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Helioseismology and the solar age

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Abstract. The problem of measuring the solar age by means of helioseismology has been recently revisited by Guenther & Demarque (1997) and by Weiss & Schlattl (1998). Different best values for $t_{\rm seis}$ and different assessment of the uncertainty resulted from these two works. We show that depending on the way seismic data are used, one may obtain $t_{\rm seis} \approx 4.6$ Gy close to the age of the oldest meteorites, $t_{\rm met} = 4.57$ Gy, like in the first paper, or above 5 Gy like in the second paper. The discrepancy in the seismic estimates of the solar age may be eliminated by assuming higher than the standard metal abundance and/or an upward revision of the opacities in the solar radiative interior.

We argue that the most accurate and robust seismic measure of the solar age are the small frequency separations, $D_{\ell,n} = \nu_{l,n} - \nu_{\ell+2,n-1}$, for spherical harmonic degrees $\ell = 0, 2$ and radial orders $n \gg \ell$. The seismic age inferred by minimization of the sum of squared differences between the model and the solar small separations is $t_{\rm seis} = 4.66 \pm 0.11$ Gy, a number consistent with meteoritic data. Our analysis supports earlier suggestions of using small frequency separations as stellar age indicators.

Key words: Sun: abundances – Sun: evolution – Sun: interior – Sun: oscillations

1. Introduction

The idea that helioseismology may be used to test the assumption that the solar age is equal to the age of the oldest meteorites is not new. Gough & Novotny (1990), who considered the problem in great detail, concluded that the accuracy of 0.3 Gy may be achieved once the seismic age indicators are measured to a precision of 0.1 μHz. The precision of current seismic data is now significantly better. However, results of recent studies of the problem yield conflicting conclusions.

Before we go to the results of these studies, let us first point out that we should expect a unique determination of the solar age from seismic data. Calculated p-mode frequencies depend on the assumed solar age but they also depend on other input solar parameters and physical quantities. All these data are subject to uncertainties. We now have at our disposal nearly 2000 accurate frequency data for solar p-modes to determine solar age – the only observable in the standard solar model (SSM) construction which we surrender. It would be indeed surprising if the answer would not depend on the way we make use of seismic data. An assessment of the uncertainty of $t_{\rm seis}$ is even more problematic.

Guenther & Demarque (1997) concluded their comparison of the solar frequencies with those for models calculated upon assuming different age with the following statement: “The best agreement with the calculated oscillation spectra is achieved for 4.5 ± 0.1 Gy”. Unfortunately, they did not explain how these numbers were obtained.

Weiss & Schlattl (1998), proceeding in a more formal way, used χ^2 minimization to determine $t_{\rm seis}$. They considered various seismic observables and corresponding parameters in the models calculated for various assumed solar ages. The observables include surface helium abundance, $Y_{\rm seis}$, depth of the convective zone, $r_{\rm cz}$, sound speed in the the radiative interior, and the radial mode frequencies. In nearly all the cases they considered, the minimum was reached for age well above 5 Gy. Typical values of $t_{\rm seis}$ they derive are in the range 5.1–5.2 Gy. Taken for granted, the high values of the solar age would mean an essential revision of our views on the evolution of the solar system. This is not what Weiss & Schlattl (1998) propose. Rather, they regard the difference between $t_{\rm seis}$ and $t_{\rm met}$ as a measure of the uncertainties in the age determination based on the state-of-art stellar evolution theory.

The main motivation for our work was to explain the large difference in the conclusions of the two papers regarding the value of $t_{\rm seis}$ and its uncertainty. Weiss & Schlattl (1998) themselves have addressed this problem but we did not find their explanation sufficient. We will begin with providing some information about new solar models calculated for the purpose of this investigation. In the main part of the paper, we review the inference about the solar age based on various seismic observables and we identify those which we believe are good age indicators.

2. New solar models

We constructed a large number of solar models taking into account diffusion of helium and heavy elements following Thoul et

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Table 1. Parameters of selected solar models

Model	t [Gy]	Z/X	OPAL	Y_0	Z_0	$Y_{\rm ph}$	$Z_{\rm ph}$	$r_{\rm cz}/R_{\rm ph}$	X_c	ρ_c[g/cm^3]	T_c [10^6 K]
0	4.57	0.0245	1996	0.2739	0.02024	0.2429	0.01811	0.7163	0.3331	157.1	15.803
1	5.00	0.0245	1996	0.2705	0.02045	0.2386	0.01821	0.7109	0.3090	164.5	15.927
2	4.57	0.0270	1996	0.2814	0.02199	0.2502	0.01971	0.7126	0.3212	157.9	15.934
3	4.57	0.0245	1992	0.2777	0.02010	0.2467	0.01801	0.7141	0.3297	157.6	15.841

