

PERIODIC LIGHT CURVE CHANGES FOR β LYRAE

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ABSTRACT

We apply a variety of classical and recently developed periodicity analyses to light curves of β Lyrae in 9 eras that extend over a range of about 150 years. Some new data are tabulated. Most periodic variations that have been reported in the literature do not pass standard significance tests, according to our adopted criteria. Certain other periods do pass the tests, although most do so only for one, or at most two, data sets. Our main findings are that a period of about 9 months is present in all data sets and that it connects extremely well in phase from era to era, all the way back to Baxendell's observations of 1840–1877. The semiamplitude is small but reasonably consistent, averaging about 2% of the flux in the light curve maxima. Any substantially larger excursions that occur are nonperiodic. Phase coherence of the 9 month period is fairly good, even with a linear ephemeris, and becomes excellent if one allows for a slow sinusoidal variation of the 9 month periodicity. There is some evidence for a monotonic decrease in the amplitude of the 9 month periodicity with increasing wavelength. © 1995 American Astronomical Society.

1. INTRODUCTION

β Lyrae is one of the most observed binary systems. It consists of a B6–8II star that probably fills its Roche lobe and corotates with the 12.9 day orbital motion, and a rather mysterious and more massive “Be” component whose spectral lines are undetectable or nearly so. This strongly interacting binary is commonly thought to be near the end of the rapid phase of mass transfer, with flow from the B6–8II to the Be component. There is a steady increase of the orbital period of 19 s per year (Klimek & Kreiner 1973). The Be gainer probably is embedded in an opaque accretion disk (Huang 1963; Wilson 1974). β Lyrae occupies a key place in the pantheon of binary stars, and is crucial to our understanding of binary star evolution. A number of review papers with extensive bibliographies have been published in the last few decades (Sahade 1980; Harmanec 1990; Harmanec & Scholz 1993). Here we cite only papers directly relevant to our immediate aim, which is to examine periodic light curve variations.

Many reports on both periodic and irregular light curve variations have appeared in the literature. Blagg (1924) collected more than 1200 visual observations of β Lyrae made by J. Baxendell between 1840 and 1877 and noticed a secondary “wave” (denoted the B wave by Blagg) after subtraction of a mean light curve over the orbital cycle. Our attention to the Blagg paper was called by Hubeny *et al.* (1994). After including additional observations made by British Astronomical Association (B.A.A.) observers between 1877 and 1920 (some hundreds in the intervals 1877–1882 and

1898–1905, and about 1500 between 1906 and 1920), Blagg (1925, 1928) fine tuned the so-called B wave characteristics to a period of 6.584 days, with a periodic variation in the time of occurrence of the minimum of 552 cycles or 3634.368 days. Danjon (1928) considered Blagg's period of 6.584 days to be fictitious and the result of a combination of the 12.9 day orbital period and the sidereal year. (Note that in Sec. 3 we directly find a main period of approximately one year for two data sets.) Danjon's paper includes 196 mag estimates of β Lyrae obtained between 1923 and 1926 with a Pickering-type photometer (for a description of this instrument, see Walsh 1965). Based on these, and older light curves by Stebbins (1916) and Guthnick & Prager (1917), Danjon found a light curve variation with semiamplitude 0.06 mag and period 156 days. Guthnick (1947) also discussed light curve changes, which then were further examined by Jokschi (1956). No periodicity was found. Jokschi's periodogram analysis of 19th century visual observations by Argelander and Schmidt also failed to show any periodicity. Jokschi's analysis was one of the first done with an electronic computer, although it is mentioned only briefly at the end of a short note, with no specifics given. Alduseva (1970) analyzed spectrophotometric observations made between 1961 and 1963 and combined those with photoelectric light curves from the 1958–1959 international β Lyrae campaign. Alduseva found a 1141 day variation that is given as 3.2 yr in a subsequent paper (Alduseva 1974). Guinan (1989) was the first to find some evidence of a 9 month light curve oscillation. Observations made with the 38 cm reflector at Villanova (Pennsylvania) University and the 25 cm automated

TABLE 1. β Lyrae data sets.

Identifier	Years	JD Range ^a	N	Type	Bandpass	Ref
BAX	1840-1877	-6838.5-06901.4	1207	vis	eye	1
LUI	1898-1916	14403.4-21149.3	1461	vis	eye	2
DAN	1923-1926	23499.5-24835.4	211	pe	eye	3
LOR	1925-1933	24318.3-27341.4	1293	vis	eye	2
GUT	1943-1944	30933.3-31384.3	190	pe	orange-red ^b	4
LLH	1969-1971	40025.6-41283.5	940	pe	U, B, V	5, 6, 7
VIL1	1970-1975	40897.5-42573.7	1335	pe	H α ^c , H β ^c	8
VIL2	1984-1989	45969.6-47512.5	1095	pe	H α ^c , γ	8
APT	1987-1994	46979.9-49443.1	2383	pe	V, R, I	8

^aJulian Date -2400000.0^bestimated "isophoton" (Guthnick's term) wavelength 600 nm^cWide-band filters which should respond mainly to the continuum rather than the Balmer lines.

Notes to Table 1.

vis: visual; pe: photoelectric.

References for Table 1.

(1) Blagg 1924; (2) Grouiller & Bloch 1935; (3) Danjon 1928; (4) Guthnick 1947; (5) Lovell & Hall 1970; (6) Lovell & Hall 1971; (7) Landis, Lovell, & Hall 1973; (8) this paper.

photoelectric telescope (APT), located on Mount Hopkins, Arizona, show an overall change in brightness at primary minimum of about 0.25 mag and a return to a previous value in about 275 days. Guinan mentioned similar concurrent changes of smaller amplitude in other parts of the light curve.

Some of the variability discussions may be influenced by biases for or against certain theoretical models, and one can be misled by looking at only part of the available data. Except for Joksche's negative result, we know of no impersonal analysis of β Lyrae light curve periodicities, although one can find remarks about periodic variability based on subjective examination of graphs, as in the papers cited above. The amplitude sometimes is given as an eye-estimate, but more commonly is not mentioned at all. The issue of phase coherence between data sets has not previously been addressed in any explicit manner. Because subjective practices can lead to incorrect inferences, searches for periodicities via time-series analysis and impersonal data fitting by the method of least squares are needed. This requirement pertains especially to the more noisy observations, for which it may be impossible to see a small-amplitude periodicity by eye inspection of graphs, even when the periodicity can readily be demonstrated by power spectral analysis. In brief, only one of the periodicities claimed by earlier workers is confirmed as consistently significant by the power spectral analyses and least-squares analyses done here, but we have traced that one (Guinan 1989) back through approximately 150 yr of observations.

TABLE 2. β Lyrae Villanova H α (wide) observations. (Sample)*

HJD	Δm	HJD	Δm	HJD	Δm	HJD	Δm
40897.51130	0.1004	40897.53412	0.0970	40876.61497	-0.0149	40879.60945	0.7373

Notes to Table 2.

*Table 2 is presented in its complete form in the AAS CD-ROM Series, Volume 5, 1995. The first page of this table is presented here for guidance regarding its form and content.

TABLE 3. β Lyrae Villanova H β (wide) observations. (Sample)*

HJD	Δm	HJD	Δm	HJD	Δm	HJD	Δm
40876.61252	0.1779	40879.60683	0.9972	40879.63781	1.0674	40883.54350	0.2012

Notes to Table 3.

*Table 3 is presented in its complete form in the AAS CD-ROM Series, Volume 5, 1995. The first page of this table is presented here for guidance regarding its form and content.

2. OLD AND NEW OBSERVATIONS

Homogeneous data sets (same observer, instrument, and bandpass) with long baselines and no major gaps are very desirable for our purposes. A remarkable data base of 19th century β Lyrae visual light curves exists in Blaggs' (1924) compilation of J. Baxendell's visual observations that span the years 1840-1877. Long baselines, good distributions in time, and large quantities of data points can somewhat compensate for the low precision of visual measures. All accessible data sets that we judged suitably well distributed in time were utilized. Restricted use was made of Guthnick's (1947) observations, which are not so well distributed. We wanted to include the excellent infrared curves for 1977-82 described by Zeilik *et al.* (1982), but did not succeed in obtaining them. Information about the data sets used here is assembled in Table 1, where approximate dates of the observations give the time spans covered. To convey an idea of the usefulness of the individual data sets, Table 1 also contains the quantity N of observations in each set, as well as the type of observation, bandpass, and references.

The photometric program from which the VIL and APT observations resulted has been described in some detail by Guinan (1989). In Tables 2-5 (these tables can also be found in the AAS CD-ROM Series, Vol. 5, 1995) we present samples of the original observations on which the VIL and APT data sets are based. All differential magnitudes are with respect to the comparison star γ Lyr. The H α and H β filters are broad, with full width at half maximum (FWHM) 29 and 18 nm, respectively. The effective wavelengths are 660 and 487 nm. The Balmer lines presumably have only minor influences on these rather wide bandpasses.

3. STRATEGY

Impersonal analysis of every accessible data set with sufficient time base has turned up essentially the same 9 month period that Guinan (1989) found by inspection of 1970-75 data, although with typical semiamplitudes of 2%-3%, rather than the 5%-10% percent shown by Guinan's Fig. 5. Although the amplitudes are small in every case, we show in

TABLE 4. β Lyrae Villanova Strömgren γ observations. (Sample)*

HJD	Δy	HJD	Δy	HJD	Δy	HJD	Δy
45969.62107	0.4988	45969.62130	0.4984	45969.62349	0.5068	45969.62372	0.5009

Notes to Table 4.

*Table 4 is presented in its complete form in the AAS CD-ROM Series, Volume 5, 1995. The first page of this table is presented here for guidance regarding its form and content.

TABLE 5. β Lyrae APT observations. (Sample)*

HJD	ΔV^a	ΔR^a	ΔI^a	ΔV_c^b	ΔR_c^b	ΔI_c^b
2446979.9548	—	—	—	—	—	1.826
2446980.9514	0.227	0.065	0.048	—	—	—
2447052.7326	0.141	—	—	—	—	—
2447064.7331	0.286	0.147	0.131	1.989	1.901	1.878
—	—	—	—	—	—	—

^aVariable - Comparison^bCheck - Comparison

Notes to Table 5.

