Astron. Astrophys. 353, L9–L12 (2000)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Possible third family of compact stars more dense than neutron stars

Norman K. Glendenning[1] **and Christiane Kettner**[2]

[1] Nuclear Science Division & Institute for Nuclear and Particle Astrophysics, Lawrence Berkeley National Laboratory, MS: 70A-3307 Berkeley, CA 94720, USA
[2] Institut für Theoretische Physik I, Universität Augsburg Memmingerstrasse 6, 86135 Augsburg, Germany

Received 22 October 1999 / Accepted 15 November 1999

Abstract. The work of J. A. Wheeler in the mid 1960's showed that for smooth equations of state no stable stellar configurations with central densities above that corresponding to the limiting mass of 'neutron stars' (in the generic sense including hybrids) were stable. Accordingly, there has been no reason to expect that a stable degenerate family of stars with higher density than the known white dwarfs and neutron stars might exist. Nevertheless, we have found a class of equations of state that describe a first order phase transition and are insufficiently smooth to obey the conditional theorem of Wheeler. We identify the attributes that give rise to a third family of stable dense stars, discuss how such a higher density family of stars could be formed in nature, and how the promising new exploration of oscillations in the X-ray brightness of accreting neutron stars might provide a means of identifying them.

Key words: equation of state – stars: neutron

1. Background

There are two known classes of compact stars, white dwarfs, which were discovered in 1910 and neutron stars in 1967. Distinct classes (or families) of compact degenerate stars originate in properties of gravity; the distinction is made rigorous by the 'turning point' theorem of Wheeler and collaborators (see Harrison et al. 1965). The theorem concerns solutions of the stellar structure equations, whether Newtonian or Relativistic: there is a change in stability of one radial mode of normal vibration whenever the mass reaches a maximum or minimum as a function of central density. The theorem is a property of gravity and does not depend on the equation of state; of course the location of the turning points does. The theorem assures that distinct families of stars, such as white dwarfs and neutron stars, are separated in central density by a region in which there are no stable configurations. Does General Relativity admit the possibility of a third distinct family of degenerate stars at higher density than neutron stars?

There have been two reasons to doubt that a third family is allowed: One is physical and the other mathematical. The physical reason is the absence of an analogous mechanism to the one that ushers in the two known families. White dwarfs are stabilized by degenerate electron pressure which fails at such density that electron capture reduces their effectiveness. Stability is reestablished at densities about five orders of magnitude higher when the baryon Fermi pressure (and ultimately the short-range nuclear repulsive interaction) supports neutron stars. When neutron stars loose stability at their maximum mass, there is no evident mechanism for stabilizing a denser family. If quark deconfinement occurs, the Fermi pressure of baryons is replaced—not supplemented—by the pressure of their quark constituents. Indeed a phase transition will generally reduce the pressure at a given energy density.

The mathematical reason for doubting a third family is based on a theorem of Wheeler et al. (see Harrison et al. 1965): They proved analytically for polytropic equations of state, that general relativistic stars have an infinity of maxima and minima of the mass as a function of central density and therefore an infinity of sequences for which the mass has positive slope. Positive slope is sometimes thought to be a sufficient condition for stability. It is not. All configurations with densities greater than the first mass limit for neutron stars were shown to be unstable to acoustical radial vibrations, and end either by exploding or imploding to a black hole.

Some non-analytic models have been tested for stability also. These include the 'Harrison–Wheeler" equation of state (Harrison et al. 1965), and more recently new families associated with higher quark flavors (Kettner et al. 1995, Prisznyak, Lukacs and Levai 1994). However, in each case, configurations above the 'neutron stars' were proven to be unstable. On the basis of this pioneering work, a conditional theorem can be inferred: For any sufficiently smooth equation of state, neutron stars are the last stable class of compact stars. Configurations of higher density oscillate in mass as a function of