al.(1994). In one model (Model 5), which we refer to in Sect. 5, diffusion was ignored. In all the models, we use OPAL equation-of-state (Rogers et al., 1996). For opacity we use the newest Livermore opacity table (OPAL96, Iglesias & Rogers, 1996) for Grevesse & Noels (1993) heavy element mixture. For comparison, we calculated one model using an earlier version of the Livermore opacities (OPAL92, Iglesias et al., 1992; Rogers & Iglesias, 1992). At low temperatures we used Alexander & Ferguson (1994) data on molecular and grain opacities. Nuclear reaction rates are calculated according to Bahcall & Pinsonneault (1995). We calculated one model (Model 4, see Sect. 5) with modified reaction rates, still within the range of uncertainties quoted by Bahcall & Pinsonneault.

We assumed the value of photospheric radius $R_{\rm ph}$ = 696.3 Mm. This value is by 0.8 Mm higher than the most recent determination of Brown & Christensen-Dalsgaard (1998). The reason for our choice is a better agreement with the seismically inferred sound-speed in the lower convective zone. The small difference is inconsequential for the conclusions of this work. The model radii were fitted to the adopted value with the precision better than 5×10^{-5}. The luminosity was assumed to be 3.844×10^{33} erg s^{-1} and models were fitted to precision better than 2×10^{-4}.

We calculated a number of models for various values of the age, t, at the standard value of the metal-to-hydrogen ratio, $Z/X = 0.0245$, and at an enhanced value of 0.027. The parameters for selected models are listed in Table 1.

A comparison between Model 0 and Model 1 shows the effect of the age on main parameters of the solar models. The older sun ($t > t_{met}$) has produced a larger amount of helium in the core. Longer evolution means also more time for the gravitational settling i.e. a larger difference between the initial helium abundance, Y_0, and the present abundance in the outer layers, $Y_{\rm ph}$. In order that the solar model accounts for the same luminosity, one has to reduce the initial helium abundance with respect to that of the SSM. With the exception of the energy production region, the helium abundance is reduced everywhere in the solar model and one can thus explain the following features:
i)The present photospheric helium abundance is lower.
ii)Matter is more opaque to radiation, so that convection starts deeper in the sun.
iii)Below the convective zone and above the energy production core the sound speed is higher, due to the reduced “mean molecular weight” μ.
iv)In the energy production core, the effect of helium accumulation should dominate, resulting in a larger μ and consequently a smaller sound speed.

Of course, the opposite occurs for a younger sun. In the next section we will discuss in greater detail the differences in the sound speed between various models.

3. Inference from seismically determined solar parameters

Solar age cannot be directly determined by means of helioseismology. In all the approaches, including this one, families of solar models with various assumed ages are calculated and $t_{\rm seis}$ is determined by means of a comparison of more direct seismic observables. The most direct are the frequencies, but with no additional assumptions one may use the density, $\rho(r)$, or the squared isothermal sound speed, $u(r)$, determined by means of the frequency inversion. These two functions are linked by the hydrostatic equilibrium condition. From u, ρ and their derivatives one may determine a number of other useful structural functions. If, in addition, we assume equation of state (EOS) data, we may infer the values of $Y_{\rm ph}$ and $r_{\rm cz}$. The last two seismic observables were used by Weiss & Schlattl (1998) in their first attempt at the solar age determination. They subsequently considered also other quantities. There are various possibilities. We regard a comparison of the sound speed as most revealing. The value of $r_{\rm cz}$ does not contain independent information and, since it is determined from the derivative of u, it is less accurate.

3.1. The sound speed

The result of the inversion for $\delta u/u$, the relative difference in $u(r)$ between the sun and model 0, is shown in Fig. 1, where $r = R$ corresponds to the temperature minimum. In the same plot we show the difference in u between some other models (see Table 1) and model 0.

The solar data are from the inversion of the frequency data obtained with the MDI instrument (Rhodes et al. 1997) and the GOLF instrument (Gabriel et al. 1997) on board of the SOHO spacecraft. The first data set contains modes with the ℓ values from 0 to 250. We ignored the f-modes, and we were left with the frequencies of 1890 p-modes with ℓ up to 184. The second set contains 153 frequency data for modes with ℓ degrees up to 5. The data were combined into a set of 1945 p-mode frequencies. The inversion was done by means of the SOLA method (Pijpers & Thompson, 1992; Dziembowski et al., 1994).

