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Stagnation-point flow in colliding-wind binary systems

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ABSTRACT

We investigate further the dynamics of colliding winds in hot-star binary systems, by concentrating in this paper on the effects of the radiation pressure of both stars on the material moving between them. Naturally, the presence of a luminous binary companion tends to inhibit the wind acceleration towards the stagnation-point because the resultant radiation force is smaller than in a single star. By also considering the countervailing influence of the companion's gravitational attraction, we derive inequalities governing the stellar mass-loss flux.

We find that, in close binary systems, such as V444 Cyg, the winds collide at much lower head-on velocities than those implied by the single-star models used in earlier calculations. This leads directly to lower expected X-ray luminosities and temperatures for colliding hot-star winds, and we suggest that this accounts for the long-standing discrepancies between theory and observation.

Key words: hydrodynamics – radiative transfer – binaries: general – stars: early-type – stars: mass-loss – X-rays: stars.

1 INTRODUCTION

The dynamics of stellar winds in binary systems is a complex and challenging problem in radiation hydrodynamics. Building on the basic understanding of the origin of the winds driven by line-radiation pressure from the surfaces of single hot stars, there are complications in binary systems that are bound to alter the dynamics fundamentally: first, there are the effects of the reduction of surface gravity caused by a companion, as described by Friend & Castor (1982) for X-ray binary systems, and Stevens (1988) for early-type binaries; secondly, the ionization of wind material by accretion-powered X-rays may suppress line-driving completely (MacGregor & Vitello 1982; Stevens 1991); and thirdly, in binary systems in which both stars are luminous and have winds, account must be taken of the dynamical impact of the companion's radiation field and any ionizing radiation produced in the shocked regions where the winds collide (Stevens, Blondin & Pollock 1992).

In this paper, we consider the consequences of a mechanism at work in early-type binary systems which, although mentioned in passing by Stevens et al. (1992) and Usov (1990, 1992), has not yet received detailed treatment: namely the fact that there are two stars exerting radiation pressure on the flow of stellar-wind material in the system. Qualitatively, it is easy to see that this is an effect of great importance, especially near the axis of symmetry of the binary system. The velocity laws of single stars show that wind acceleration occurs over distances up to 10 stellar radii from the stellar surface, although, according to the Castor, Abbott & Klein (1975, hereafter CAK) line-absorption theory, the mass-loss rate is determined by conditions very close to the surface. Consider an ideal close circular binary system of two identical hot stars, for which a typical few-day orbital period would imply a separation of only a few stellar radii. The mass loss from each star is now driven by a radiation force that is the vector sum of contributions from both stars, whose general effect near the centre will be to push material sideways out of the system, as shown in Figs 1 and 2. The region between the stars near the axis of symmetry is the most interesting, as the stars are pushing in opposite directions. For example, at the mid-point, where the single stellar wind might have reached 0.6 or 0.7 of its terminal velocity, the resultant radiation force disappears and reverses direction. As a consequence, acceleration is inhibited and the two winds are certain to collide at lower velocities than expected from single stars. Even though only a small fraction of the wind volume is involved – even close companions fill only a few per cent of each other's skies – the region between the two stars is the focus of X-ray observations that have been undertaken of colliding-wind systems. The overall wind properties, on the other hand, which are observed at longer wavelengths, are not likely to be seriously affected.

We have investigated this idea as a potential solution to the persistent, and rather mysterious, problem that most colliding-wind systems are observed as weaker and cooler X-ray sources than expected from current theoretical considerations. It has been a consistent feature of calculations, ranging from the early order-of-magnitude estimates of Cooke, Fabian & Pringle (1978), for example, to Stevens et al.'s (1992) more sophisticated hydrodynamic code, that the model X-ray luminosities and temperature are both considerably too high. The fact that single stars are X-ray sources of broadly similar luminosities should not confuse the issue: if all the X-rays were to come without enhancement from the individual stars, the problem would be worse. Nevertheless, Pollock (1987) came to the firm conclusion that Wolf-Rayet (WR) binaries are a few times brighter than single stars, and Chlebowski, Harnden & Sciortino (1989) and Chlebowski & Garmany (1991) found the same trend in O stars. There are a few much brighter X-ray sources, notably WR 140 (Williams et al. 1990; Koyama et al. 1990), WR 125 (Williams et al. 1992) and WR 147 (Caillault et al. 1985). All are certainly, or are likely to be, distant binaries: WR 140 (HD 193793), for example, has a period of nearly 8 yr.

Although the recent treatments of colliding winds, by Luo, McCray & MacLow (1990) and Stevens et al. (1992), have improved agreement between theory and observation for these long-period binaries, the discrepancies remain for the close systems; the theoretical models of V444 Cyg (WN + O), for example, imply X-ray fluxes and temperatures a few times higher than observed. Also, Chlebowski et al. (1989) found that O-star binaries also showed uncomfortably low X-ray temperatures. It is clear now that the single-star wind boundary conditions imposed in front of the shock in earlier models of close binaries, including our own, were incorrect. In this paper, we have made an initial attempt to calculate the consequences of a better understanding of the wind acceleration between the stars, by concentrating on the solution along the line-of-centres only, where 1D methods may be applied. More detailed 2D models will come later. We note that Usov (1990) tried to reconcile observations and theory of V444 Cyg using a rough energy-conservation law based on a similar idea.

In strong, even conclusive, support of our basic assumptions that the wind is massive and reasonably homogeneous is the deep X-ray absorption observed in accretion-powered X-ray sources (Pollock 1987), and in WR 140 by Williams et al. (1990). In spite of this, and the more indirect support of a large number of other data, there are advocates of other ideas. For example, Cherepashchuk (1990) has proposed that, when very inhomogeneous winds collide, high-density clumps pass through the shock without contributing significantly to the X-ray flux. It is our intention here to see if more conventional models, with a more accurate description of the wind acceleration near the centre of the system, are better able to reproduce the lower luminosities and temperatures observed in colliding-wind binaries. In Section 2, we derive the equations governing the dynamics and use them in Section 3 to model the flow along the axes of typical systems. In Section 4, we concentrate on V444 Cyg, while Section 5 contains a discussion of some wider implications for colliding-wind binary systems.

