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Magnetic field dragging in accretion discs

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ABSTRACT
We consider a thin accretion disc of half-thickness H, vertically threaded by a magnetic field. The field is due to contributions from both the disc current and an external current (giving rise to a uniform external field). We derive an integro-differential equation for the evolution of the magnetic field, subject to magnetic diffusivity η and disc accretion with radial velocity v_r. The evolution equation is solved numerically, and a steady state is reached. The evolution equation depends upon a single, dimensionless parameter $\mathscr{D}=2\eta/(3H|v_r|)=(R/H)(\eta/\nu)$, where the latter equality holds for a viscous disc having viscosity ν. At the disc surface, field lines are bent by angle i from the vertical, such that $\tan i=1.52\mathscr{D}^{-1}$. For values of $\mathscr{D}$ somewhat less than unity, the field is strongly concentrated towards the disc centre, because the field lines are dragged substantially inwards.

Key words: accretion, accretion discs – magnetic fields – MHD – ISM: jets and outflows.

1 INTRODUCTION

We investigate the time evolution of an axisymmetric magnetic field that threads an accretion disc. The field is assumed to be weak in the sense that the rotation velocity in the disc remains approximately Keplerian. It is also assumed that the magnetic field does not contribute to angular momentum transport in the disc, in the sense that the field outside the disc remains force-free, and that each field line intercepts the disc at only one radius.

Previous work in this area was carried out by van Ballegooijen (1989), who considered the time-evolution of an axisymmetric magnetic field which was self-generated by azimuthal currents within an accretion disc in a close binary system. In this case, of course, the fields decay, and van Ballegooijen was able to estimate decay time-scales and magnetic field structures of the various modes of decay. A more interesting problem arises when the accretion disc is initially threaded by an externally generated field. In this case, for a steady disc, there can be a final, steady configuration of magnetic field, in which the inward dragging of field lines by the disc is balanced everywhere by the outward movement of field lines due to magnetic diffusivity. This is of particular relevance to models of magnetically generated outflows or jets which presuppose the existence of a global magnetic field threading an accretion disc (Blandford & Payne 1982; Pudritz & Norman 1986; Lovelace, Wang & Sulkanen 1987; Königl 1989; see also the review by Pringle 1993).

In his approach to the problem, van Ballegooijen used $B_z^{\rm d}(R,t)$ as the dependent variable which describes the vertical component of the magnetic field in the disc plane ($z=0$), as a function of radius R and time t. For this approach to succeed it is necessary, in the induction equation, to obtain an estimate of the quantity $\partial B_R/\partial z$ at $z=0$, and this is simply approximated as $\partial B_R/\partial z|_{z=0}\simeq B_R^{\rm s}/H$, where $B_R^{\rm s}$ is the radial field component on the upper disc surface, and $H(\ll R)$ is the disc semithickness. The problem now is to write $B_R^{\rm s}$ in terms of $B_z^{\rm d}$. Van Ballegooijen's solution to this (see also Tagger et al. 1990) is to treat the solution for the field structure external to the disc as a potential problem. This turns out to be an acceptable approximation locally within the disc. However, in a global sense, a problem arises outside the disc radial boundaries. In adopting the approach, we write (outside the disc for $z>0$, say) $\boldsymbol{B}=-\nabla\Phi$, with $\nabla^2\Phi=0$. If the disc fills the $z=0$ plane from $z=0$ to $z=\infty$, then the problem can be simply solved in the upper half-plane, using the 'magnetic surface charge' boundary condition that $B_z^{\rm d}=-\partial\Phi/\partial z$ is known on $z=0^+$, and using appropriate symmetry to obtain the relevant solution in $z<0$. However, a problem arises if the disc does not completely fill the $z=0$ plane, as the solution now involves mixed boundary conditions, and we must set, by symmetry, $B_R=-\partial\Phi/\partial R=0$ on $z=0$ in those regions where there is no disc. Moreover, if the disc has a central hole (i.e., if the disc extends only in the

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range $R_{in} \lesssim R \lesssim R_{out}$, with $R_{in} > 0$), the solution to the potential problem is no longer unique, as the solution space is now multiply connected. Indeed, in this case, the uniqueness of the solution now hinges on the knowledge of the net azimuthal current through the disc. Van Ballegooijen manages to sidestep these problems, by assuming boundary conditions on $z=0$ which, while not fully appropriate to the problem he considered, do not substantially affect the estimates of the decay time-scales he obtains.