*Table 5 is presented in its complete form in the AAS CD-ROM Series, Volume 5, 1995. The first page of this table is presented here for guidance regarding its form and content.

the next section that good phase coherence extends all the way back to Baxendell's time (i.e., about 150 yr).

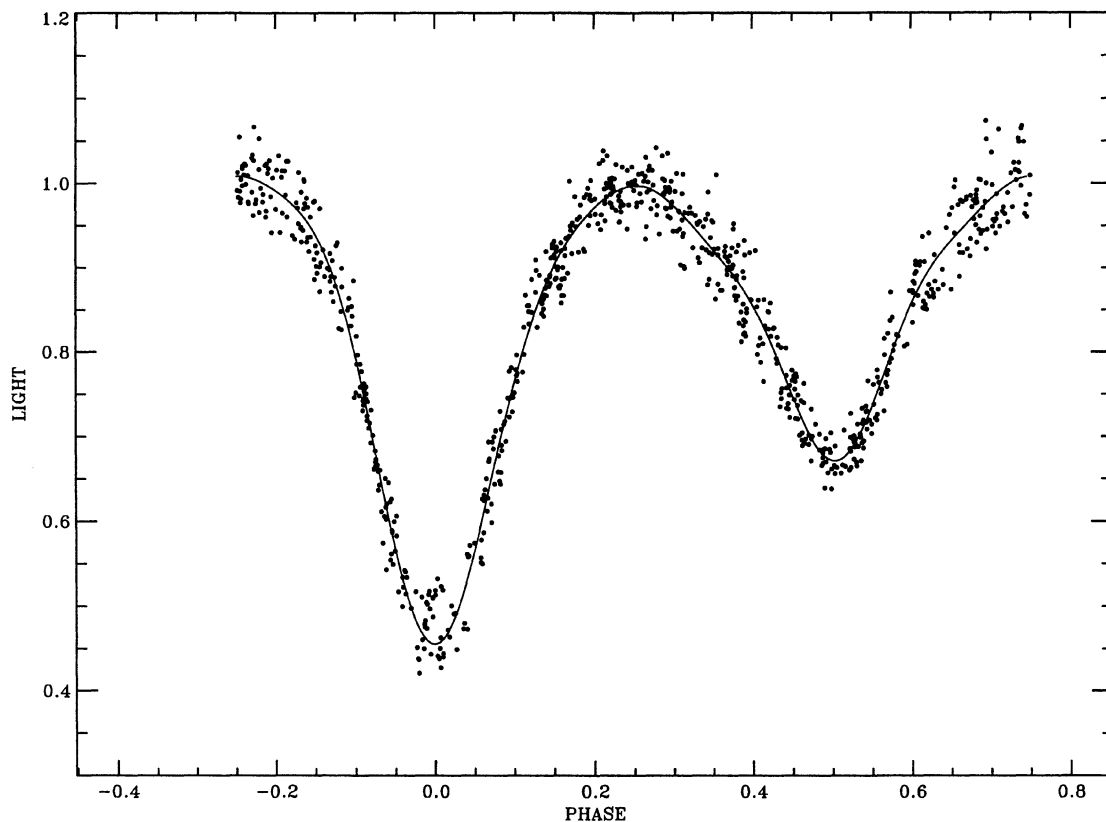
We did the analyses in light rather than magnitude. The reason is that fixed amplitude in light (essentially the simplest situation) will show greater variation in minima (eclipses) than in maxima on a magnitude scale, with the effect being only an artifact of the magnitude scale. Since β Lyrae is reduced to about half-brightness at primary minimum, the variation in magnitude, for fixed amplitude in light, will be twice as large there as in the maxima. Reports that the minima vary more than the maxima are at least partly (but perhaps not entirely, see further) due to working in magnitudes. Ideally, all data sets that we subject to any

one analysis should be on the same photometric system, but that is unrealistic for obvious reasons. Therefore we worked with residuals from mean curves, so as to eliminate offsets due to the various photometric systems of the existing data, in the first approximation. In order to form proper mean curves, we needed to remove the effects caused by β Lyrae's change in orbital period. The main periodic light curve signal will be on the 12.9 day orbital period that advances at a remarkably constant rate of 19 s/yr (Klimes & Kreiner 1973; Harmanec & Scholz 1993). Typically, the data sets span several consecutive years, or, for the Baxendell observations, even several decades. The effects of the increase in orbital period were removed by computing phases with the quadratic ephemeris

$$T(\text{min}) = \text{HJD } 2408247.966 + 12.913780E + 3.87196 \times 10^{-6}E^2, \quad (1)$$

derived by Harmanec & Scholz (1993). This ephemeris is in good agreement with that by Klimes & Kreiner (1973), and predicts times of minima well for the past 100 years. Phases for the Baxendell data were obtained with Blagg's (1928) revised ephemeris

$$T(\text{min}) = \text{HJD } 2398590.57 + 12.908006E + 3.914 \times 10^{-6}E^2. \quad (2)$$

FIG. 1. Observed and mean APT V light curve of β Lyrae.

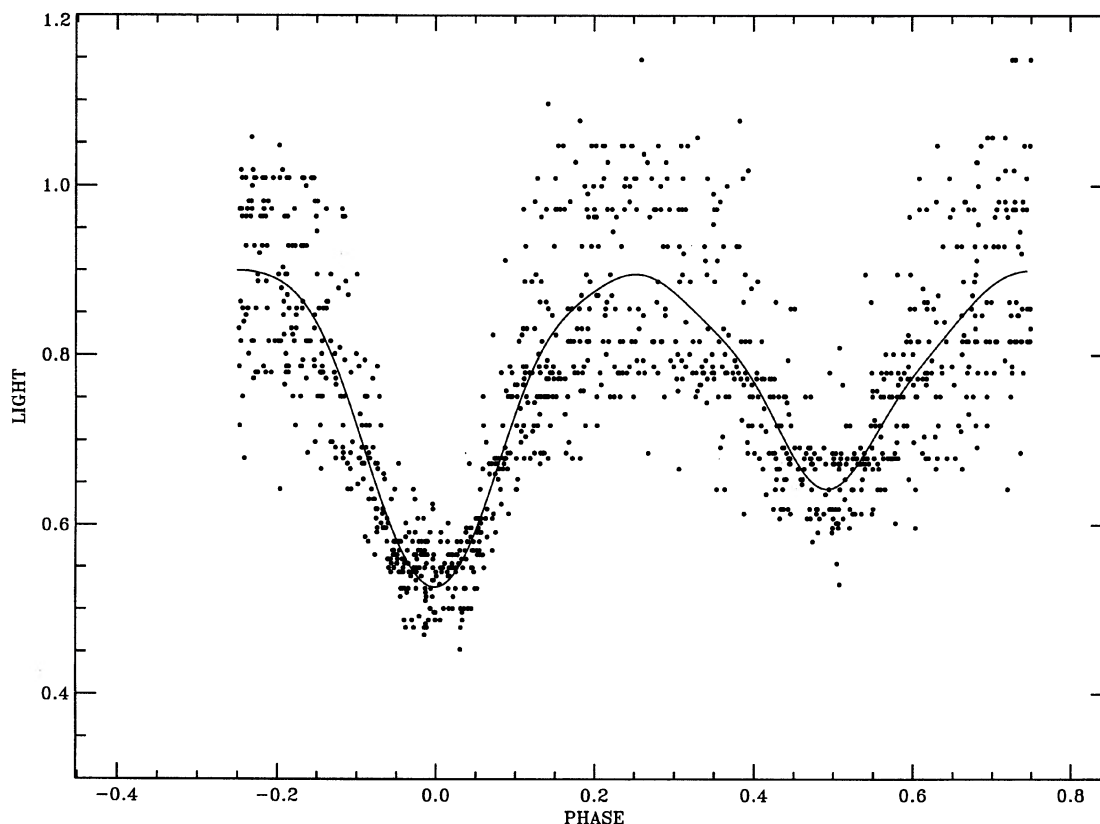
FIG. 2. Baxendell's light curve of β Lyrae with a fitted mean curve.

TABLE 6. Method I Results also found with Method II.

Set/Bandpass	$z_{\max}$	σ_z	$z_{\lim}^a$	M	P	z	F
BAX	8.67	1.30	12.57	2460 ± 207	278	18.12	$0.33 \cdot 10^{-4}$
					6.69	13.96	$0.21 \cdot 10^{-2}$
					283	15.04	$0.25 \cdot 10^{-2}$
					274	12.01	$0.33 \cdot 10^{-2}$
DAN	7.32	1.23	11.02	542 ± 57	278	30.75	$0.26 \cdot 10^{-9}$
LOR	9.42	1.41	13.64	5950 ± 560	654	18.69	$0.46 \cdot 10^{-4}$
					29.6	18.34	$0.64 \cdot 10^{-4}$
					6.43	17.43	$0.16 \cdot 10^{-3}$
					6.47	14.94	$0.19 \cdot 10^{-2}$
LLH U	7.13	1.36	11.19	461 ± 40	296	13.35	$0.73 \cdot 10^{-3}$
LLH B	7.59	1.41	11.81	837 ± 93	21.7	11.74	$0.37 \cdot 10^{-2}$
					296	30.57	$< 10^{-10}$
LLH V					2.47	13.23	$0.15 \cdot 10^{-2}$
					287	34.31	$< 10^{-10}$
VIL1 H β	7.50	1.56	12.19	802 ± 93	266	113.8	$< 10^{-10}$
					2.51	32.63	$< 10^{-10}$
					1.92	30.02	$< 10^{-10}$
					19.1	25.29	$0.83 \cdot 10^{-8}$
VIL1 H α					266	118.7	$< 10^{-10}$
					4.85	25.49	$0.68 \cdot 10^{-8}$
VIL2 y	7.03	1.67	12.04	563 ± 83	281	65.85	$< 10^{-10}$
					2.15	49.45	$< 10^{-10}$
VIL2 H α					283	73.89	$< 10^{-10}$
					10.2	42.56	$< 10^{-10}$
					1.52	42.34	$< 10^{-10}$
					3.10	40.15	$< 10^{-10}$
APT V	8.76	1.37	12.89	2630 ± 220	281	122.3	$< 10^{-10}$
					340	54.35	$< 10^{-10}$
					214	13.52	$0.35 \cdot 10^{-2}$
					281	101.2	$< 10^{-10}$
APT R					346	48.48	$< 10^{-10}$
					281	85.21	$< 10^{-10}$
APT I					340	48.47	$< 10^{-10}$
					31.7	19.11	$0.13 \cdot 10^{-4}$
					4.38	14.56	$0.13 \cdot 10^{-2}$

^a $z_{\lim} = z_{\max} + 3\sigma_z$

All observations were converted to light and normalized to unity around the maxima. A mean light curve for a standard phase range of 0–1 was obtained by fitting a Fourier series (up to 7 θ terms) to each set of normalized observations. Residual light versus time data sets were then formed by calculating phases according to the above ephemerides and subtracting the concordant mean curve values from the original light values. Two light curves are shown as examples. Figure 1 is the APT V light curve, with the solid line indicating the mean curve. Figure 2 shows Baxendell's visual observations. Not unexpectedly, the scatter is very large in the visual observations. However, as we will demonstrate further, the same 9 month periodicity is present in all of the visual observations.