One sees in Fig. 1 that the difference in u through most of the sun interior seems to favor higher age. However, the quantitative answer depends on the choice of the location in the sun’s interior. In the region $0.1R < r < 0.35R$, u is almost independent of age. In the inner core the dependence on age is the

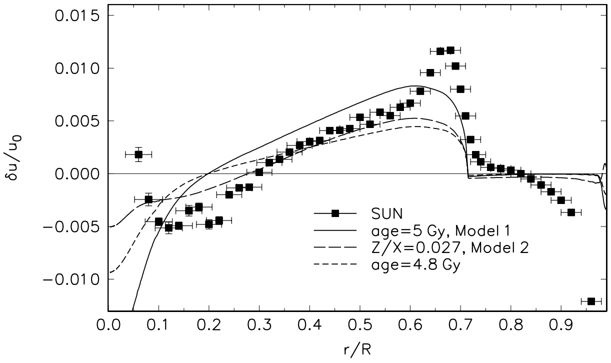

Fig. 1. Relative differences in u between the sun and Model 0 determined by means of helioseismic inversion. Also shown are the differences between different models and Model 0. The vertical error bars (visible only for the inner most points) reflect only measurement errors. True uncertainty of the inversion is much greater (Degl'Innocenti et al., 1997).

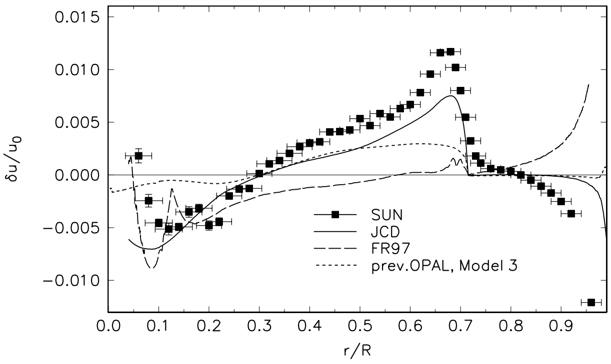

Fig. 2. Relative differences in u between the sun and Model 0 determined by means of helioseismic inversion are compared with the differences between different models and Model 0. Model JCD (Christensen-Dalsgaard et al., 1996) and Model 3 were calculated with OPAL92 and that denoted FR97 with OPAL96 opacities.

strongest. Older models have higher helium abundance, hence higher mean molecular weight. This effect dominates the sound speed behavior. Unfortunately, results of seismic sounding of the inner core are unreliable.

An assessment of the solar age based on $u(r)$ is sensitive to the assumed metal abundance in the model. An increase of the Z/X parameter by 10% has a similar effect on the sound speed in the outer part of the radiative interior as a 6% increase of age.

The implication about the age based on δu depends also on other ingredients of the solar model construction such as opacity, nuclear reaction rates and diffusion coefficients. We will not consider all these effects in detail. In Fig. 2 we show few examples of the difference in u between models calculated assuming $t = t_{\rm seis}$. Model JCD (Christensen-Dalsgaard et al., 1996) is the closest to the sun. The improvement in the opacity data spoils this good agreement. However, as the comparison with Model 3 shows, the difference in opacity does not explain the whole difference between JCD and model 0. We suspect that the remaining difference in u may be caused by the difference in the treatment of the element settling. The difference between the model denoted FR97 (Ciacio et al., 1997) and model 0 in the outer part of the radiative interior is very small. A comparison of the plots in Figs. 1 and 2 shows that the revision the OPAL has resulted in changes of u similar to lowering Z/X by 6%. Thus, with earlier OPAL opacities we will get solar age lower by 3.6% (0.16 Gy).

In all the cases, values of $\delta u/u$ in the outer part of the radiative interior point to $t_{\rm seis} > t_{\rm met}$. The difference is model dependent. We will quantify it in Sect. 3.1. Finally, let us point out that the result of inversion shown in Figs. 1 and 2 looks very similar to that of Brun et al. (1998) except for $r < 0.1$. The implication concerning the solar age based on δu from their inversion would therefore be similar to ours.

3.2. Helium abundance

The value of $Y_{\rm ph} = Y_{\rm seis}$ as determined from the same data and with the same reference model is 0.249. It is by 0.006 larger than in our standard model and by 0.010 larger than in the model with age 5 Gy. The age inferred from $Y_{\rm seis}$ would be about 4 Gy. The number is in a reasonable agreement with Weiss & Schlattl (1998). Clearly, there are conflicting conclusions about $t_{\rm seis}$ from $u(r)$ and $Y_{\rm seis}$. Not surprisingly Weiss & Schlattl (1998) find rather large minimum values of χ^2 in their multi-parameter fits.