In view of the difficulties inherent in previous methods for dealing with this problem, we discuss here an alternative approach which allows us to obtain a solution to the problem in a more direct manner, without problems with the choice of appropriate boundary conditions. Our method of solution is described in Section 2, and the key to the method lies in the choice of dependent variable (equation 5). In Section 3, we describe how the method can be implemented to obtain numerical solutions. In Section 4, we illustrate the method by solving this problem of a steady accretion disc initially threaded by a uniform vertical field. In Section 5, we discuss the light these solutions shed on the various attempts at modelling magnetically powered bipolar outflows.

2 THE BASIC EQUATIONS

We start with the usual MHD equations in the form

$$\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{1}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J}, \tag{2}$$

and

$$\boldsymbol{J} = \sigma\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right). \tag{3}$$

The conductivity, σ, is related to the magnetic diffusivity, η, by the usual equation,

$$\eta = c^2/4\pi\sigma, \tag{4}$$

and is non-zero only within the disc. We take the disc to extend between $z = \pm H(R)$, using cylindrical polar co-ordinates (R, ϕ, z), and assume the thin-disc approximation, i.e. that $H \ll R$. At finite radii outside the disc, the current density $\boldsymbol{J}$ is zero, producing a so-called potential field which is force-free.

We assume the magnetic field to be purely poloidal (i.e. $B_\phi = 0$). Such a situation is, strictly speaking, impossible, because the radial component of the field will be sheared into an azimuthal component by the differential rotation of the disc. However, the rate at which the process occurs depends on the disc thickness and vanishes in the limit of a zero-thickness disc. This effect adds considerable complexity to the problem, and we ignore it here. Thus we may write the field in terms of a stream function $\psi(R, z)$, which is identical to the azimuthal component, A_ϕ, of the usual vector potential, in the form

$$\boldsymbol{B} = \nabla \times [\psi(R, z)\, \boldsymbol{e}_\phi], \tag{5}$$

where $\boldsymbol{e}_\phi$ is the unit vector in the azimuthal direction. In component form we then have

$$B_R = -\frac{\partial \psi}{\partial z}, \tag{6}$$

and

$$B_z = \frac{1}{R}\frac{\partial}{\partial R}(R\psi). \tag{7}$$

In the current context, we note that ψ is symmetric in z, and that a useful physical interpretation of ψ is to note that the value of $R\psi$ at radius R on the disc mid-plane is directly proportional to the magnetic flux passing through the disc interior to R, i.e.

$$R\psi(R, 0) = \int_0^R B_z(R, 0)\, R\, \mathrm{d}R. \tag{8}$$

Using equations (1), (3), (4) and (5), we obtain

$$\frac{\partial}{\partial t}(\psi \boldsymbol{e}_\phi) = \boldsymbol{v} \times \boldsymbol{B} - \frac{4\pi\eta}{c}\boldsymbol{J}. \tag{9}$$

The equation we require is the ϕ-component of this equation, which can be written as

$$\frac{\partial}{\partial t}(R\psi) = -v_R \frac{\partial}{\partial R}(R\psi) - \frac{4\pi\eta}{c}RJ_\phi, \tag{10}$$

where v_R is the radial velocity of the disc, and we have used the thin-disc approximation to neglect v_z. Formally, at this stage, this requires $|v_z/v_R| \ll |B_z/B_R|$, but this condition can be relaxed when we average vertically through the disc (see below). Again, in a formal sense, we should mention the poloidal components of equation (9), especially since the dominant component of the $\boldsymbol{v} \times \boldsymbol{B}$ term is presumably due to the azimuthal disc velocity v_ϕ. In the absence of a wind emanating from the disc, we shall require the poloidal current to be zero, and so require some effect to counterbalance the induced emf $(\boldsymbol{v} \times \boldsymbol{B})$. The assumption here is that, although small currents are induced by this emf, the net effect of these currents is to set up a charge distribution within the disc which provides an electric field to balance exactly the $\boldsymbol{v} \times \boldsymbol{B}$ term.