Ordinary Fourier analysis requires evenly spaced data but we have unequally spaced data, so we applied two appropriate methods. Method I is the Numerical Recipes (Press *et al.* 1992) program for spectral analysis of unevenly spaced data developed by Lomb (1976) and Scargle (1982) (hereafter referred to as LS). It computes a normalized periodogram of spectral power $P(f)$ as a function of frequency f , in a frequency interval $[f_{\text{lo}}, f_{\text{hi}}]$, where f_{lo} is the inverse of the total span of the data set ($1/\Delta T$), and f_{hi} a selected multiple of the frequency $f_c = N/2\Delta T$. Here N is the total number of observations in the set and f_c is analogous to the Nyquist frequency in the case of equally spaced data. The program also provides information about the significance of any peak in the periodogram. On the assumption that the data are pure

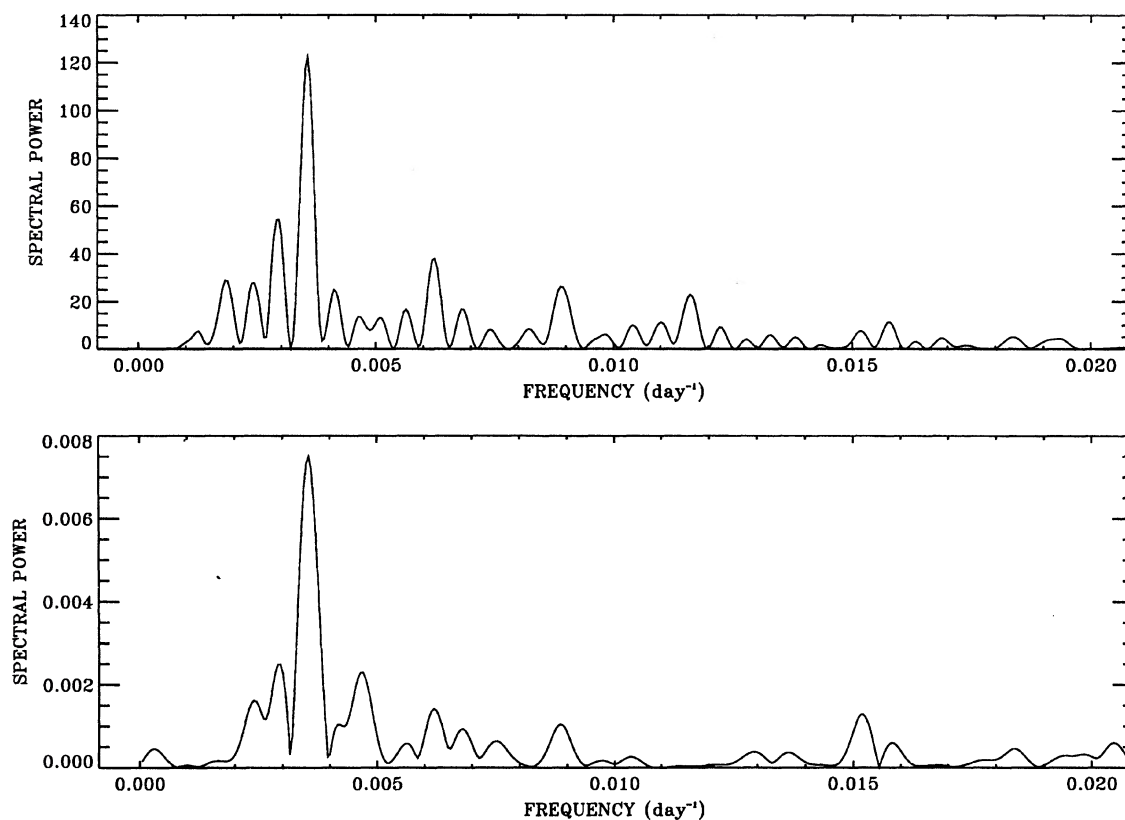


FIG. 3. LS periodogram (top) and Fourier spectrum (bottom) for the APT V residual light curve.

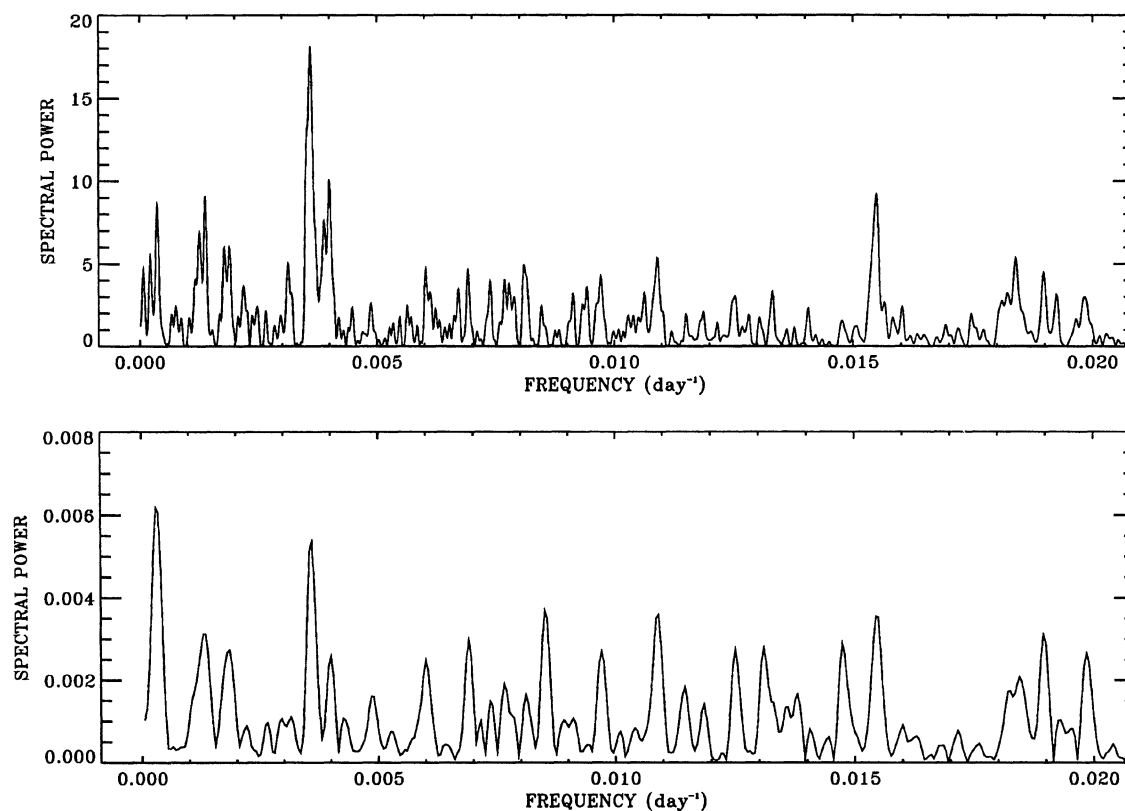


FIG. 4. LS periodogram (top) and Fourier spectrum (bottom) for the Baxendell residual light curve.

TABLE 7. Fitted parameters.

Set/Bandpass	<i>N</i>	Δt_0	<i>A</i>	<i>t</i> ₀	<i>P</i>
BAX	1207	0.0013 ± 0.0022	0.0188 ± 0.0032	-1357.9 ± 7.4	278.52 ± 0.95
LUI	1461	-0.0008 ± 0.0018	0.0141 ± 0.0025	17031.1 ± 8.3	282.1 ± 1.2
DAN	211	-0.0029 ± 0.0032	0.0278 ± 0.0054	24129.7 ± 6.4	275.1 ± 4.3
LOR	1293	0.0004 ± 0.0021	0.0229 ± 0.0028	25551.2 ± 5.9	277.2 ± 1.8
GUT	190	-0.0087 ± 0.0038	0.0129 ± 0.0043	30970.3 ± 11	281.8*
LLH <i>U</i>	212	-0.0035 ± 0.0037	0.0303 ± 0.0055	40631.6 ± 8.8	291.9 ± 5.5
LLH <i>B</i>	365	0.0023 ± 0.0019	0.0255 ± 0.0030	40941.0 ± 4.8	298.5 ± 4.4
LLH <i>V</i>	363	0.0018 ± 0.0017	0.0244 ± 0.0027	40930.0 ± 4.3	290.5 ± 4.0
VIL1 H β	658	-0.0036 ± 0.0013	0.0285 ± 0.0016	41477.7 ± 2.8	267.4 ± 2.7
VIL1 H α	677	0.0086 ± 0.0013	0.0280 ± 0.0015	41478.8 ± 2.7	269.7 ± 2.6
VIL2 <i>y</i>	540	-0.0006 ± 0.0011	0.0197 ± 0.0015	46861.2 ± 3.9	281.9 ± 2.4
VIL2 H α	555	-0.0150 ± 0.0010	0.0200 ± 0.0014	46848.9 ± 3.2	282.2 ± 2.1
APT <i>V</i>	819	-0.0009 ± 0.0008	0.0204 ± 0.0011	48232.1 ± 2.3	282.0 ± 1.0
APT <i>R</i>	799	-0.0010 ± 0.0008	0.0186 ± 0.0011	48234.3 ± 2.6	282.2 ± 1.1
APT <i>I</i>	765	-0.0011 ± 0.0008	0.0178 ± 0.0012	48236.5 ± 2.9	281.8 ± 1.2

*not adjusted for reasons explained in the text.

