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Stability of a relativistic rotating electron–positron jet

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ABSTRACT
In the force-free approximation, an electron–positron jet is shown to be stable to axisymmetric perturbations for all velocities of longitudinal motion and rotation. The stability of the jet is a result of the shear of the magnetic field, which prohibits the convective motion of a charged fluid in the radial direction. The dispersion curves $\omega=\omega(k_{\parallel})$ have a minimum for $k_{\parallel_0}\simeq 1/R$, where R is the jet radius. This results in the accumulation of perturbations inside the jet with wavelengths of the order of the jet radius. This type of oscillatory structure is observed in kiloparsec jets, in particular in 3C 273, which is evidently an electron–positron beam.

Key words: instabilities – MHD – galaxies: jets – quasars: individual: 3C 273.

1 INTRODUCTION

One of the important problems in the physics of extragalactic jets is their stability. The stability of jets has been investigated in many previous works, with different assumptions for the velocity of matter in a jet and for the influence of the magnetic field on the dynamics of the flow. Perturbed modes have been considered both for the cylindrical geometry of the jet, and for the plane boundary between the jet and the outside medium if the wavelength is small enough [see, for example, the paper by Torricelli-Ciamponi & Petrini (1990) and the literature cited therein]. Many papers devoted to jet stability are cited by Begelman, Blandford & Rees (1984), and the recent paper by Appl & Camenzind (1992) also discusses the stability of a jet carrying electrical current in the magnetohydrodynamics (MHD) approximation. The effect, however, of a strong electric field on the stability of a relativistic flow of magnetized plasma is not yet clear. Indeed, at the speed of the hydrodynamic flow, $v\simeq c$, the electric field in a plasma with a high conductivity is very close to the magnetic field, i.e. $\boldsymbol{E}=-\boldsymbol{v}\times\boldsymbol{B}/c$. The charge density ϱ is equal to $\nabla\cdot\boldsymbol{E}/4\pi\simeq E/L$ and the current density $\boldsymbol{j}$ in a stationary flow, or for a small non-stationarity, is $\boldsymbol{j}=c\cdot(\nabla\times\boldsymbol{B})/4\pi\simeq cB/L$. The ratio of the electric force per unit volume, $\varrho\boldsymbol{E}$, to the magnetic force, $\boldsymbol{j}\times\boldsymbol{B}/c$, is therefore of the order of $E^2/B^2\simeq v^2/c^2\simeq 1$ for the relativistic case. When considering relativistic models with a significant magnetic field it is thus necessary to involve the electric force and, for the non-stationary case, to add the displacement current.

On the other hand, stationary axisymmetric hydrodynamic flows with strong electric currents have long been investigated in connection with the problem of the central engines in active galactic nuclei (Blandford 1976; Blandford & Znajek 1977; Macdonald 1984; Camenzind 1987) and also in connection with the structure of neutron-star magnetospheres (Michel 1969, 1973). It is of interest to examine the stability of the solutions obtained.

The simplest case for investigation is apparently the so-called force-free approximation, i.e. the case where the energy density of electromagnetic fields is much greater than the energy density of matter (including the rest energy). The terms in the momentum equation, which are proportional to the mass and pressure of the liquid, are therefore small compared to the electromagnetic force $\varrho\boldsymbol{E}+\boldsymbol{j}\times\boldsymbol{B}/c$, so it is possible to obtain $\varrho\boldsymbol{E}+\boldsymbol{j}\times\boldsymbol{B}/c=0$. The force-free approximation, together with the ideal hydrodynamics approximation (which represents an infinite conductivity of the plasma and consequently an absence of electric field in the frame moving with the element of the medium), can be applied to the neighbourhood of a massive black hole, which is thought to be the central engine of an active galactic nucleus. Such an approach was developed by Blandford & Znajek (1977) and Macdonald (1984) [see also chapter 4 of the book 'Black Holes: The Membrane Paradigm' by Thorne, Price & Macdonald (1986) and chapter 7 of the book 'The Physics of Black Holes' by Novikov & Frolov (1986), and references therein]. The force-free approximation is apparently also valid for the inner parts of the jet, which are close to the axis of symmetry and are connected with the black hole (Lovelace, Wang & Sulkanen 1987). Here the strong magnetic field is expected to be of the order of 10^4 G, large enough for the electron density to be 1 cm^{-3} and to screen the longitudinal (along the magnetic field) electric field component so that the MHD approximation is valid. We assume the black hole to have typical values for its mass and its rotation parameter, namely $M \simeq 10^8\ \mathrm{M}_\odot$ and $a = J/M \leq 1$, respectively. In this case the energy density of the fields is 10^{13} times greater than the rest-energy density of e^+e^- pairs, and so the force-free approximation is adequate. In the inner part of the flow, which is connected to the black hole by magnetic field lines, the particle density cannot exceed the value ϱ/e significantly because the particles do not escape the black hole, and e^+e^- pair production is only made possible by the existence of the longitudinal electric field, which vanishes for $n \gg \varrho/e$.

This paper investigates the stability of a force-free axisymmetric MHD jet. The geometry of the flow is cylindrical. We suggest that the jet propagates in a medium whose density is greater than that of the jet but whose temperature and pressure are small, so the condition of impermeability is fulfilled and the boundary is at rest. The poloidal magnetic field is assumed to be uniform and parallel to the jet axis. The fluid moves along spirals as a result of the radial electric field. We will show that under such conditions the relativistic flow is stable for axisymmetrical modes ($m=0$, where m is azimuthal wavenumber). We find the dispersion curves for such modes, $\omega = \omega(k)$, and investigate the appearance of the standing wave when $v_{\mathrm{g}} = \mathrm{d}\omega/\mathrm{d}k = 0$.

