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The Nature of the Period Changes in RV Tauri Stars

JOHN R. PERCY, MICHAEL BEZUHLY, AND MONICA MILANOWSKI
Erindale College, University of Toronto, Mississauga, Ontario, Canada L5L 1C6
Electronic mail: jpercy@erin.utoronto.ca

ENDRE ZSOLDOS
Konkoly Observatory, P.O. Box 67, Budapest XII, 1525 Hungary
Electronic mail: zsoldos@ogyálla.konkoly.hu

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ABSTRACT. We have studied the period changes in 15 RV Tauri stars, using the Eddington–Plakidis hypothesis, which assumes that the (O−C) diagrams can be interpreted as a superposition of random errors in the measured times of minimum, and random cycle-to-cycle fluctuations in the period. Except for three stars for which the data are sparse, the hypothesis fits the (O−C) data very well, suggesting that the assumptions are correct. The magnitude of the random period fluctuations does not appear to correlate with any obvious physical property of the stars.

1. INTRODUCTION

According to the fourth edition of the *General Catalogue of Variable Stars* (Kholopov 1985), the RV Tauri variable stars are:

> "pulsating supergiants having spectral types F to G at maximum light, and K to M at minimum light. The light curves are characterized by the presence of double waves with alternating primary (deep) and secondary (shallow) minima, which can vary in depth so that primary minima may become secondary ones, and *vice versa*; the complete light amplitude may reach 3 to 4 magnitudes in V. The periods between two adjacent primary minima (called the "double" or formal periods) lie in the range 30 to 150 days. Two sub-types may be isolated: RVa—variables which do not vary in mean magnitude; RVb—variables which periodically vary in mean magnitude with periods of 600 to 1500 days (or more), and with amplitudes up to 2 magnitudes in V."

RV Tauri stars are found among the old disk and Population II stars, and are thought to be post-asymptotic-giant-branch (post-AGB) stars (Jura 1986). In this case, the stars should be contracting from the AGB to the white dwarf stage on a time scale of thousands of years. The periods should be decreasing at a measurable rate, and this might be used as a test of the evolution models.

The most extensive study of period changes in RV Tauri stars is by Erleksova (1971), who compiled data and presented (O−C) diagrams for 21 such stars. In each case, the (O−C) diagram was interpreted in terms of a series of abrupt period changes, which appeared to be as often positive as negative. Percy et al. (1991) studied the periods of U Mon and R Sct over a century or more, and found single abrupt-decreases which, when averaged over the interval of observation, were in good agreement with the predictions of pulsation-evolution models. Zsoldos has carried out a series of comprehensive studies of the periods of several RV Tauri stars (Zsoldos 1988, 1991, 1993, 1995, 1996; Zsoldos and Kollath 1991). Usually, the (O−C) diagrams appear cyclic, though the time span of the data is not much greater than the cycle length. In AC Her, the (O−C) variations appear strictly periodic, with a period of 9323 days. Zsoldos (1995) has also looked for correlations between the amplitude of these cycles, and other properties of the stars; he finds some evidence for a relation between (O−C) amplitude, and color, at least in RVa stars.

Mira stars are asymptotic-branch (AGB) stars pulsating with V ranges of >2.5 mag. They also show quasicyclic variations in (O−C), and it is known that these are due, in almost every case, to the effect of random cycle-to-cycle fluctuations in period (Eddington and Plakidis 1929; Isles and Saw 1987, 1989; Percy et al. 1990; Lloyd 1992; Koen and Lombard 1993). This raises the possibility that the (O−C) diagrams of RV Tauri stars are also due to such fluctuations. In that case, it would be difficult to extract the evolutionary period changes from the random fluctuations. Percy et al. (1992) showed, by simulations, that the (O−C) diagrams produced by such a process are very similar to the observed (O−C) diagrams of RV Tauri stars. In the present paper, we analyze the times of minimum of 15 RV Tauri stars, to see if this model fits the observations.

2. OBSERVATIONS

We have analyzed the times of minima of the 15 RV Tauri stars listed in Table 1. They are the stars for which Zsoldos has accumulated times of minimum from the literature. The references to the times of minimum are given in Zsoldos (1988, 1991, 1993, 1995, 1996), Zsoldos and Kollath (1991), or in the lists at the end of the paper. Both deep and shallow minima are included. The accuracy of these timings varies; it can be estimated from the intercept of the $\langle u(x) \rangle$ diagrams (as described in Sec. 3), or from minima for which there is more than one timing. The two estimates appear to be consistent.