Notes to Table 7.

Errors are internal standard deviations of the least squares fit.

noise, Scargle (1982) (see also Horne & Baliunas 1986) shows that when periodogram power is normalized by the total variance, the probability $P(f)$ that any peak is of height z or higher is e^{-z} . Consequently, the probability that every sampled frequency f will have a power lower than z is $(1 - e^{-z})^M$, where M is the number of sampled independent frequencies, and the probability that at least one peak has height z or greater is $F = 1 - (1 - e^{-z})^M$. The more significant a peak, the lower the corresponding F value. To evaluate F we need to know the number M of independent fre-

quencies. The LS program uses a default value $M = (f_{\text{hi}}/f_c) \times N$. However, following Horne & Baliunas (1986), we determined M by means of Monte Carlo simulations. We generated 1000 sets of pseudo-Gaussian noise for each data set, holding fixed the number and locations of the data points, and computed the LS periodogram with the f_{hi} value of the original set. We then selected the highest peak in each periodogram. This yielded a set of 1000 deviates z to which we fitted a probability function $(1 - e^{-z})^M$. Our results are presented in Table 6, where columns 2–5 are the results of the Monte Carlo simulations. They are the mean values z_{max} and dispersions σ_z of the sample of 1000 peak values, the limiting value for listing a peak in Table 6, $z_{\text{lim}} = z_{\text{max}} + 3\sigma_z$ (which is somewhat arbitrary), and M . Entries without Monte Carlo numbers indicate sets that have essentially the same times of observation as the preceding set so that the same Monte Carlo numbers apply. Finally, the last three columns list the period, $P = 1/f$, the power z , and the corresponding F value for peaks higher than z_{lim} . Method II calculates a Fourier transform of the observations and applies the CLEAN algorithm (Roberts *et al.* 1987), which removes any artifacts introduced by missing data (i.e., that makes the Fourier line spectrum meaningful for unequally spaced data). The final result of the CLEAN program is a spectrum containing the real and imaginary parts of the cleaned Fourier trans-

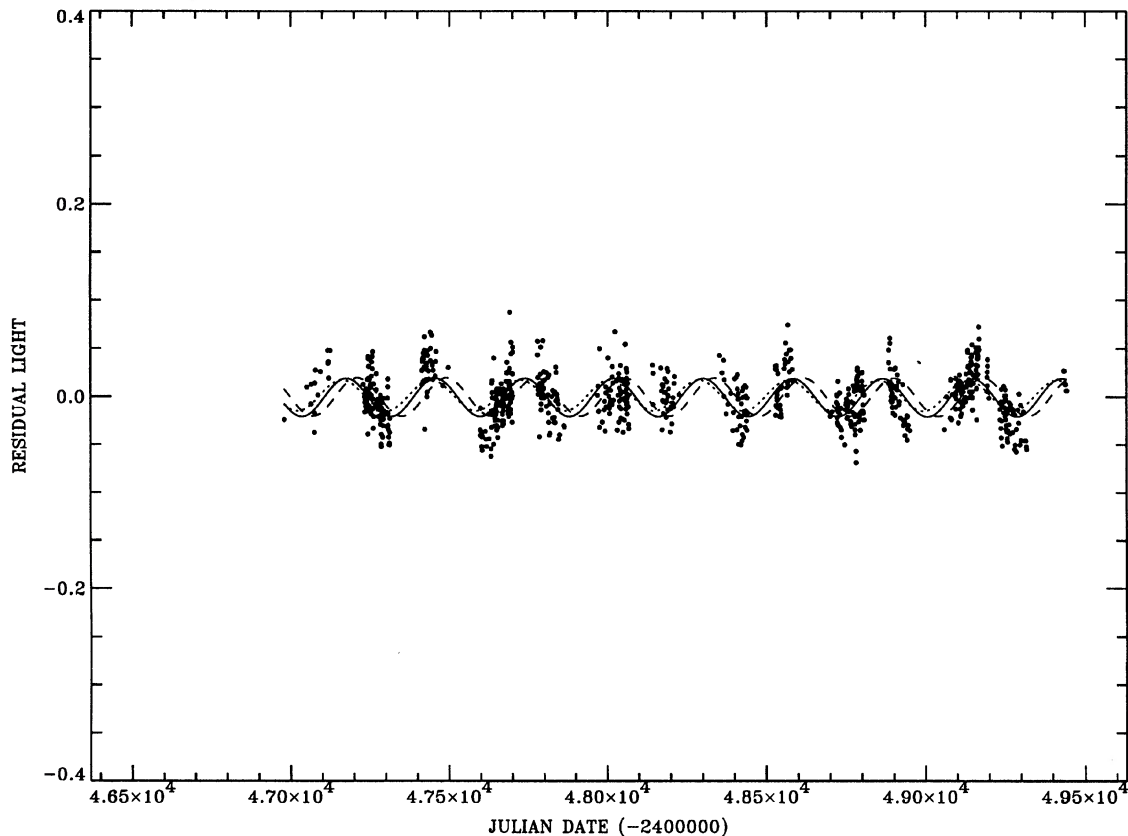


FIG. 5. APT *V* residual light curve. The solid curve represents a variable-period sinusoid (mean period 283.6 days) fitted to the data. The dashed and dotted curves are variable-period sinusoids extended from the VIL *y* and Baxendell residual light curves respectively to illustrate phase coherence. The illustrated phase mismatches in Figs. 5–7 are among the largest for any of the pairs of data sets.

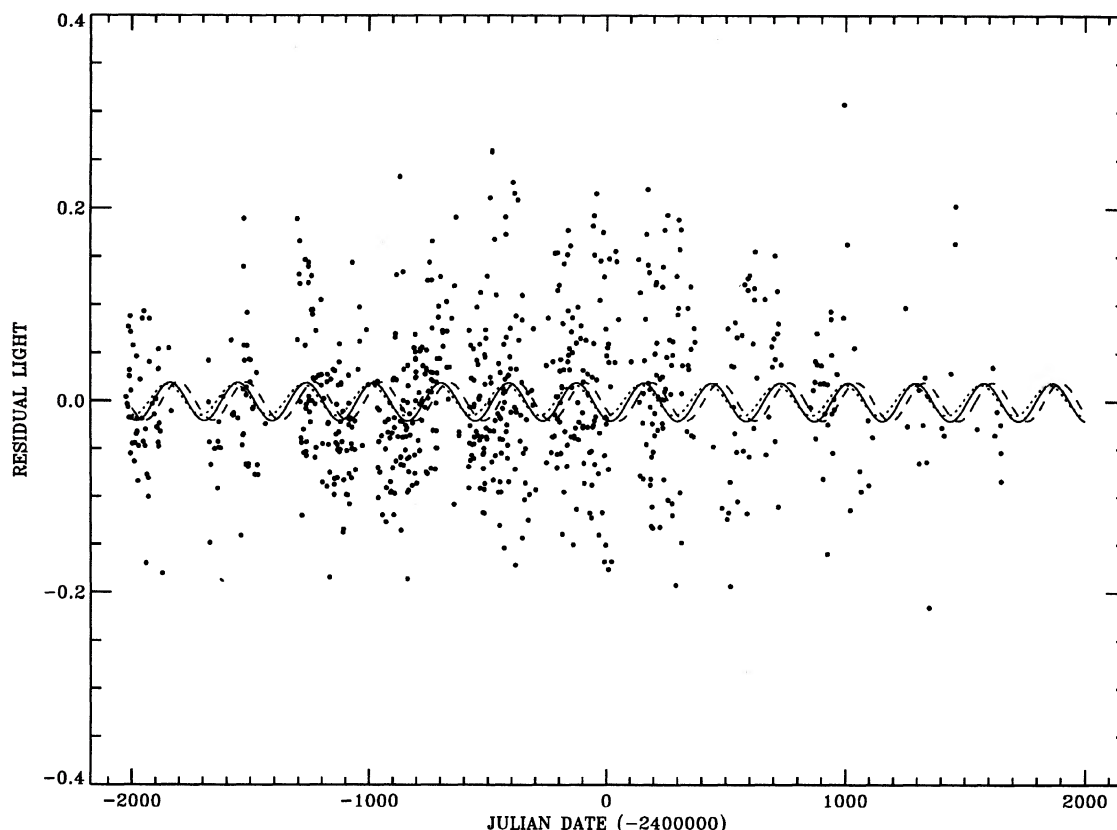


FIG. 6. A section of Baxendell's residual light curve. The dotted curve represents a variable-period sinusoid fitted to all of Baxendell's data. The solid and dashed curves are variable-period sinusoids extended from the APT V and VIL y residual light curves respectively to illustrate phase coherence. The 9 month sinusoid cannot be seen subjectively in the noisy observations, even after it is known to exist according to several kinds of analysis. However, it essentially agrees in period, phase, and amplitude with periodicities in the other 14 data sets.

form at a number of frequencies that depend on the frequency resolution and selected upper limit. The square root of the sum of the real and imaginary parts squared of the Fourier transform is a measure of spectral power. Method II provides some assurance in the validity of periods identified by Method I. We imposed a second criterion for a period, indicated by a peak higher than z_{lim} in the LS periodogram, to be listed in Table 6. The second criterion is that the period must be confirmed by a peak in the Fourier spectrum higher than the typical highest peak in a spectrum for Gaussian deviates located at the times of observation. A few typical periodograms are shown in Figs. 3 and 4. For the Baxendell observations (Fig. 4), the highest peak in the Fourier spectrum is at $f=0.00030 \text{ day}^{-1}$ ($P=3300 \text{ day}$). This period is not listed in Table 6 because the corresponding peak in the LS periodogram did not exceed the limiting value, z_{lim} .