Send offprint requests to: Norman K. Glendenning

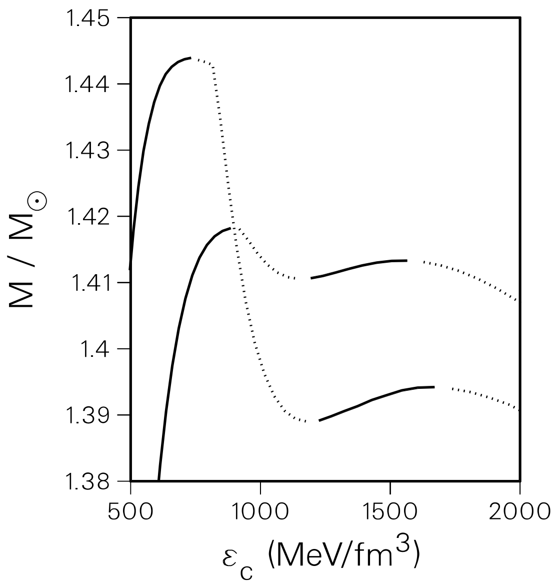

Fig. 1. Two out of many examples of stellar sequences for which Neutron stars and a higher density stable family of 'non-identical twins' exist. Stability is proven in connection with the discussion of Fig. 5 and is indicated by solid lines.

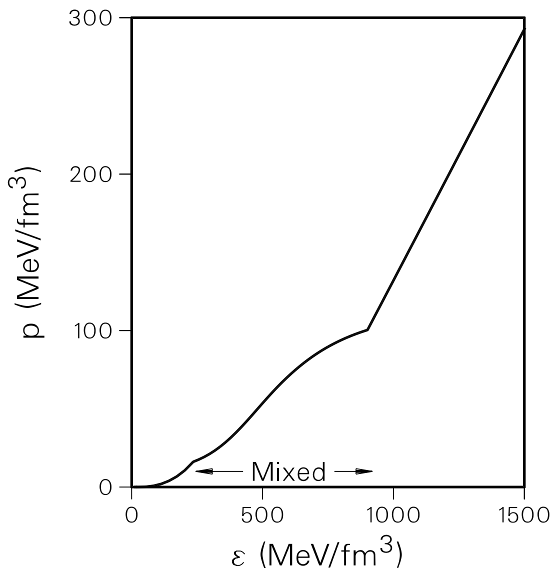

Fig. 2. Equation of state from the low density normal nuclear matter through the mixed phase and into the pure quark matter phase at high density (The normal density of nuclear matter is $140\ \mathrm{MeV/fm^3}$.)

density and acquire an additional unstable mode of vibration at each maximum and minimum.

2. Cause of stability

Nevertheless, we show that the equations of stellar structure do admit stable solutions above neutron stars in density and we discuss the attributes required of the equation of state. The examples we have of a third family occur for the deconfinement phase transition given certain plausible combinations of parameters defining the nuclear and quark deconfined equations of state. Two of many examples of stellar sequences with a 'neutron star' branch and another stable higher density branch are shown in Fig. 1. The 'neutron star' sequence is terminated by the softening in the equation of state in the mixed phase when a substantial core of mixed phase is attained. A new sequence at higher density is stabilized by replacement of the mixed phase by a pure quark phase core. The stars near and at the termination of the "neutron star" branch and those of the third family are both hybrids in the sense that they have quark matter in the core, whether it be in mixed or pure phase, surrounded by confined nuclear matter. In this case there are stars of the same mass but radically different quark content and also of size.

The equation of state for the sequence with maximum neutron star mass $\sim 1.42M_{\odot}$, is shown in Fig. 2. The form is general for first order phase transitions of whatever origin in any substance with two or more independent components (Glendenning 1992). (For neutron stars the two independent components are electric and baryonic charge.) The three parts of the equation of state that are separated by a discontinuity in slope correspond to the pure confined or nuclear matter phase, the mixed coexistence phase of nuclear and quark matter, and the pure deconfined quark matter phase.