Adopting higher Z/X values allows us to reduce the contradiction. We see in Table 1 that in model with $Z/X = 0.027$, $Y_{\rm ph}$ is close to $Y_{\rm seis}$, and in Fig. 1 we see that $u(r)$ is closer to one inferred by the inversion. A similar, though smaller, effect is obtained by adopting the previous version of the OPAL opacities. Still, the most significant difference in u in the outermost part of the radiative interior cannot be removed by higher $Z/X = 0.027$. Modification in opacity is an option but it must be quite different from the return to earlier version of OPAL. Gough et al. (1996) suggested that the spike of $\delta u/u$ at $r \approx 0.68R$ may be a consequence of neglecting a macroscopic mixing below the base of convective zone in the standard solar models. Models including this effect have been constructed by Richard et al. (1996). Such models explain the deficit of Li abundance in the sun's photosphere and yield better agreement with seismic determination of u near the base of convective zone. The effects leads also to an increase of Y in the envelope. Macroscopic mixing is a hypothetical effect and its description involves free parameters, so it is not included in the standard models. The effect most likely takes place. For present application this means that $Y_{\rm seis}$ and u in the outer part of the envelope are not safe probes of the solar age. In addition, there are difficulties with estimating uncertainties in seismic determination of Y following from inadequacies in the thermodynamical parameters.

3.3. Estimates of $t_{\rm seis}$ based on selected values of u and $Y_{\rm ph}$

For the sake of illustration of the discrepancies we will give estimates of $t_{\rm seis}$ based on different observables. Unlike Weiss & Schlattl (1998), we will not try to fit simultaneously more

Table 2. Selected seismic observables and their 1σ uncertainties, $\Delta Q/Q$.

Q	α_Q	$Q_\odot$	$\Delta Q/Q$
ũ(0.3)	0.03	0.4782	±0.1%
ũ(0.4)	0.05	0.3618	±0.1%
ũ(0.5)	0.07	0.2820	±0.12%
ũ(0.6)	0.09	0.2218	±0.14%
ũ(0.65)	0.08	0.1952	±0.14%
Y_{ph}	−0.20	0.249	±1.4%

Table 3. Values of $\tilde{u}$ and $Y_{\rm ph}$

Q_i	JCD	model 0	FR97
ũ(0.3)	0.4781	0.4781	0.4772
ũ(0.4)	0.3612	0.3607	0.3603
ũ(0.5)	0.2812	0.2805	0.2803
ũ(0.6)	0.2214	0.2203	0.2204
ũ(0.65)	0.1945	0.1932	0.1932
Y_{ph}	0.245	0.243	0.238

Table 4. Helioseismic estimate of solar age (Gy), as inferred from the differences $Q - Q_i$, calculated for different SSMs.

Q_i	JCD	model 0	FR97
ũ(0.3)	4.60 ± 0.15	4.60 ± 0.15	4.90 ± 0.14
ũ(0.4)	4.72 ± 0.09	4.86 ± 0.10	4.96 ± 0.10
ũ(0.5)	4.76 ± 0.08	4.93 ± 0.08	4.98 ± 0.08
ũ(0.6)	4.66 ± 0.07	4.93 ± 0.08	4.90 ± 0.08
ũ(0.65)	4.78 ± 0.08	5.20 ± 0.09	5.20 ± 0.09
Y_{ph}	4.21 ± 0.29	4.04 ± 0.28	3.64 ± 0.25

than one parameter because our aim is only to quantify the problems with the assessment of the solar age with the method reviewed in this section. Furthermore, the meaning of the formal χ^2-minimization procedure is problematic in the present case, as in fact Weiss & Schlattl (1998) emphasized.

In Table 2 we provide a list of the selected observables, Q with errors, its estimated 1σ uncertainty $\Delta Q/Q$, and the quantity

$$\alpha_Q = \frac{d\ln Q}{d\ln(t/t_{\rm met})} \quad , \tag{1}$$

which measures sensitivity of each observable to the solar age. The values of $\tilde{u} = uR/GM$ and $Y_{\rm ph}$ are from the inversion described in Sect. 3.1. The estimates of uncertainties, $\Delta Q/Q$, are from Degl'Innocenti et al. (1997).

In Table 2 we list the values of the selected observables calculated in the three standard solar models.

In Table 4 we provide the values of t inferred from the differences between the sun and the models by using the various observables Q. The numbers mostly quantify only the effects discussed earlier in this section.

4. Direct and almost direct use of measured frequencies

It is unfortunate that the parameters of seismic models which exhibit greatest sensitivity to solar age are, for various reasons, unreliable. The sound speed in the inner core cannot be precisely measured because the inversion is not accurate enough. Other parameters are formally very accurate but we cannot trust model predictions. Since the nature of the uncertainties is so diversified, we are reluctant to quote any quantity as a best value of $t_{\rm seis}$ and its errors.