2 EQUILIBRIUM CONFIGURATION

Let us consider a flow of liquid in a force-free cylindrical jet. The Maxwell equations are

$$\nabla \cdot \boldsymbol{E} = 4\pi\varrho, \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{3}$$

$$\nabla \times \boldsymbol{B} = 4\pi \boldsymbol{j} + \frac{\partial \boldsymbol{E}}{\partial t}. \tag{4}$$

Here we use units for which $c=1$. The condition of an ideal flow is

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}, \tag{5}$$

where $\boldsymbol{v}$ is the plasma velocity. The force-free approximation is guided by the relation

$$\varrho \boldsymbol{E} + \boldsymbol{j} \times \boldsymbol{B} = 0. \tag{6}$$

First we will find the stationary configuration of the jet. In this case $\nabla \times \boldsymbol{E} = 0$ and the velocity $\boldsymbol{v}$ can be written as

$$\boldsymbol{v} = K\boldsymbol{B} + \Omega^F r \boldsymbol{e}_\phi. \tag{7}$$

Here and below, r, z and ϕ are cylindrical coordinates, $\boldsymbol{e}_r$, $\boldsymbol{e}_z$ and $\boldsymbol{e}_\phi$ are unit vectors in the cylindrical coordinate frame, $K = K(r)$, and $\Omega^F = \Omega^F(r)$. Then

$$\boldsymbol{E} = -\Omega^F r (\boldsymbol{e}_\phi \times \boldsymbol{B}) \tag{8}$$

and Ω^F can be treated as the angular rotation velocity of the magnetic field lines (Thorne et al. 1986). In the cylindrical configuration, $\boldsymbol{B} = B_z(r)\,\boldsymbol{e}_z + B_\phi(r)\,\boldsymbol{e}_\phi$, so equation (2) holds automatically. Substituting into equation (6) the ϱ-value from (1) and $\boldsymbol{j}$ from (4), we obtain

$$\boldsymbol{E}(\nabla \cdot \boldsymbol{E}) - \boldsymbol{B} \times (\nabla \times \boldsymbol{B}) = 0. \tag{9}$$

According to (8) the only non-zero component of equation (9) is the r-component. This implies

$$\Omega^F B_z \frac{\mathrm{d}}{\mathrm{d}r}(\Omega^F r^2 B_z) = B_z \frac{\mathrm{d}B_z}{\mathrm{d}r} + \frac{1}{r} B_\phi \frac{\mathrm{d}}{\mathrm{d}r}(r B_\phi). \tag{10}$$

Integration of (10) gives

$$\Omega^{F^2} r^4 B_z^2 = r^2 B_\phi^2 + \int_0^r r'^2 \frac{\mathrm{d}}{\mathrm{d}r'}(B_z^2)\,\mathrm{d}r'. \tag{11}$$

Equation (11) defines all possible solutions for the force-free electromagnetic fields for a cylindrical configuration of the magnetic tubes. If we know any two of $\Omega^F(r)$, $B_z(r)$ and $B_\phi(r)$, we can find the third from equation (11). Thus in the cylindrical geometry the solution for the fields depends on two arbitrary functions.

Equation (11) can be rewritten in terms of the charge of the jet per unit length $Q(r)$ inside a cylinder of radius r, the current $I(r)$ flowing in the negative direction along the z-axis, and the poloidal magnetic field $B_z(r)$. From equations (1) and (4) we obtain

$$Q = \tfrac{1}{2} r^2 \Omega^F B_z, \qquad I = 2\pi r B_\phi.$$

Thus equation (11) takes the form

$$4Q^2 = \frac{I^2}{4\pi^2} + \int_0^r r'^2 \frac{\mathrm{d}}{\mathrm{d}r'}(B_z^2)\,\mathrm{d}r'. \tag{12}$$

We integrate equation (11) considering the fields as continuous functions and taking into account equation (8):

$$r^2(B^2 - E^2) = 2\int_0^r r' B_z^2 \mathrm{d}r',$$

where $B^2 = B_z^2 + B_\phi^2$ and $E = E_r = -\Omega^F r B_z$. We see that the condition $|E| < |B|$ is fulfilled, i.e. there is no light surface inside the jet on which $|E| = |B|$ and $|v| = 1$. For all solutions of (11) the velocity of the plasma flow v does not therefore exceed the velocity of light. Let us note that in the force-free approximation the velocity $\boldsymbol{v}$ is formally defined by equation (5), and for the solution of equations (1)–(6) it can be greater than unity. The light surface exists, for example, in the force-free configuration found by Blandford (1976) and in the pulsar magnetosphere described by Beskin, Gurevich & Istomin (1983).

We shall make an additional remark. The case where the total charge of the jet is equal to zero is probably the most natural, i.e. $Q(R) = 0$, where R is the jet radius. If the jet has a charge that is not equal to zero, the electric field penetrates into the surrounding medium. This results in charge motion in the plasma and in a decrease of the charge of the jet. The absence of an electric current through the surrounding medium means that $I(R) = 0$. The current is closed inside the jet: it flows in different z-directions for various r-values. The conditions $Q(R) = 0$ and $I(R) = 0$ impose limitations upon the functions $\Omega^F(r)$ and $B_z(r)$. From (12) it is seen that the following condition must be fulfilled:

$$R^2 B_z^2(R) = 2\int_0^R r' B_z^2(r')\,\mathrm{d}r'. \tag{13}$$

The demand that $Q(R) = 0$ means that either $\Omega^F(R) = 0$ or $B_z(R) = 0$. The latter possibility is in contradiction with the

$B_z(r)\neq 0$ found from relation (13). Consequently, in addition to (13) we need

$$\Omega^F(R)=0. \tag{14}$$

For such a jet the boundary conditions are $B_z(R)\neq 0$, $B_\phi(R)=0$.

For $B_z=$ constant, from equation (11) we have

$$B_\phi=\pm\Omega^F rB_z. \tag{15}$$

Relation (13) is fulfilled. The criterion of zero total charge and current for the jet is given by relation (14). If $\Omega^F(R)=0$ then $I(R)=0$ and also $Q(R)=0$, but if $\Omega^F(R)\neq 0$ then $I(R)\neq 0$ and $Q(R)\neq 0$. When we later consider fluctuations, we adopt the limiting condition $B_z(r)=$ constant. We shall not demand that condition (14) be satisfied, and the results of our investigation do not depend on whether or not the requirements $Q(R)=0$ and $I(R)=0$ are fulfilled.

The equilibrium stationary configuration of the jet is shown in Fig. 1.