Features of the stellar sequence shown in Fig. 1 can be identified with features in the equation of state shown in Fig. 2. One can see that near the end of the mixed phase $dp/d\epsilon$ becomes small; therefore also the adiabatic index, $\Gamma = d\ln p/d\ln\rho = (p+\epsilon)/p \cdot dp/d\epsilon$ (where $p,\ \epsilon,\ \mathrm{and}\rho$ denote pressure, energy density and baryon density). In this upper region of the mixed phase, the pressure is too weak a function of energy density to sustain stability: The canonical neutron star family terminates. The weakening of the adiabatic index referred to is characteristic of equilibrium between two phases. The adiabatic index is larger in the pure phases, and the increase in the pure quark phase restores stability over a small range of central densities. We prove stability for our examples below. However, while the behavior of the adiabatic index described above is a quite general attribute for phase transitions in multicomponent substances, a stable third family is not. Stability is an integral property that depends, more or less, on the configuration of the bulk of the star and therefore on the equation of state over a broad density range. Consequently, the appearance of a third family will depend on the density region in which the critical behavior occurs.

3. Neutron star twin

We discuss now the ingredients of the equation of state. Because empirical data for the equation of state does not exist above saturation density, we wish to base our extrapolation to higher density in a causal manner that is related to known saturation properties of nuclear matter. We employ the so-called relativistically covariant mean-field-model, introduced originally in the mid 1950's (Johnson and E. Teller 1955, Duerr 1956) and (Walecka 1974). The model was later extended to better describe nuclear properties (Boguta and Bodmer 1977),

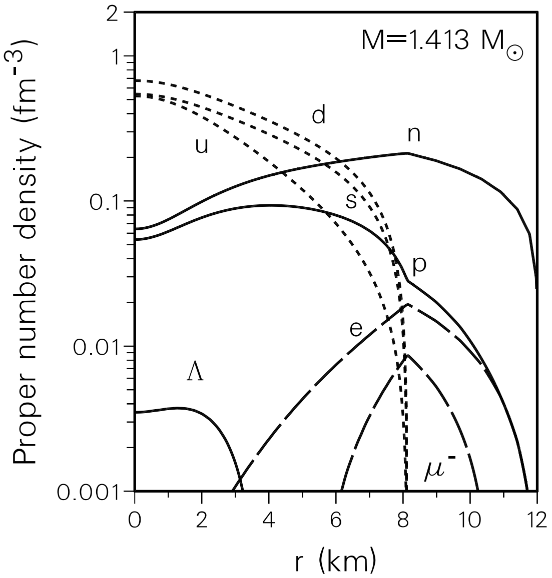

Fig. 3. Particle populations of a lower-density twin.

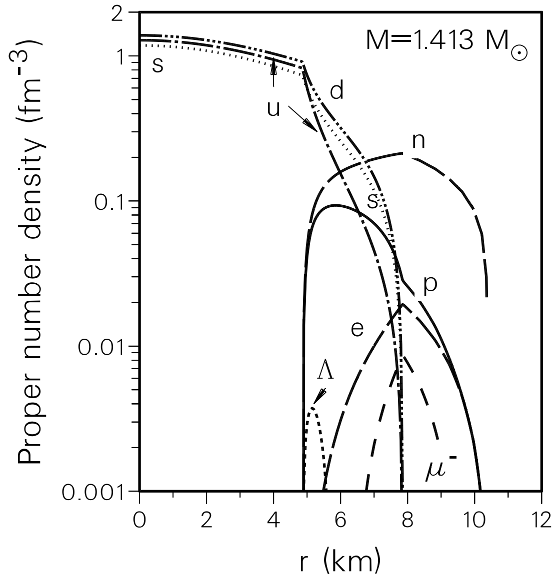

Fig. 4. Particle populations in a higher-density twin of the same mass as that in Fig. 3.

and to incorporate higher mass baryons as may exist in dense matter (Garpman, Glendenning, & Karant 1979, Glendenning 1985, 1997). The coupling constants can be related algebraically to five symmetric nuclear matter properties (Glendenning 1997): saturation density ($0.153\ \mathrm{fm}^{-3}$), binding (16.3 MeV) and symmetry energy coefficient (32.5 MeV) which are accurately known and the compression modulus and effective nucleon mass at saturation, which are less well known. In Fig. 1, we have illustrated two examples of a third family for which we have proven stability. The particular parameters are $K = 290$ MeV and $m^\star/m = 0.57$ and 0.66. Quark matter is described by the MIT bag model. The bag constant is $B^{1/4} = 180$ MeV and the quark model equation of state is given by Farhi & Jaffe (1984) with $\alpha_s = 0$. We found many such examples of third families. There is at least one region of parameters for which stability exists for a third family that is spanned in one direction by $K = 290$ MeV and $m^\star/m = 0.55$ to 0.68 and in the other by $m^\star/m = 0.62$ and $K = 250$ to 350 MeV and beyond.