2 THE DYNAMICS OF WINDS IN EARLY-TYPE BINARY SYSTEMS

Castor, Abbott & Klein's (1975) line-driven wind theory and subsequent important improvements by Pauldrach, Puls & Kudritzki (1986) provide the basis of current understanding of the winds observed from single hot stars: the wind is accelerated by the transfer of momentum from outwardly directed photospheric radiation following UV resonance-line absorption by matter. Readers should refer to those two papers for a description of the formalism, methodology and assumptions that we adopt below. We take a broadly similar approach to the adaptations of CAK theory by Friend & Castor (1982) and Stevens (1991) in their studies of the wind dynamics in massive X-ray binary systems in which a hot star has a compact, neutron-star companion.

We are interested, rather, in binary systems of two hot stars as illustrated in Fig. 1. While we expect the overall stagnation-point flow found in earlier work still to apply, as two oppositely facing shocks are generated between the stars as the outwardly accelerated winds collide, the winds in regions A and B are driven in a binary system by line-absorption of radiation from two stars. In regions C and D between the shocks, separated by a contact discontinuity, the hot gas is too highly ionized to suffer any more acceleration. Indeed, we expect both the hot shocked gas and the oppositely moving companion wind to be optically thin to the continuum at the relevant driving frequencies. We designate the masses M_1 and M_2; radii R_1 and R_2; effective temperatures T_1 and T_2; luminosities L_1 and L_2; binary separation D_{sep}, and corresponding orbital period P_{orb}.

The radiative force able to drive cool material in a sample binary system is illustrated schematically in Fig. 2. Near the stars, the force is dominated by the outward push from the nearby photosphere. Over most of the stellar surfaces, in directions away from the system centre, the winds develop much like their single-star counterparts. However, for material moving towards the stagnation point, along the line-of-centres in particular, the retarding effect of star 2's radiation becomes increasingly important, until it would become dominant in its turn if the shock did not intervene. In general, where material is cool enough to be driven, the force components perpendicular to the line-of-centres combine to push material near the centre sideways out of the system, while the acceleration force along the line-of-centres is reduced as the components cancel. In close binary systems, development of the wind in these central regions is inhibited, and the winds collide at lower velocities than otherwise expected. In addition, the combined effects of the radiation fields, to push material perpendicularly out of the region between the two stars, will probably have the effect of reducing the amount of X-ray emission from colliding-wind systems, although to confirm this will require more detailed multidimensional calculations. Over most of the stellar surfaces, however, and in more distant binaries, the winds develop like their single-star counterparts. Although we have been preoccupied with the effects of radiation, it is necessary not to forget the countervailing effect of the companion's gravity, which reduces the work required to accelerate

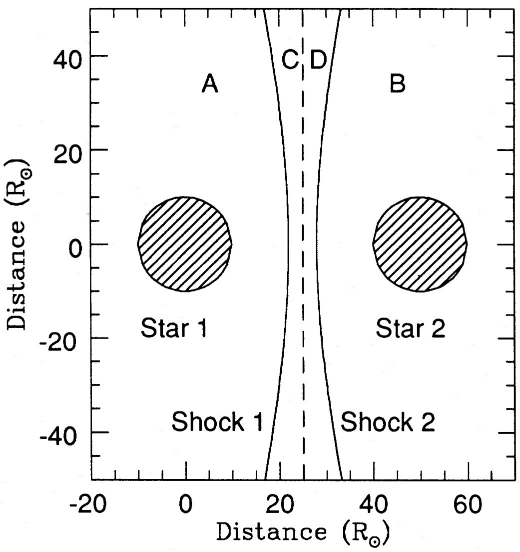

Figure 1. Sketch of a colliding-wind binary system of identical stars, each with an isotropic wind. The unshocked parts of the winds of stars 1 and 2, labelled A and B respectively, are bounded by two opposing shock fronts, represented by the solid lines. In the hot region in between, regions C and D, of shocked material from stars 1 and 2, respectively, are separated by a contact discontinuity, represented by the dashed line.

material towards the stagnation point. Using the equations governing the dynamics of binary winds, we show below that, as expected, it is the balance between radiative and gravitational forces that largely determines the nature of the flow, leading to quantitative predictions of the behaviour of colliding-wind systems and their X-ray properties in particular.

2.1 The hydrodynamic equations of motion

Consider flow along the axis of cylindrical symmetry between the stars towards the stagnation point, where the winds collide. We are particularly interested in two quantities, which we shall refer to as the stagnation mass-loss and the stagnation velocity law, in order to emphasize that they refer only to motion along the line-of-centres between the stars, and in their comparison with the single-star flow that applies to those parts of each star that are little affected by the companion. The equations governing the motion of the wind of star 1 along the axis between the stars are the laws of conservation of mass and momentum:

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}=\rho z^2 v, \tag{1}$$

$\mathscr{F}(z, v, \mathrm{d}v/\mathrm{d}z)=0$, where

$$\mathscr{F}(z, v, \mathrm{d}v/\mathrm{d}z)=\left(v-\frac{a^2}{v}\right)\frac{\mathrm{d}v}{\mathrm{d}z}+\frac{\mathrm{d}\Phi}{\mathrm{d}z}-\frac{2a^2}{z}-g_\mathrm{R}. \tag{2}$$

In these equations, $\mathrm{d}\dot{M}/\mathrm{d}\Omega$ is the stagnation mass-loss, ρ is the wind density, z is the distance from the centre of star 1 along the line-of-centres, v is the wind velocity, a is the speed of sound, Φ is the gravitational potential and g_R is the radiative line-force. For the sake of simplicity, we have neglected the effects of both rotation and tidal distortions caused by the presence of the companion star and have assumed that the stellar radius remains constant. Although it would have been reasonably straightforward to do so, their inclusion would not alter the conclusions reached below. Indeed, the tidal distortions would reduce the wind velocity even more than the radiative effect that is our main concern. Two effects come into play: first, reduced gravity causes the photosphere to expand towards the companion, reducing in its turn the size of the region available for wind acceleration, and, secondly, gravity darkening, which demands that the radiation intensity is locally proportional to the gravity, reduces the radiative driving force. The calculations below are thus likely to underestimate, somewhat, the differences between binary and single-star models. In these circumstances,

$$\Phi=-\frac{GM_1(1-\Gamma_1)}{z}-\frac{GM_2(1-\Gamma_2)}{(D_\mathrm{sep}-z)}, \tag{3}$$

where Γ_i is the Eddington ratio $\Gamma_i=\sigma_\mathrm{e}L_i/4\pi GM_ic$. We assume that the flow is isothermal, of temperature

$T_\mathrm{wind}=0.8\ T_\mathrm{eff}$.