3 STABILITY

Let us start with an analysis of the stability of the jet configuration described above. In subsequent formulae, values referring to a non-perturbed solution will be denoted by the subscript '0', while those referring to perturbation will be denoted by the subscript '1'. After removing the quantities ϱ, $\boldsymbol{E}$ and $\boldsymbol{j}$ from the initial system of equations (1)–(6), there remain only three resultant equations:

$$\nabla\cdot\boldsymbol{B}=0, \tag{16}$$

$$\frac{\partial\boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B}), \tag{17}$$

$$(\boldsymbol{v}\times\boldsymbol{B})\nabla\cdot(\boldsymbol{v}\times\boldsymbol{B})-\boldsymbol{B}\times(\nabla\times\boldsymbol{B})-\boldsymbol{B}\times\frac{\partial}{\partial t}(\boldsymbol{v}\times\boldsymbol{B})=0. \tag{18}$$

The quantities $\boldsymbol{B}$ and $\boldsymbol{v}$ can be represented as $\boldsymbol{B}=\boldsymbol{B}_0+\boldsymbol{B}_1$ and $\boldsymbol{v}=\boldsymbol{v}_0+\boldsymbol{v}_1$. In first-order perturbation theory, equations (16)–(18) then imply

$$\nabla\cdot\boldsymbol{B}_1=0, \tag{19}$$

$$\frac{\partial\boldsymbol{B}_1}{\partial t}=\nabla\times(\boldsymbol{v}_1\times\boldsymbol{B}_0)+\nabla\times(\boldsymbol{v}_0\times\boldsymbol{B}_1), \tag{20}$$

$$\begin{aligned}&(\boldsymbol{v}_0\times\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}_1\times\boldsymbol{B}_0)+(\boldsymbol{v}_0\times\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}_0\times\boldsymbol{B}_1)\\&\quad+(\boldsymbol{v}_1\times\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}_0\times\boldsymbol{B}_0)-\boldsymbol{B}_0\times(\nabla\times\boldsymbol{B}_1)\\&\quad+(\boldsymbol{v}_0\times\boldsymbol{B}_1)\nabla\cdot(\boldsymbol{v}_0\times\boldsymbol{B}_0)-\boldsymbol{B}_0\times\frac{\partial}{\partial t}(\boldsymbol{v}_0\times\boldsymbol{B}_1)\\&\quad-\boldsymbol{B}_0\times\frac{\partial}{\partial t}(\boldsymbol{v}_1\times\boldsymbol{B}_0)-\boldsymbol{B}_1\times(\nabla\times\boldsymbol{B}_0)=0.\end{aligned} \tag{21}$$

We consider perturbations of the form

$$\boldsymbol{B}_1=\boldsymbol{b}_1(r)\cdot\exp(-\mathrm{i}\omega t+\mathrm{i}kz+\mathrm{i}m\phi),$$

$$\boldsymbol{v}_1=\boldsymbol{a}_1(r)\cdot\exp(-\mathrm{i}\omega t+\mathrm{i}kz+\mathrm{i}m\phi),$$

where m is an integer. Substituting this expression into equations (19)–(21) and using relation (7) for $\boldsymbol{v}_0$, equation (19) becomes

$$\mathrm{i}kB_{z1}+i\frac{m}{r}B_{\phi 1}+\frac{\partial B_{r1}}{\partial r}+\frac{1}{r}B_{r1}=0. \tag{22}$$

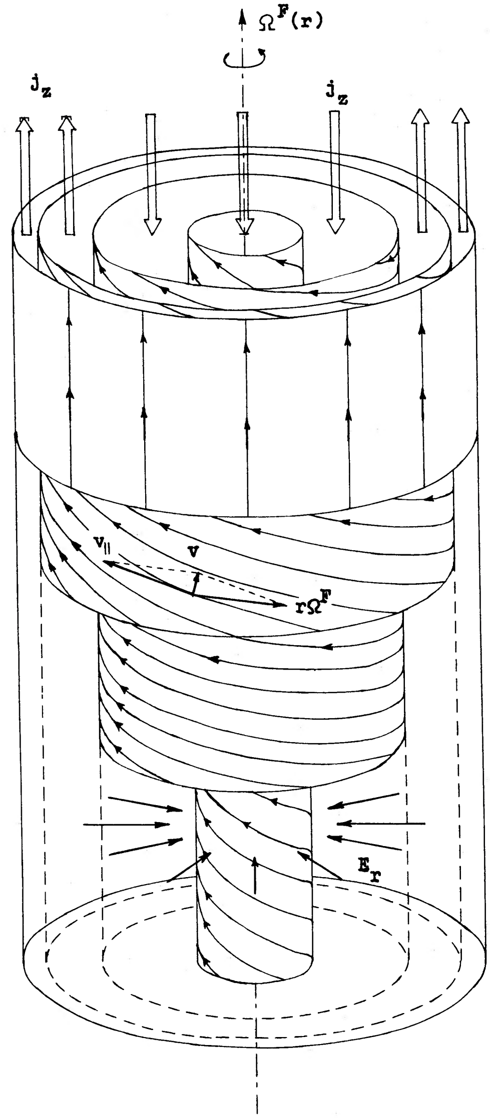

Figure 1. The equilibrium stationary configuration of a jet with a uniform poloidal magnetic field B_z. The frequency of rotation in the dimensionless units described in Section 3 is $\Omega^F=27r(1-r)$. The jet boundary for $r=1$ and three magnetic tubes for $r=1/4$, $2/3$ and $9/10$ are shown. The magnetic field lines are spiralling on a magnetic tube. Since $\Omega^F(1)=0$, the total current through the jet is equal to zero and the magnetic field is purely poloidal both at the boundary and at the axis of symmetry. The curling of magnetic field lines is a maximum for $r=2/3$, decreasing for smaller and larger radii. The density of the poloidal current j_z is negative when $r<3/4$ and positive when $1>r>3/4$. The electric field $\boldsymbol{E}$ induced by jet rotation is radial. The plasma velocity $\boldsymbol{v}$ along the magnetic tube consists of two components: rotation with angular velocity $\Omega^F(r)$, and motion along the magnetic field lines with a speed $\boldsymbol{v}_\parallel=K\boldsymbol{B}$. We see that the rotation velocity $r\Omega^F$ can exceed the speed of light c (in our case the maximum value of $r\Omega^F$ is $4c$ at $r=2/3$); nevertheless, the quantity v is restricted by c due to the existence of a predominantly toroidal magnetic field. The dispersion curves $\omega=\omega_n(k)$ for perturbations of this equilibrium state are plotted in Fig. 3.

