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STUDY OF SPECTROSCOPIC BINARIES WITH TODCOR. I. A NEW TWO-DIMENSIONAL CORRELATION ALGORITHM TO DERIVE THE RADIAL VELOCITIES OF THE TWO COMPONENTS

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ABSTRACT

We propose a generalization of the cross-correlation technique, to obtain *simultaneously* the Doppler shift of the two components of composite spectra. The technique—TODCOR—computes the correlation of an observed spectrum against combinations of the two template spectra, with all possible shifts. Thus, the correlation is a two-dimensional function whose two independent variables are the two shifts of the two components. The location of the maximum of this function corresponds to the actual shifts of the two components. The technique is made feasible through the use of an FFT algorithm. TODCOR improves the analysis of binary spectra with small velocity differences, which are difficult to analyze with the original cross correlation. The new algorithm was tested with numerous simulated spectra. We were able to derive the correct velocities of the two components in all cases, one of which is presented.

Subject headings: binaries: close — techniques: radial velocities

1. INTRODUCTION

Cross correlation is a frequently used technique to obtain the Doppler shifts of digitized celestial spectra. This technique, first introduced by Simkin (1974) and further developed by Tonry & Davis (1979), cross-correlates the observed spectrum against an assumed template and obtains the relative radial velocity shift by locating the correlation maximum (Wyatt 1985). The technique can find the correct radial velocity even for extremely low signal-to-noise (S/N) spectra (e.g., Latham 1985) and was therefore applied extensively to the study of spectroscopic binaries (e.g., Latham 1992). A thorough review of the application of the cross correlation to various kinds of binaries was recently presented by Hill (1993).

Stellar spectra composed of two components with comparable intensity yield a cross correlation with two peaks, which correspond to the different velocities of the components. However, whenever the relative velocity of the two components is small, the two peaks cannot be resolved. To overcome this difficulty, we developed TODCOR—a new two-dimensional correlation algorithm, which can *simultaneously* obtain the Doppler shifts of the two components. A short version of this work has been presented in Mazeh & Zucker (1992).

The new algorithm assumes that the observed spectrum is a combination of two known spectra with unknown shifts. The algorithm calculates the correlation of the observed spectrum against combinations of two templates, with all possible shifts. The correlation, thus, is a two-dimensional function whose two independent variables are the radial velocities of the two components. The location of the maximum of this function corresponds to the actual Doppler shifts of the two components. The two radial velocities are independent variables, and, therefore, there is no special significance to their difference. Consequently, TODCOR is able, in principle, to resolve even components of equal velocities.

A straightforward implementation of the algorithm would require an enormous amount of computation, making the approach impractical with present computers. We therefore developed an efficient method to perform the algorithm, reducing the computation needed to analyze spectra containing 1000 points by three orders of magnitude.

This paper presents the new algorithm and points out its potential. Section 2 demonstrates the difficulties of the one-dimensional original technique. Section 3 presents the basic ideas of the new algorithm, while some tedious algebraic details are deferred to the Appendix. Section 4 demonstrates the capability of TODCOR with one example, and § 5 discusses briefly the potential of the new algorithm.

2. LIMITS OF THE ONE-DIMENSIONAL TECHNIQUE

The one-dimensional technique utilizes the correlation of an observed stellar spectrum against a known template to derive the stellar velocity shift of the observed spectrum. A double-line spectroscopic binary manifests itself through the one-dimensional cross correlation by displaying two local peaks in the correlation function. Usually, the higher peak represents the velocity shift of the primary star, while the secondary peak is considered to indicate the velocity of the secondary. However, this approach to composite spectrum is possible only if the cross correlation displays two well separated peaks. Whenever the velocity difference between the primary and the secondary is comparable to the intrinsic width of the correlation peak, the two peaks cannot be resolved, and the secondary velocity cannot be derived. Moreover, even the primary velocity cannot be obtained correctly, as the combined peak is shifted due to the presence of the secondary.