The radiative line-force g_R in equation (2) has a component from each star acting in opposite directions:

$$g_\mathrm{R}=\frac{\sigma_\mathrm{e}\mathscr{M}(t)}{c}[F_1K_1-F_2K_2], \tag{4}$$

where $F_1(z)$ and $F_2(z)$ are the radiative fluxes, $K_1(z, v, \mathrm{d}v/\mathrm{d}z)$ and $K_2(z, v, \mathrm{d}v/\mathrm{d}z)$ are the finite disc factors (Pauldrach et al. 1986), and t is the Sobolev optical depth parameter $t=\sigma_\mathrm{e}\rho v_\mathrm{th}(\mathrm{d}v/\mathrm{d}z)^{-1}$. We have used a single force multiplier, $\mathscr{M}(t)$, to represent the dynamical influence of both radiation fields. Following Abbott (1982), it is represented in its simplest form as a power law in t:

$$\mathscr{M}(t)=kt^{-\alpha}, \tag{5}$$

where k and α are constants. We have neglected any electron-density or geometric-dilution-factor dependences on the force multiplier, which Abbott (1982), anyway, showed were small.

Our use of a single force multiplier to represent the dynamical influence of both radiation fields needs some justification. In a single star, the force multiplier depends on both the local radiation field and the local ionization balance, which is itself determined, in part, by the radiation field (cf. Abbott 1982). In a colliding-wind system, the radiation field is made up of three contributions of different temperatures, two stellar UV components combined with X-rays from the shock. A proper calculation of the ionization balance and associated force multiplier would be very complicated and out of place in the simple approach we have adopted here. Nevertheless, we are reasonably confident that our approximation is not likely to be in serious error for the systems we have considered, because, first, the X-ray luminosity is more or less negligible in comparison with the UV flux and is likely to affect only trace ions and, secondly, the three systems below have stars of roughly similar temperatures, even if their luminosities are different, leading us to expect relatively modest ionization changes.

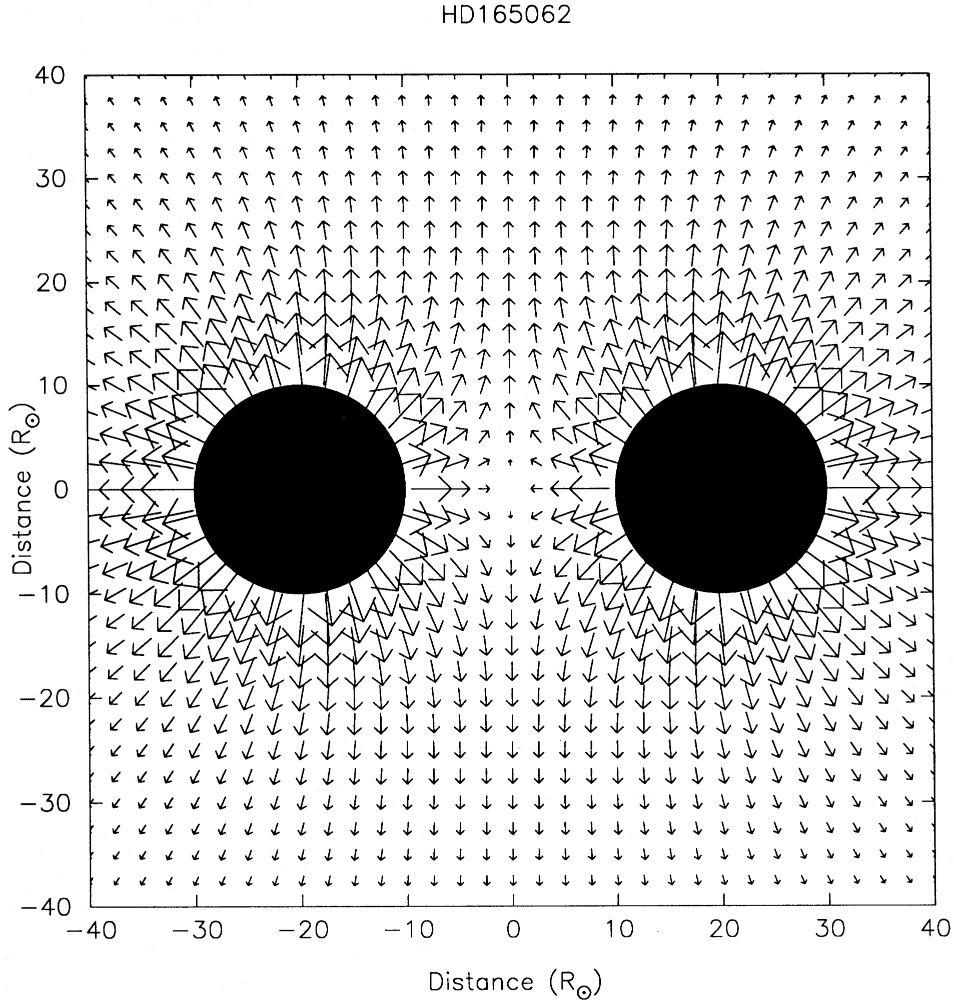

Figure 2. The magnitude and direction of the radiative flux vector in a sample binary system of identical hot stars. At the centre, the radiative force goes to zero and reverses direction, causing poor development of the wind in this vicinity. There is also a combined outward force of cylindrical symmetry that tends to divert material away from collision with the companion wind. These effects will tend to reduce both the temperature and luminosity of the X-ray emission observed from a system of this type. The radiative force of a single star would be the same as that acting outwards along the line-of-centres.

