the five previously detected frequencies (f_1, f_2, f_3, f_5, f_8) was computed and pre-whitened from the data. The power spectrum of the residuals shows three close peaks near 5.500, 5.532 and 5.576 d^{-1}. We now know that the central peak is the correct peak and that the triplet is caused in part by amplitude variability and an unfortunate coincidence of the alias structure of additional modes. This is demonstrated in Fig. 2: the triple structure (top panel) is almost eliminated by pre-whitening a seven-frequency solution including the two correct peaks, f_6 and f_7 (middle panel). The next dominant peak is another previously known frequency, f_{11}, while pre-whitening an eight-frequency solution shows an additional expected peak, f_9.

The presence of eight frequencies is verified by both methods. Two additional modes are shown by only one of the two methods. One of these modes is f_4 at 6.117 d^{-1}, for which in this data set an aliasing problem with f_7 at 6.190 d^{-1} exists. Consequently, its detection (and amplitude of 1.7 mmag) is regarded as uncertain. The mode at f_9, on the other hand, is affected less by aliasing and included in our best solution with nine frequencies. Table 2 lists the amplitudes, which also allows for amplitude variation between 1983 and 1984.

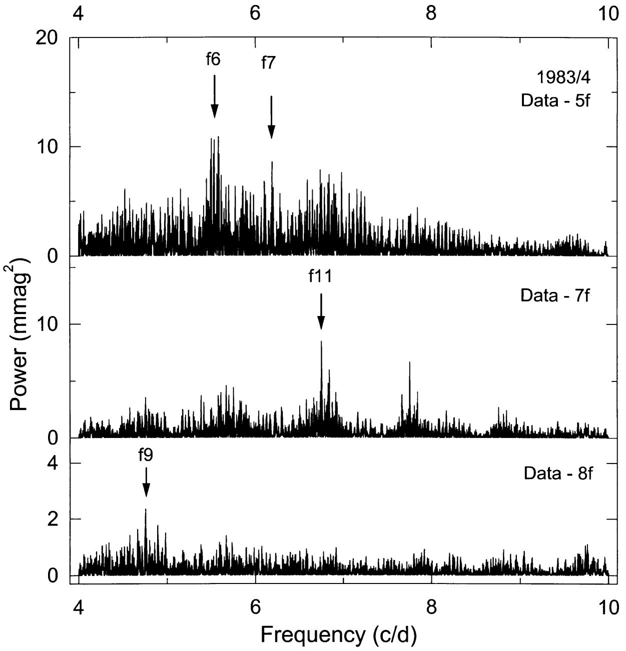

Figure 2. Power spectrum of the residuals of the 1983/4 multisite campaign of 4 CVn after removing the five dominant frequencies (f_1 to f_3, f_5, f_8, see Table 2). The additional peaks corresponding to the modes known from the 1996/7 campaign are shown by arrows. Altogether nine correct frequency peaks can be found in the data, but only if the central peak of the 5.50–5.58 d^{-1} triplet is chosen. Without this knowledge, only the five previously reported pulsation modes can be correctly extracted from the data.

2.2 1974–1978 data

Photometric data, which are mostly unpublished, are available for

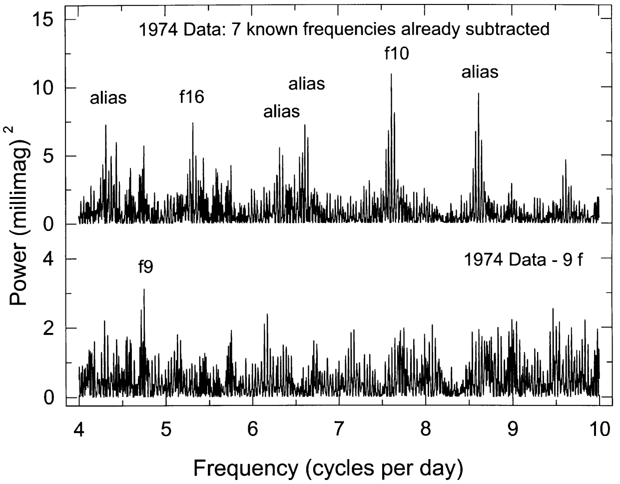

Figure 3. Power spectrum of the residuals of the 1974 data of 4 CVn after removing the seven dominant frequencies. The additional peaks corresponding to the modes known from the 1996/7 campaign are marked. In the lower panel, the two frequencies detected in the top panel have also been pre-whitened.

the years 1974, 1976, 1977 and 1978. We have reanalysed the data for these years and a summary of the available photometry is listed in Table 2. Our knowledge of the multifrequency spectrum of 4 CVn allows us to recognize a number of additional frequencies in the 1974–1978 data and to obtain insight into the amplitude variability during these years.