Choosing, instead, a direct use of frequency differences we face another problem. The formal approach to determination of $t_{\rm seis}$ is the minimization of

$$\chi^2 = \frac{1}{J}\sum_{j=1}^{J}\left(\frac{\nu_\odot - \nu_{\rm model}(t)}{\sigma}\right)^2_j , \tag{2}$$

where the sum includes all $J = 1945$ p-modes in the set, and σ are measurement errors. The problem is revealed in Fig. 2, in which we may see that χ^2 depends only very weakly on age. There is a minimum near 5.2 Gy, but it is very shallow and does not allow a trustworthy estimate of $t_{\rm seis}$.

This problem is a consequence of the fact that the main part of the frequency differences between the sun and the model has nothing to do with the differences in the internal structure but rather it is caused by inadequacies in the treatment of oscillations in the outer layers, where the neglect of nonadiabatic effects and dynamical effects of convection is not justified. These inadequacies are significant in the outermost layers above $r = 0.99R$, i.e., above the lower turning point of all the p-modes in the set. The lower turning point is determined by the parameter $(\ell + 0.5)/\nu$. Its maximum value for modes in our set is 0.1 and corresponds to the turning point $r_t = 0.99R$. Sufficiently far above the turning point, the relevant eigenfunctions, except for normalization, are ℓ-independent. Therefore, we may expect that the part of the frequency differences due to the effects in the layers above $r = 0.99R$ scale as $F(\nu)/I_j$, where I_j is the mode inertia calculated upon assuming the same normalization of the eigenfunctions in the photosphere.

In order to eliminate these near-surface contaminations, we fitted $F(\nu)$ in a polynomial form to the frequency differences $\nu_{\odot,j} - \nu_{{\rm model},j}$ and considered only the residual part of the differences

$$\nu_{F,j} = \nu_{\odot,j} - \frac{F(\nu_{\odot,j})}{I_j} . \tag{3}$$

The quantity $\nu_{F,j} - \nu_{{\rm model},j}$ is the part of the frequency difference that may be attributed only to the difference in the internal structure. In Fig. 3 we plot two $\chi^2_{F,s}(t)$ functions, which is a modified χ^2 with $\nu_\odot$ replaced ν_F. The parameter s is the maximum value of the quantity $(\ell + 0.5)/\nu$ (ν in μHz), which determines the lower turning point allowed in the set of modes. The case $s = 0.1$ corresponds to including all 1945 p-modes. The case $s = 0.02$ corresponds to a truncated set which includes only 956 modes with $r_t < 0.8R$ In the latter case, we additionally remove effects of inadequate treatment of convection which are responsible for large values of $\delta u/u$ above 0.9. The minima

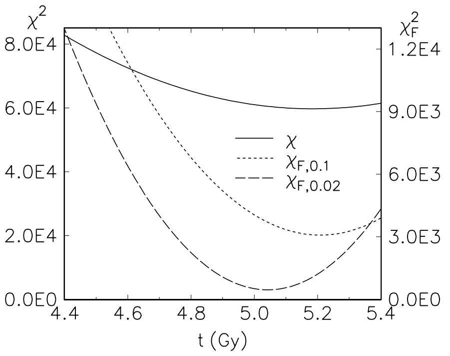

Fig. 3. Determination of the solar age by fitting p-mode frequencies. Values of χ^2 (left vertical axis, solid line) are calculated with Eq. 2. Values of $\chi^2_{F,s}$ for $s = 0.1$ and $s = 0.02$ (right vertical axis, dashed lines) are calculated also with Eq. 2, but with $\nu_\odot$ replaced by ν_F (see Eq. 3). The choice $s = 0.1$ implies use of all p-mode frequencies and elimination of the near surface differences between the sun and the model. With $s = 0.02$ we use only modes with the lower turning point above $0.8R$ and we additionally eliminate effects of inadequacies in the treatment of convection.

Table 5. Seismic age from p-mode frequencies

	$s = 0.1$		$s = 0.02$	
Z/X	$t_{\rm seis}$	χ^2	$t_{\rm seis}$	χ^2
0.0245	5.22 ± 0.40	3.16×10^3	5.04 ± 0.13	5.2×10^2
0.0270	4.91 ± 0.34	3.45×10^3	4.77 ± 0.13	5.3×10^2

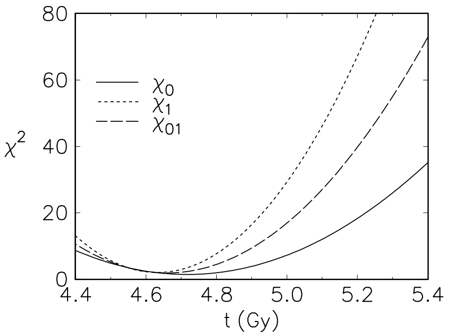

Fig. 4. Determination of the solar age by fitting small frequency separations (see Eq. 4). The quantity χ_0 is defined in Eq. 5, χ_1 and $\chi_{0,1}$ are defined immediately after.