A consistent periodic variation common to all residual light curves is clearly shown by Table 6. The period differs somewhat among data sets, but is of the order 280 days, in agreement with Guinan's (1989) estimate of 275 days. Every data set, without exception, contains this periodic signal. Except for the Baxendell and Luizet observations, the 9 month period has the highest spectral power, with the corresponding peak in the LS periodogram or Fourier spectrum being significantly higher than any other. For the Baxendell and Lu-

izet data, the periods with the most power were 360.9 and 340.0 day, respectively. These values are close to a year and might be due to some annual observational effect. Harmonic signals with these periods, up to the seventh harmonic, were subtracted respectively from the BAX and the LUI data and new periodograms were made. The harmonic coefficients came from least-squares fits of Fourier series whose fundamental frequencies corresponded to the above periods. The next strongest period for each of these two data sets (cf. Table 6) appears to be the same 9 month period found in the other sets. The 9 month period is not among those suspected in previous papers, except Guinan's, and we find only a few of the suggested periods of earlier papers at a significant level, given our adopted criteria (see above). The few previously suggested periods that we do find appear only in one or two data sets.

In order to refine our estimates of the periods and amplitudes and to find their uncertainties, we fitted a periodic function

$$\Delta I = \Delta I_0 + A \sin[2\pi f(t - t_0)] \quad (3)$$

to each data set. The adjusted parameters were the intercept ΔI_0 , semi-amplitude A , frequency $f=1/P$, and time of zero-phase t_0 (in Julian date -2400000). Equation (3) is nonlinear in some of the adjustable parameters, so a special least-

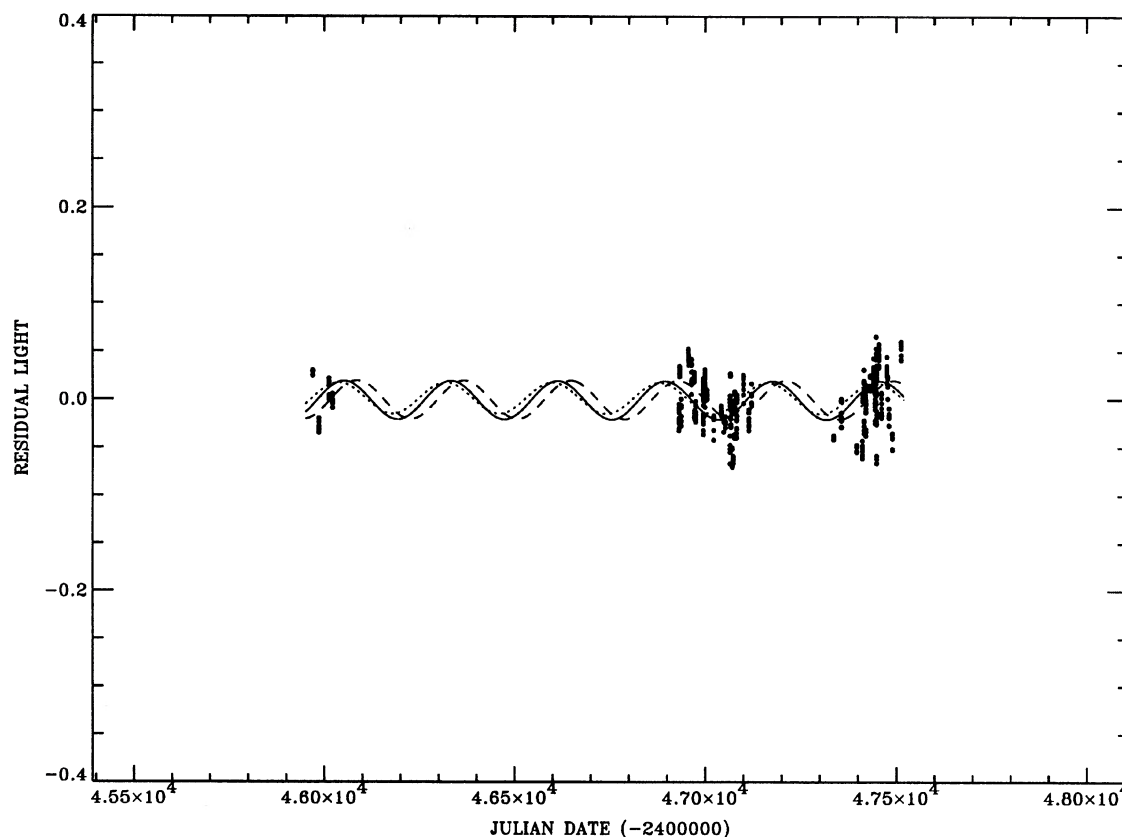


FIG. 7. VIL y residual light curve. The dashed curve represents a variable-period sinusoid fitted to the data. The solid and dotted curves are variable-period sinusoids extended from the APT V and Baxendell residual light curves respectively to illustrate phase coherence.

squares program was applied. We selected the Numerical Recipes (Press *et al.* 1992) program based on ideas developed by Levenberg and Marquardt. Table 7 lists the fitted parameters with their standard deviations. For each dataset, the mean time was subtracted from each datapoint so that t_0 is the time of zero-phase closest to this mean. Although Guthnick's (1947) observations span almost 2 yr, they have a very large gap that makes them unsuitable for an independent sinusoidal fit with an anticipated 282 day period. Consequently f was not adjusted for that data set. Figures 5, 6, and 7 show the residual light curves for the sets APT V , BAX, and VIL y , together with fitted sinusoids that will be discussed in Sec. 4. The APT V solution is represented by a solid line, the BAX fit by a dotted line, and the VIL y fit by a dashed line. All three sinusoids are displayed in each plot in order to illustrate the phase coherence of the 9 month signal (see Sec. 4).

The least-squares semiamplitude of the 9 month signal varies among data sets with a mean of 0.0220 in our normalized light unit (so a little over 2% of the light in the maxima). The amplitude is substantially smaller than the width of the scatter band in the photoelectric cases, and very small indeed compared to the scatter of all visual observations. There is some evidence for a decrease of the amplitude with wavelength. Figure 8 is a graph of the semiamplitudes versus wavelength (not those of Table 7, but somewhat re-

financed values given in Table 10). A linear downward trend appears to be present. The solid line represents a weighted linear least-squares fit. The intercept is 0.0285 ± 0.0053 light units and the slope $-1.24 \pm 0.82 \times 10^{-5}$ light units/nm. (All of our error estimates are standard deviations.)

We investigated quantitatively Guinan's (1989) comment that the amplitude is larger in primary minimum than around the maxima. Since that statement is based on a graphical display of the 1970–1975 $H\alpha$ observations (our VIL1 set) on a magnitude scale, we examined the dependence of the amplitude on phase for the same data set, as well as for the $H\beta$ and APT V observations. For each of these sets we made three subsets consisting of observations at phases in the intervals $[-0^{\circ}06, 0^{\circ}06]$, $[0^{\circ}15, 0^{\circ}35]$, and $[0^{\circ}65, 0^{\circ}85]$, corresponding to the bottom of primary minimum, maximum I, and maximum II, respectively. Table 8 lists the semiamplitudes from a least-squares fit of Eq. (3) to these subsets, keeping $P=280.373$ day (see further) fixed. Guinan's inference at $H\alpha$ is verified at about the 3σ level for the minimum versus max II and at the 1σ level for the minimum versus max I. Results are somewhat different for $H\beta$ and APT V , as shown by the table. In overview, there is some indication of an enhanced variation in primary minimum, but the effect is not very pronounced. Figure 9 shows the $H\alpha$ observations in the bottom of primary minimum together with the fitted curve.

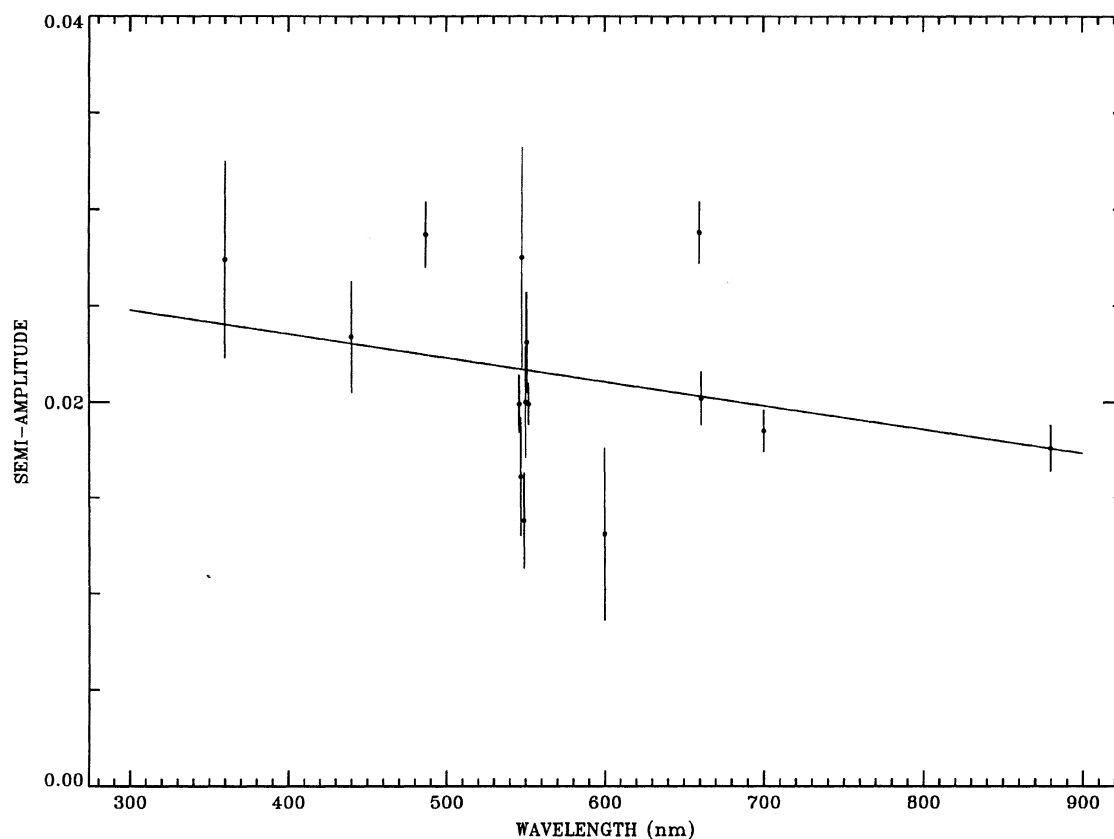


FIG. 8. Semi-amplitude of fitted sinusoids (in units of light in the light curve maxima) as a function of wavelength. The solid line represents a weighted linear least-squares fit. Dots representing data sets at 550 nm have been shifted slightly in the figure in order to enhance error bar visibility.