Our first approach was the application of the Brute Force Method of fitting the 34 known modes. The application to the combined 1974–1978 data is not completely straightforward, since amplitude variability of the years needs to be included: we allow the amplitudes of the seven dominant modes to vary. Furthermore, the frequency values of the 1996/7 campaign might need to be adjusted slightly because of very small frequency variability.

The Brute Force Method confirms the previously known seven frequencies (f_1 to f_3, f_5 to f_8) in the literature. For the detection of additional modes the results are unsatisfying. The two most promising additional modes are f_4 and f_{16} with amplitudes of 2 mmag, followed by a large number of frequencies with amplitudes near 1.5 mmag. However, we regard the solution as unstable due to the strong sensitivity of the resulting amplitudes on small frequency adjustments. This may not be surprising if one considers the small number of nights available for each of the years 1976, 1977 and 1978. Applying the method to only the more extensive 1974 data leads to even more unstable solutions.

Next we apply the more conservative Fourier Peak Method. Here the extensive 1974 data appear the most promising. The top panel of Fig. 3 shows the power spectrum after pre-whitening the best seven-frequency solution. Two peaks with their 1 d^{-1} aliases are evident. Both agree with the expected positions of f_{10} and f_{16}. Pre-whitening the resulting nine-frequency solution leaves f_9.

A similar analysis for the 1976–1978 data reveals f_{11} and f_4, which is demonstrated for 1977 in Fig. 4.

We will now turn to the fascinating amplitude and phase changes of f_2 (7.375 d^{-1}): the amplitude dropped from 15.2 mmag in 1974 to 3.6 mmag during 1976 and 1.4 mmag in 1977, while it increased again to 7.5 mmag during 1978. The amplitude drop is the most rapid change observed so far in 4 CVn. Previously, the study of this change was hampered by the lack of knowledge of the exact frequency of this mode, but with the 1996/7 data the earlier behaviour of this mode falls into place. We find:

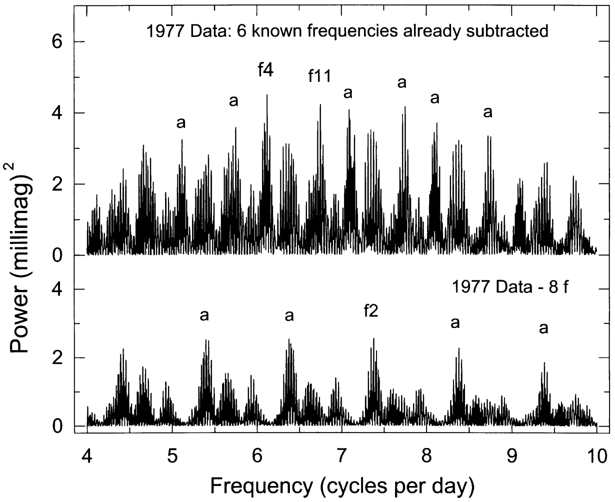

Figure 4. Power spectrum of the residuals of the 1977 data of 4 CVn after removing the six dominant frequencies, while leaving f_2. The additional peaks corresponding to the modes known from the 1996/7 campaign are marked, while the 1 d^{-1} aliases are marked with ‘a’. In the lower panel, the two frequencies detected in the top panel have also been pre-whitened. The bottom panel also shows that f_2, which has dropped severely in amplitude from 1974, is still present with a small amplitude.

(i) The mode is present during all four observing seasons. Even at minimum amplitude during 1977 (1.4 mmag), it shows a definite peak in the power spectrum (bottom panel of Fig. 4).

(ii) No single frequency value can represent the four years without a phase jump. A numerical frequency optimization is based mainly on the 1974 and 1978 data because of the relatively large amplitudes in those years. Such an optimized frequency is unable to fit the intervening years due to incorrect phasing, unless negative amplitudes are allowed. A negative amplitude is numerically identical to a positive amplitude with a 0.5 cycle phase shift. The fact that the 1974 data was taken through a *B*, rather than a *V* filter, introduces a negligible phase error of only ~0.01 cycles (see Breger et al. 1999) and can be ignored.

(iii) Phasing the 1974 to the 1976 data gives a frequency of 7.3752 d^{-1}. This value is in *exact* agreement with the value derived from the large 1996/7 data set and the value found from the long 1974 data set alone, which covers 123 d. The 1977 to 1978 data also give 7.3752 d^{-1}, but shifted in phase by 0.48 ± 0.02 cycles relative to 1974 to 1976.