To demonstrate the limits of the one-dimensional technique, we simulated a double-line spectrum composed of calculated A and G star spectra (Kurucz 1991). We used a 45 Å spectral band centered around 5190 Å, which is observed in the Center for Astrophysics for routine stellar work (Latham 1985). The rotational velocities of the two stars were chosen to be 40 km s^{-1}, with intensity ratio of 0.25. To mimic real observed spectra we added normally distributed noise with S/N ratio of 20, for each of the 2048 pixels of the spectra. We chose three different values for the velocity difference between the two

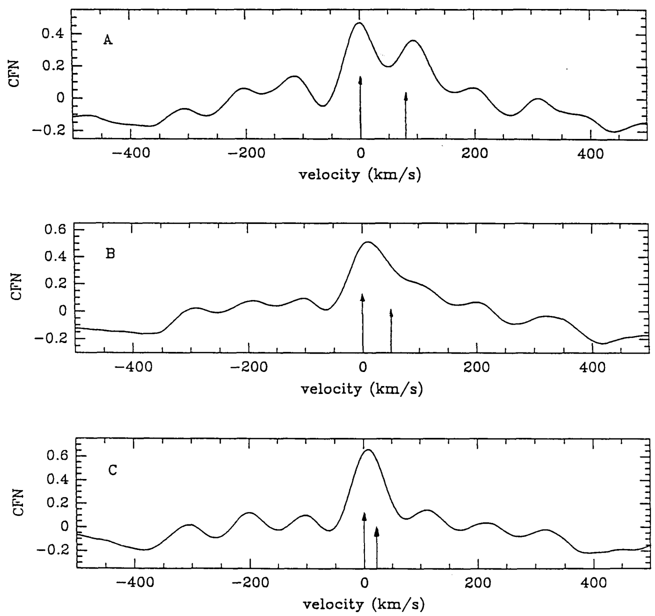

FIG. 1.—One-dimensional cross correlation functions of the composite spectra against the A star spectrum. The arrows indicate the velocities used in the simulation.

components: 80, 50, and 20 km s^{-1}, respectively. The radial velocity of the primary was chosen in all three cases to be 0.

We have applied the one-dimensional cross correlation technique to the simulated spectra using the A star template, the results of which are displayed in Figure 1. The two correlation peaks are well separated in the first case, where the velocity difference is 80 km s^{-1}. In the second case, where the velocity difference is only 50 km s^{-1}, the second peak can barely be noticed, and it is very difficult to derive the secondary velocity. Actually, the primary peak does not yield the correct velocity of the primary, because of the presence of the secondary. In the last case, where the velocity difference is only 20 km s^{-1}, the two peaks are not even distinguishable. Figure 2 presents the correlation of the same three simulated spectra with the secondary template, that of the G star. The ability to resolve the two stars is similar. Section 4 will show that even the most difficult case is easily resolved by TODCOR.

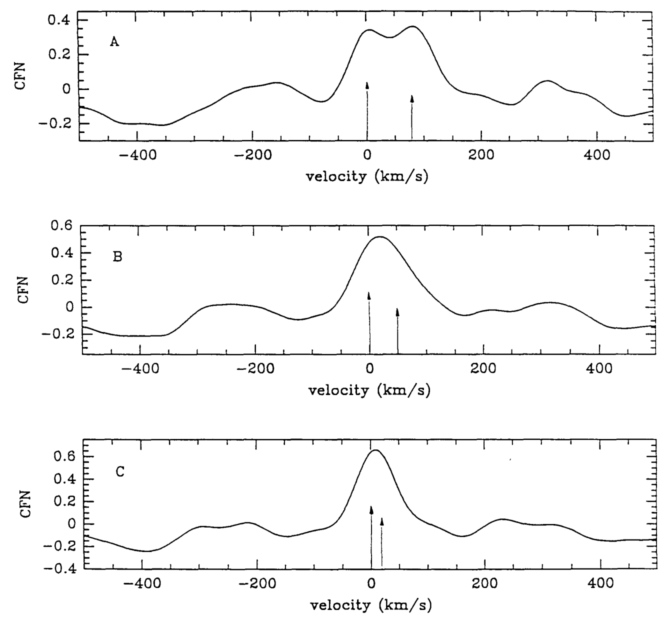

FIG. 2.—One-dimensional cross correlation functions of the composite spectra against the G star spectrum. The arrows indicate the velocities used in the simulation.