Table 6. Seismic age from small separations

	$\ell = 0$		$\ell = 1$		$\ell = 0\&1$	
Z/X	$t_{\rm seis}$	χ_0^2	$t_{\rm seis}$	χ_1^2	$t_{\rm seis}$	χ_{01}^2
0.0245	4.71 ± 0.14	1.40	4.64 ± 0.08	1.34	4.66 ± 0.11	1.52
0.0270	4.63 ± 0.14	1.48	4.54 ± 0.08	1.44	4.57 ± 0.11	1.68

of the modified χ are pronounced and therefore we may, at least formally, determine the solar age and its uncertainty. Not surprisingly, the minimum is deeper for $s = 0.02$. Still, the minimum value is $\gg 1$. One may see in Fig. 1 that $\delta u/u$ in the radiative interior cannot be compensated by an adjustment of the age.

In Table 5, we list the values of $t_{\rm seis}$ (in Gy) determined as the minima of $\chi_{F,0.1}$ and $\chi_{F,0.02}$. The errors are determined as the distances from $t_{\rm seis}$, where $\chi^2 = 2\chi^2(t_{\rm seis})$.

The results shown in Table 5 are consistent with implications from $\delta u(r)$ discussed in the previous section. There are only few modes sensitive to u in the inner core, where δu is not consistent with high $t_{\rm seis}$. Also, even with $s = 0.1$ there are not many modes sensitive to $Y_{\rm ph}$. The results agree with those of Weiss & Schlattl (1998). All this does not mean that we should treat $t_{\rm seis}$ given in Table 5 as realistic estimates of solar age. Rather, we think, the high values obtained for models with the standard metal abundance reflect an attempt to compensate such deficiencies of the model as too low opacity and/or neglect of macroscopic mixing beneath the base of convective envelope. With $Z/X = 0.027$ we obtained $t_{\rm seis}$ which is still higher but, within the error, consistent with $t_{\rm met}$.

5. Solar age from small separations

The inner core is the region where the sound speed is most sensitive to the age. Inversion for u in this region is unreliable but this does not mean that oscillation frequencies are not affected by the sound speed modifications near the center. The quantities which are most sensitive to changes in the inner core are small separations

$$D_{\ell,n} = \nu_{\ell,n} - \nu_{\ell+2,n-1} \tag{4}$$

for $\ell = 0$ and 1. In fact it has been recognized long time ago that data on $D_{\ell,n}$ may be used for measuring stellar ages (Ulrich, 1986; Christensen-Dalsgaard, 1988; Gough & Novotny, 1990).

In our set we have data on $D_{0,n}$ for n from 10 to 32 and on $D_{1,n}$ for n from 10 to 27. We now form three age indicators,

$$\chi_0^2 = \frac{1}{23}\sum_{n=10}^{32}\frac{(D_{0,n,\odot} - D_{0,n,{\rm model}}(t))^2}{\sigma_{0,n}^2 + \sigma_{2,n-1}^2}, \tag{5}$$

χ_1^2, which is defined is the same way as χ_0^2 but for $\ell = 1$, and χ_{01}^2, which includes small separations both for $\ell = 0$ and $\ell = 1$.

The behavior of the three indicators is shown in Fig. 4. The χ^2 minima occur now at the ages which are only somewhat larger than $t_{\rm met}$ and have values only somewhat higher than 1. Table 6 summarizes information about the minima for models with the standard and the enhanced value of Z/X. In the latter case the minima occur still closer to $t_{\rm met}$, but the difference is small and cannot be regarded as significant. The ages $t > 5$ Gy are clearly disfavored. There is a rough agreement of our result with that of Guenther & Demarque (1997), who relied on comparison of frequencies for ℓ up to 100 and small separations for ℓ up to 10. Also in their comparisons the strong case for $t_{\rm seis} \approx t_{\rm met}$ comes from small separations at $\ell = 0$ and 1.

