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MINING ENERGY IN AN EXPANDING UNIVERSE

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ABSTRACT

In principle, the expansion of the universe can be harnessed to provide energy. In a gedankenexperiment, energy is gained by connecting together widely separated bodies with strings. The tension and the energy generated are calculated for single strings. Mining energy in an expanding universe in this way raises unresolved issues concerning the conservation of energy. Apparently, the tethered-body experiment delivers "nascent" energy that previously did not exist in any identifiable and quantifiable form. It is argued that energy in a homogeneous and unbounded universe, in general, is not conserved on the cosmic scale.

Subject headings: cosmology: theory — relativity

1. INTRODUCTION

In a unit of mass, nuclear reactions release energy $\sim 10^{-2}c^2$, accretion disks around black holes release gravitational energy $\sim 10^{-1}c^2$, but expansion of the universe in certain cosmological models—such as the "free-lunch" inflationary model (Guth 1981)—can in principle release energy exceeding c^2. Where does this energy come from?

Photons traveling in expanding space between comoving galaxies lose energy. This is the cosmological redshift effect (see Harrison 1981), and the natural question is what happens to the lost energy? Energy is conserved for the Doppler and gravitational redshifts in spatially bounded systems, and in these cases the "lost" energy is manifest in identifiable alternative forms. But in a spatially unbounded homogeneous and isotropic universe that conforms to the Robertson-Walker metric, the lost energy fails to manifest identifiable alternative forms. If we argue that the lost energy of individual photons transforms into metric disturbances (i.e., deformations of the Robertson-Walker metric), these disturbances, as they propagate, will also lose energy because of the cosmological redshift. The question now becomes, what happens to the lost energy of the gravitational waves?

Uniform radiation, such as the cosmic background radiation, is subject to the cosmological redshift effect, and in this instance the adiabatic form of the first law applies, but leaves unresolved the problem of the lost internal energy in an expanding, homogeneous, and unbounded universe. Does the energy totally vanish, or does it reappear, perhaps in some global dynamic form? The tentative answer based on standard relativistic equations is that the vanished energy does not reappear in any other form, and therefore it seems that on the cosmic scale energy is not conserved.

The possibility of mining energy in a de Sitter space has been discussed by Davies (1984). Here I indicate how new energy in a gedankenexperiment can be mined, either in unlimited amounts in certain accelerating universes or in finite amounts in decelerating universes, and argue that the energy gained is "nascent" in that it did not exist previously in any identifiable form.

2. THE TETHERED BODY EXPERIMENT

Imagine a comoving body of mass m tethered by an inextendable string of negligible mass to an unwinding mechanism located on a second comoving body of much larger mass at distance L. Initially, with bodies comoving, the string unwinds and increases in length at the rate $\dot{L} = HL$, in accordance with the velocity-distance law, where $H = \dot{R}/R$ is the Hubble term and $R(t)$ is the scale factor. The experimenter on the larger body applies tension to the string and reduces the recession velocity of the smaller body. The tension in the string, from the equation of motion and the Robertson-Walker metric, is

$$T = -m \frac{1}{R} \frac{d(\gamma R U)}{dt}, \tag{1}$$

where U is the peculiar velocity of the smaller tethered body in the comoving frame, $\gamma = (1 - U^2/c^2)^{-1/2}$, and the string unwinds at the rate $\dot{L} = HL + U$. When the peculiar velocity reaches the value $U = -HL$, the tension vanishes in a decelerating (but not necessarily an accelerating) universe, and the energy $E = \int T\,dL$ gained, assuming that R changes little during the experiment, is

$$E \approx mc^2\{[1 - (HL/c)^2]^{-1/2} - 1\} \approx \tfrac{1}{2}m(HL)^2, \tag{2}$$

for $L \ll L_H$, where $L_H = c/H$ is the Hubble distance. If, now, the tethered body is released ($T = 0$), it will eventually relax back to the comoving state ($U = 0$) in accordance with equation (1). Thus the energy E released at the unwinding mechanism, given by equation (2), derives from the expansion of the universe, and, oddly enough, is created by increasing the kinetic energy of mass m in the comoving frame by an amount equal to E. In effect, the cosmic internal energy (in kinetic form) increases, and an equal amount of mined energy is released at the unwinding mechanism (in thermal or other form) and is available for immediate use by the experimenter. This process may be repeated, although not indefinitely, by detaching the string, rewinding it, and reattaching it to a succession of comoving bodies.