Following Pauldrach et al. (1986), we approximate the finite disc correction factors for each star, K_1 and K_2, by purely radial functions, so that $K_1(z, v, \mathrm{d}v/\mathrm{d}z) \sim K_1(z)$, with

$$K_1(z) = \frac{1-[1-(R_1/z)^2]^{1+\alpha}}{(1+\alpha)(R_1/z)^2}, \tag{6}$$

and $K_2(z, v, \mathrm{d}v/\mathrm{d}z) \sim K_2(z)$, with

$$K_2(z) = \frac{1-\{1-[R_2/(D_{\mathrm{sep}}-z)]^2\}^{1+\alpha}}{(1+\alpha)[R_2/(D_{\mathrm{sep}}-z)]^2}. \tag{7}$$

The usual coordinate transformation from (z, v) to (u, w), where

$$u(z) = -\frac{2GM_1(1-\Gamma_1)}{za^2} \tag{8}$$

and

$$w(v) = \frac{v^2}{a^2}, \tag{9}$$

gives

$$w' = \frac{1}{GM_1(1-\Gamma_1)}\left(z^2 v \frac{\mathrm{d}v}{\mathrm{d}z}\right). \tag{10}$$

The equation of motion for the stellar wind, $\mathscr{F}(u, w, w') = 0$, becomes

$$\mathscr{F}(u, w, w') = \left(1-\frac{1}{w}\right) w' + h(u) - CB(u)\, w'^{\alpha}, \tag{11}$$

where

$$B(u)=K_1(u)-\left(\frac{M_2\Gamma_2}{M_1\Gamma_1}\right)K_2(u)A(u), \tag{12}$$

and the constant C is given by

$$C=GM_1\Gamma_1 k[GM_1(1-\Gamma_1)]^{\alpha-1}(\sigma_e v_{th}\,\mathrm{d}\dot{M}/\mathrm{d}\Omega)^{-\alpha}. \tag{13}$$

In addition,

$$h(u)=1+\frac{4}{u}-\frac{M_2(1-\Gamma_2)}{M_1(1-\Gamma_1)}A(u) \tag{14}$$

and

$$A(u)=\left(\frac{u_D}{u-u_D}\right)^2, \tag{15}$$

where $u_D=u(D_{sep})$.

It is simply the different radial dependence of the functions $h(u)$ and $B(u)$ that distinguishes our equation (11) from the standard CAK formulation. The equation of motion has a critical point in the supersonic portion of the flow, and the solution at this point is determined by three conditions that hold here, namely (a) the equation of motion $\mathscr{F}(u, w, w')=0$, (b) the singularity condition $\partial\mathscr{F}/\partial w'=0$, and (c) the regularity condition $\mathrm{d}\mathscr{F}/\mathrm{d}u=\partial\mathscr{F}/\partial u+w'\partial\mathscr{F}/\partial w=0$.

Substitution of the equation of motion (equation 11) into the regularity and singularity conditions leads to the following critical-point quantities:

$$w_c'=\left[\frac{h_c}{(1-\alpha)}\frac{1}{B_c}\frac{\mathrm{d}B}{\mathrm{d}u}\bigg|_c-\frac{\mathrm{d}h}{\mathrm{d}u}\bigg|_c\right]^{1/2}+\frac{\alpha h_c}{(1-\alpha)}, \tag{16}$$

$$w_c=1+\frac{\alpha h_c}{(1-\alpha)}\left[\frac{h_c}{(1-\alpha)}\frac{1}{B_c}\frac{\mathrm{d}B}{\mathrm{d}u}\bigg|_c-\frac{\mathrm{d}h}{\mathrm{d}u}\bigg|_c\right]^{-1/2}, \tag{17}$$

$$C=\frac{1}{B_c}\frac{h_c}{(1-\alpha)}(w_c')^{-\alpha}, \tag{18}$$

with $h_c=h(u_c)$ and $B_c=B(u_c)$. In the absence of a companion, $M_2=k_2=0$, and these expressions reduce to Pauldrach et al.'s (1986) single-star equations in the isothermal limit. Having derived values for w_c, w_c' and C, for an assumed value of u_c, equation (18) gives the stagnation mass-loss $\mathrm{d}\dot{M}/\mathrm{d}\Omega$, while the complete solution $v(z)$ is recovered by integrating the equation of motion from the critical point inwards to the photosphere, and outwards as far as possible. Finally, the proper value of u_c is fixed, by requiring that the total electron-scattering optical depth in the wind is unity.

2.2 The stagnation mass-loss

Some simple expressions can be derived from the critical-point conditions for the dependence of the stagnation mass-loss on the binary parameters, including the binary separation in particular:

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}\propto C^{-1/\alpha}\propto\left[\frac{B_c}{h_c}\right]^{1/\alpha}w_c'. \tag{19}$$

Inspection shows that, generally, at the critical point

$$\frac{\alpha h_c}{(1-\alpha)}\gg\left[\frac{h_c}{(1-\alpha)}\frac{1}{B_c}\frac{\mathrm{d}B}{\mathrm{d}u}\bigg|_c-\frac{\mathrm{d}h}{\mathrm{d}u}\bigg|_c\right]^{1/2}, \tag{20}$$

so that

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}\propto[B_c]^{1/\alpha}[h_c]^{1-1/\alpha}. \tag{21}$$

The competition between radiation and gravity is clear in the two terms of this equation. Considering only gravitational forces, the mass-loss flux increases as the binary components approach and the potential function $h(u_c)$ drops, giving $\mathrm{d}\dot{M}/\mathrm{d}\Omega\propto h_c^{-0.6}$. This was the effect considered by Stevens (1988). The value of h_c can be approximated by

$$h_c\sim 1-c_1 A_c \tag{22}$$

where $c_1=M_2(1-\Gamma_2)/[M_1(1-\Gamma_1)]$, and $A_c=A(u_c,\ D_{sep})$ (equation 15). The opposing effect, $\mathrm{d}\dot{M}/\mathrm{d}\Omega\propto B_c^{1/\alpha}$, is the tendency of the mass-loss flux to decrease in the face of the reduction due to the companion of the *net* radiative flux at the critical point, which is proportional to $B(u_c)$. Writing B_c as

$$B_c=K_1(r_c)[1-c_2A_c(D_{sep})], \tag{23}$$

with $c_2=[L_2K_2(r_c)]/[L_1K_1(r_c)]$, and assuming finite disc correction factors at the critical point ($r_c\sim 1.05\,R_1$) of $K_1(r_c)\sim 0.66$ and $K_2(r_c)\sim 1$, we can expand equation (21) in powers of the small quantity A_c to obtain

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}=\left[\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}\right]_\infty\{1-\alpha[c_2-(1-\alpha)c_1]A_c\}, \tag{24}$$

where $[\mathrm{d}\dot{M}/\mathrm{d}\Omega]_\infty$ is the single-star mass-loss rate. Defining

$$\mathscr{S}=\frac{L_1}{L_2}\frac{M_2}{M_1}=\frac{\Gamma_1}{\Gamma_2}, \tag{25}$$

we find that the stagnation mass-loss rate is more than the single-star rate $\mathrm{d}\dot{M}/\mathrm{d}\Omega\geq[\mathrm{d}\dot{M}/\mathrm{d}\Omega]_\infty$ when $c_2<(1-\alpha)c_1$ or

$$\mathscr{S}\gtrsim\frac{3}{2(1-\alpha)}\sim 4. \tag{26}$$

This relationship implies that, when the ratio of the Eddington factors is greater than approximately 4, the reduction in mass-loss caused by the radiation field of the secondary is more than compensated for by the enhanced mass-loss resulting from the gravitational presence of the companion. The magnitude of the increase or decrease is, however, also sensitively dependent on the binary separation.