3. THE TWO-DIMENSIONAL CORRELATION

To overcome the difficulties presented by the above last two examples, we suggest here a new algorithm—TODCOR—to analyze the spectra of double-line spectroscopic binaries. In Tonry & Davis's (1979) approach, $f(n)$ is the observed spectrum whose Doppler shift is to be found by correlating it against $g(n)$—the "template" of zero shift. Both the stellar spectrum and the template are assumed to be given as a function of n, where

$$n = A \ln \lambda + B .$$

Thus, the Doppler shift results in a uniform linear shift of the spectrum.

By shifting the template by s, and calculating the correlation between the spectrum and the shifted template, $g(n - s)$, we get the correlation as a function of s:

$$C_{f,g} = C_{f,g}(s) .$$

The actual Doppler shift is estimated by the location $\hat{s}$ of the maximum of $C_{f,g}$.

We suggest to correlate the observed spectrum $f(n)$ against a template composed of two, possibly different, templates $g_1(n)$ and $g_2(n)$, shifted by s_1 and s_2, respectively:

$$g_1(n - s_1) + g_2(n - s_2) .$$

The correlation function is now a function of the *two* shifts:

$$R_{f,g_1,g_2} = R_{f,g_1,g_2}(s_1, s_2) .$$

The actual Doppler shifts are now estimated by the location $(\hat{s}_1, \hat{s}_2)$ of the maximum of R_{f,g_1,g_2}.

To find the global maximum of the correlation, TODCOR performs a search over a grid of points in the two-dimensional space of the two variables (s_1, s_2). This is done, as explained in the Appendix, by using the Fourier transform of $f(n)$, $g_1(n)$ and $g_2(n)$. The use of the transforms speeds up the computation by a large factor and makes TODCOR applicable on present workstations for any presently observed stellar spectra.

The insertion of the second template into the analysis introduces an additional complication to the problem. The one-dimensional technique is insensitive to any scale factor of the template. That means that the absolute calibration of the template is irrelevant to the correlation and therefore for finding its maximum. However, when we use two templates, their *relative* intensity becomes very important. It seems as if the user of TODCOR has to supply the algorithm with this information, in addition to the two templates.

Whenever the intensity ratio of the two spectra is not known, we can put its unknown value as an additional variable of the correlation function and find its best estimate. In such a case, the correlation is between the observed spectrum and

$$g_1(n - s_1) + \alpha g_2(n - s_2) ,$$

and, therefore, the correlation R_{f,g_1,g_2} is actually a function of s_1, s_2, and α:

$$R_{f,g_1,g_2} = R_{f,g_1,g_2}(s_1, s_2, \alpha) .$$

To find the maximum correlation in the three-dimensional parameter space, TODCOR utilizes the fact that the correlation is an elementary function of α. Therefore, it is possible to derive analytically the value of α which maximizes the correlation for any given value of s_1 and s_2. The algorithm then searches over a grid of values of the two shifts to find the position of maximum correlation, as is explained in detail in the Appendix.

4. TESTING THE ALGORITHM

We have performed numerous tests in which TODCOR was applied to simulated spectra, prepared with the same method as those presented in § 2. All tests show a very good agreement between the true values and the estimates obtained by TODCOR.

For example, we applied TODCOR to the three simulated spectra discussed in § 2. Figure 3 displays the two dimensional correlation as a function of s_1 and s_2 for the most difficult case, where the velocity difference between the components is only 20 km s^{-1}. We used as our templates the same A and G type stellar spectra, with which the composite spectra were simulated. To mimic a real case, we have applied TODCOR without any specification of the value of the intensity ratio. The maximum was found at 0.0 ± 0.7 and 20.5 ± 0.7 km s^{-1} velocity shift of the primary and the secondary, respectively, with an intensity ratio of 0.27 ± 0.04. This is in a very good agreement with the true parameters used in the simulation.