A similar situation exists in stellar structure theory. In the absence of nuclear reactions, the emission of luminous energy is accompanied by an increase in internal energy (Eddington 1926), and both forms of energy are at the expense of gravitational energy. In a uniform unbounded universe, however, the explanation is much less obvious.

In more detail, consider a body of mass m_1 at distance L_1 from the experimenter, and a second body of mass m_2 in the opposite direction at distance L_2, and that both are connected together by an inextendable string of length[1] $L = L_1 + L_2$. When the peculiar velocities are small compared with the velocity of light c, the tension is

$$T = -m(\ddot{L} + qH^2L) , \tag{3}$$

where $m = m_1 m_2/(m_1 + m_2)$ and $q = -\ddot{R}R/\dot{R}^2$ is the deceleration term.[2] The string stays in tension when $\ddot{L} < -qH^2L$. In the particular case when either L or $\dot{L}$ is constant, the tension is positive in an accelerating universe of $q < 0$ and negative in a decelerating universe of $q > 0$. By unwinding the string at either body at the rate $\dot{L}$, the generated power is

$$\dot{E} = -m\dot{L}(\ddot{L} + qH^2L) . \tag{4}$$

2.1. *Nonrelativistic Solutions*

Assume that the string unwinds at rate $\dot{L} = \alpha HL$, hence $U = -(1 - \alpha)HL$, with α constant (hence $L \propto R^\alpha$) and less than unity. The tension now has the value

$$T = (1 - \alpha)(\alpha - q)mH^2L , \tag{5}$$

and the power generated becomes

$$\dot{E} = \alpha(1 - \alpha)(\alpha - q)mH^3L^2 . \tag{6}$$

If $1 > \alpha > q$, the tension stays positive and energy is generated[3] in an expanding ($H > 0$) universe. From

$$Rd(HL)/dR = -HL(1 - \alpha + q) , \tag{7}$$

it can be seen that in the nonrelativistic approximation the power generated is unlimited in accelerating universes of $q \le -(1 - \alpha)$ and limited in all other universes of $q > -(1 - \alpha)$.

With m in solar-mass units, L in megaparsecs, $H = 100\,h$ km s^{-1} Mpc^{-1}, a finite amount of energy is mined at the rate

$$\dot{E} = 2 \times 10^{30} FmL^2 J y^{-1} , \tag{8a}$$

where

$$F = \alpha(1 - \alpha)(\alpha - q)h^3 = 3 \times 10^{-3} \tag{8b}$$

for $q = 0.5$ (Einstein–de Sitter model), $\alpha = 0.6$, and $h = 0.5$. A body, such as the Moon ($m = 3.7 \times 10^{-8}$ solar masses), tethered at 1 Mpc distance, generates power of the order of the present-day world consumption by the human race.

In the power-law models $R \propto t^n$, $H = n/t$, $q = (1 - n)/n$, equation (6) integrates to give an energy gain

$$E = \tfrac{1}{2}m(H_0 L_0)^2(1 - \alpha)(\alpha - q)(\alpha - q - 1)^{-1}(y^2 - 1) , \tag{9}$$

where $y = HL/H_0 L_0$. Energy is continually mined in accelerating universes of $q \le -(1 - \alpha)$, or $n > \alpha^{-1}$, but only in limited amounts $\sim \tfrac{1}{2}m(H_0 L_0)^2$ in all other models. In the open ($k = -1$) Friedmann model, q tends to zero ($n \to 1$) and

$$E \to \tfrac{1}{2}m\alpha^2(H_0 L_0)^2 , \tag{10}$$

and in the flat Einstein–de Sitter model of $n = \tfrac{2}{3}$, $q = \tfrac{1}{2}$,

$$E \to \tfrac{1}{2}m\alpha(1 - \alpha)(2\alpha - 1)(3 - 2\alpha)^{-1}(H_0 L_0)^2 . \tag{11}$$