3 RESULTS FOR SAMPLE BINARY SYSTEMS

Using the equations developed above, we have calculated solutions for the flow towards the stagnation point in two different binary systems: 'binary A', modelled on HD 165052 (O6.5V + O6.5V), in which the components are identical, and 'binary B', modelled on the eccentric system Iota Orionis (O9III + B1III), in which the components are not the same.

HD 165052 was used as an example by Luo et al. (1990). Its component stars are both classified O6.5V and have roughly equal luminosities. We assume that $M_1 = M_2 = 30$ $M_\odot$, $R_1 = R_2 = 10$ $R_\odot$ and $T_1 = T_2 = 40\,000$ K (Howarth & Prinja 1989), giving $L_1 = L_2 = 2.3 \times 10^5$ $L_\odot$, $\Gamma_1 = \Gamma_2 = 0.2$, and escape velocities of $v_{\rm esc} = 960$ km s^{-1}. From Abbott (1982), we assume $k = 0.174$ and $\alpha = 0.606$. The single-star model wind calculated with these parameters gives values of $v_\infty = 2766$ km s^{-1}, which implies $v_\infty \sim 3v_{\rm esc}$, as expected, and $\dot{M} \sim 8 \times 10^{-7}$ $M_\odot$ yr^{-1}, in reasonable agreement with Howarth & Prinja's (1989) observations. While HD 165052's period of 6.14 d gives a binary separation of ~ 55 $R_\odot$, we have calculated solutions for a range of values of $D_{\rm sep}$. For this system, $\mathscr{S} = 1$ and

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}\right]_\infty (1 - 0.07A_{\rm c}), \tag{27}$$

so that the stagnation mass-loss is expected to decrease with $D_{\rm sep}$.

By contrast, the components of Iota Orionis are quite different. Following Stickland et al. (1987), we assume that $M_1 = 40$ $M_\odot$ and $M_2 = 20$ $M_\odot$, $R_1 = 16$ $R_\odot$ and $R_2 = 10$ $R_\odot$, and $T_1 = 30\,000$ K and $T_2 = 20\,000$ K, giving $L_1/L_2 \sim 13$. For the dominant wind of the primary, with $k = 0.17$ and $\alpha = 0.59$ (Pauldrach et al. 1986), the single-star model wind calculated with these parameters gives values of $v_\infty = 2515$ km s^{-1} and $\dot{M} \sim 4 \times 10^{-7}$ $M_\odot$ yr^{-1}, again reasonably consistent with those found by Howarth & Prinja (1989).

Because of its highly eccentric orbit ($e \sim 0.76$), the binary separation of Iota Orionis decreases, without any intervention from us, from 270 $R_\odot$ at apastron to 35 $R_\odot$ at periastron. With mass and luminosity ratios considerably different from unity, the wind of the primary dominates, and $\mathscr{S} \sim 6.5$ and

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega}\right]_\infty (1 + 0.07A_{\rm c}), \tag{28}$$

so that, on approaching periastron, the stagnation mass-loss should increase.

Fig. 3 shows a comparison between the values for the stagnation mass-loss from full numerical calculations using the equations for Section 2 and those predicted by equation (24) for binaries A and B. There is good agreement. The parameter $\mathscr{S}$ determines the direction of change of the stagnation mass-loss with $D_{\rm sep}$, while the rate of change depends on both $\mathscr{S}$ and $D_{\rm sep}$. Naturally, systems where $\mathscr{S}$ is large because the primary is dominant are less affected by the companion's presence.

The expected increase in the stagnation mass-loss as a system like Iota Orionis approaches periastron was first discussed from a theoretical point of view by Stevens (1988), and has since received observational backing by Gies, Wiggs & Bagnuolo (1993). Assuming that $L_* \propto M_*^{3.5}$ for hot stars in general, equation (26) shows, in crude terms, that, for all binary systems of large enough mass-ratios $M_1/M_2 \gtrsim 1.75$, the stagnation mass-loss should exceed the single-star rate. We wait with interest to see whether analysis of eccentric and other hot-star binary systems conforms to the predictions of the theory derived above.

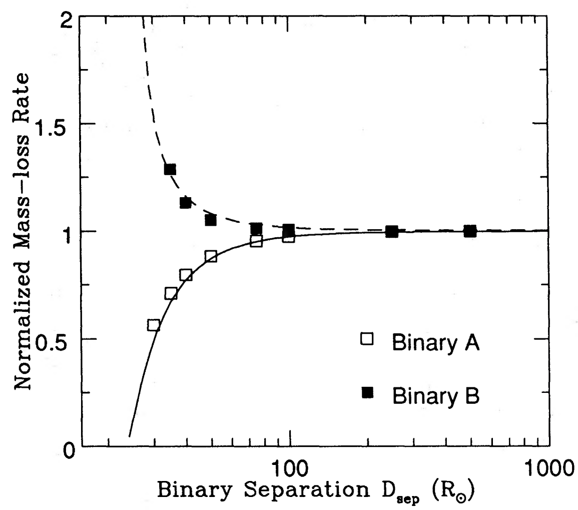

Figure 3. Comparisons between the numerical stagnation mass-loss as a function of binary separation and those predicted by the analytical formula of equation (24) for 'binary A' (equal stellar components, $\mathscr{S} = 1$) and 'binary B' (unequal stellar components, $\mathscr{S} = 6.5$). The analytical formulae are shown for binary A and binary B by the solid and dashed lines, respectively. The numerical results for several values of $D_{\rm sep}$, shown by the open squares for binary A and filled squares for binary B, are in good agreement.