We obtain the following expressions for components of equations (20) and (21):

$$\mathrm{i}B_{r1}\left(\omega-kKB_{z0}-\frac{m}{r}KB_{\phi0}-m\Omega^F\right)=-\mathrm{i}kB_{z0}v_{r1}-\mathrm{i}\frac{m}{r}B_{\phi0}v_{r1} \tag{23}$$

for the r-component of (20),

$$\begin{aligned}\mathrm{i}(\omega-kKB_{z0})B_{\phi1}&=\frac{\partial}{\partial r}(v_{r1}B_{\phi0}-B_{r1}KB_{\phi0}-B_{r1}\Omega^F r)-\mathrm{i}k(B_{z0}v_{\phi1}-B_{\phi0}v_{z1})\\&\quad-\mathrm{i}k(KB_{\phi0}+\Omega^F r)B_{z1}\end{aligned} \tag{24}$$

for the ϕ-component of (20),

$$\begin{aligned}\mathrm{i}B_{z1}\left(\omega-\frac{m}{r}KB_{\phi0}-m\Omega^F\right)&=\frac{1}{r}\frac{\partial}{\partial r}(rB_{z0}v_{r1}-rKB_{z0}B_{r1})\\&\quad+\mathrm{i}\frac{m}{r}(B_{z0}v_{\phi1}-B_{\phi0}v_{z1}-KB_{z0}B_{\phi1})\end{aligned} \tag{25}$$

for the z-component of (20),

$$\begin{aligned}&\frac{1}{r^2}\frac{\partial}{\partial r}[\Omega^F r^3 B_{z0}(B_{z0}v_{\phi1}-B_{\phi0}v_{z1}\\&\quad+KB_{\phi0}B_{z1}-KB_{z0}B_{\phi1}+\Omega^F rB_{z1})]\\&\quad+\mathrm{i}v_{r1}[B_{z0}^2(\omega-m\Omega^F)+B_{\phi0}(\omega B_{\phi0}+kB_{z0}\Omega^F r)]\\&\quad+\mathrm{i}B_{r1}\left[-KB_{z0}^2(\omega-m\Omega^F)-B_{\phi0}(KB_{\phi0}+\Omega^F r)\right.\\&\quad\left.\times\left(\omega+k\frac{B_{z0}}{B_{\phi0}}\Omega^F r\right)+kB_{z0}+\frac{m}{r}B_{\phi0}\right]\\&\quad-\frac{\partial}{\partial r}(B_{z0}B_{z1})-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2B_{\phi0}B_{\phi1})=0\end{aligned} \tag{26}$$

for the r-component of (21) after some reduction,

$$\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}(\Omega^F r^2 B_{z0})(-v_{r1}+KB_{r1})B_{z0}+B_{r1}\frac{1}{r}\frac{\partial}{\partial r}(rB_{\phi0})\\&\quad+\mathrm{i}\omega B_{z0}(B_{z0}v_{\phi1}-B_{\phi0}v_{z1})+\mathrm{i}B_{z0}B_{\phi1}(k-\omega KB_{z0})\\&\quad+\mathrm{i}B_{z0}B_{z1}\left(\omega\Omega^F r+\omega KB_{\phi0}-\frac{m}{r}\right)=0\end{aligned} \tag{27}$$

for the ϕ-component of (21), and

$$\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}(\Omega^F r^2 B_{z0})(v_{r1}B_{\phi0}-KB_{\phi0}B_{r1}-\Omega^F rB_{r1})+B_{r1}\frac{\partial B_{z0}}{\partial r}\\&\quad-\mathrm{i}\omega B_{\phi0}(B_{z0}v_{\phi1}-B_{\phi0}v_{z1})-\mathrm{i}B_{\phi0}B_{\phi1}(k-\omega KB_{z0})\\&\quad-\mathrm{i}B_{\phi0}B_{z1}\left(\omega\Omega^F r+\omega KB_{\phi0}-\frac{m}{r}\right)=0\end{aligned} \tag{28}$$

for the z-component of (21).

Of the seven equations (22)–(28), only five are independent. If we add equation (27), multiplied by $B_{\phi0}$, to equation (28), multiplied by B_{z0}, and take into account the relation (10) between $B_{\phi0}$ and B_{z0}, we obtain that this sum vanishes identically. Thus equations (27) and (28) are linearly dependent. Expression (19) follows from (20), so we need not consider (22). Thus we have a system of the five equations (23)–(27) for five variables B_{r1}, B_{z1}, $B_{\phi1}$, v_{r1} and $B_{z0}v_{\phi1}-B_{\phi0}v_{z1}$. From equations (23), (27) and (25), we express v_{r1} in terms of B_{r1} and B_{z1}, and $B_{z0}v_{\phi1}-B_{\phi0}v_{z1}$ in terms of B_{r1} and $B_{\phi1}$. Equation (24) then gives an expression for $B_{\phi1}$ in terms of B_{r1}:

$$\begin{aligned}B_{\phi1}&=\frac{\mathrm{i}}{\omega^2-k^2-m^2/r^2}\left[-\frac{k}{r}\frac{\partial}{\partial r}(\Omega^F r^2 B_{z0})B_{r1}\right.\\&\quad\times\frac{\omega-m\Omega^F}{kB_{z0}+mB_{\phi0}/r}-\frac{k}{r}B_{r1}\frac{1}{B_{z0}}\frac{\partial}{\partial r}(rB_{\phi0})\\&\quad\left.+\omega\frac{\partial}{\partial r}\left(B_{r1}\frac{\Omega^F rkB_{z0}+\omega B_{\phi0}}{kB_{z0}+mB_{\phi0}/r}\right)-\frac{m}{r^2}\frac{\partial}{\partial r}(rB_{r1})\right].\end{aligned} \tag{29}$$

Substituting into equation (26) the expressions for $B_{z0}v_{\phi1}-B_{\phi0}v_{z1}$, B_{z1} and v_{r1}, and making some reductions, we obtain

$$\begin{aligned}&\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\Omega^F r^3 B_{z0}\left[-\frac{\omega}{k}B_{\phi1}+\frac{\mathrm{i}}{k}\frac{\partial}{\partial r}\left(B_{r1}\frac{\Omega^F rkB_{z0}+\omega B_{\phi0}}{kB_{z0}+mB_{\phi0}/r}\right)\right]\right\}\\&\quad+\frac{m}{k}\frac{\partial}{\partial r}\left(\frac{1}{r}B_{z0}B_{\phi1}\right)-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2B_{\phi0}B_{\phi1})\\&\quad-\frac{\mathrm{i}}{k}\frac{\partial}{\partial r}\left(B_{z0}\frac{1}{r}\frac{\partial}{\partial r}(rB_{r1})\right)-\mathrm{i}B_{r1}\frac{1}{kB_{z0}+mB_{\phi0}/r}\\&\quad\times\left[B_{z0}^2(\omega-m\Omega^F)^2+B_{\phi0}^2\left(\omega+k\frac{B_{z0}}{B_{\phi0}}\Omega^F r\right)^2\right.\\&\quad\left.-\left(kB_{z0}+\frac{m}{r}B_{\phi0}\right)^2\right]=0.\end{aligned} \tag{30}$$

