Publications of the Astronomical Society of the Pacific

Vol. 104 **1992 November** **No. 681**

Publications of the Astronomical Society of the Pacific
104: 981–1034, 1992 November

Binaries in Globular Clusters[1]

PIET HUT

Institute for Advanced Study, Princeton, New Jersey 08540
Electronic mail: piet@guinness.ias.edu

STEVE MCMILLAN

Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104
Electronic mail: steve@zonker.drexel.edu

JEREMY GOODMAN[2]

Princeton University Observatory, Princeton, New Jersey 08544
Electronic mail: jeremy@astro.princeton.edu

MARIO MATEO[3]

The Observatories of the Carnegie Institution of Washington, Pasadena, California 91101
Electronic mail: mateo@ociw1.caltech.edu

E. S. PHINNEY[2,4]

Theoretical Astrophysics, 151-33, California Institute of Technology, Pasadena, California 91125
Electronic mail: esp@tapir.caltech.edu

CARLTON PRYOR[5–7]

Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855
Electronic mail: pryor@pryor.rutgers.edu

HARVEY B. RICHER[5]

Department of Geophysics and Astronomy, University of British Columbia, Vancouver, BC V6T 1Z4, Canada
Electronic mail: h.richer@mtsg.ubc.ca

FRANK VERBUNT

Astronomical Institute, University of Utrecht, P.O. Box 80000, 3508 TA Utrecht, The Netherlands
Electronic mail: verbunt@fys.ruu.nl

MARTIN WEINBERG

Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
Electronic mail: weinberg@owl.phast.umass.edu

Received 1992 August 9; accepted 1992 August 19

ABSTRACT. Recent observations have shown that globular clusters contain a significant binary population. This is a dramatic change from the conventional view of even a decade ago, which held that globular clusters formed without any binaries at all, since the observed X-ray binaries were understood to be formed through dynamical capture. Over the last few years, a number of different observational techniques have resulted in the detection of a substantial number of binaries most of which are believed to be primordial. When the many selection effects are taken into account, these detections translate into a binary abundance in globular clusters that may be somewhat smaller than those in the Galactic disk and halo, but not by a large factor. Within the current uncertainties, it is even possible that the primordial binary abundance in globular clusters is comparable to that in the Galactic disk. We discuss different successful optical search techniques, based on radial-velocity variables, photometric variables, and the positions of stars in the color–magnitude diagram. In addition, we review searches in other wavelengths, which have turned up low-mass X-ray binaries and more recently a variety of radio pulsars. On the theoretical side, we give an overview of the different physical mechanisms through which individual binaries evolve. We discuss the various simulation techniques which recently have been employed to study the effects of a primordial binary population, and the fascinating interplay between stellar evolution and stellar dynamics which drives globular-cluster evolution.

TABLE OF CONTENTS

1. INTRODUCTION

Only a dozen years ago, there was no evidence for a substantial population of binaries in any globular cluster. On the contrary, a systematic search for radial-velocity variables by Gunn and Griffin (1979) did not discover any spectroscopic binaries. This was interpreted by them to imply that globular clusters are significantly deficient in binaries with respect to a younger Galactic population.

Because we still have only a rudimentary understanding of the process of binary formation, it is not possible to predict theoretically what fraction of stars should have been formed in binaries in globular clusters, even in the idealized case where we knew the initial conditions under which the clusters formed. Therefore, it even seemed possible, if perhaps not plausible, that globular clusters could have been born without any binaries whatsoever. In this case, the 12 observed X-ray sources and the 2 novae might have formed through the dynamical capture by main-sequence stars of neutron stars and white dwarfs, respectively.

[1]Invited review paper.
[2]A. P. Sloan Foundation Fellow.
[3]Hubble Fellow.
[4]Presidential Young Investigator.
[5]Visiting Astronomer, Canada–France–Hawaii Telescope, operated by the National Research Council of Canada, the Centre National de la Recherche Scientifique of France, and the University of Hawaii.
[6]Work based partly on observations made at the Multiple Mirror Telescope, a joint facility of the Smithsonian Astrophysical Observatory and the University of Arizona.
[7]Visiting Astronomer, Cerro Tololo Inter-American Observatory and Kitt Peak National Observatory, National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc. (AURA), under cooperative agreement with the National Science Foundation.

As a consequence, nearly all theorists happily modeled globular clusters as if indeed all stars had started off single. This assumption was made partly for reasons of simplicity and economy. However, another reason was that theorists were occupied in the early 1980s with the then unresolved question concerning the fate of globular clusters during and after core collapse. As long as this problem was not solved for a population of single stars, and as long as there were no compelling observational reasons, it did not seem necessary to worry about primordial binaries.

After a coherent picture of post-collapse evolution began to emerge in the mid-1980s, simulations of cluster evolution became increasingly refined and detailed. Around the same time, a number of different observational techniques began to offer a more bountiful picture of the globular-cluster binary population. The last few years, especially, have witnessed a rich harvest of direct and indirect binary detections.

In Sec. 2 we discuss the main observational techniques that have recently been successful in identifying and analyzing binaries in globular clusters. In Sec. 2.1 we review how red-giant radial-velocity measurements have led to the discovery of a number of relatively wide binaries. In Sec. 2.2 we discuss the identification of (much shorter period) photometrically variable binaries. In Sec. 2.3 we describe how the presence of a binary population has been deduced and quantified from analyses of color–magnitude diagrams. Finally, in Secs. 2.4 and 2.5 we review the X-ray binary and binary pulsar content of globular clusters.