Equation (4) can be integrated with the Friedmann-Lemaître equations of zero cosmological constant with an equation of state $P = f\rho c^2$. On substituting in equation (4) the expressions

$$\dot{L}\ddot{L} = \alpha^2(H_0 L_0)^2 \dot{y}y , \tag{12a}$$

$$qH^2L\dot{L} = q_0(H_0 L_0)^2 \dot{x}x^{1-3(1+f)/\alpha} , \tag{12b}$$

where $x = L/L_0$, $y = HL/H_0 L_0$, we find that the integrated energy mined is

$$E = \tfrac{1}{2}m(H_0 L_0)^2[\alpha^2(1 - y^2) + a^{-1}\Omega_0(1 + 3f)(1 - x^a)] , \tag{13}$$

where $a = 2 - 3(1 + f)\alpha^{-1}$, $q_0 = \Omega_0(1 + 3f)/2$, $\rho_0 = \Omega_0 \rho_c$, and $\rho_c = 3H_0^2/8\pi G$ is the critical density. In accelerating universes defined by $f < -\tfrac{1}{3}$, the energy mined is continuous for $0 < \alpha < 1$; generally, however, the energy gained tends to a fixed amount $\sim \tfrac{1}{2}m(H_0 L_0)^2$.

In the inflationary universe (Guth 1981) of $q = -1$, $H =$ constant, equation (6) becomes

$$\dot{E} = \alpha(1 - \alpha^2)mH^3L^2 , \tag{14}$$

and this integrates to give a total energy that increases with L^2:

$$E = \tfrac{1}{2}(1 - \alpha^2)mH^2(L^2 - L_0^2) , \tag{15}$$

in agreement with equation (12).

2.3. *A Relativistic Solution*

The relativistic expression for the power generated in the inflationary universe is

$$\dot{E} = -m\frac{\dot{L}}{R}\frac{d(\gamma RU)}{dt} = m\frac{\alpha}{1 - \alpha}\frac{U}{R}\frac{d(\gamma RU)}{dt} ,$$

from equation (1) and the relation $\dot{L} = -\alpha(1 - \alpha)^{-1}U$. Hence

$$\dot{E} = m\frac{\alpha}{1 - \alpha}U^{(\alpha-1)/\alpha}\frac{d(\gamma U^{(\alpha+1)/\alpha})}{dt} , \tag{16}$$

and integrating from $U = 0$, we find

$$E = m\left[\frac{\alpha}{1 - \alpha}\gamma U^2 + c^2(1 - \gamma^{-1})\right] . \tag{17}$$

[1] Practical considerations would suggest that the length of the string is large compared with either the scale of astronomical irregularities (such as clusters of galaxies) or $[2G(m_1 + m_2)/\Omega H^2]^{1/3}$, whichever is the larger. The mass of the string, its properties, the time taken to propagate tension along the string, and other practical matters are ignored, and it is stressed that mining energy in the manner proposed lies beyond the bounds of foreseeable technology.

[2] The relativistic form of eq. (3) for a string of $\beta_i \le c$ is

$$T = -\gamma_i m_i\{\gamma_i^2 \ddot{L}_i - (\gamma_i^2 - 1)H\dot{L}_i + [\gamma_i^2(1 + q) - 1]H^2L_i\}$$

where $\gamma_i = (1 - \beta_i^2)^{-1/2}$, $\beta_i = U_i/c$, and $i = 1, 2$.

[3] The relativistic forms of eqs. (5) and (6), for a mass m tethered to a much larger mass, are

$$T = \gamma(1 - \alpha)[\gamma^2(\alpha - q - 1) + 1]mH^2L ,$$

$$\dot{E} = \alpha\gamma(1 - \alpha)[\gamma^2(\alpha - q - 1) + 1]mH^3L^2 ,$$

and the tension is positive when $1 > \alpha > (q + \beta^2)$.

The energy mined inside the Hubble sphere (in this case the event horizon) of radius $L_H = c/H$, in which $-U < c(1-\alpha)$, is finite. As $-U \rightarrow c$, $L \rightarrow L_H/(1-\alpha)$, and $\dot{L} \rightarrow c\alpha/(1-\alpha)$, the energy mined goes to infinity. It is interesting that a tethered body at distance $L_H < L < 2L_H$ beyond the event horizon, may still have $-U < c$ and $\dot{L} < c$.

It must be stressed that throughout this paper all practical issues, such as the mass and other physical properties of the strings, are totally ignored, and may well involve insuperable problems. In the relativistic treatment, when the tethered body approaches the Hubble distance, the propagation of tension in the string at a speed no greater than light-speed raises additional issues that also have been ignored. Does a tethered body beyond the Hubble distance L_H violate causality? Apparently not, provided the peculiar speed U of the tethered body is less than c relative to the local comoving frame and to the distant comoving frame of the unwinding mechanism. The cosmological event horizon in this case is relative to the unwinding mechanism and refers to comoving bodies and not the tethered body.