3.1 The stagnation velocity law

The stagnation velocity law is calculated by integration from the critical point and is thus not open to simple analytical study. The velocities at which the winds collide are especially important for estimates of the X-ray temperature and luminosity of the system (Section 3.3). In general terms, because the types of stars present in binary systems are individually capable of accelerating winds of their own, material from a companion star will be slowed down, whatever happens to the stagnation mass-loss. We have calculated the velocity laws of the primaries of binaries A and B for a variety of $D_{\rm sep}$. Some details of the results for binary A are shown in Fig. 4 and summarized in Table 1, taking the calculations out to the stagnation point at $z_{\rm s} = D_{\rm sep}/2$, where winds of equal strength are expected to collide. It is clear that the wind velocity can be dramatically reduced by the presence of a companion star and that this is likely to have important repercussions for some observational characteristics of these binaries, including the expected X-ray behaviour.

In the calculations, we have used, for convenience, velocities that would have been attained at the stagnation point, rather than the proper immediate pre-shock velocity about half a shock thickness away. In close binaries, however, where the shocked gas is able to cool, the shock is very thin (Stevens et al. 1992), while in the wide binary systems, where the post-shock gas is adiabatic, the half shock-thickness is about $D_{\rm sep}/8$ (Luo et al. 1980), but the acceleration is weak and the difference between velocities also small.

Fig. 5 shows some calculated values of the pre-shock velocities for binaries A and B, where, for binary B, with its unequal stellar components, we have assumed the stagnation point is at the surface of star 2, $z_{\rm s} = D_{\rm sep} - R_2$. Also shown are the corresponding single-star velocities, making it clear

Table 1. The hydrodynamics of stagnation-point flow in a colliding-wind binary of identical stars. Shown are values of the binary separation $D_{\rm sep}$, the stagnation mass-loss rate $\dot{M}$, the velocity at the stagnation-point $z_{\rm s}=D_{\rm sep}/2$, and the functions $h_{\rm c}$ and $B_{\rm c}$, defined in Section 2.1.

D_{sep} ($R_\odot$)	$4\pi d\dot{M}/d\Omega$ ($M_\odot$ yr^{-1})	$v(z_s)$ (km s^{-1})	h_c	B_c
∞	$8.0.10^{-7}$	2766	0.996	0.662
500	$8.0.10^{-7}$	2600	0.996	0.661
100	$7.8.10^{-7}$	1949	0.983	0.649
55	$7.3.10^{-7}$	1332	0.942	0.601
40	$6.4.10^{-7}$	854	0.871	0.546

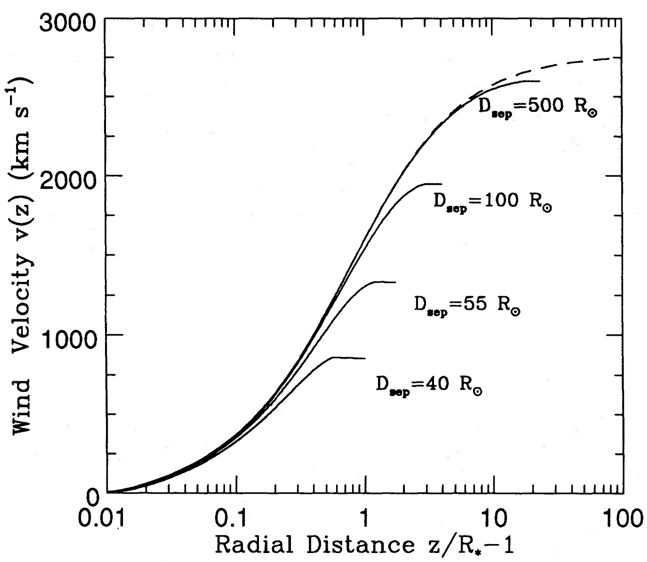

Figure 4. The inhibition of radiative acceleration caused by the presence of a companion star. We show the calculated stagnation velocity law for our model 'binary A' of identical stars, for several binary separations: $D_{\rm sep}=500$ R$_\odot$, 100 R$_\odot$, 55 R$_\odot$ – the actual value for HD 165052 – and 40 R$_\odot$. The single-star solution is shown, for comparison, by the dashed line. The velocity of the flow towards the stagnation-point at $z_{\rm s}=D_{\rm sep}/2$ is suppressed by the radiation pressure of the companion.

by how much the shock-front velocity may be overestimated by using a standard velocity law, although, as expected at very large $D_{\rm sep}$, the shock-front velocity is equal to the single-star terminal velocity.

For binary A, with $\mathscr{S}=1$, radiative deceleration effects are pronounced, and the shock-front velocity tends to fall away sharply from the corresponding single-star velocity with decreasing $D_{\rm sep}$. At HD 165052's $D_{\rm sep}=55$ R$_\odot$, the respective predicted wind collision velocities are $v=1332$ km s^{-1} for the model calculations and $v=1930$ km s^{-1} for the single-star law, a difference of 30 per cent.

As for the mass-loss rate discussed above, the effects are less serious in binary B where the primary is much more massive and luminous. However, use of the incorrect single-star law for Iota Orionis would still lead to difficulties. While at apastron, where $D_{\rm sep}=270$ R$_\odot$, the binary model calculation gives $v=2310$ km s^{-1} which is in reasonable agreement

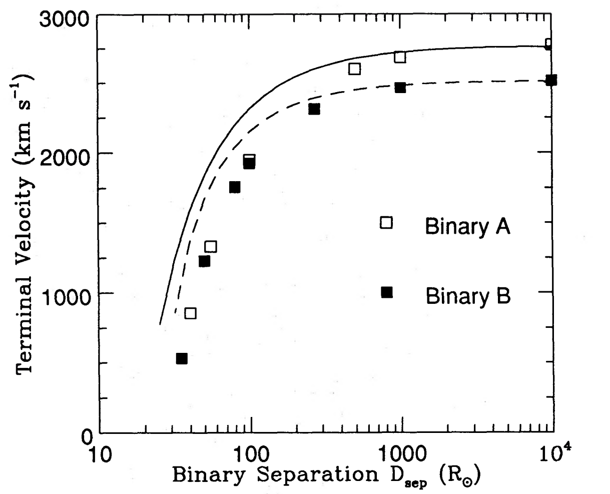

Figure 5. The reduction in terminal wind velocity as a function of binary separation $D_{\rm sep}$, for model systems binary A, $\mathscr{S}=1$, and binary B, $\mathscr{S}=6.5$. For binary A, the terminal velocities are taken at $z_{\rm s}=D_{\rm sep}/2$ and are plotted as open squares, and, for binary B, the terminal velocities are taken at $z_{\rm s}=D_{\rm sep}-R_2$, at the surface of the fainter and less massive secondary, and are plotted as filled squares. Also shown are the corresponding velocities from the single-star wind laws for binary A (full line) and binary B (dashed line).

with the standard law's value of $v=2390$ km s^{-1}, at periastron, on the other hand, where $D_{\rm sep}=35$ R$_\odot$, the respective values $v=530$ km s^{-1} and 1110 km s^{-1} differ by about a factor of 2.