When (29) is used to express $B_{\phi1}$ in terms of B_{r1}, equation (30) becomes the second-order ordinary differential equation of the variable B_{r1}. Solving this equation, we can find the radial dependence of perturbations propagating along the jet. $B_{r1}(r)$ must fulfil the boundary conditions for $r=0$ and $r=R$. For $r=0$, B_{r1} must be regular. If $|m|\neq1$, this condition can be strengthened to become $B_{r1}|_{r=0}=0$. The latter expression follows from the continuity of the vector $\boldsymbol{B}_1$ near the symmetry axis, and must be automatically fulfilled for solutions in which B_{r1} is regular when $r=0$. Straightforward analysis of the behaviour of the solutions of the system (29), (30) near $r=0$ confirms the two kinds of boundary conditions mentioned above. The boundary condition for $r=R$ must be derived from the rigidity of the jet wall $v_{r1}|_{r=R}=0$, which has been assumed above. Taking into account (23), this gives $B_{r1}|_{r=R}=0$. Thus we have to investigate the solutions of the system (29), (30) taking into account the boundary conditions

$$\begin{aligned}&B_{r1}|_{r=0}\quad\text{is regular,}\\&B_{r1}|_{r=R}=0.\end{aligned} \tag{31}$$

It is noteworthy that more real boundary conditions must be considered at the perturbed surface of the jet by writing the pressure equality on both sides of the jet surface and the continuity of the displacement of any fluid elements across this surface, which is similar to the treatment by Appl & Camenzind (1992) of the non-relativistic flow without an electric field. When considering a complete system of hydrodynamic equations, however, we need at least one non-force-free medium on both sides of the jet surface in order that such a boundary condition might be written. This is beyond the scope of this paper. In the framework of the force-free approximation, the conditions noted above will be automatically fulfilled for all solutions of equations (29) and (30). For this reason one cannot investigate the possible development of Kelvin–Helmholtz instability on the boundary between a jet and ambient material if the force-free approximation is adopted.

In equations (29) and (30) there is no parameter $K(r)$ which determines the component of $\boldsymbol{v}_0$ parallel to $\boldsymbol{B}_0$. This is why the results of our investigation of stability do not depend on the values and profiles of the longitudinal velocity.

We further restrict ourselves to the jet model with a homogeneous magnetic field $B_{z0}=$constant, and hence use expression (15) for $B_{\phi 0}$. The choice of the sign in formula (15) depends on the choice of the direction of the poloidal component of the flow in the jet. This keeps equations (29) and (30) unchanged under reversal of the signs of $B_{\phi 0}$ and k. For definiteness we adopt the sign '+' in (15) and consider arbitrary values of k. A substitution of (29) into (30) using (15) then leads to the following second-order differential equation, where the prime denotes differentiation with respect to r:

$$B''_{r1}+B'_{r1}\left(\frac{1}{r}-\frac{2m^2}{r^3}\frac{1}{\omega^2-k^2-m^2/r^2}-m\frac{\mathrm{d}\Omega^F}{\mathrm{d}r}\frac{1}{k+m\Omega^F}\right)$$
$$+B_{r1}\left[\omega^2-k^2-\frac{m^2}{r^2}-m(k+m\Omega^F)\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{(k+m\Omega^F)^2}\frac{\mathrm{d}\Omega^F}{\mathrm{d}r}\right)\right]$$
$$+\frac{1}{k-\omega+2m\Omega^F-\Omega^{F^2}r^2(\omega+k)}(A_1B'_{r1}+A_2B_{r1})=0. \quad (32)$$

The quantities A_1 and A_2 are

$$A_1=-2\Omega^{F^2}r(\omega+k)+\frac{\mathrm{d}\Omega^F}{\mathrm{d}r}\frac{\omega+k}{k+m\Omega^F}$$
$$\times(m-m\Omega^{F^2}r^2-2kr^2\Omega^F), \quad (33)$$

$$A_2=2m\left(\frac{\mathrm{d}\Omega^F}{\mathrm{d}r}\right)^2\frac{\Omega^F r^2(\omega+k)-m}{k+m\Omega^F}+\frac{\mathrm{d}\Omega^F}{\mathrm{d}r}\frac{1}{k+m\Omega^F}$$
$$\times\left[\!\left[\frac{2m}{r}\frac{1}{\omega^2-k^2-m^2/r^2}\left\{\frac{m^2}{r^2}(3m\Omega^F-\omega+k)\right.\right.\right.$$
$$\left.-\Omega^F r^2(\omega^2-k^2)\left[\Omega^F(\omega+k)+\frac{m}{r^2}\right]\right\}+\frac{m}{r}(\omega-k)$$
$$+2\Omega^F rk(\omega+k)+7\Omega^{F^2}rm(\omega+k)\right]\!\!\right]$$
$$+\frac{2m}{r^2}\Omega^F+\frac{1}{r^2}(\omega-k)+3\Omega^{F^2}(\omega+k)+2\frac{m^2}{r^4}\frac{1}{\omega+k}$$
$$+2\frac{m^2}{r^2}\frac{1}{\omega^2-k^2-m^2/r^2}\left[\frac{m^2}{r^4(\omega+k)}-\Omega^{F^2}(\omega+k)\right]. \quad (34)$$

In the case of axisymmetric modes, when $m=0$, this equation is substantially simplified and can be reduced to the form

$$\frac{\omega+k}{\omega-k}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[\Omega^{F^2}r^5\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}B_{r1}\right)\right]+\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rB_{r1})\right]$$
$$+B_{r1}[\omega^2-k^2+\Omega^{F^2}r^2(\omega+k)^2]=0. \quad (35)$$

We will demonstrate that the edge problem for equation (35) with the boundary conditions (31) has a trivial solution when the imaginary part of ω does not vanish and k is real. This implies that the jet is stable with respect to the axisymmetric modes. In order to prove this, we multiply (35) by the complex conjugated value B^*_{r1} and integrate the expression obtained over r with a weight r from $r=0$ to $r=R$.