2.4. *A Network of Strings*

The nonconservation of internal energy is impressively demonstrated by constructing on the cosmic scale an imaginary homogeneous and isotropic network of strings in tension (Vilenkin 1985; Kibble & Turok 1986; Turok 1988). The network has the same effect as the cosmological term or as a negative-pressure fluid, and Gott & Rees (1987) have shown that the internal energy in a comoving volume tends to increase and the expansion of the universe tends to accelerate.

3. NONCONSERVATION OF ENERGY

The following discussion demonstrates the nonconservation of energy on the cosmic scale.

3.1. *Lemaître-Robertson Equations*

According to general relativity, a universe conforming to the Robertson-Walker metric has density ρ and pressure P given by (Lemaître 1931 and Robertson 1933):

$$8\pi G\rho = 3(H^2 + K) - \Lambda \equiv 3\Omega H^2 , \qquad (18)$$

$$8\pi G P c^{-2} = H^2(2q - 1) - K + \Lambda , \qquad (19)$$

where $K = k/R^2$ ($k = 0, \pm 1$) is the curvature, Λ is the cosmological constant, q is the deceleration term, and Ω is the density parameter. These equations combine to give the well-known result

$$d(\rho c^2 R^3)/dt + P\,dR^3/dt = 0 . \qquad (20)$$

3.2. *Thermodynamic First Law in Cosmology*

For a comoving volume V proportional to R^3, containing total internal energy $E = \rho c^2 V$, equation (20) takes the adiabatic (conserved entropy[4]) form of the first law of thermodynamics

$$dE/dt + P\,dV/dt = 0 . \qquad (21)$$

In the laboratory, an expanding cell of volume V, containing internal energy E at pressure P, loses energy adiabatically to the external world at the rate

$$dE/dt = -P\,dV/dt . \qquad (22)$$

and the sum of energies inside and outside the cell remains constant.

In an expanding, homogeneous, unbounded universe, all large-scale comoving regions are alike in content, and each, by detailed balancing, may be regarded as a closed system having no external world to which the lost energy $-P\,dV$ can be transferred. We may imagine the whole universe partitioned into macroscopic cells, each of comoving volume V, and all having contents in identical states. The $-P\,dV$ energy lost from any one cell cannot reappear in neighboring cells because all cells experience identical losses. The usual idea of an expanding cell performing work on its surroundings cannot apply in this case.

3.3. *Pressure Decelerates and Tension Accelerates*

Examination of equations (18) and (19) suggests that the lost internal energy is not balanced by an equivalent gain in energy in dynamic form. On perturbing equation (18) at constant volume ($\delta R = 0$), and using $\delta E = Vc^2\,\delta\rho$, we find

$$\delta E = 3VH\,\delta H/4\pi G . \qquad (23)$$

Thus, in the relativistic treatment, an increased (decreased) internal energy corresponds to an increased (decreased) rate of expansion, and a decrease in internal energy is not balanced by an increase in the bulk kinetic energy, as in normal bounded thermodynamic systems. On perturbing equation (19) at constant volume ($\delta R = 0$) and constant expansion rate ($\delta H = 0$), we find

$$\delta P = c^2 H^2\,\delta q/4\pi G , \qquad (24)$$

and thus, in the relativistic treatment, an increased (decreased) pressure corresponds to an increased (decreased) deceleration, which again runs counter to the behavior of normal bounded thermodynamic systems.

From equation (20), using the equation of state $P = f\rho c^2$, we find that E varies as V^{-f}, and an expanding universe under tension ($f < 0$) creates internal energy, and under pressure ($f > 0$) loses internal energy. In the inflationary universe ($f = -1$), energy is continually created at a sufficient rate to sustain constant energy density (McCrea 1951, 1964); the universe is thrown into a state of accelerated expansion, and the new internal energy is not at the expense of dynamic energy. In this remarkable application of the relativistic equations, a closed universe can in principle inflate from close to zero size and zero mass to a vast system containing billions of galaxies spanning billions of megaparsecs. Nowhere in the equations can we find this immense energy in potential form in the initial state, and clearly, in this instance total energy is not conserved.

4. NEWTONIAN ANALOGIES

Can total energy (internal and dynamic) be defined in an expanding universe on the basis of equations (18) and (19)?

[4] Generation of entropy by bulk viscosity or other means requires a more elaborate energy-momentum tensor (Tolman 1934, pp. 321–323, 339). The nonconservation of energy argument, however, remains essentially unchanged.