Within a thin disc, we expect the following approximations to hold:

$$B_z(R, z) \simeq B_z(R, 0), \tag{11}$$

$$B_R(R, z) \simeq B_R^{\mathrm{s}}(R) \times (z/H), \tag{12}$$

and

$$\psi(R, z) \simeq \psi(R, 0)[1 + \mathcal{O}(z^2/L^2)]. \tag{13}$$

Here B_R^{s} is the radial field on the $z=H$ disc surface, and $L \gg H$ is a length-scale defined approximately by $L^2 \sim HRB_z/B_R^{\mathrm{s}}$. Note that we assume here that $\partial\psi/\partial R|_{z=0} \sim \psi(R, 0)/R$. Given this, we may average equation (10) vertically through the disc and obtain an approximate equation for $\psi_0(R) \equiv \psi(R,0)$ in the form

$$\frac{\partial}{\partial t}(R\psi_0) = -\bar{v}_R \frac{\partial}{\partial R}(R\psi_0) - \frac{4\pi\bar{\eta}}{c} R \frac{J^s_\phi}{2H}, \tag{14}$$

where the approximation requires $B^s_R \ll B_z(R/H)$. Here $\bar{v}_R$ and $\bar{\eta}$ are appropriate vertical averages of these quantities, and we henceforth drop the bar. $J^s_\phi(R)$ is now an azimuthal surface current density defined by

$$J^s_\phi(R) \equiv \int_{-H}^{H} J_\phi(R,z)\,dz. \tag{15}$$

We note further that, by applying Stokes's theorem to equation (5), we obtain

$$B^s_R(R) = \frac{2\pi}{c} J^s_\phi(R). \tag{16}$$

Finally, to obtain an equation for $\psi_0(R)$ in closed form, we need to relate $\psi_0(R)$ and $J^s_\phi(R)$. Now $\psi_0(R)$ is produced by two contributions: first, $\psi_d(R)$ due to currents in the disc, $J^s_\phi(R)$, and, secondly, $\psi_\infty(R)$ due to an externally imposed field, which may be regarded as being due to currents at infinity.

Thus we may write

$$\psi_0(R) = \psi_d(R) + \psi_\infty(R), \tag{17}$$

where, formally (see, for example, Jackson 1975),

$$\psi_d(R) = \frac{1}{c}\int_{R_{in}}^{R_{out}}\int_0^{2\pi} \frac{J^s_\phi(R')\cos\phi'\,d\phi' R' dR'}{(R'^2 + R^2 - 2RR'\cos\phi')^{1/2}}. \tag{18}$$

As an example, we note that, if the disc is subjected to a uniform imposed external field $\boldsymbol{B} = (0, 0, B_0)$, then

$$\psi_\infty(R) = \frac{1}{2} B_0 R. \tag{19}$$

In principle, equation (18) can be inverted to obtain $J^s_\phi(R)$ as an integral involving $\psi_d(R)$. Substituting this equation for $J^s_\phi(R)$ into equation (17), we then have a linear integro-differential equation for $\psi_0(R)$.

3 NUMERICAL METHOD OF SOLUTION

We now consider the solution of the evolution equation derived for $\psi_0(R)$ by numerical means. We shall assume that the radial velocity $v_R(R,t)$ is a given function of radius and time, but note that it is possible to solve for $v_R(R,t)$ self-consistently by solving the disc-evolution equation (e.g. Pringle 1981) in parallel. We postpone consideration of this possibility.

The numerical method we use is simple first-order explicit for the quantity $R\psi$, with the advective term evaluated by upstream differencing (the Lelevier method, see Potter 1973) written in a way that conserves $R\psi$. The main complication arises in evaluating the second term on the right-hand side of equation (14), using the integral relation given by equation (18).

We define grid points in the disc at radii R_i, $i = 1, \ldots N$, and evaluate the quantities $(R\psi)$ at the grid points. We represent the sheet current $J^s_\phi(R)$ by a series of ring currents I_i evaluated at the grid points where we write

$$J^s_\phi(R_i) = I_i/dR_i, \tag{20}$$

where

$$dR_i = \frac{1}{2}(R_{i+1} - R_{i-1}). \tag{21}$$

We may now evaluate $(R\psi_d)_i$ (equation 17) in terms of the contribution from each ring current, i.e. formally (Jackson 1975),

$$(R\psi_d)_i = \sum_{j=1}^{N} Q_{ij} I_j, \tag{22}$$

where

$$Q_{ij} = \frac{4}{c}\frac{R_i R_j}{R_i + R_j}\left[\frac{(2-k^2)K(k) - 2E(k)}{k^2}\right], \tag{23}$$