Let us introduce the new dimensionless variables $\omega'=\omega R$, $k'=kR$ and $r'=r/R$, which will be written henceforth without the primes. We obtain the following equation:

$$\frac{\omega+k}{\omega-k}\int_0^1 B^*_{r1}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[\Omega^{F^2}r^5\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}B_{r1}\right)\right]\mathrm{d}r$$
$$+\int_0^1 B^*_{r1}r\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rB_{r1})\right]\mathrm{d}r$$
$$+\int_0^1 r|B_{r1}|^2[\omega^2-k^2+\Omega^{F^2}r^2(\omega+k)^2]\,\mathrm{d}r=0.$$

After partial integration of the first and second terms, and taking into consideration expression (31) in its stronger form $B_{r1}|_{r=0}=0$, we arrive at the expression

$$-\frac{\omega+k}{\omega-k}I_1-I_2+(\omega^2-k^2)\,I_3+(\omega+k)^2I_4=0,$$

where

$$I_1=\int_0^1\Omega^{F^2}r^5\left|\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}B_{r1}\right)\right|^2\mathrm{d}r,$$

$$I_2=\int_0^1\frac{1}{r}\left|\frac{\mathrm{d}}{\mathrm{d}r}(rB_{r1})\right|^2\mathrm{d}r,$$

$$I_3=\int_0^1 r|B_{r1}|^2\,\mathrm{d}r,$$

$$I_4=\int_0^1\Omega^{F^2}r^3|B_{r1}|^2\,\mathrm{d}r.$$

Transformation of this equation to make it cubical with respect to ω leads to

$$\omega^3-\omega^2k\frac{1-\beta}{1+\beta}-\omega\left(k^2+\alpha\frac{1+\gamma}{1+\beta}\right)+k^3\frac{1-\beta}{1+\beta}+\alpha k\frac{1-\gamma}{1+\beta}=0, \quad (36)$$

where $\alpha = I_2/I_3$, $\beta = I_4/I_3$ and $\gamma = I_1/I_2$. The integrals I_1, I_2, I_3 and I_4 are real positive values depending on ω and k by means of B_{r1}. If there exist a ω and a k at which the edge problem (35), (31) has a non-trivial solution, then this ω must be the root of the cubical equation (36) with real coefficients, and, moreover, none of the integrals $I_{1,2,3,4}$ can be equal to zero. We need to prove that the cubical equation (36) has no roots with non-zero imaginary parts when α, β, $\gamma > 0$ and k is any real number. Let us replace ω by $\omega' + k(1-\beta)/3(1+\beta)$. Expression (36) then becomes

$$\omega'^3 + p\omega' + q = 0,$$

where the coefficients p and q have the forms

$$p = -\frac{1}{3}k^2\left(\frac{1-\beta}{1+\beta}\right)^2 - k^2 - \alpha\frac{1+\gamma}{1+\beta},$$

$$q = \frac{2}{3}k^3\frac{1-\beta}{1+\beta}\left[1 - \frac{1}{9}\left(\frac{1-\beta}{1+\beta}\right)^2\right] + \frac{2}{3}\frac{\alpha k}{(1+\beta)^2}(1-2\gamma+2\beta-\gamma\beta).$$

The reduced cubical equation obtained above would have only real roots if $Q \le 0$, where

$$Q = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3.$$

Substitution of the expressions for p and q, after some simple but cumbersome calculations, gives

$$Q = \frac{k^6}{27(1+\beta)^5}[-s^3(1+\gamma)^3(1+\beta)^2 - (1+\beta)16\beta^2 + \frac{8}{3}sA + s^2(1+\beta)B], \quad (37)$$

where $s = \alpha/k^2 > 0$,

$$A = 3\beta(1-\beta) - 12\beta^3 - 6\beta^4 - 6\gamma - 12\gamma\beta - 3\gamma\beta^2 + 3\gamma\beta^3,$$

$$B = -1 - 20\gamma - 38\gamma\beta + 8\beta + 8\beta^2 + 8\gamma^2 + 8\beta\gamma^2 - \gamma^2\beta^2 - 20\beta^2\gamma.$$

The sign of Q depends on the sign of $Q' = 27(1+\beta)^4 Q/k^6$. The expression for Q' can be transformed to the form

$$Q' = -(1+\beta)s[\gamma(s\gamma-4)^2 + (s-4\beta)^2 + 3s^2\gamma(1+\gamma)] - (s\gamma\beta + s - 4\beta)^2 - s^2\gamma(20 + 36\beta + 20\beta^2). \quad (38)$$

For all s, γ, $\beta > 0$, one can see from (38) that Q' is negative, as required.

4 DISPERSION CURVES

The boundary conditions for B_{r1} cannot be satisfied for all values of ω and k. It is necessary that some dispersion relation $\omega = \omega(k)$ is obeyed, where ω and k are real, in accordance with the existence (proved above) of a non-trivial solution of the edge problem for equation (35). When $\Omega^F = 0$, equation (35) reduces to the Bessel equation

$$B''_{r1} + \frac{1}{r}B'_{r1} - \frac{1}{r^2}B_{r1} + B_{r1}(\omega^2 - k^2) = 0.$$

Its solutions that satisfy the boundary conditions have the form (in dimensionless variables)

$$B_{r1} = CJ_1(\lambda_n r),$$

where C = constant, and λ_n is the nth root of the first-order Bessel function $J_1(x)$. In this case the dispersion relation is

$$\omega^2 = k^2 + \lambda_n^2.$$

Every number n corresponds to the nth branch of the dispersion curve $\omega_n(k) = \omega$. In the case $\Omega^F(r) \ne 0$, the dispersion curves have been calculated numerically by solving equation (35). The results of calculations for two distinct profiles $\Omega^F(r)$ are shown in Figs 2 and 3, where the first three branches of the function $\omega_n(k)$ are plotted. Equation (35) does not change when ω and k are simultaneously replaced by $-\omega$ and $-k$, so we have plotted the dispersion curves for positive frequencies of ω only. The branches situated in the lower half of the (ω, k) plane can be obtained from the ones situated in the upper half by means of central symmetry reflection with respect to the origin. $\Omega^F(r) = \Omega(1-r^2)$ in Fig. 2, while $\Omega^F(r) = \Omega r(1-r)$ in Fig. 3. Here Ω is a parameter that can vary from 0 to $+\infty$. The curves are plotted for three values of Ω.

The notable feature of these curves is the existence of a minimum in the dependence $\omega = \omega(k)$, where $k \ne 0$. This is a common phenomenon for all the branches, and is caused by the terms in equation (35) depending linearly on ω and k. Because of this, perturbations with $k \approx k_{\min}$ do not propagate, since the group velocity $\mathrm{d}\omega/\mathrm{d}k$ vanishes for $k = k_{\min}$. This wave packet undergoes only diffuse broadening due to the finite value of $\mathrm{d}^2\omega/\mathrm{d}k^2$ for $k = k_{\min}$. Let us consider an example of disturbance propagation along the jet. Suppose that at the moment $t = 0$ and at the point $z = 0$ a source of perturbations with a spectrum $A(\omega)$ starts to operate. A mode with a dispersion relation $\omega = \omega(k)$ is excited by this