We conclude that the frequency of f_2 was essentially constant from 1974–1978 with its standard value of 7.3752 d^{-1}. Between 1974 and 1976 the amplitude dropped by a factor of four. A year later, the mode was observed again with an even smaller amplitude and phase-shifted by about half a cycle. The amplitude then increased rapidly to the observed value of 8 mmag in 1978. This observed behaviour cannot be explained by the observational uncertainties, since these are much smaller than the observed effects. Note also that the large gaps (~ a decade) in the 1966–1997 data prevent the derivation of unique, accurate frequency values from the analysis of the combined data.

2.3 1966–1970 data

A reanalysis of the 1966/7 and 1969/70 data sets with the new

34-frequency solution (Brute Force Method) yields unstable amplitudes, even if constant amplitudes for the time period 1966–1970 are assumed. This is a reflection of a severe overinterpretation of the data. The poor suitability of the data (28 nights covering five years) is also demonstrated by a terrible spectral window.

We now turn to the Fourier Peak Method. The presence of the previously known seven frequencies, f_1 to f_3 and f_5 to f_8 is indeed confirmed, but no additional frequencies can be detected. We were able to subdivide the data into two groups, 1966–1967 and 1969–1970, to examine amplitude variability. Table 2 lists the new solutions. The amplitudes are in good agreement (within ±1 mmag) with those listed in Breger (1990a).

3 DISCUSSION

Table 2 lists the annual amplitudes we have determined for the detected modes in the previous sections. The 1996 values were taken from the results of the 1996 DSN campaign. For the 1997 APT data, increasing the number of frequencies from the previously known 19 to 34 produced only minor changes in the derived amplitudes. The amplitude variability is displayed graphically in Fig. 5.

3.1 Statistics

There exist enough data covering three decades to calculate statistics on the amount of amplitude variability for the eight dominant frequencies shown in Table 2. We have computed a power spectrum of the amplitudes as a function of time for each of the eight modes. These eight power spectra were then normalized and averaged. The result is shown in Fig. 6. We note that the use of the power spectra, as opposed to direct Fourier transforms, avoids problems with the phasing of the amplitude variations of different modes.

The amount of variability is a strong function of the length of the time base. Consequently, we have considered two time-scales: one and ten years. The decades actually vary from seven to thirteen years caused by the availability of data, but this flaw does not affect the results. These time-scales were calculated for each mode as follows: The annual difference is the average difference in amplitude between the years 1976/7, 1977/8, 1983/4 and 1996/7, while the 1966–1967/1969–1970 changes (divided by three because of the longer time-scale) and 1974/1976 changes (divided by two) were also included. The average annual variation was normalized by dividing by the average amplitude of each mode. Variations over a decade relate the average amplitudes in the time periods 1966–1970, 1974–1978, 1983–1984, and