Most discussions resort to Newtonian analogies that, when examined closely, are found to be misleading more than enlightening.

With the pressure set equal to zero (and the cosmological term omitted), the "Newtonian" argument (Milne 1934; McCrea & Milne 1934; Bondi 1960) yields

$$8\pi G\rho = 3(H^2 + K) \equiv 3\Omega H^2 , \tag{25}$$

$$d(\rho R^3)/dt = 0 , \tag{26}$$

in agreement with the relativistic equations (18) and (19). The Newtonian treatment, which followed and did not precede the relativistic treatment, violates basic classical concepts and is strictly only quasi-Newtonian. The argument goes as follows. A unit of mass at an arbitrary comoving coordinate distance r and proper distance $R(t)r$, can be considered to have "potential energy" $-GM/Rr$ and "kinetic energy" $\frac{1}{2}(RHr)^2$ relative to the center $r = 0$ of the mass $M = 4\pi\rho R^3 r^3/3$ of radius Rr. The ratio of these two so-called energies is $-\Omega$. If we take twice their sum and divide by r^2, we find $(HR)^2(1 - \Omega) = -k$, where k is a dimensionless constant of the motion that is usually identified with the curvature constant of relativity theory. Because the pressure is zero, the internal energy $E = Mc^2$ of the expanding mass M remains constant. It can therefore be claimed that both the dynamic and internal forms of energy are separately conserved and the total energy (their sum) is a constant of the expansion. According to this argument, energy in "Newtonian cosmology" seems nicely conserved. This appearance, however, is purchased at a stiff cost.

First, the pressure is not always zero. In the early universe, $HR \gg 1$, and hence the density parameter Ω is very close to unity. By making the unrealistic assumption that the pressure is always negligible, we are free to claim that the total energy is zero. Thus Tryon (1973) has proposed that the universe is a zero-energy creation.

Second, quite apart from its unrealistic nonrelativistic approximations and Euclidean assumptions, the treatment applies to a particular region with a fictitious center and boundary. The potential and kinetic energies of the given unit of mass at the fictitious boundary are not single valued and definitive, as in a bounded system, but arbitrary and relative to the fictitious center. In a centerless and unbounded universe, the energies, as defined, are nonphysical and not even meaningful.

Third, the "potential energy" per unit mass is the gravity potential ψ. The determination of ψ in an unbounded system encounters the insolubility of the Dirichlet problem (Kellogg 1929; Ramsey 1964). A unique single-valued gravity potential function does not exist in a uniform and unbounded distribution of matter, and the Newtonian gravitational force is indeterminate.[5]

In relativistic systems, energy can be defined only in asymptotically flat spacetime (Tolman 1934; Arnowit, Deser, & Misner 1960; Brill & Deser 1968; Schoen & Yau 1979; Witten 1981). Simple familiar Newtonian analogies in cosmological studies, as in Newtonian cosmology, cannot be accepted as reliable guides in discussions concerning the conservation of energy.

4. CONCLUSION

The tethered-body experiment demonstrates that in principle energy can be extracted from the expansion of the universe. This raises into prominence the question of energy conservation. Apparently energy mined in an expanding universe, as in the tethered body experiment, is nascent and unlike energy normally extracted from spatially bounded systems. The tentative conclusion of this discussion is that energy in recognizable forms (kinetic, potential, and internal) in an expanding, spatially unbounded, homogeneous universe is not conserved.

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[5] The solution of the Poisson equation $\nabla^2\psi = 4\pi G\rho$, when ρ is a uniform density, is $\psi = \phi + 2\pi G\rho r^2/3$, where ϕ is a solution of the Laplace equation $\nabla^2\phi = 0$. Thus in addition to the limitation $\psi \ll -c^2$, ψ in an unbounded space is not a uniquely defined single-valued function (because the origin of the radial coordinate r can be located anywhere) and, like the harmonic function ϕ, is indeterminate. This problem dates back in various forms to the Bentley-Newton correspondence (Harrison 1986) and has been dubbed the "gravity paradox" by Jaki (1969). In relativistic cosmology, the symmetrized Robertson-Walker form of the metric tensor g_{ik} evades the Dirichlet existence problem besetting the Newtonian gravity potential. See remarks by Einstein (1923, p. 380), Layzer (1954), McCrea (1955), North (1965), and Harrison (1981).

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