To show the significance of the detection we plot in Figure 4 two cuts of the two-dimensional correlation. The cuts run parallel to the parameter axes of Figure 3 and go through the maximum. In fact, the two plots of Figure 4 present the correlation as a function of the shift of one of the templates when the other shift is frozen out.

We also applied TODCOR to the two other, easier, simulated spectra of § 2. Here, again, the parameters found were very similar to the true ones.

The three examples considered here, together with numerous other simulations, demonstrate the potential of TODCOR, and show that the new algorithm can resolve spectra of many double-line spectroscopic binaries.

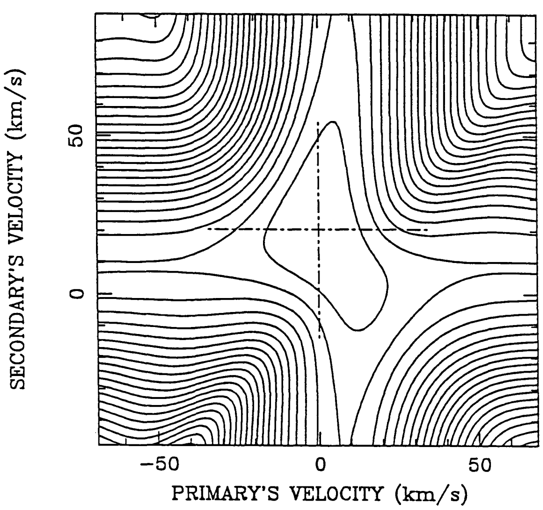

FIG. 3.—Contour plot of the two-dimensional correlation function around the maximum. The dashed lines are parallel to the axes and go through the maximum.

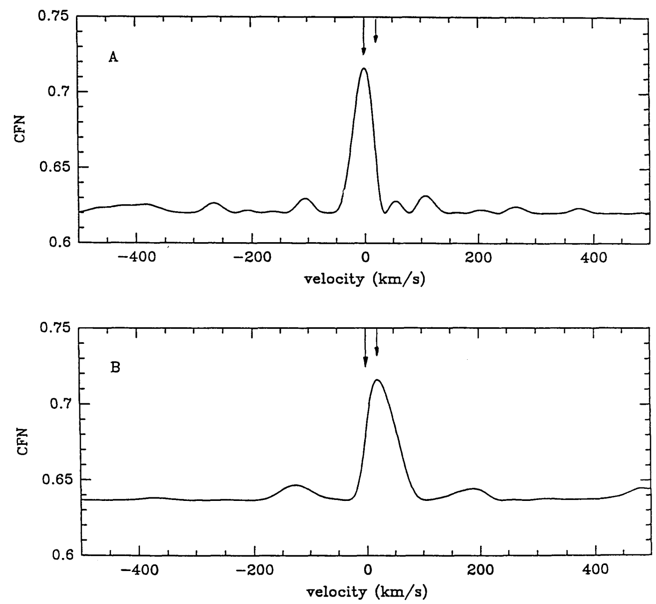

FIG. 4.—Cross sections of the two-dimensional correlation function, taken along the dashed lines of Fig. 3. The arrows indicate the velocities used in the simulation.

5. DISCUSSION

We have demonstrated that the algorithm presented here is capable of extracting the radial velocities of the two components of spectroscopic binaries. However, the examples considered here are free of any characteristics which might introduce some difficulties in the analysis. Such features can be high stellar rotation, which can cause smearing of the cross correlation function, or a mismatch between the templates and the true spectra, which results in a low correlation peak (Hill 1993). These problems restrict the capability of any correlation technique, including TODCOR. However, TODCOR does improve the ability to measure stellar velocities in binary systems by eliminating the error caused by the blending of the two peaks.