4. PHASE COHERENCE

The consistent presence of a 9 month period in the residual light curves over the past 150 years leads to the question of whether this oscillation shows any phase coherence, i.e., how well does the oscillation connect in phase from one data set to another? To examine phase coherence, we first combined all data sets into one set of 10 115 points. We assigned weights equal to the inverse variance of the set to which each datum belongs, so as to account for the unequal precision of the sets. A weighted least-squares fit of Eq. (3) to all the data yields parameters $\Delta t_0 = -0.00069 \pm 0.00034$, $A = 0.01666 \pm 0.00049$, $t_0 = 2432532.8 \pm 1.7$ JDH, and $P = 280.373 \pm 0.034$ days. Subsequently, keeping the parameters Δt_0 , A , and t_0 fixed at the values listed above, we stepped Eq. (3) in frequency from $f = 0.0025 \text{ day}^{-1}$ ($P = 400$ day) to $f = 0.0067 \text{ day}^{-1}$ ($P = 150$ day) in increments of 10^{-6} day^{-1} , and calculated, for each value of f , the sum of the weighted squared residuals between the observations and estimates of Δt from Eq. (3). After dividing by $N - 1$, with N

the total number of points, and taking the square root, we obtain the analog of a standard deviation, σ . A plot of σ versus period is shown in Figs. 10 and 11. The first figure is for the more recent data (later than 1969, $E_{\text{cycle}} \geq -21$) and the second for all the data. Each graph shows the quality of fit for all periods within its specified range. Clearly, the deepest minimum is located around $P = 282$ day for both figures (Figs. 8 and 9). We also stepped Eq. (3) in zero-phase time t_0 , from -140 to $+140$ days away from the above t_0 value, in steps of 20 days. For every t_0 value, the frequency-stepping procedure was repeated. This yielded a two-dimensional array of σ 's which could be examined for possible local minima. Several nearly equally deep local minima exist, and this fact will be important for our further discussion. The above solution (with $P = 280.373$ day) does correspond to an absolute minimum in the variance. So, with an assumed 280.373 day oscillation for every individual set, we proceeded by assigning a cycle number E_{cycle} to each time of zero phase. Also, the times of zero phase were redetermined by making a least-squares fit of Eq. (3) to each set, adjusting Δt_0 , A , and t_0 , while keeping P fixed at 280.373 day. The results are listed in Table 9. To see if the reference epochs of the individual data sets advance linearly in time, we fitted a function

$$t_0 = t_0^0 + P_0 E \quad (4)$$

TABLE 8. Residual light semi-amplitude for different phase intervals.

Set/Bandpass	Min	Max I	Max II
VIL1 H β	0.0373 ± 0.0040	0.0228 ± 0.0050	0.0281 ± 0.0030
VIL1 H α	0.0381 ± 0.0040	0.0327 ± 0.0027	0.0296 ± 0.0039
APT V	0.0246 ± 0.0040	0.0145 ± 0.0022	0.0270 ± 0.0026

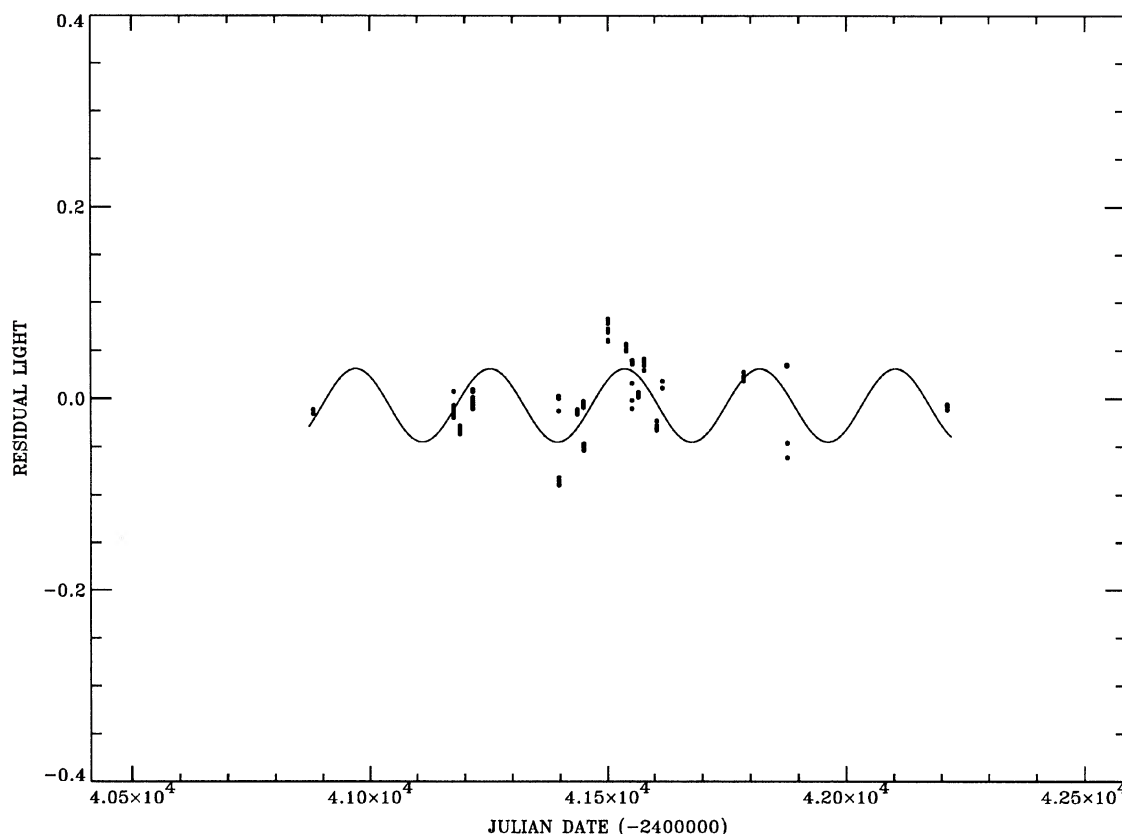


FIG. 9. Residual light in H α at the bottom of primary minimum and a fitted sinusoid with period 281.4 day.

to the Table 9 times of zero phase versus cycle number. We find $t_0^0 = 46834.5 \pm 8.2$ (JD -2400000.0) and $P_0 = 281.24 \pm 0.26$ day, but the residuals (listed in Table 9 in the Res1 column) are large, including -113 days for GUT, -101 days for BAX, and $+90$ days for LOR. Since the largest possible residual would be about 141 days (half the 282 day period), residuals of 113, 101, and 90 days are not small, so no real phase coherence has been demonstrated over the entire interval of 150 years, up to this point in the analysis. Still, phase coherence over the last 20 yr looks convincing, with the largest residual being only 28 days, or 10% of the 282 day period. Especially remarkable is that the recent coherence stands out despite our having to work with a variety of photometric systems. It should also be noticed that in assigning cycle numbers, the relative number of cycles between t_0 values prior to 1933 ($E \leq -75$), and between those after 1943 ($E \geq -56$), is firmly established, but some uncertainty arises when jumping from cycle -56 to cycle -76 . If we now ask why recent data should show very strong phase coherence while older data do not, an obvious and plausible reason is simply that some of our whole cycle assignments may be wrong. Indeed, when the cycle numbers of the data sets prior to 1933 are increased by one unit, a similar fit gives $t_0^0 = 46840.2 \pm 7.8$ days and $P_0 = 283.41 \pm 0.25$ days, and now the largest residuals are much smaller, being -46 days for LUI and $+57$ days for LLH $_B$. Although such residuals seem larger than one would expect in the case

of perfect phase coherence (cf. standard errors of t_0 in Table 9), the largest is only 20% of the 283.4 day period. It would seem essentially impossible to find a complete set of 15 residuals this small in the absence of phase coherence, since the larger residuals should then be of the order of half the period. Still, we must deal with the issue of why the residuals are as large as 57 days, when the phasing of the individual data sets has standard errors of only 2–11 days. Before we addressed this point, we restarted the analysis at the 283.41 day period. [Interestingly, although $P_0 = 283.41$ days; $t_0 = 46840.2$ days is not the absolute global minimum in the solution space of Eq. (3), it is essentially at a neighboring, nearly equally deep minimum.] New least-squares fits were made of Eq. (3) to all data sets, adjusting Δt_0 , A , and t_0 , while keeping P fixed at 283.41 days. The fitted parameters are presented in Table 10, together with the new cycle numbers and the residuals (Res2) of the zero-phase times with respect to the ephemeris

$$t_0 = 46840.1 (\pm 7.7) + 283.39 (\pm 0.26) E_{\text{cycle}}, \quad (5)$$

which resulted from a linear least-squares fit to the new times of zero phase versus cycle number [Eq. (4)]. Clearly, the new t_0 values, as well as the fitted ephemeris, are very close to those determined previously and no further iterations are necessary. A plot of the residuals versus cycle number is shown in Fig. 12. Although the new representation seems the best that one can do under a fixed period, it is rather a strin-