TABLE 1
Tauri Stars: Properties and Results of Analysis

Star	Period	Te		log g		e	a
	days	W	D	W	D	days	days
EZ Aql	38.64					0.11	0.74
TW Cam	87.44	5000	4800	1.1	0.9	0.93	2.84
IW Car	67.5					1.9:	1.0:
DF Cyg	49.81	4750	5050	1.5	1.4	0.44	1.60
SS Gem	89.16	5250	5250	2.0	2.5:	0.60	2.36
SU Gem	50.12	5750	4800:	1.1	1.2	0.68	0.53
AC Her	75.41	6000	5000	1.9	2.2	0.71	1.38
BT Lac	40.50					0.47	0.43
EP Lyr	83.43	5250	6000:	1.5	4.3:	0.23	2.88
V Mon	92.23	4750	5500	1.9	1.8	1.15	2.87
TT Oph	61.08	5000	5000:	1.5	2.0	0.49	0.75
AR Pup	75					2.8:	—
R Sge	69.74	4750	5100	1.9	1.6	0.65	2.37
RV Tau	75.41	4250	4650	1.1	1.3	0.46	2.06
V Vul	75.35	4750	5400	1.1	1.5	1.33	1.2:

W: From Wahlgren (1992); D: From Dawson (1979)

3. ANALYSIS

There are several methods for testing the hypothesis of random cycle-to-cycle fluctuations, starting with the pioneering work of Eddington and Plakidis (1929) and of Sterne and Campbell (1937), continuing through the more recent work of Isles and Saw (1987, 1989), and Lloyd (1992), to the elegant work of Koen and Lombard (1993). We have used the formalism of Eddington and Plakidis (1929).

In this approach, the hypothesis is made that the (O-C) variations are due to random fluctuations (e) in period from one cycle to the next. There are also random errors (a) in the measured times of minimum light. These quantities are each assumed to be accidental and uncorrelated. We define $z(r)$ as the (O–C) of the rth minimum, compared with the ephemeris, and $u(x,r)=z(r+x)-z(r)$ is the accumulated delay in x periods, which, according to the hypothesis, is the sum of x uncorrelated random fluctuations. Allowing also for the random errors in the measured times of minimum, we find that the average value $\langle u(x)\rangle$ over all values of r is given by

$$\langle u(x)\rangle^2=2a^2+xe^2$$

so a plot of $\langle u(x)\rangle^2$ against x should be a straight line with slope e^2 and intercept $2a^2$. Note that x is measured in cycles, not days.

We therefore first determined the $z(r)$ from the best ephemerides of the stars, and then used these to determine $\langle u(x)\rangle^2$ for all possible pairs of minima x cycles apart. The $\langle u(x)\rangle$ values were plotted against x. Using the linear region of the diagram (which turned out to be $x=0$ to 40, except for the stars mentioned below), the slope and intercept were determined by least squares. We used the half-periods or single periods, as opposed to the formal or double periods—the intervals between adjacent deep or shallow minima.

4. RESULTS

In Table 1, we list the properties of the RV Tauri stars analyzed, and the results of our analysis. The properties are taken from compilations by Dawson (1979) and Wahlgren (1992). The poor correlation between Dawson's and Wahlgren's values is not surprising, in view of the fact that Dawson's values are only partially phase averaged, and Wahlgren's are generally based on a single phase point; the temperatures of RV Tauri stars can vary by 1000 K around a cycle, and the log g can vary by up to 2.0. The parameters e (the period fluctuation per cycle) and a (the scatter in the measured times of minima) are also given.

In general, the Eddington and Plakidis model fits the stars well, except in the stars for which the data are very sparse. For all the stars except those mentioned below, the model fitted the data from $x=0$ to 40 or beyond. Eddington and Plakidis explain why the model breaks down for larger values of x—especially in stars with a limited time span of data.

Figure 1 shows the $\langle u(x)\rangle$ graphs for four stars with extensive data, namely, AC Her, EP Lyr, RV Tau, and V Vul. These results are entirely typical except for the four stars discussed below. The graphs are linear and well defined from $x=0$ to $x=40$. In V Vul, there is a slight curvature as the graph approaches $x=0$. Figure 2 shows the graphs for four stars for which the model does not produce a straight, well-defined line, namely:

AR Pup. There is considerable scatter in the $\langle u(x)\rangle$ graph, but the slope is well defined.

BT Lac. The $\langle u(x)\rangle$ graph is only linear to $x=15$.

EZ Aql. The $\langle u(x)\rangle$ graph shows alternating high and low values, indicating that the secondary minima are not exactly halfway between the primary ones. The graphs for the even and odd values of x are, however, linear, and have the same slope.