Using TODCOR, it may also be possible to estimate the secondary velocity for binaries with small mass ratio. In such systems, the secondary spectrum can be substantially different from the primary one. Therefore, the correlation of the observed spectra with a combination of two *different* templates is highly advantageous, to measure the radial velocity of the secondary in particular. Measuring the radial velocity of the secondaries is important for studying the mass-ratio distribution of short-period binaries and the mass distribution of the secondaries (Mazeh & Goldberg 1992 a, b, c; Mazeh et al. 1992). It may also prove to be productive to use TODCOR iteratively with tomographic separation of spectra (Bagnuolo & Gies 1991), in order to obtain both more precise radial velocities and precise spectral classification of the components. Another possible application of the algorithm is to detect very faint secondaries and to measure their velocity, using very high S/N spectra (Mazeh & Zucker 1993). This application will be discussed in a separate further work.

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APPENDIX

The one-dimensional cross correlation function of Tonry & Davis (1979) is

$$C_{f,g}(s) = \frac{\Sigma_n f(n)g(n-s)}{N\sigma_f \sigma_g}\,, \tag{A1}$$

where N is the number of bins in the spectra, and σ_f and σ_g are the rms of the spectra:

$$\sigma_f^2 = \frac{1}{N}\sum_n f(n)^2\,.$$

Actually, since the sums do not include exactly N summands but, rather, the number of overlapping bins, N in the denominators should be changed to the overlap length. The calculations do not differ much, so we chose to keep N for simplicity.

As Tonry & Davis (1979) point out, one can compute the numerator in equation (A1) efficiently with the FFT algorithm. We denote the discrete Fourier transforms (DFT) of $f(n)$ and $g(n)$ by $F(k)$ and $G(k)$, respectively. The DFT of $\Sigma_n f(n)g(n-s)$ is

$$F(k)G(k)^*,$$

where $G(k)^*$ denotes the complex conjugate of $G(k)$. Using FFT, we can calculate the whole cross correlation function in an $O(N \log N)$ process, instead of $O(N^2)$.

Within the new two-dimensional algorithm, we have to correlate $f(n)$ against a combination of *two* templates, with two *different* Doppler shifts:

$$g_1(n-s_1) + \alpha g_2(n-s_2)\,,$$

where α is the intensity ratio of the two stars, and we assume it is known. Later we relax this assumption.

As a direct extension of equation (A1) we get

$$R_{f,g_1,g_2}(s_1, s_2, \alpha) = \frac{\Sigma_n f(n)[g_1(n-s_1) + \alpha g_2(n-s_2)]}{N\sigma_f \sigma_g(s_1, s_2)}\,, \tag{A2}$$

in which

$$\sigma_g^2(s_1, s_2) = \frac{1}{N}\sum_n [g_1(n-s_1) + \alpha g_2(n-s_2)]^2\,.$$

The numerator in equation (A2) can be written as:

$$\sum_n f(n)g_1(n-s_1) + \alpha \sum_n f(n)g_2(n-s_2)\,,$$

that is, two summands which can both be computed efficiently using FFT, like the numerator of equation (A1). However, the denominator in equation (A2) includes σ_g, which is a function of s_1 and s_2, unlike the one in equation (A1), which was constant. To compute σ_g, we note that

$$\sigma_g^2(s_1, s_2) = \frac{1}{N}\left[\sum_n g_1^2(n-s_1) + 2\alpha \sum_n g_1(n-s_1)g_2(n-s_2) + \alpha^2 \sum_n g_2^2(n-s_2)\right] = \sigma_{g1}^2 + 2\frac{\alpha}{N}\sum_n g_1(n-s_1)g_2(n-s_2) + \alpha^2\sigma_{g2}^2\,.$$

The first and the third summands include the rms of the individual templates. The second summand has exactly the same form as the numerator of equation (A1), which makes it easy to compute via FFT. Thus, we get:

$$R_{f,g_1,g_2}(s_1, s_2, \alpha) = \frac{\Sigma_n f(n)g_1(n-s_1) + \alpha\Sigma_n f(n)g_2(n-s_2)}{N\sigma_f\sqrt{\sigma_{g1}^2 + 2\alpha/N\Sigma_n g_1(n)g_2[n-(s_2-s_1)] + \alpha^2\sigma_{g2}^2}}\,.$$