We believe that only in the case of inference based on the small separations it is justified to speak about "age determination" because only with these observables we attain $\chi^2 \sim 1$. Furthermore, only in this case the inference is truly robust to

Table 7. Seismic age indicators from small separations in various models at $t = 4.57$ Gy and Z/X=0.0245

MODEL	χ_0^2	χ_1^2	χ_{01}^2
0	2.89	2.32	2.64
3	2.31	1.69	2.04
JCD	1.82	0.99	1.46
4	2.75	2.01	2.43
5	20.06	44.84	30.94

Model 4 is the same as model 0, but with a 3.2% increase of the ^{3}He+^{4}He reaction cross-section and a 6% decrease of the ^{3}He+^{3}He reaction cross-section. Model 5 is the same as Model 0 but ignoring the effect of gravitational settling.

other uncertainties still present in the standard model construction. The overall uncertainty of the seismic measurement of the solar age with the data on small separations is not significantly larger than the formal errors quoted in Table 6. The effect of the Z/X uncertainty, as we see in this table, is ≤ 0.1 Gy. Now we will review other uncertainties that may affect small separations.

Effect of uncertainties on the age indicators χ_0,χ_1, and χ_{01} may be assesed from data in Table 7. The effect of the opacity is revealed by comparison of models 0 with models 3 and JCD and we see that it is small. As we discussed in Sect. 3, the difference in opacity does not explain the whole difference in the sound speed between the models 0 and JCD. We alluded that the treatment of the element settling may contribute. In any case the implication for $t_{\rm seis}$ is certainly within the uncertainties quoted in Table 6. We should note that JCD model which is characterized by the lowest value of χ_{01}^2 yields also the values of $t_{\rm seis}$ which are the closest to $t_{\rm met}$ on the basis of the seismic observables listed in Table 4.

Ignoring gravitational settling altogether (see Model 5 in Table 7) has a significant effect on small separations. However, the effect is now part of the physics included in the standard modeling of the sun.

Calculated values of the small separations are affected by the nuclear reactions cross-sections. The most important effect is expected from changes in the branching ratio of the ^{3}He+^{4}He to the ^{3}He+^{3}He reaction. Its increase implies more neutrino energy losses, less economic hydrogen burning, and consequently less hydrogen in the center of the sun. Such models mimic ones with $t > t_{\rm seis}$. However with currently adopted uncertainties in the cross-sections (see Model 4 in Table 7) the consequences for the age indicators are not significant.

Mixing of hydrogen and helium reduces the μ-gradient in the core and thus has a similar effect as a lower age. This is not a standard effect and we feel that there is not enough justification to consider it as a source of uncertainty. Certainly macroscopic mixing at the base of the convective zone is of more concern because we have some evidence for it. The mixing affects gravitational settling and therefore may have an appreciable effect on small separations.

Small separations are influenced by the centrifugal and magnetic distortion (Dziembowski & Goupil, 1998). The effect of centrifugal distortion in the sun is small because it is a very slowly rotating star. However, at a rotation rate five times higher, the values of $D_{0,n}$ for $n \approx 20$ are reduced by $\approx 0.5\,\mu$Hz, which corresponds to about 0.5 Gy. Thus the effect has to be kept in mind when we will have small separations data for other stars. For the sun, the magnetic effects in high activity years are significant but they may easily be purged (Dziembowski & Goode, 1997). The problem does not concern the frequencies used in the present paper because we used data from 1996/97 season when the solar activity was at its minimum.

6. Conclusions

There is no evidence from helioseismology that the solar age is different from $t_{\rm met}$. Though it is true that by adopting higher values for the solar age one may achieve a better agreement for most of seismic data, this cannot be regarded as an argument that the sun is older than the meteoritic data indicate. What seismic testing of current standard solar models reveals is the need for increase of the sound speed in the outer part of the radiative zone by about half percent (~ 0.01 in u). The required change may be partially achieved by an age increase above 5 Gy but it may also be caused by an increase of opacity in the relevant region. The required opacity increase may result from some still ignored effects in the OPAL calculations but also may indicate that the metal content in the outer part of the radiative zone is higher. We showed that adopting $Z/X = 0.027$, which is by 10% higher than the standard one but still within the error bars of determination, we derive $t_{\rm seis}$ below 5 Gy and only marginally inconsistent with $t_{\rm met}$.

In fact, helioseismology provides a strong support for the assumption $t = t_{\rm met}$. We showed that small frequency separations $\nu_{0,n} - \nu_{2,n-1}$ and $\nu_{1,n} - \nu_{3,n-1}$ determined from the data are in a drastic disagreement with the models older than 4.9 Gy and they are in a good agreement with the models calculated assuming $t = t_{\rm met}$. The age of the sun determined from the best seismic data and with use of our standard models, which were calculated with the latest OPAL opacity data and the standard metallicity parameter $Z/X = 0.0245$, is

$$t_{\rm seis} = 4.66 \pm 0.11\,{\rm Gy}.$$

Outside the error range, we have $\chi^2 > 2\chi^2_{\rm min}$.

The small separations are only weakly affected by uncertainties in the opacity. Still, models with enhanced opacity in the outer part of the radiative zone yield values of $t_{\rm seis}$ even closer to $t_{\rm met}$. Thus we conclude that the inadequacies of the current solar models cannot be reconciled by departing from the standard assumption about solar age but the resolution must be searched in opacity enhancement.