IW Car. The data are very sparse. The $\langle u(x)\rangle$ graph

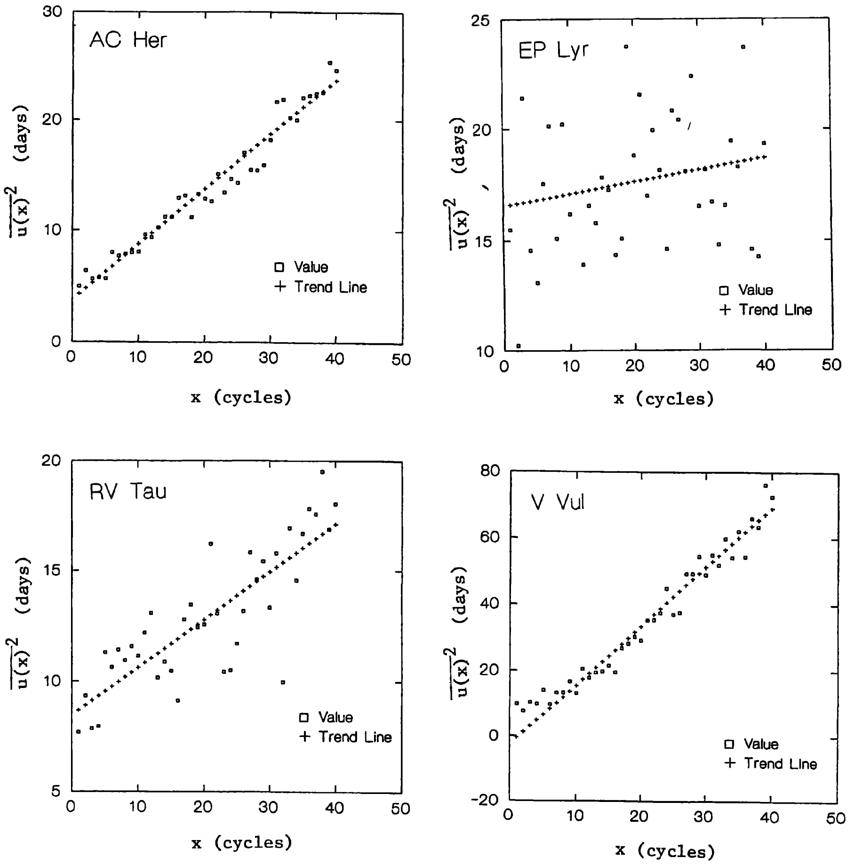

FIG. 1—The $\langle u(x)\rangle$ graphs ($(\langle u(x)\rangle)^2$ vs x) for four of the stars for which the Eddington–Plakidis model fits well: AC Her, EP Lyr, RV Tau, and V Vul. The slope is related to the magnitude of the cycle-to-cycle period fluctuation, and the intercept is related to the average error of the times of minimum magnitude. In V Vul, there is a slight curvature of the graph as it approaches $x=0$.

shows cycles with values of x which are multiples of 20, which mirror similar cycles in the (O–C) diagram.

5. DISCUSSION

The average fluctuation in period, e, varies from 0.1 day in EZ Aql to almost 3 days in AR Pup (though the data for this star are very sparse). The average scatter in the times of minimum, a, range from about 0.5 day in SU Gem and BT Lac, to almost 3 days in EP Lyr and U Mon. This scatter may reflect both measurement error, and random fluctuations in the times of minimum. The ratio e/P varies from 0.005 to 0.02, as compared with about 0.01 to 0.05 for Mira stars.

As might be expected, the values of e correlate with the amplitudes of the (O–C) cycles determined by Zsoldos (1995), especially when the length of the dataset is also considered. There is a slight correlation between e and period, which might be expected; this suggests that e/P might be a better measure of the period fluctuation.

There is no obvious correlation between the values of e, and any other parameter of the stars. Zsoldos (1995) found some correlation between the amplitude of the (O–C) cycles, and color, in RVa stars. There is no obvious correlation between the values of e, and the effective temperatures, nor is there any correlation with log g. One might expect that low-gravity stars might have less stable pulsation than high-gravity ones. There is also no correlation with the regularity of the alternating deep and shallow minima (Percy 1993), though the data on the regularity of this phenomenon are rather sparse. It is difficult to look for correlations when the number of stars is so small, and the number of possible parameters is so large.

Our results indicate that the (O–C) diagrams of RV Tauri stars are dominated by the effects of random, cycle-to-cycle fluctuations in period. This means that true evolutionary changes in period will be more difficult to determine, unless they are greater than the accumulated effects of the random fluctuations. The evolutionary changes in period are sensitive to the exact parameters and evolutionary state of the model; they range from 0.02 to 0.20 days per year or more in the models examined by Percy et al. (1991). The lower rates would not likely be observable amidst the random fluctuations, but the higher rates should be detectable in the better-studied stars.