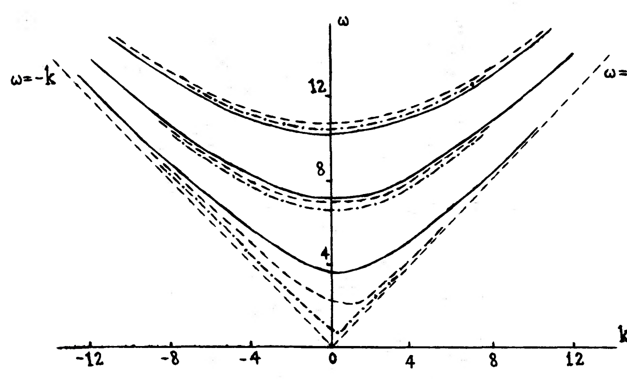

Figure 2. The dispersion curves $\omega = \omega_n(k)$ for $\Omega^F(r) = \Omega(1-r^2)$. The first three branches are shown. The solid line corresponds to $\Omega = (3\sqrt{3})/4$, the dashed line to $\Omega = 6\sqrt{3}$, and the dot-dashed line to $\Omega = 60\sqrt{3}$. For $\Omega = (3\sqrt{3})/4$ the values of $k_{\min}$ and $\Delta\omega/k^2_{\min}$ are, respectively, in the dimensionless units described in the text, 0.23 and 0.12 for the first branch, -0.07 and 0.07 for the second branch, and -0.09 and 0.049 for the third branch. For $\Omega = 6\sqrt{3}$, the values are 0.89 and 0.24 for the first branch, 0.21 and 0.07 for the second branch, and 0.02 and 0.15 for the third branch; for $\Omega = 60\sqrt{3}$, they are 0.35 and 0.77 for the first branch, 0.07 and 0.06 for the second branch, and 0.06 and 0.056 for the third branch. The values of the parameter Ω are chosen such that the maximum rotational velocities of the magnetic field lines inside the jet are $c/2$, $4c$ and $40c$, respectively.

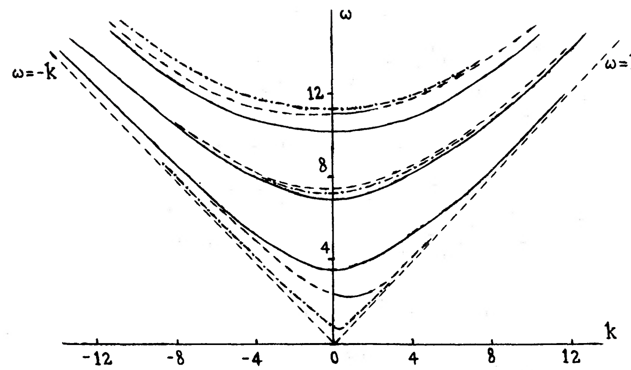

Figure 3. The dispersion curves $\omega=\omega_n(k)$ for $\Omega^F(r)=\Omega r(1-r)$. The solid line corresponds to $\Omega=27/8$, the dashed line to $\Omega=27$, and the dot-dashed line to $\Omega=270$. The values of the parameter Ω are chosen such that the maximum rotational velocities of the magnetic field lines inside the jet are $c/2$, $4c$ and $40c$, respectively. For $\Omega=27/8$ the values of $k_{\rm min}$ and $\Delta\omega/k^2_{\rm min}$ are, respectively, in the dimensionless units described in the text, 0.11 and 0.12 for the first branch, -0.06 and 0.08 for the second branch, and -0.035 and 0.05 for the third branch. For $\Omega=27$, the values are 0.95 and 0.23 for the first branch, 0.23 and 0.07 for the second branch, and 0.02 and 0.14 for the third branch; for $\Omega=270$, they are 0.35 and 0.80 for the first branch, 0.06 and 0.07 for the second branch, and 0.04 and 0.06 for the third branch.

source, i.e. at the point $z=0$ the disturbance can be written as

$$B_{r1}=\int_{-\infty}^{+\infty}A(\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega, \tag{39}$$

where the r-dependence of B_{r1} is omitted for simplicity. Strictly speaking, expression (39) is valid only for fixed r, and $A(\omega)$ is the coefficient of the expansion of a harmonic with a given frequency ω to the radial modes, which are determined by solving the edge problem (35), (31). When the branch $\omega=\omega(k)$ is given, the main contribution to B_{r1} is provided by the frequencies near $\omega_{\rm min}$, so the dependence $\omega=\omega(k)$ can be approximated near $k_{\rm min}$ by the quadratic law

$$\omega=\omega_0+\Delta\omega\left(\frac{k-k_0}{k_0}\right)^2.$$

The minimum value of $\omega(k)$ is attained for $k=k_0$ and equals ω_0. In the neighbourhood of the minimum point,

$$\frac{\mathrm{d}\omega}{\mathrm{d}k}=2\frac{\Delta\omega}{k_0}\frac{k-k_0}{k_0},\qquad \frac{\mathrm{d}^2\omega}{\mathrm{d}k^2}=2\frac{\Delta\omega}{k_0^2}.$$

For perturbations propagating in the direction of positive z, we may write

$$B_{r1}=\int_{-\infty}^{+\infty}A(\omega)\exp\left(-\mathrm{i}\omega t+\mathrm{i}k_0z+\mathrm{i}k_0z\sqrt{\frac{\omega-\omega_0}{\Delta\omega}}\right)\mathrm{d}\omega$$

and, when $\omega=\omega_0+\omega_1$ is substituted, the integral becomes

$$B_{r1}=\int_{-\infty}^{+\infty}A(\omega_0+\omega_1)\exp\left(-\mathrm{i}\omega_1t+\mathrm{i}k_0z\sqrt{\frac{\omega_1}{\Delta\omega}}\right)\mathrm{d}\omega_1 \times\exp(-\mathrm{i}\omega_0t+\mathrm{i}k_0z). \tag{40}$$

In expression (40) we should take the branch that has the positive imaginary part of the square root when $\omega_1<0$, so that the waves with $\omega<\omega_0$ decay as they propagate to $z>0$. $A(\omega)$ must have no irregularities at $\mathrm{Im}\,\omega>0$ when considered as a function of the complex argument of ω. Only in such a case, according to (39), does the source induce no perturbations at $t<0$. When $t<0$ one can obtain from (40) that $B_{r1}(t,z)=0$, while at $t>0$

$$\begin{aligned}B_{r1}={}&2\mathrm{i}\int_0^{\infty}A(\omega_0+\omega_1)\exp(-\mathrm{i}\omega_1t)\sin\left(k_0z\sqrt{\frac{\omega_1}{\Delta\omega}}\right)\mathrm{d}\omega_1\\&\times\exp(-\mathrm{i}\omega_0t+\mathrm{i}k_0z)-2\pi\mathrm{i}\exp(-\mathrm{i}\omega_0t+\mathrm{i}k_0z)\\&\times\sum_{\mathrm{Im}\,\omega'<0}\mathrm{res}|_{\omega_1=\omega'}\Bigg[A(\omega_1+\omega_0)\\&\times\exp\left(-\mathrm{i}\omega_1t+\mathrm{i}k_0z\sqrt{\frac{\omega_1}{\Delta\omega}}\right)\Bigg],\end{aligned} \tag{41}$$