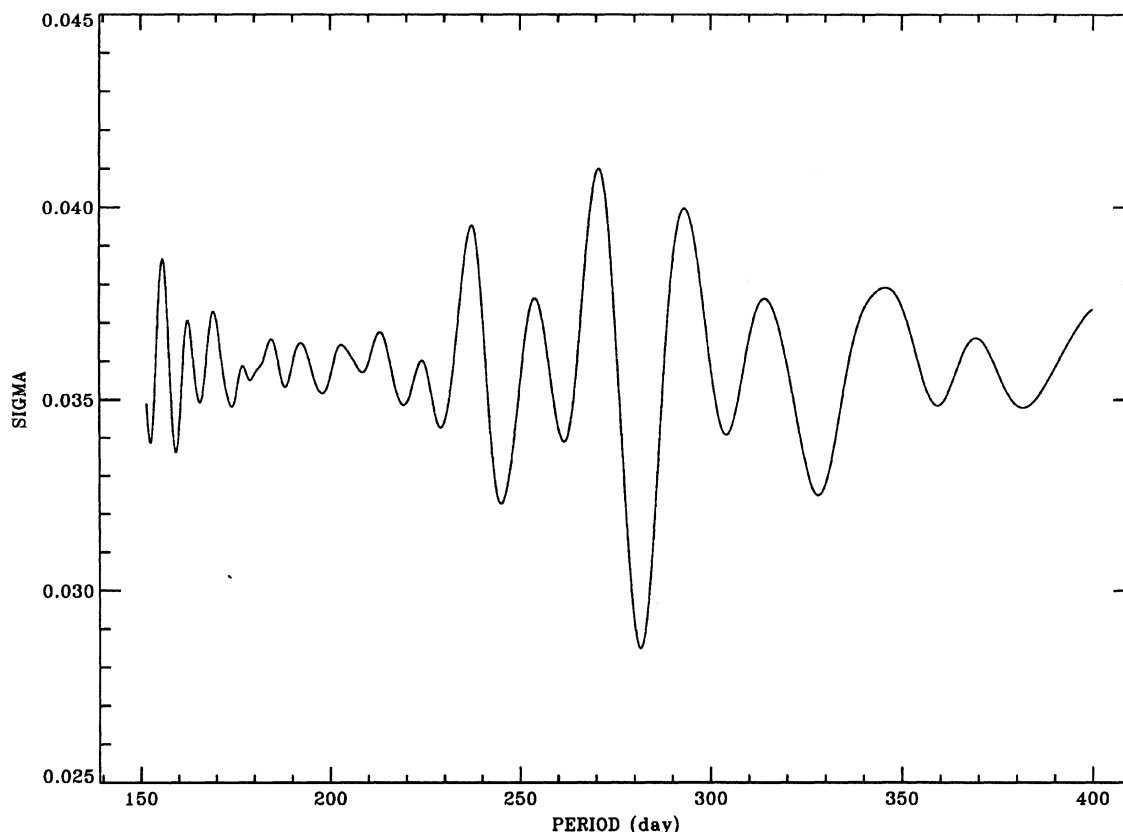


FIG. 10. Frequency-step test standard deviation for data sets later than 1969.

gent condition to assume the 9 month period to be constant over 150 yr. We therefore expected some curvilinear behavior in the residuals, and curvilinear behavior did emerge in the form of an approximate sinusoid, as one can readily see in the figure. A Levenberg–Marquardt fit [see above remarks on Eq. (3)] of a function

$$t_0(\text{observed}) - t_0(\text{computed}) = a + B \sin[2\pi f_c(E - E_0)] \quad (6)$$

to the residual times of zero phase versus cycle number of Table 10 gives $a = -10.1 \pm 9.8$ day, $B = -42.5 \pm 8.6$ day, $P_c = 1/f_c = 119 \pm 11$ cycles, and $E_0 = -116.3 \pm 9.3$. The corresponding sinusoid through the t_0 residuals in Fig. 12 is shown as a solid line and the remaining residuals are listed in the Res3 column of Table 10. The fit appears to be quite good. Alternatively, this graphical behavior suggests that the frequency f in Eq. (3) is not constant but varies sinusoidally with a period of approximately 119 cycles or 92 yr. We examined this possibility further by fitting a function

$$\Delta l = \Delta l_0 + A \sin \Phi, \quad (7)$$

to the global set of 10 115 residual light versus Julian date points, with the phase Φ given by the integral

$$\Phi = \int_{t_0}^t 2\pi f dt, \quad (8)$$

and the frequency f varying sinusoidally according to the equation

$$f = f_0 + C \sin[2\pi f_c(t - t_c)]. \quad (9)$$

Evaluation of Eq. (8) leads to

$$\Phi = 2\pi[f_0(t - t_0)] - \frac{C}{f_c} \{ \cos[2\pi f_c(t - t_c)] - \cos[2\pi f_c(t_0 - t_c)] \}, \quad (10)$$

which, after insertion in Eq. (7), leads to an expression containing seven adjustable parameters. The fit gives $\Delta l_0 = -0.00211 \pm 0.00035$, $A = 0.01867 \pm 0.00049$, $t_0 = 32406.4 \pm 4.2$ (Julian date $-2400000.$), $P_0 = 1/f_0 = 283.60 \pm 0.05$ day, $C = -2.55 \pm 0.15 \times 10^{-5}$ day $^{-1}$, $P_c = 1/f_c = 33601 \pm 950$ (Julian date $-2400000.$), and $t_c = 20280 \pm 570$ (Julian date $-2400000.$), finding a secondary cycle with a period of 92.0 ± 2.6 yr. In terms of the 283.60 day period, the semiamplitude of the sinusoidal variation is 2.05 days. The Levenberg–Marquardt fit to Eq. (7) must be carried out with some care, as a simultaneous solution for all seven parameters is unstable in a certain sense. The problem is that the “solution” will jump around among various local minima, only one of which is the properly identified one that exhibits long term phase coherence. Our remedy was to use the Method of Multiple Subsets or MMS (Wilson & Biermann 1976). We found that any of various choices of subset combinations (say 3 and 4 or even 6 and 1 parameters) immediately produces stable, quickly converging solutions. Thus, although one cannot put all seven pa-

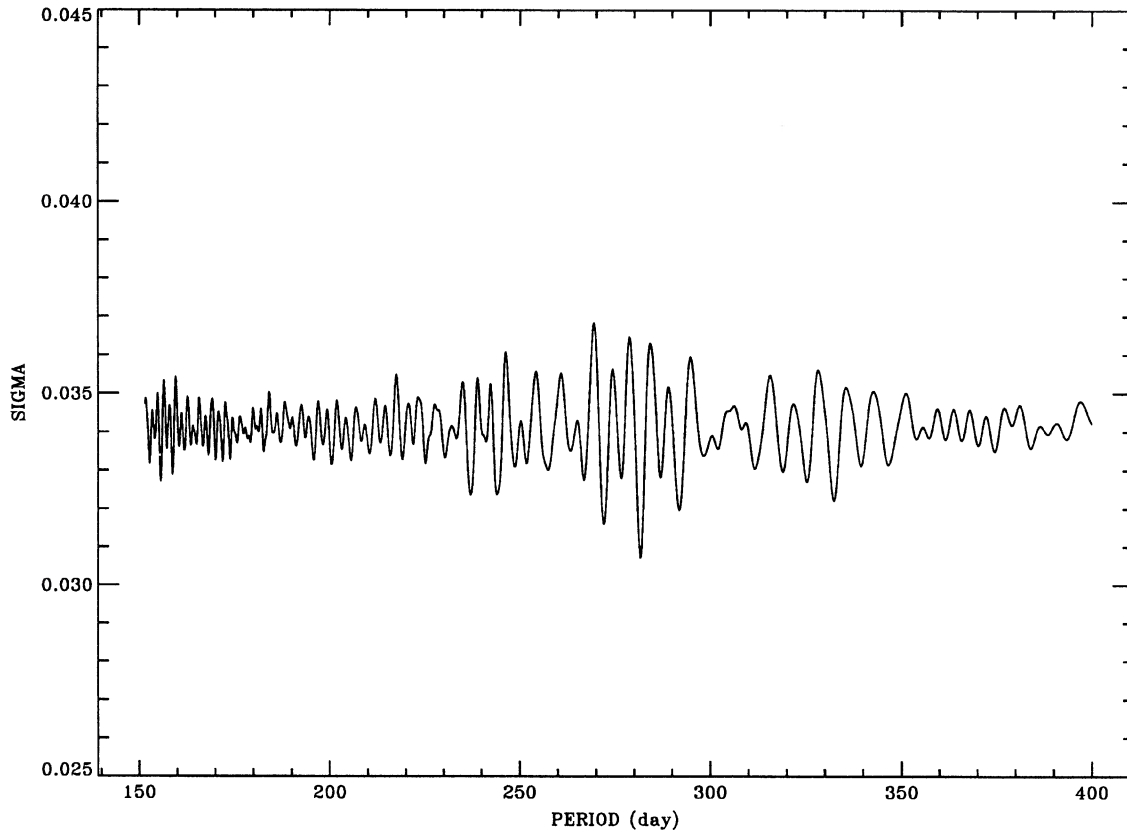


FIG. 11. Frequency-step test standard deviation for all data sets combined.

rameters into the process together, almost any breakdown into subsets saves the situation. The listed standard errors are from a final run with all seven parameters adjusted together, but with only the error estimates utilized and the parameter corrections ignored. With the 9 month period now allowed to have a small variation, we can compute the residuals of the reference epochs (t_0) of Table 10 with respect to those predicted by Eq. (7). These residuals are listed in the Res4 column of Table 10. Their typical size is about that expected from ordinary observational error. Although two of the LLH

residuals are larger than expected (26 and 13 days, compared to about 5 days standard error), the average residual of all three LLH points is only 11 days, or below 2 standard deviations. A few residuals still are considerably larger than expected from observational uncertainty, but the largest are only 8% and 4% of the 283.6 day period. The phase coherence thus demonstrated seems to us very remarkable.

5. OVERVIEW AND CONCLUSIONS

A statistically significant 9 month periodicity is present in every suitably long and well distributed light curve data set

TABLE 9. Phase coherence test results for $P=280.373$ day.