6. CONCLUSIONS

The Eddington and Plakidis model fits the times of minima of the RV Tauri stars analyzed, except for those stars for which the data are very sparse. For the others, the $\langle u(x)\rangle$

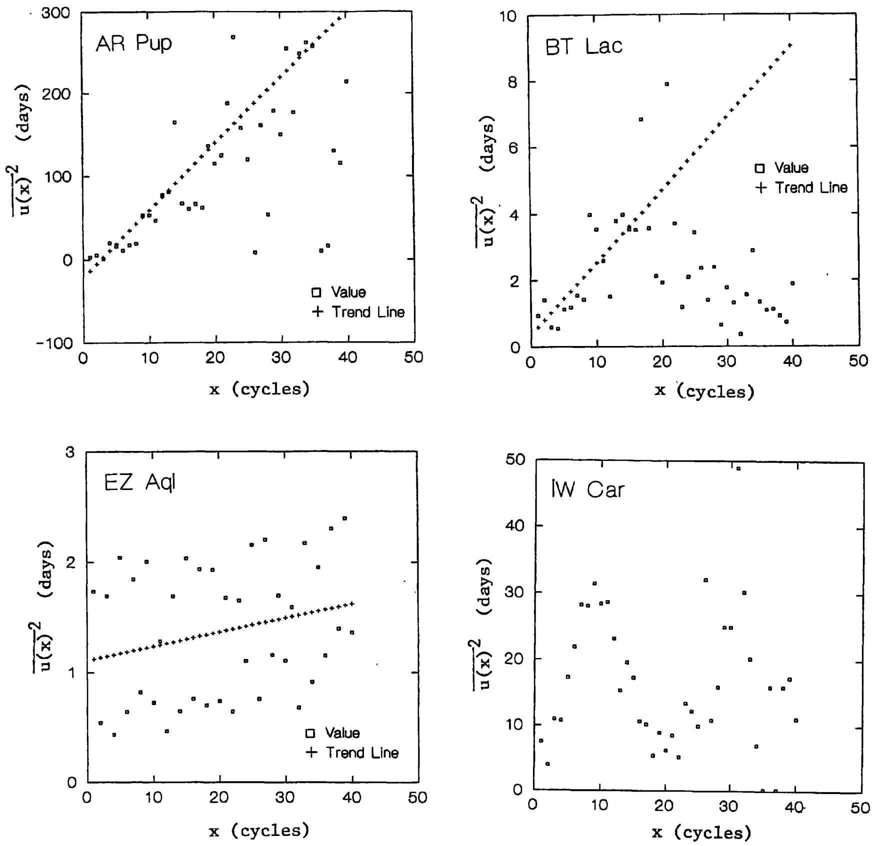

FIG. 2—The $\langle u(x)\rangle$ graphs for the four stars for which the Eddington–Plakidis model does not give a good fit to the data. In most cases, this is due to the sparseness of the data, as explained in the text.

graph is linear and well defined, indicating that the (O–C) diagrams of the RV Tauri stars are affected by random cycle-to-cycle fluctuations in period. This means that evolutionary changes in period will be more difficult to detect. Since they are expected to be monotonic, however, they will eventually be detectable over the random effects. The types of approach developed by Koen and Lombard (1993), for instance, will probably be most effective for detecting them.

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APPENDIX: REFERENCES ON SPECIFIC STARS

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IW Carinae
O'Connell, D. 1946, Riverview Publ., 2, 46
(plus estimates from photoelectric light curve)

DF Cygni
Beyer, M. 1948, Astron. Abhandl., 11, No. 4
Erleksova, G. E. 1971, Perem. Zvezdy, 18, 53
Harwood, M., and Gerasimovic, B. P. 1927, Harvard Bull., No. 849, 15
Jacchia, L. 1931, Beob.-Zirk, 46
Koval, G. T. 1953, Odessa Izv., 3, 313
(plus estimates from photoelectric light curve)

SS Geminorum
Zsoldos, E. 1991, Ap&SS, 181, 203

SU Geminorum
Beyer, M. 1948, Astron. Abhandl., 11, No. 4
Beyer, M. 1976, Bamberg Veroff., 10, No.
Erleksova, G. E. 1971, Perem. Zvezdy, 18, 53
Jacchia, L. 1931, Beob.-Zirk., 13, 30
Nabokov, M. 1927, Astron. Nach., 230, 233
(plus estimates from photoelectric light curve)

AC Herculis
Zsoldos, E. 1988, Inf. Bull. Var. Stars, No. 3192

BT Lacertae
Tempesti, P. 1955, Mem. Soc. Astron. Ital., 26, 125

EP Lyrae
Zsoldos, E. 1995, A&A, 296, 122

U Monocerotis
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RV Tauri
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V Vulpeculae
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