$$k^2 = 4R_i R_j/(R_i + R_j), \tag{24}$$

and $E(K)$, $K(k)$ are the elliptic integrals

$$E(k) = \int_0^{\pi/2} (1 - k^2\sin^2\alpha)^{1/2}\,d\alpha, \tag{25}$$

and

$$K(k) = \int_0^{\pi/2} (1 - k^2\sin^2\alpha)^{-1/2}\,d\alpha. \tag{26}$$

The matrix with elements Q_{ij} is real and symmetric, and has positive eigenvalues. It is just a property of the grid, and so need only be evaluated once during the calculation. Formally, the diagonal terms ($i = j$) as written above are singular, and in fact, for $i = j$, we replace $Q_{ij} = Q(R_i, R_j)$ by

$$Q_{ii} = \frac{1}{2}[Q(R_i, R_i + \lambda\,dR_{i+(1/2)}) + Q(R_i, R_i + \lambda\,dR_{i-(1/2)})], \tag{27}$$

where

$$dR_{i+(1/2)} = R_{i+1} - R_i. \tag{28}$$

Because the singularity is only a logarithmic one, the precise value of λ in equation (27) is not too important, and we used $\lambda = 1/2$.

Having obtained the matrix elements Q_{ij}, we then evaluate the inverse matrix elements Q^{-1}_{ij} by numerical means. The method used was LU decomposition using routines from Press et al. (1989), which proved to be accurate to machine accuracy. We note again that, although the inversion process is lengthy for large grids, the inverse matrix is a property of the grid alone, and so the inversion need only be carried out once. We then evaluate the second term on the right-hand side of equtaion (14) numerically at grid point i, by use of equations (20), (22) and (17), to obtain

$$J^s_\phi(R_i) = (dR_i)^{-1}\sum_{j=1}^{n} Q^{-1}_{ij}[(R\psi_0)_i - (R\psi_\infty)_i]. \tag{29}$$

For accuracy and for numerical stability, we need to take a small enough time-step. For the advective term, the usual

Courant–Friedrich condition is required. For the diffusivity term, we need to consider equation (14) with the advective term set to zero, namely

$$\frac{\partial}{\partial t}(R\psi_0) = -\frac{4\pi\bar{\eta}}{c} R \frac{J^s_\phi}{2H}. \tag{30}$$

In addition, we may take $\psi_\infty(R)$ to be zero, since this acts like a source term and does not affect diffusion time-scales. We notice that $\psi_0 \cos\phi$ is then analogous to a gravitational potential due to a surface density $-J^s_\phi \cos\phi/cG$, with G being the usual gravitational constant.

If we consider localized perturbations for which $\psi_0 = \psi_{0k}(t)\exp(\mathrm{i}kR)$ and $J^s_\phi = J^s_{\phi k}(t)\exp(\mathrm{i}kR)$, with $kR \gg 1$, we then have, using the standard WKB approximation in density-wave theory (Lin & Shu 1964),

$$\psi_{0k} = \frac{2\pi J^s_{\phi k}}{c|k|}. \tag{31}$$

Equation (30) then gives

$$\frac{\partial}{\partial t}(\psi_{0k}) = \frac{4\pi\bar{\eta}}{c}\frac{J^s_{\phi k}}{2H} = -\psi_{0k}\frac{|k|}{H}\eta. \tag{32}$$

This shows that a mode with wavenumber k decays at a rate γ_k, given by

$$\gamma_k = \eta|k|/H. \tag{33}$$

In order to avoid numerical instability, if a first-order Euler method is used to integrate forward in time (as is the case here), we require that $\Delta t \lesssim 2/\gamma_k$, with Δt being the time-step. This gives $\eta|k|\Delta t \lesssim 2H$. The most restrictive case arises when $|k|$ takes on the maximum possible value represented on the grid. To within a factor of order unity, this is $|k| \simeq \pi/\Delta R$, ΔR being the smallest step-size on the grid. In this case, we require for stability that

$$\Delta t \lesssim 2H\Delta R/(\pi\eta). \tag{34}$$

This condition serves as an effective Courant condition involving the diffusion velocity $\pi\eta/(2H)$.