We refer to stars of the denser sequence in Fig. 1 as non-identical twins of neutron stars because for both families it is the Fermi pressure of particles carrying baryon number that supports the star against gravity in addition to repulsion at short distance between any nucleons that may be present. In some cases, the denser family has a larger limiting mass than the 'neutron' star family. In the present examples, both families contain deconfined quark matter, but in the first, only in the mixed phases. The particle populations are shown in Figs. 3 and 4 for a selected common mass. Both contain a region in which confined and deconfined matter coexist. The lower-mass twin has an 8 km radius core of mixed phase. The inner 4 km of the higher-mass star is in the pure quark phase. However, it is not the particular content of the star that creates the additional family, but rather a particular way in which the adiabatic index changes with density.

4. Stability test

We now demonstrate stability to radial vibrations that would otherwise bring about collapse or explosion of a star. Stars on both segments of the stellar sequence shown in Fig. 1 that have positive slope $dM/d\rho_c > 0$ satisfy the necessary but insufficient condition for stability. Stability can be tested by an analysis of the radial modes of oscillation (Chandrasekhar 1964). The squared frequency ω^2 of the fundamental mode is plotted in Fig. 5. Positive values indicate stability and correspond to the segments with positive slope in Fig. 1. The analysis shows that the fundamental (nodeless $n = 0$) oscillation becomes unstable at the first maximum, as is usual, but unusually, stability of this mode is restored at the following minimum, to be lost again at the next maximum. The usual pattern is that the fundamental mode becomes unstable at the maximum in the neutron star family and a higher mode in order $n = 1, 2, 3 \cdots$ becomes unstable at each higher minimum and maximum. Undoubtedly the usual pattern resumes at densities higher than our third stable family. At such high densities that matter is in the pure quark phase, asymptotic freedom is likely to assure that the equation of state is smooth like a polytrope. Indeed, that is precisely the asymptotic behavior of the MIT bag equation of state. From some density above the point where the equation of state is smooth, we are assured of the denumerable infinity of turning points and ever increasing number of unstable normal modes such as was found by Wheeler and collaborators (Harrison et al. 1965).

Pressure oscillations bring particle populations instantaneously out of equilibrium. But it is not necessary to know the time-scale for equilibration of the transition between quark matter and nuclear matter and how it compares with typical periods of oscillations to analyze stability. The turning points in the stellar sequence between stability and instability occur only at the zeros of ω^2, and because the stellar oscillations are

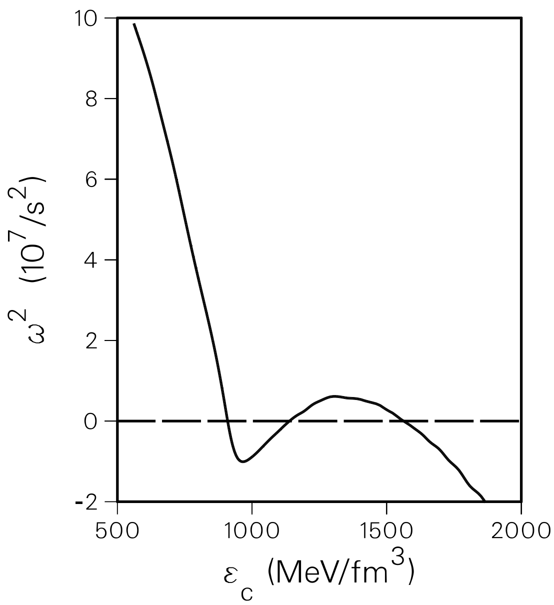

Fig. 5. The square of the frequency of the fundamental ($n = 0$) radial vibration as a function of central density. Perturbations behave as $\exp(i\omega t)$ so they are unstable with diverging amplitude $\exp(|\omega|t)$ when $\omega^2 < 0$.

infinitely slow at the turning points, their location can be determined exactly. Thus, independent of the reaction time-scales, the stability or instability of all members of the stellar sequence can be determined.