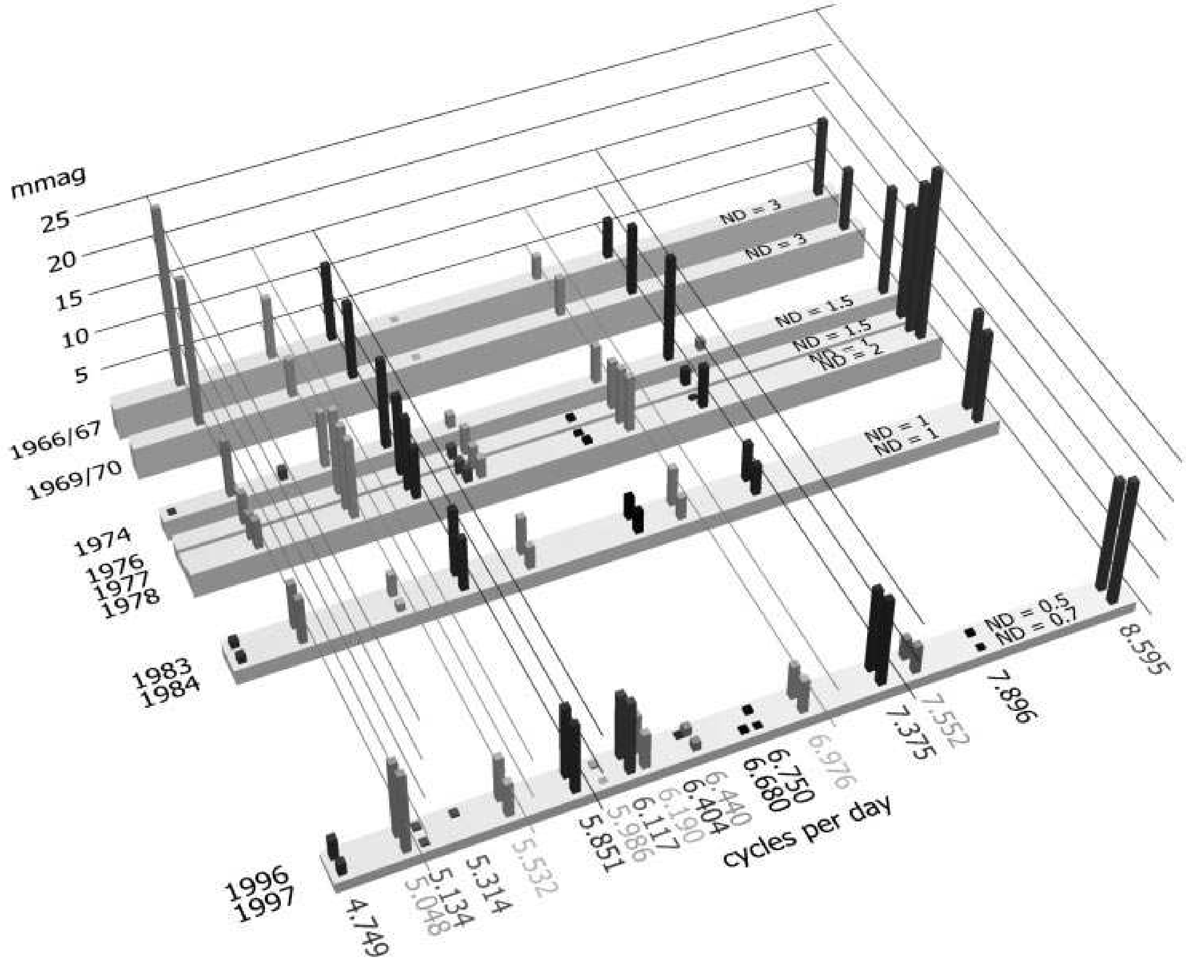

Figure 5. Amplitude variability of 4 CVn from 1966 to 1997. The amplitude is given by the height of the peaks. The different thresholds for the different years are shown and marked with 'ND'. Amplitudes smaller than these limits are not shown.

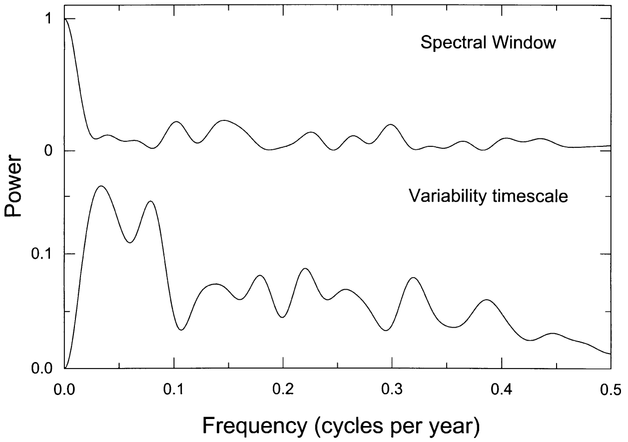

Figure 6. Power spectrum of the amplitude variability of 4 CVn. This represents the average power of the eight dominant modes with the amplitudes normalized for each mode. The main peak suggests a time-scale near 30 years. Since this is also the length of the available data, the peak can only be a rough estimate. Furthermore, the amplitude variability of the different frequencies is not well correlated.

Table 3. Amplitude variability on different time-scales.

Frequency in cycles per day		Amplitude variability in per cent	
4 CVn		Annual	Decade
f_1	8.595	12	34
f_2	7.375	34	42
f_3	5.048	8	62
f_4	6.117	11	102
f_5	5.851	10	20
f_6	5.532	12	60
f_7	6.190	14	17
f_8	6.976	11	20
FG Vir			
f_1	12.716	5 (5)	4
f_2, f_4 to f_8		12 (10)	29
f_3	23.403	58 (57)	in noise

Notes: Amplitude variability was corrected for photometric uncertainties of the amplitude determination using formulae given in Breger et al. (1999). For FG Vir we have also calculated a worst-case error scenario assuming that all amplitude variability in modes with amplitudes less than 1 mmag is due to photometric noise. The noise figure can then be calculated from the variability of the low-amplitude modes. These worst-case results are given in brackets.

1996–1997 with each other. For each mode, three differences are derived which were normalized by the average amplitude during the four time blocks. For the annual amplitudes, the formal values of the uncertainties can be calculated from the residuals (see Breger et al. 1999). These values may be an underestimate if the observational errors are correlated. The correction due to the formal uncertainties ranged from 0 to 2 per cent variability. Consequently, even doubling these corrections would not seriously affect the conclusions.