4 A PARTICULAR PROBLEM

As an example of the method given above, we apply the approach to a particular problem. We start with a disc that is threaded by a uniform vertical field strength B_0. We then allow the disc to evolve with a flow field corresponding to a steady disc with kinematic viscosity ν and ask what is the final form of the magnetic field. We take the disc to extend from $R_{\rm in} = 1$ to $R_{\rm out} = 100$, in arbitrary units. For simplicity, we assume the relative disc thickness, H/R, the kinematic viscosity, ν, and the magnetic diffusivity, η, to be constant with radius. We assume that the steady disc flow is set up by adding matter at radius $R_{\rm add} = 75$, and by removing it at $R_{\rm sink} = 1.5$. Thus the velocity field of the disc can be written

$$v_{\rm R}(R) = \begin{cases} 0 & 1 \le R \le 1.5 \\ -3\nu/2R & 1.5 < R < 75 \\ 0 & 75 \le R \le 100. \end{cases} \tag{35}$$

We use a logarithmic grid with $N = 100$ grid points extending from $R = 1$ to $R = 100$. Sufficient accuracy and numerical stability are obtained by adjusting the time-step to be at most 0.2 of the Courant–Friedrich limit and by allowing the diffusivity term to change $(R\psi)$ at any grid point by at most 2 per cent in a time-step.

Starting with the initial conditions given above, we then allow the computation to run until a steady state is achieved. Physically, what is happening is that the vertical field component is advected inwards on a time-scale

$$t_\nu \sim R^2/\nu, \tag{36}$$

and diffusivity allows this component to move outwards on some time-scale t_η. By examination of the terms on the right-hand side of equation (14), and by noting equation (16), we find that

$$t_\eta \sim \frac{R^2}{\eta}\frac{H}{R}\frac{B_z}{B^s_R}. \tag{37}$$

Initially, when B^s_R is small, this time-scale is large, but as B_z is advected inwards B^s_R grows until a steady state is achieved with $t_\nu \sim t_\eta$. By combining equations (36) and (37), we find that we may expect this to happen when

$$\frac{B^s_R}{B_z} \sim \frac{H}{R}\mathscr{P}^{-1}, \tag{38}$$

where $\mathscr{P} \equiv \eta/\nu$ is the magnetic Prandtl number.

It is further evident from equation (14), by noting that J^s_ϕ is related linearly to ψ_0 by quantities independent of η, ν or H/R (equation 18), that, if we scale the time in units of, say, the viscous time-scale (or equivalently set $\nu = 1$), the equation has only one free parameter, namely the quantity $\mathscr{D}$ where

$$\mathscr{D} \equiv (R/H)\mathscr{P}. \tag{39}$$

The results of the computations are as follows. First, we confirm the estimate given above in equation (38) and find that, if the field emerging from the disc makes an angle i with the vertical, so that $\tan i = B^s_R/B_z$, then

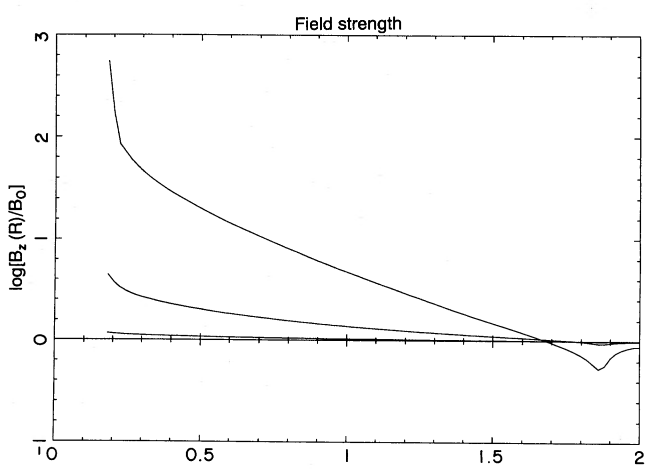

Figure 1. Plot of $\log[B_z(R)/B_0]$ versus $\log R$. Each curve corresponds to a constant value of $\mathscr{D}$. The highest curve for small R corresponds to $\mathscr{D} = 0.2$, the intermediate curve to $\mathscr{D} = 2.0$, and the flattest curve to $\mathscr{D} = 20$.