where the sum is taken at irregular points of $A(\omega)$ when $\mathrm{Im}\,\omega<0$. Suppose that the spectrum $A(\omega)$ is regular for $\omega_1=0$; then the coefficients in (41) will decay exponentially when $t\to\infty$ and $z\to\infty$. It is therefore only necessary to consider the first term on the right-hand side of (41) to obtain B_{r1} at large t and z. Equation (41) transforms here to the form

$$B_{r1}=-\frac{2}{t}\exp(\mathrm{i}p^2)\int_{-\infty}^{+\infty}A\left[\omega_0+\frac{1}{t}(s-p)^2\right] \times(s-p)\exp(-\mathrm{i}s^2)\,\mathrm{d}s, \tag{42}$$

where $p=k_0z/2\sqrt{\Delta\omega t}$. Only values of $|s|\simeq1$ contribute to the integral in (42). This corresponds to the fact that only the frequency interval from ω_0 to $\omega_0+\max\{1,p^2\}/t$ is involved in the calculation of B_{r1} in expression (42). When $t\gg1/\omega_0$ and $z/t\ll\sqrt{4\Delta\omega\omega_0}/k_0$, the width of this frequency interval is much smaller than ω_0, so one can use an approximate expression for $\omega(k)$, as was done in equations (40)–(42). When $p\gg1$, the frequencies near ω_0+p^2/t make the main contribution to (42). This corresponds to the propagation of perturbations with a group velocity of $z/t=\mathrm{d}\omega/\mathrm{d}k$. When t increases at a fixed z, p diminishes and the frequencies making the main contribution to B_{r1} approach ω_0; $\mathrm{d}\omega/\mathrm{d}k$ diminishes too. When $p\ll1$, the frequencies involved approach ω_0 with a width of the order of $1/t$, and the point z is situated in a region of diffuse broadening of the perturbations produced by the source at $z=0$. Thus the parameter p determines the source of the contribution to $B_{r1}(t,z)$: if $p\gg1$, the wave propagates with a group velocity of $\mathrm{d}\omega/\mathrm{d}k$, and if $p\ll1$ it gradually broadens with a diffusion coefficient of $\Delta\omega/k_0^2$ for a wave packet with a frequency ω_0.

Let us, for example, consider the propagation of the disturbances when $B_{r1}|_{z=0}$ is given by the following expression:

$$B_{r1}|_{z=0}=\begin{cases}0 & t<0,\\ A_0\exp(-\mathrm{i}\omega_0 t) & 0<t<t_0,\\ 0 & t>t_0\end{cases}$$

for $B_{r1}|_{t=0}=0$. One can obtain for $B_{r1}(t,z)$

$$B_{r1}(t,z)=A_0\exp(-\mathrm{i}\omega_0 t+\mathrm{i}k_0 z)\times\left\{1-\frac{2}{1+\mathrm{i}}\left[C\left(\frac{k_0 z}{\sqrt{2\pi\Delta\omega t}}\right)+\mathrm{i}S\left(\frac{k_0 z}{\sqrt{2\pi\Delta\omega t}}\right)\right]\right\}$$

when $0<t<t_0$, and

$$B_{r1}(t,z)=\frac{2A_0}{1+\mathrm{i}}\exp(-\mathrm{i}\omega_0 t+\mathrm{i}k_0 z)\times\left\{C\left[\frac{k_0 z}{\sqrt{2\pi\Delta\omega(t-t_0)}}\right]-C\left(\frac{k_0 z}{\sqrt{2\pi\Delta\omega t}}\right)+\mathrm{i}S\left[\frac{k_0 z}{\sqrt{2\pi\Delta\omega(t-t_0)}}\right]-\mathrm{i}S\left(\frac{k_0 z}{\sqrt{2\pi\Delta\omega t}}\right)\right\}$$

when $t>t_0$.

Here

$$C(x)=\int_0^x\cos\left(\frac{\pi}{2}x^2\right)\mathrm{d}x$$

and

$$S(x)=\int_0^x\sin\left(\frac{\pi}{2}x^2\right)\mathrm{d}x$$

are Fresnel integrals. This example clearly illustrates the behaviour of small perturbations, as described above.

In general, the source produces disturbances that correspond to all branches of the dispersion curve $\omega(k)$. For the branches with higher frequencies the diffusion coefficient $\Delta\omega/k_0^2$ diminishes, and the amplitude of perturbation must decrease too because of simultaneous excitation of several modes, of which only one has $\mathrm{d}\omega/\mathrm{d}k=0$ for a certain value of k.

The quantities $k_{\min}$ and $\Delta\omega/k_0^2$ for the first three branches are shown in Figs 2 and 3 (in the dimensionless units introduced above). The characteristic values of $k_{\min}$ are of the order of $1/R$. The wavelength of the stagnant disturbance accumulated inside the jet is therefore of the order of R. Such a phenomenon is observed in the optical band for a jet in 3C 273, where a modulation of the brightness with a wavelength comparable to the jet radius is clearly visible (see the paper by Morrison & Sadun 1992 and fig. 2 therein). The model of the spectrum of the jet radiation in a broad frequency range (from X-rays to radio) described by Morrison & Sadun (1992) implies that the 3C 273 jet is an electron–positron beam. The results of our consideration of the behaviour of small perturbations can be applied to such beams.

5 SUMMARY

We have shown that a jet with a longitudinal electric current is stable within the force-free approximation for all velocities of motion and rotation. The current flowing along the jet creates a toroidal magnetic field and the field lines become spiral. The curling of the magnetic field lines changes with radius. We therefore have a shear of the magnetic field which prevents a possible instability. The important point is that the fluid pressure is low compared to that of the electromagnetic field. Even a small shear stabilizes the convective motion. We also find that the dispersion curves $\omega=\omega_n(k_\parallel)$ have minima for certain values of $k_\parallel=k_{\parallel_0}$. This means that such oscillations form a standing wave with a wave vector $k_{\parallel_0}$. The amplitude of the standing wave will be larger than the amplitudes of other waves because it experiences a dispersion spreading only. This phenomenon is caused by the fact that oscillations with wave vectors less than and greater than $k_{\parallel_0}$ propagate in opposite directions. A standing wave with a wavelength of the order of the jet radius is obviously observed in 3C 273.

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