3.2 Colliding-wind binaries – X-ray characteristics

The proper testing ground for the ideas discussed above is their ability to account for observations better than current theory, and we have had particularly in mind the measurements of X-ray temperatures and luminosities that provide the most direct evidence of wind collisions. Looking once more at Fig. 1, the region of hot gas is bounded by two shocks between the stars, in which X-rays are produced by conversion of the wind's kinetic energy into thermal energy. The post-shock gas temperature is given by

$$kT_{\rm s}=\frac{3}{16}\,\overline{m}v^2, \tag{29}$$

or

$$kT_{\rm s}\sim 1.2v_8^2\ \mathrm{keV}, \tag{30}$$

for material of solar abundance, where the mean particle mass $\overline{m}\sim 10^{-24}$ g, and v_8 is the wind velocity in front of the shocks in units of 10^8 cm s^{-1}. This temperature is the maximum post-shock temperature, and material away from the line-of-centres will tend to radiate less, because of the outward radiative push of the companion, and at lower temperatures, as the shock becomes more oblique. Enough of the X-ray emission, however, comes from near the stagnation point for this single-point temperature to characterize the emission reasonably well. Clearly, it is necessary to use as good a value of the velocity as possible, and we concluded above that use of the single-star velocity law gives velocities and temperatures that are too high.

For HD 165052, whose $D_{sep} = 55\ R_{\odot}$, the binary calculations above give $kT_s = 2$ keV, compared with $kT_s = 4.5$ keV from the single-star velocity law, while kT_s varies considerably around Iota Orionis's eccentric orbit between periastron and apastron, from 0.35 to 6 keV and from 1.5 to 7 keV, according to the binary and single-star models, respectively.

It is difficult to estimate the X-ray luminosity of the binary model properly without undertaking 2D hydrodynamic calculations along the lines of Stevens et al. (1992), incorporating a more complete description of the radiation field such as that implicit in Fig. 2. As we have noted, the combined effect of both radiation fields will be to slow down the shock-front velocity and preferentially to drive material perpendicularly out of the region between the two stars (Section 2), both effects affecting the X-ray characteristics of the system. Stevens et al. (1992), however, did derive two asymptotic scaling relationships for the X-ray luminosity which we can apply here. For close binaries, where the isothermal limit applies, $L_X = f\dot{M}v^2$; here f is the fraction of the wind's kinetic energy converted into radiation. For similar stars, $f \sim 1/6$. In the other, adiabatic limit,

$$L_X \propto \dot{M}^2 v^{-3/2} D^{-1} \frac{(1+\mathscr{R})}{\mathscr{R}^4}, \tag{31}$$

where $\mathscr{R}$ is the momentum ratio.

Luo et al. (1990) showed that the adiabatic limit applies to HD 165052 and discussed models of this type that are able to reproduce the observed X-ray luminosity. Assuming a collision velocity of 1900 km s^{-1} and a mass-loss rate for each star of $2.0 \times 10^{-7}\ M_{\odot}\ yr^{-1}$, they derived a luminosity of $L_X(0.5 < E < 3.0\ keV) \sim 1.8 \times 10^{33}$ erg s^{-1} consistent with the *Einstein* measurement of $\log_{10} L_X = 33.02 \pm 0.26$ (Chlebowski et al. 1989). Our binary flow model above gives a velocity reduced to ~ 1332 km s^{-1}, which has the effect of increasing the total luminosity by a factor of 3, while reducing its temperature by about 50 per cent. The lack of a published X-ray spectrum, however, makes it impossible to assess the relative merits of the model.

Things are better with Iota Orionis: Chlebowski et al. (1989) estimated a characteristic temperature of 0.5 keV for a luminosity of $\log_{10} L_X = 32.47 \pm 0.21$. A major shortcoming of models by Luo et al. (1990) and Stevens et al. (1992), for example, has been the unacceptably high temperatures of a few keV, rather than those ≤ 1 keV often observed. Our calculations above show that a close companion can slow down the wind sufficiently to produce the low temperatures observed. For Iota Orionis around periastron, the temperature is consistent with that observed. It is obvious, however, that much more detailed observations are required accurately to constrain the X-ray temperatures and luminosities.

4 THE WN + O SYSTEM V 444 CYG

Central to the whole discussion are the close Wolf-Rayet binaries exemplified by V444 Cyg (WN5 + O6III), whose orbital period is $P_{orb} = 4.2$ d. Recent *ROSAT* observations by Corcoran et al. (in preparation) confirm the luminosity $L_X \sim 10^{33}$ erg s^{-1} and temperature $T \sim 1$ keV of earlier *Einstein* measurements, which are both too low to reconcile with the high velocities and mass-loss rates thought to apply to the WR component.

Here, we have followed Schmutz, Hamann & Wessolowski (1989) and assumed that $M_{WR} = 10\ M_{\odot}$ and $M_O = 25\ M_{\odot}$, $R_{WR} = 10\ R_{\odot}$ and $R_O = 10\ R_{\odot}$, and $T_{WR} = 35\,000$ K and $T_O = 40\,000$ K, giving $L_{WR}/L_O \sim 0.6$ and $\mathscr{S} \sim 1.5$, instead of the alternative smaller $R \sim 3\ R_{\odot}$ and hotter $T \sim 90\,000$ K WR star inferred by Cherepashchuk, Eaton & Khaliullin (1984) from a light-curve analysis. Although Schmutz et al. detailed the reasons why their larger, cooler WR should be preferred, St-Louis et al. (1993) have recently argued in favour of the smaller, hotter star on the basis of new measurements of the polarization eclipse. Pending the outcome of this debate, we have adopted the cooler alternative as a convenient parametrization of the general agreement in the literature that the O-star is brighter at the relevant line-driving frequencies. St-Louis et al. (1993) summarize the variety of $\dot{M}$ measurements, which range from their polarimetric value of $\dot{M} = 0.75 \times 10^{-6}\ M_{\odot}\ yr^{-1}$, through $\dot{M} = 1.0 \times 10^{-6}\ M_{\odot}\ yr^{-1}$ from the observed orbital period change, to a radio value of $\dot{M} = 2.4 \times 10^{-6}\ M_{\odot}\ yr^{-1}$. The terminal velocity is roughly 1800 km s^{-1} (Prinja, Barlow & Howarth 1990).