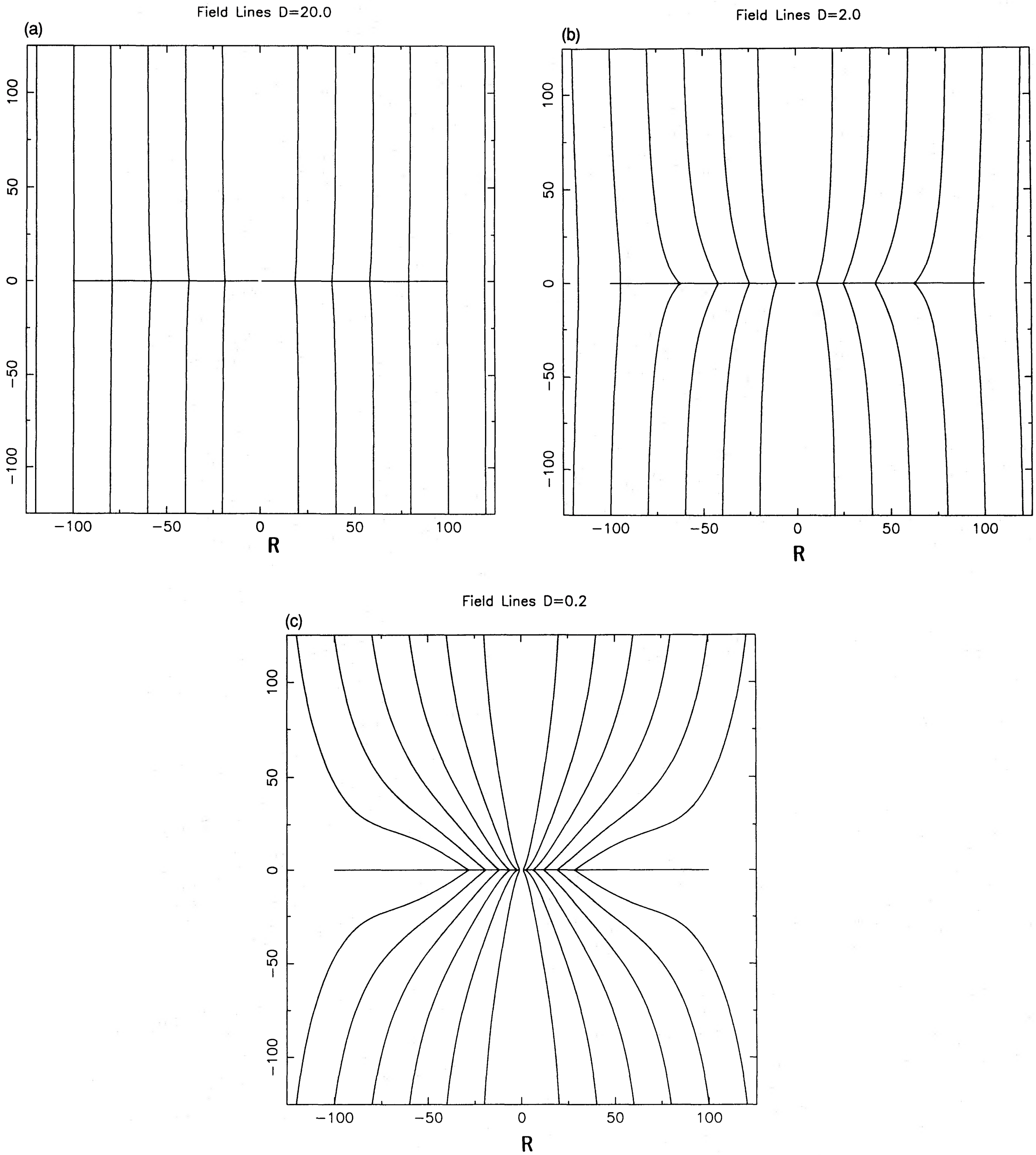

Figure 2. Plot of field lines in the equilibrium disc for the three values of $\mathscr{D}$ in Fig. 1. In each plot, the horizontal line marks the horizontal region occupied by the disc.

$$\tan i = 1.52\mathscr{D}^{-1}. \tag{40}$$

In Fig. 1, we plot the equilibrium value of B_z/B_0 against R for various values of $\mathscr{D}$. This figure shows the expected result that, as the diffusivity is decreased, the external field is more easily swept inwards. We also note that for $\mathscr{D} \gtrsim 1$ the field is barely swept inwards at all, whereas for $\mathscr{D} \lesssim 1$ the field is strongly carried inwards, and the value of the field at the central grid point is a sensitive function of $\mathscr{D}$. This is reminiscent of the behaviour found by Clarke & Pringle (1988) with respect to upstream diffusion of contaminant in an accretion disc. In Fig. 2, we plot the equilibrium magnetic field lines for three values of $\mathscr{D}$. The figure shows that, for the relatively high magnetic diffusivity case of $\mathscr{D} = 20.0$, the field lines are straight and uniform, as the external field is hardly affected. For smaller values of $\mathscr{D}$, the field becomes more bent near the disc plane, and also becomes more compressed towards the disc centre.