Set/Bandpass	Δt_0	A	t_0	E_{cycle}	Res1 ^a
BAX	0.0011 ± 0.0022	0.0177 ± 0.0031	-1359.1 ± 8.0	-171	-101
LUI	-0.0008 ± 0.0018	0.0137 ± 0.0025	17035.9 ± 8.5	-106	13
DAN	-0.0025 ± 0.0032	0.0283 ± 0.0056	24131.5 ± 6.2	-81	78
LOR	0.0002 ± 0.0021	0.0220 ± 0.0028	25550.4 ± 6.0	-76	90
GUT	-0.0084 ± 0.0037	0.0128 ± 0.0042	30972.1 ± 11	-56	-113
LLH U	-0.0053 ± 0.0037	0.0262 ± 0.0049	40908.7 ± 9.0	-21	-20
LLH B	0.0016 ± 0.0019	0.0227 ± 0.0029	40945.4 ± 5.0	-21	17
LLH V	0.0013 ± 0.0017	0.0228 ± 0.0026	40932.6 ± 4.4	-21	4.1
VIL1 H β	-0.0055 ± 0.0013	0.0289 ± 0.0017	41475.5 ± 2.8	-19	-15
VIL1 H α	0.0068 ± 0.0012	0.0288 ± 0.0016	41475.6 ± 2.6	-19	-15
VIL2 y	-0.0007 ± 0.0010	0.0195 ± 0.0015	46862.8 ± 3.3	0	28
VIL2 H α	-0.0152 ± 0.0009	0.0197 ± 0.0014	46850.4 ± 2.9	0	16
APT V	-0.0010 ± 0.0007	0.0200 ± 0.0011	48231.6 ± 2.3	5	-9.1
APT R	-0.0011 ± 0.0008	0.0185 ± 0.0011	48234.3 ± 2.6	5	-6.5
APT I	-0.0011 ± 0.0008	0.0177 ± 0.0012	48236.6 ± 2.9	5	-4.1

^aZero-phase Julian Date (t_0) residuals, see text for details.

Notes to Table 9.

Errors are internal mean errors of the least squares fit.

TABLE 10. Phase coherence results for $P=283.4$ day.

Set/Bandpass	Δt_0	A	t_0	E_{cycle}	Res2 ^a	Res3 ^b	Res4 ^c
BAX	0.0012 ± 0.0022	0.0161 ± 0.0031	-1352.0 ± 8.9	-170	-17	-19	-9.2
LUI	-0.0007 ± 0.0018	0.0138 ± 0.0025	17027.1 ± 8.6	-105	-58	-24	-8.4
DAN	-0.0022 ± 0.0032	0.0275 ± 0.0057	24132.4 ± 6.5	-80	-37	13	4.3
LOR	0.0001 ± 0.0021	0.0200 ± 0.0029	25549.1 ± 6.7	-75	-37	7.8	-4.8
GUT	-0.0091 ± 0.0040	0.0131 ± 0.0045	30968.2 ± 11	-56	-2.4	5.7	-12
LLH U	-0.0049 ± 0.0036	0.0274 ± 0.0051	40913.3 ± 8.5	-21	24	-5.8	-5.5
LLH B	0.0017 ± 0.0019	0.0234 ± 0.0029	40944.5 ± 4.9	-21	55	25	26
LLH V	0.0014 ± 0.0017	0.0231 ± 0.0026	40931.7 ± 4.2	-21	43	13	13
VIL1 H β	-0.0059 ± 0.0013	0.0287 ± 0.0017	41475.1 ± 2.9	-19	19	-9.2	-8.0
VIL1 H α	0.0063 ± 0.0013	0.0288 ± 0.0016	41474.8 ± 2.7	-19	19	-9.5	-8.3
VIL2 y	-0.0005 ± 0.0010	0.0199 ± 0.0015	46859.7 ± 3.2	0	20	24	24
VIL2 H α	-0.0149 ± 0.0009	0.0202 ± 0.0014	46847.9 ± 2.8	0	7.8	12	12
APT V	-0.0009 ± 0.0007	0.0199 ± 0.0011	48232.4 ± 2.4	5	-25	-8.8	-12
APT R	-0.0009 ± 0.0008	0.0185 ± 0.0011	48234.1 ± 2.6	5	-23	-7.1	-10
APT I	-0.0010 ± 0.0008	0.0176 ± 0.0012	48236.2 ± 2.9	5	-21	-4.9	-7.9

^aZero-phase Julian Date (t_0) residuals with respect to ephemeris (5).

^bZero-phase Julian Date (t_0) residuals with respect to Eqn. (6)

^cZero-phase Julian Date (t_0) residuals with respect to Eqn. (7)

Notes to Table 10.

Errors are internal mean errors of the least squares fit.

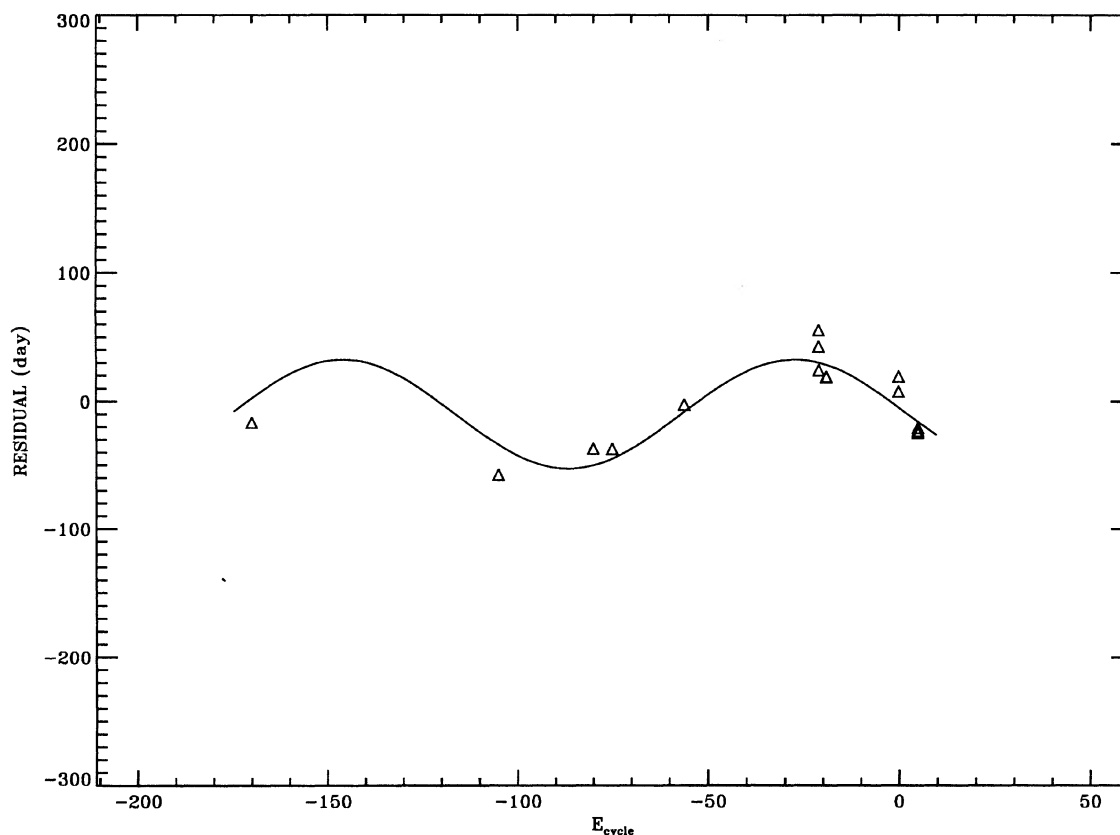


FIG. 12. Residuals of the zero-phase Julian dates (t_0) with respect to ephemeris (5) for every β Lyrae data set and the fitted sinusoid.

that we have been able to access. We analyzed 15 data sets (with multibandpass sets from the same era counted separately) in 9 eras that span about 150 yr. The derived semi-amplitudes are of similar magnitude for all data sets, with the extremes being about 0.013 and 0.030 in units of the flux at light curve maximum. There are other significant periodicities in some of the data sets, and perhaps some of these will turn out to be astrophysically interesting, particularly if they reappear in future observations. As shown by Table 6, there already may have been some reoccurrences, such as a period of about 6.5 days in two data sets and one of about 30 days in two others. However, our most immediate interest is in the consistent 9 month period. Most periods reported in the mainly subjective analyses of earlier papers do not pass standard significance tests at our adopted confidence levels.

If one does not consider directly the issue of phase coherence, multiple least-squares solutions exist for the (9 month) period and reference epoch of the full 150 yr data set. For example, solutions at periods of 280.4, 281.6, and 283.4 days differ very little in variance. The 283.4 day period is special however, because whole cycle assignments can be made such that all of the individual data sets agree closely in phase with the “global” (i.e., 150 yr) phasing, whereas that cannot be done at the other periods. An even much tighter agreement in phase of all data sets is found if one does not require the 9 month period to be strictly fixed, but allows for a small si-

nusoidal excursion with a period of 92 yr. (Such a variation in period would be indistinguishable from a small oscillation in the reference epoch, given the available patchy data base.) In that case, most of the residuals from perfect phase coherence are not greatly larger than one expects from observational error. Given the limited span of observations, we do not claim that a 92 yr periodicity has been clearly demonstrated, but only that the 9 month periodicity is not constant. There is some evidence that the amplitude of the 9 month periodicity decreases monotonically with wavelength. Guinan (1989) has mentioned some possible physical mechanisms for a 9 month period. Wilson & Van Hamme (1995) investigate other plausible physical mechanisms.

We emphasize that judgment of the reality of the 9 month period does not hinge on it being apparent to the eye. Quite to the contrary, the most remarkable finding may be the solid demonstration of the periodicity despite its being too small to appear in timewise plots in a visually convincing way. The roughly 2% semi-amplitude is indeed far smaller than the random excursions to be seen even in the best behaved photoelectric data. A beautiful example of the power of the Lomb-Scargle algorithm is in Fig. 13.8.1 of Press *et al.* (1992), where a tiny periodic component is recovered from noisy data. The periodicity is not at all detectable by the eye, even when one knows the period. We base our actual judgment on finding the 9 month period in 15 independent data

sets (9 eras) and on our demonstration of phase coherence over a century and a half.

Our findings emphasize the importance of long term monitoring of particularly interesting binaries such as β Lyrae. Such monitoring could uncover information of fundamental importance that cannot be recovered from short observational time windows. With automatic photometric telescopes (APTs), continual long term monitoring is now feasible, and can provide not only long baselines in time, but also consistent photometric systems over those long baselines.

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