The simplest CAK models of the type we have used above do not apply to WR winds (Lucy & Abbott 1993), because the single-scattering momentum limit is exceeded, $\dot{M}v_{\infty} \gg L_*/c$, and diffuse radiation is ignored. Lucy & Abbott used a Monte-Carlo technique to overcome these difficulties and construct successful models. We have adopted a purely empirical approach and have used a CAK model whose line-force parameters $k = 0.55$ and $\alpha = 0.6$ are physically inconsistent but do have the merit of reproducing reasonably well the overall properties of the WR wind, giving $\dot{M} = 1.05 \times 10^{-5}\ M_{\odot}\ yr^{-1}$ and $v_{\infty} = 1526$ km s^{-1}.

Despite these limitations, we have modelled the stagnation-point flow of the WR component using the parameters above, which imply $\mathscr{S} \sim 1.5$. In these circumstances, we would have expected the companion O-star to have a significant effect on the WR wind's dynamics and this indeed turns out to be the case, as illustrated in Fig. 6. Because of the comparative strength of the WR wind, we have assumed that the stagnation point is at the surface of the O-star. The binary model has a shock-front velocity of $v(z_s) = 625$ km s^{-1}, compared with the corresponding single-star value of $v(z_s) = 1525$ km s^{-1}, while the stagnation mass-loss is close to the single-star value.

The considerably lower shock-front velocity of the binary model leads immediately to lower X-ray temperatures. As discussed by Stevens et al. (1992), the post-shock region in V444 Cyg is likely to be isothermal, with most of the thermal energy radiated away on a time-scale that is short compared to the dynamical time-scale. For gas of WN abundances, $kT_s = 2.5 v_8^2$ keV, so that the expected temperature is about 0.7 keV compared with 3.5 keV. This is likely to be the explanation of the low characteristic temperature of the X-ray emission of V444 Cyg.

5 DISCUSSION

In the very wide WR binary systems, such as HD 193793 (WR 140, Williams et al. 1990) and AS 431 (WR 147, Caillault et al. 1985), the X-ray emission is both luminous and hot, with temperatures of a few keV, and almost certainly from strongly adiabatic colliding winds. Normal theory based

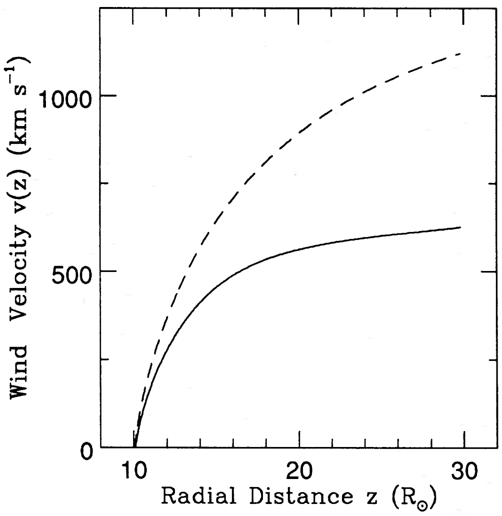

Figure 6. Comparison between the stagnation velocity laws for the WR component of V444 Cyg of the binary model, shown by the solid line, and the single-star model, shown by the dashed line. The solutions are shown up to the stagnation-point at the O-star surface at $z_s = 30\ R_\odot$, where the wind's kinetic energy is shock-converted into X-rays. With a shock-front velocity of roughly half the single-star value, the post-shock gas temperature of the binary model is about a factor of 3 lower.

on the observed properties of single stars serves reasonably well because the component stars are far apart.

Theory and observation sit less happily together in the close binary systems where the temperatures and luminosities implied by the single-star models are too high. It is near the centres of these systems that the gravitational and radiation conditions under which the wind develops are significantly different from those encountered by the material flow away from a single star. It is clearly necessary to assess how wind models are affected. The results we have found generate some optimism that properly constructed binary wind models could account for some observational properties of close binaries in which colliding winds are supposed to operate. The results are also in line with Leer & Holzer's (1980) general principles of wind models. In the most successful single-star CAK models, the critical point, at which the mass-loss rate is determined, lies very close to the photosphere and only a very close binary companion is able to change conditions here enough to affect mass-loss rate significantly. Over the large region of the stagnation-point flow outside the critical point, however, development of the velocity law is considerably affected for all the binary systems we have considered. The slower stagnation-point flow calculated for V444 Cygni probably resolves previous difficulties reconciling X-ray observations with theory.

These considerations could be of special interest to the interpretation of other close WR binaries. For example, Lewis et al. (1993) described a qualitative model of the spectroscopic properties of the 2.13-d binary CX Cephei (WN5 + O5V), in which emission lines generated in material flowing towards the stagnation-point play a significant role. The slower development of the wind we have found is certain to alter the strength and shape of the high-ionization N IV and N V lines they have observed.

There are many limitations to the work we have done, which applies only to the stagnation-point flow. For the flow at large within a binary system, and for a proper estimate of X-ray spectra and luminosities, 2D calculations are required along the lines of those reported by Luo et al. (1990) and Stevens et al. (1992) and others, although they will involve the extra difficulty in incorporating the proper binary force fields. Nor have we made any attempt to consider non-steady winds or instabilities discussed, for example, by Owocki, Castor & Rybicki (1988). Much work is also needed on the observational side. We would make an appeal, in particular, for as complete an X-ray coverage as possible of the orbits of colliding-wind binaries. The observations made so far have very inadequate phase coverage and demonstrate that only a proper investment of observing time will allow worthwhile constraints to be made on the physical parameters of systems of this type.

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