In this connection, we remark that the decay rate for a disturbance with wavenumber k is given by equation (33) as $\gamma_k \simeq \eta |k|/H$, whereas the advection rate is $|kv_R|$. The condition for advection to occur before decay is then just $|v_R| \gtrsim \eta/H$, or $\mathscr{D} \lesssim 3/2$.

5 DISCUSSION

We have put forward a method for calculating the time-evolution of an axisymmetric magnetic field that threads an accretion disc, under the assumptions that the field is sufficiently weak and so structured that it does not affect the flow, and that the only dissipative process for the field is magnetic diffusivity, η, within the disc. We have demonstrated the method by applying it to the evolution of an initially uniform, and externally imposed, magnetic field under the influence of a steady flow within an accretion disc. The steady flow was represented by a radial flow velocity (equation 35),

$$v_R = -\frac{3\nu}{2R}, \qquad (41)$$

and it was shown that significant inward dragging of field lines occurs only if

$$\mathscr{D} = (R/H)(\eta/\nu) \lesssim 1. \qquad (42)$$

We note that, from equation (40), the condition for the viability of a centrifugally driven magnetic wind (Blandford & Payne 1982), namely that $\tan i \gtrsim 1/\sqrt{3}$, reduces to a similar condition, i.e. that $\mathscr{D} \lesssim 0.88$.

In an accretion disc in which both the effective viscosity and effective magnetic diffusivity are due to (magneto-) hydrodynamic turbulent processes, it seems likely that neither of these conditions can be satisfied. In isotropic turbulence in which the largest eddy sizes are $\sim l$, with turnover velocities $\sim v_t$, it is to be expected (see, for example, the discussion in Parker 1979, chapter 17) that $\nu \sim \eta \sim l v_t$, and that $\mathscr{D} \sim R/H \gg 1$. Moreover, in an accretion disc, where the flow is strongly constrained by rotation, it may be that the turbulence is anisotropic, with turbulent cells having a shorter length-scale in the radial direction (l_R) than in the vertical direction (l_z) by factors of the order of the inverse of the Rossby number R_o (Meyer & Meyer-Hofmeister 1983; see also Rüdiger 1989, Tuominen & Rüdiger 1989). This would imply that $\mathscr{P} \sim R_o^2 \gg 1$, and so would have the effect of increasing $\mathscr{D}$ still further.

However, the above estimates are valid only if the turbulent viscosity is the main source of angular momentum loss from the disc, and, for the case of magnetic winds, it has been pointed out that the main angular momentum loss mechanism might be the wind itself. In this case, the definition of $\mathscr{D}$ must be modified to become

$$\mathscr{D} = \frac{2\eta}{3H|v_R|}, \qquad (43)$$

and then the same conditions for flux dragging and field angle still apply. Thus for any particular disc-wind model, the condition on $\mathscr{D}$ is translated into a condition on η, namely

$$\eta \lesssim \frac{3}{2} H|v_R|, \qquad (44)$$

where v_R is the radial velocity predicted by the model. As a specific example, we note that the detailed model by Wardle and Königl (1993; see also Königl 1989, 1993) yields $|v_R| \sim c_s$, where c_s is the sound speed in the disc, and $\eta \sim Hc_s$, and so is self-consistent with regard to any flux-dragging constraints.

We remark, finally, that given a particular model for the wind [that is, a model which, for given $B_z(R)$ and disc parameters, yields a mass loss and angular momentum loss per unit mass of the disc], we are now in a position to put together a comprehensive model for the disc evolution, in which wind losses play their part in driving disc evolution, and disc evolution plays its part in driving magnetic field evolution, and hence evolution of the wind structure. Once this has been accomplished, it will be possible to achieve a fully self-consistent model of magnetic winds from accretion discs.

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