For simplicity, let us define

$$C_1(s_1) \equiv \frac{1}{N\sigma_f\sigma_{g1}}\sum_n f(n)g_1(n-s_1)\,,$$

$$C_2(s_2) \equiv \frac{1}{N\sigma_f\sigma_{g2}}\sum_n f(n)g_2(n-s_2)\,,$$

$$C_{12}(s_2-s_1) \equiv \frac{1}{N\sigma_{g1}\sigma_{g2}}\sum_n g_1(n)g_2[n-(s_2-s_1)]\,.$$

Now we have

$$R_{f,g_1,g_2}(s_1, s_2, \alpha) = \frac{\sigma_{g1}C_1(s_1) + \alpha\sigma_{g2}C_2(s_2)}{\sqrt{\sigma_{g1}^2 + 2\alpha\sigma g_1\sigma_{g2}C_{12}(s_2-s_1) + \alpha^2\sigma_{g2}^2}} = \frac{C_1(s_1) + \alpha' C_2(s_2)}{\sqrt{1 + 2\alpha' C_{12}(s_2-s_1) + \alpha'^2}}\,, \tag{A3}$$

in which $\alpha' \equiv (\sigma_{g2}/\sigma_{g1})\alpha$.

We see that the final expression includes only three cross correlations: $C_1(s_1)$, $C_2(s_2)$, and $C_{12}(s_2 - s_1)$. These three functions are the correlations between the observed spectrum and each of the templates and the one between the two templates. This fact preserves the $O(N \log N)$ nature of the calculation. In fact, since we can get a rough estimate of the two shifts using the usual cross correlation, we can evaluate equation (A3) only for a small domain of the (s_1, s_2) plane.

In order to obtain error estimates for each of the shifts, say s_2, we can fix the other shift, s_1, to its value at the maximum, $\hat{s}_1$, and look at the function:

$$P(s_2) = R_{f,g_1,g_2}(\hat{s}_1, s_2, \alpha) .$$

Close examination of this function of s_2 shows that it is actually the cross correlation of f against g_2, after subtracting g_1 from f at the appropriate weight. Thus, an error estimate can be obtained using the error analysis of the one-dimensional cross correlation (e.g., Kurtz et al. 1992).

So far we have assumed that the relative weight of the two templates, α, is known. We move now to discuss the case where α is not known. We wish to choose, for each s_1 and s_2, the value of α which maximizes the correlation between $f(n)$ and the linear combination of g_1 and g_2. That means we choose, for each s_1 and s_2, the value of α for which $R_{f,g_1,g_2}(s_1, s_2, \alpha)$ reaches its maximum. After differentiating and equating to zero, we find that the value of α which gives the maximal correlation is

$$\hat{\alpha}(s_1, s_2) = \left(\frac{\sigma_{g1}}{\sigma_{g2}}\right)\left[\frac{C_1(s_1)C_{12}(s_2 - s_1) - C_2(s_2)}{C_2(s_2)C_{12}(s_2 - s_1) - C_1(s_1)}\right], \quad \text{(A4)}$$

and the correlation for this value of α turns out to be

$$R_{f,g_1,g_2}[s_1, s_2, \hat{\alpha}(s_1, s_2)] = \sqrt{\frac{C_1^2(s_1) - 2C_1(s_1)C_2(s_2)C_{12}(s_2 - s_1) + C_2^2(s_2)}{1 - C_{12}^2(s_2 - s_1)}} .$$

We have got a compact expression for the correlation in which the two templates play a similar role. Once again, we have to calculate three cross correlations and then apply equation (A4) to a subdomain of the (s_1, s_2) plane. This formula is a simple case of a more general formula known in statistics as multiple correlation. Triple or multiple stellar systems may be an application of the more general formula.

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