5. Formation and detection

From the above proof of stability we have shown that in principle a third family of stable degenerate stars could exist. Each or some of the stars on the high density branch have a non-identical twin on the low-density branch, having the same mass but different composition and radius. Can such twins be distinguished? One possible avenue is through observations on the so-called quasi-periodic oscillations in the X-ray brightness of accreting neutron stars. According to theory, mass and radius determinations may be possible (Miller and Lamb 1998, Miller, Lamb and Psaltis 1998a, 1998b). If twins exist, then the mass-radius curve will exhibit two segments of stable stars instead of one, and observed stars will fall on one or the other of the two distinct segments. The discovery of only about two stars on each branch with a radius resolution of a kilometer in our example would suffice to establish the existence of twin branches.

How could a high density twin be made in nature? The likely path to the high density twins is through the initial core collapse of a star, in which the core implodes through the normal star to the high density twin. Since no two supernova are likely to be identical, there being many variables that effect the outcome, like mass, rotation, symmetry, chemical composition of the progenitor, and the chaotic process of convection, it seems plausible that either twin could be produced. A second possible formation mechanism of the high density twin is through accretion onto a member of the low-density sequence of Fig. 1 followed by a minor explosion.

The existence of a third stable class of degenerate stars makes special demands on the equation of state—demands that cannot be met if dense matter simply evolves in a continuous way with density. Our particular model is limited in both its description of nuclear and quark matter so that the examples we have found are presumably limited in the range of masses and densities for which a third class may exist. We have no knowledge from experiment of a single point on the equation of state above nuclear density. But we do have expectations of phase transitions; asymptotic freedom of quarks would appear to elevate one of them to a law of nature. A phase transition can produce the requisite structure discussed above into the equation of state so as to restore stability for a finite density range after stability has been lost by the canonical neutron stars. The possible existence of a third family of compact stars hinges on such details that we may never determine in laboratory experiments. Therefore, the actual discovery of members of the third family would reveal, however imperfectly, a non-smooth behavior of the equation of state possibly caused by a change in phase of dense matter—that we may never know by laboratory experimentation. If some of them were X-ray sources, QPO observations may permit the use of the inverse theorem of Lindblom (1992) to gain rather particular knowledge of the equation of state.

Acknowledgements. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

References

Boguta, J., & Bodmer, A. R. 1977, Nucl. Phys. A292, 413.
Chandrasekhar, S. 1964, Phys. Rev. Lett. 12, 114, and op. cit. p. 437.
Duerr, H. P. 1956, Phys. Rev. 103, 469.
Farhi, E. & Jaffe, R. L. 1984, Phys. Rev. D 30, 2379.
Garpman, S. I. A., Glendenning, N. K. & Karant, Y. J. 1979, Nuc. Phys. A322, 382.
Glendenning, N. K. 1985, ApJ, 293, 470.
Glendenning, N. K. 1992, Phys. Rev. D 46, 1274.
Glendenning, N. K. 1997, Compact Stars, nuclear physics, particle physics, and general relativity (Springer–Verlag New York).
Harrison, B. K., Thorne, K. S., Wakano M. & Wheeler, J. A. 1965, Gravitation Theory and Gravitational collapse, (University of Chicago Press, Chicago)
Johnson M. H., & Teller, E. 1955, Phys. Rev. 98, 783.
Kettner, Ch., Weber, F., Weigel M. K., & Glendenning, N. K. 1995, Phys. Rev. D. 51, 1440.
Lindblom, L. 1992, ApJ, 398, 569.
Miller, M. C. & Lamb, F. K. 1998, ApJ, 499, L37.
Miller, M. C. Lamb, F. K. & Psaltis, D. 1998, The Active X-ray Sky ed. by L. Scarsi, H. Bradt, P. Giommi, and F. Fiore, preprint(astro-ph/9801140).
Miller, M. C., Lamb, F. K. & Psaltis, D. 1998, ApJ, 508, 791.
Prisznyak, M., Lukacs, B., & Levai, P. 1994, preprint(astro-ph/9412052).
Walecka, J. D. 1974, Ann. of Phys. 83, 491.