Both the derived amplitude variability and the error calculation can be checked by analysing the star FG Vir (Breger et al. 1998; Breger et al. 1995; Mantegazza, Poretti & Bossi 1994; Dawson, Breger & Lopez de Coca 1995). Here we have calculated the 'annual' variation by comparing the 1992, 1993 and 1995 data, and the long-term variation by comparing the 1985/6 with the 1993/5 photometry. The large number of low-amplitude modes known for FG Vir allows us to calculate a worst-case scenario for the photometric errors: we assume that all observed amplitude variability for the low-amplitude modes is caused by observational errors. This noise can then be calculated from the data and applied to the modes with larger amplitudes. The worst-case results are also presented in Table 3 (where the frequency numbering scheme of FG Vir is taken from Breger et al. 1998). It can be seen that even the worst-case scenario cannot explain the observed amplitude variability.

Most modes of 4 CVn show very similar behaviour: an amplitude variation of about 12 per cent from one year to the next. These variations are part of a longer variation with a probably cyclic behaviour on a time-scale of decades. Only f_2 shows large variations on an annual time-scale: even outside the large, single change from 1974 to 1976 (see previous section), the amplitudes are more volatile than for the other modes.

A dramatic change is also shown by f_4 (6.12 d^{-1}), which grew from non-detection in 1983/4 to a large V amplitude of 10 mmag in 1996/7. This is not a newly excited frequency: it is seen with a small amplitude of 3 mmag during 1976–1978.

The most regular and well-documented variation is shown by f_1 (8.60 d^{-1}), which slowly doubled its amplitude from 1966 to 1978 and then returned to its 'favourite' value near 14 mmag during the next two decades.

3.2 The rapid change of f_2 at 7.375 d^{-1} from 1974–1978

We have seen above that the pulsation mode at 7.375 d^{-1} exhibited the most rapid decrease seen in 4 CVn so far: the V amplitude dropped from the highest known value of 15 mmag in 1974 to 4 mmag in 1976 and to 1 mmag in 1977. After that the mode has been increasing in amplitude. The 1974–1976 and 1977–1978 data can be fit by the independently determined frequency of 7.3752 d^{-1}, but there exists a phase jump of 0.48 ± 0.02 d^{-1} between 1976 and 1977.

Two hypotheses could explain the observed behaviour of f_2:

(i) *Simple Beating Model.* The observed phase shift between 1976 and 1977 suggests beating of two close frequencies of similar, constant amplitudes. We have modelled this possibility with the least-squares multifrequency fitting program PERIOD90 (Breger 1990b) by using all the available data from 1974 to 1978. The model failed to reproduce the observed behaviour because of high residuals. This might, at first sight, appear surprising since the model has six unknowns (two each of frequency, amplitude and phase zero-point), while the observations seem to contain only an amplitude and phase for each of the four years, i. e. eight constants. Such an argument is not applicable for the present data set, since the annual observations are not Delta functions in the time domain: the 1974 data set, for example, covers 123 d. Hence the data contain more than eight constants. The poor fit, therefore, leads to the rejection of the Simple Beating Hypothesis.

(ii) *Re-excitation.* In this model f_2 decayed in amplitude and died out completely between 1976 and 1977. The mode was re-excited very shortly thereafter, but with a randomly different phase. At this time this hypothesis is the most attractive explanation, although it does not explain the phase change of almost exactly half a cycle.

3.3 Final notes

The strongly variable f_2 mode also shows the strongest mode coupling of all modes, as witnessed by the presence of combination frequencies, $f_i \pm f_j$ (see Breger et al. 1999). The amplitudes of these combination frequencies are also variable from year to year. Regrettably, values of such small amplitudes near 1 mmag are available only for the years 1996 and 1997. Consequently, correlations between the amplitude variability of the combination and parent frequencies cannot yet be studied. However, the presence of both strong amplitude variability and strong mode coupling suggests that the amplitude variability, leading up to decay or re-excitation may be caused by power transfer between modes.

The most surprising result has been the lack of correlation between the amplitude variability of the different pulsation modes, both in the time-scales and sizes of the variations. The four modes with large amplitudes and variability (f_1, f_2, f_3, f_7) can be identified with $\ell = 1$ modes of successive radial orders (Breger et al. 1999). However, the amplitudes do not vary together and no obvious transfer of pulsation amplitude between different orders is seen. The fact that the amplitude changes reverse again excludes an evolutionary origin.

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