Our answer to the question how accurately we can determine age of the sun using stellar evolution theory and helioseismic data, posed by Paczyński (1997), is more optimistic than the answer of Weiss & Schlattl (1998).

The error bars given above may be somewhat underestimated. Taking into account the uncertainties beyond those included in the formal errors, the accuracy of the astrophysical estimate the solar age is, in our opinion, ~ 0.2 Gy or 4%, which is significantly better than 0.5 Gy, as suggested by Weiss & Schlattl (1998).

The cause for the discrepant estimates is in the use of different observables. We believe that only the small separations are good probes of the solar age based on p-mode frequency data. Other ones, like frequencies themselves, seismically inferred sound speed, and photospheric helium abundance are too sensitive to the opacity to be regarded as reliable tools for measuring solar and stellar ages.

We examined various effects that may contribute to the uncertainty of the age determination from the small separations. None of the uncertainties in the physics included in modern standard solar models was found very significant. However, we identified few effects beyond standard model that may influence the small separations. Perhaps the most important is a macroscopic mixing in the outermost part of the radiative interior. We considered also the effects of the centrifugal and magnetic forces and we pointed out that while they are not important for our seismic estimate of the solar age they must be kept in mind in interpretation of data on the small separations from years of high magnetic activity as well as the data for stars rotating more rapidly than the sun.

All the seismic observables we discussed here are still available only for the sun. The observables that we are likely to have in not too distant future for other stars are the small separations. Measuring these parameters is one of the main goals of the three currently prepared or planned space asteroseismic missions. It is very fortunate that, as we have shown, the small separations are the best seismic age indicators derived from p-mode frequencies. There is a potential for measuring stellar ages based on g-modes, which are excited in a number of stars. However, also in this case it is essential to check robustness of the seismic dating.

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References

Alexander D.R., Ferguson J.W., 1994, ApJ 437, 879
Bahcall J.N., Pinsonneault M.H., 1995, Rev. Mod. Phys. 67, 781
Brown T.M., Christensen-Dalsgaard J., 1998, ApJ 500L, 195
Brun A.S., Turck-Chieze S., Morel P., 1998, ApJ 506, 913
Christensen-Dalsgaard J,. 1988, In: Rolfe E.J.(ed.) Seismology of the Sun and Sun-like stars. ESA SP-286, p. 431
Christensen-Dalsgaard J., Däppen W., Ajukov S.V., et al., 1996, Sci 272, 1286
Ciacio F., Degl'Innocenti S., Ricci B., 1997, A&AS 123, 449
Degl'Innocenti S., Dziembowski W.A., Fiorentini G., Ricci B., 1997, Astroparticle Phys. 7, 77
Dziembowski W.A., Goode P.R., Pamyatnykh A.A., Sienkiewicz R., 1994, ApJ 432, 417
Dziembowski W.A., Goode P.R., 1997, A&A 317, 919
Dziembowski W.A., Goupil M.-J., 1998, In: Bedding T.R., Kjeldsen H. (eds.) Workshop on Science with Small Space Telescopes. University of Aarhus, p. 69
Gabriel A.H., Charra J., Grec G., et al., 1997, Solar. Phys. 175, 207
Gough D.O., Kosovichev A.G., Toomre J., et al., 1996, Sci 272, 1299
Gough D.O., Novotny E., 1990, Solar Phys. 128, 143
Grevesse N., Noels A., 1993, Phys. Scripta T47, 133
Guenther D.B., Demarque P., 1997, ApJ 484, 937
Iglesias C.A., Rogers F.J., Wilson, B.G., 1992, ApJ 397, 717
Iglesias C.A., Rogers F.J., 1996, ApJ 464, 943
Paczyński B., 1997, In: Turok N.(ed.) Critical Dialogues in Cosmology. Princeton University Press, p. 169
Pijpers F.P., Thompson M.J., 1992, A&A 262, L33
Rhodes E.J. Jr., Kosovichev A.G., Schou J., Scherrer P.H., Reiter J., 1997, Solar Phys. 175, 287
Richard O., Vauclair S., Charbonnel C., Dziembowski W.A., 1996, A&A 312, 1000
Rogers F.J., Iglesias C.A., 1992, ApJS 79, 507
Rogers F.J., Swenson F.J., Iglesias C.A., 1996, ApJ 456, 902
Thoul A.A., Bahcall J.N., Loeb A., 1994, ApJ 421, 828
Ulrich R.K., 1986, ApJ 306, L37
Weiss A., Schlattl H., 1998, A&A 332, 215