In response to all these detections, theorists finally began to include primordial binaries in their simulations. As expected on the grounds of earlier analytic estimates, even a modest fraction of primordial binaries turned out to be sufficient to dramatically alter the evolution, as well as some of the observational properties, of globular clusters after core collapse. In Sec. 3 we first summarize the current physical picture of cluster evolution (Sec. 3.1), and of the evolution of individual binaries (Sec. 3.2). We then review the different types of recent simulations of globular-cluster evolution, using direct N-body techniques (Sec. 3.3), Fokker–Planck methods (Sec. 3.4), and stochastic (Monte Carlo) models (Sec. 3.5).

Finally, Sec. 4 sums up, and presents an outlook for developments in the near future. It was this summary and outlook, arrived at during an informal gathering at the Institute for Advanced Study in the Summer of 1991, which stimulated us to write the present review. Originally, we had planned that small round-table meeting as a one-day consulting session, to let the observers tell the theorists what is out there to be explained, and to let the theorists tell the observers what they thought would still be out there waiting to be discovered. Since all of us learned so much from this exchange, we decided by the end of the day to bundle our contributions into a review.

The authorship of the following sections and subsections cannot be disentangled exactly, since the manuscript has gone through a large number of revisions with mutual comments, and has seen whole blocks of text being shuffled around to increase readability. However, the main authors of the subsections of Secs. 2 and 3 can be listed as follows: Sec. 2.1 by C.P.; Sec. 2.2 by M.M.; Sec. 2.3 by H.B.R. and M.W.; Sec. 2.4 by F.V.; Sec. 2.5 by E.S.P.; Sec. 3.1 by P.H.; Sec. 3.2 by S.M., E.S.P., and F.V.; Sec. 3.3 by P.H. and S.M.; Sec. 3.4 by J.G.; Sec. 3.5 by S.M. and E.S.P.

2. OBSERVATIONS

Recently, a variety of techniques has revolutionized our view of the stellar populations of globular clusters. Between them, they cover a wide range of binary types and orbital parameters. New CCD cameras and fiber-fed spectrographs have greatly improved our capabilities in the optical, while radio observations have complemented X-ray observations in giving us detailed insight into the neutron star population, both as single stars and as members of binaries. In this section, we describe these observational techniques. For each we present the most recent results concerning the binary population in globular clusters.

2.1 Radial-Velocity Variables

2.1.1 Radial-Velocity Searches For Binary Stars In Globular Clusters

What is the frequency of primordial binaries in globular clusters today? Does the frequency correlate with other cluster properties? What is the radial distribution of binaries in clusters? Have primordial binaries been destroyed in globular clusters by dynamical processes? One tool that can address these questions is a survey for radial-velocity variables.

The most luminous giants in well-studied globular clusters typically have $V=12$–13, which is already faint for making radial-velocity measurements with a precision of 1 km s^{-1}. As a result, most surveys have examined stars within several magnitudes of the tip of the giant branch. Such surveys have three useful features: complete radial coverage, the ability to derive some properties of the binary orbits, and selection effects that are relatively easy to calculate on the probability of discovering a binary.

Even at ground-based observatories, the brighter cluster stars can be observed anywhere in the cluster, so complete, magnitude-limited samples are obtainable. Such samples determine the binary frequency without uncertainties about the radial distribution of binaries and allow that distribution to be explicitly determined. With accurate color–magnitude diagrams, radial velocities, and, in some cases, proper-motion studies, the cluster membership of the binaries discovered is relatively secure.

Continuing radial-velocity observations can determine each system's orbital period and eccentricity. The distribution of periods is itself interesting, since dynamical processes preferentially eliminate the wider systems (cf. Heggie 1975; Hut 1983). When the period is known, it is also possible to discriminate between tidal capture and primordial binaries and to place some constraints on the system's mass. Some of these points and the selection effects are discussed in more detail in Sec. 2.1.2.

Velocity surveys of luminous giants also have problems. The large radii of these stars, 0.1–0.4 AU, impose a bias on the periods detectable. Binary systems with periods shorter than about 40 days (separations less than about 0.25 AU) will not reach the luminosities required to be included in magnitude-limited samples because they will have been involved in mass transfer that either truncates the evolution of the giant or leads to coalescence through a common-envelope stage (e.g., see the discussion in Pryor et al. 1988, hereafter PLH, and in Sec. 3.2.4). This bias towards long periods and low orbital velocities also means that higher velocity precisions and longer time baselines are needed to discover those binaries that do make it into the sample.

A binary containing 0.8 and 0.4 $M_\odot$ stars separated by 0.25 AU has a period of 42 days and, if the orbit is circular, the more massive star has an orbital velocity of 22 km s^{-1}. Increasing the period by a factor of 10 increases the separation to 1.2 AU and decreases the velocity to 10 km s^{-1}. Thus detecting binaries over a decade or larger range in period requires a study lasting years and velocities accurate to 1 km s^{-1} or better. Velocities with this precision have been attainable for globular-cluster stars only for about the last 20 years and many surveys for radial-velocity variability have extended for less than 5 years.

Today, velocity precisions of 1 km s^{-1} can be achieved for giants in the closest clusters with efficient instruments on small telescopes (CORAVEL on the 1.5-m Danish telescope at ESO, the radial-velocity scanner on the Dominion Astrophysical Observatory 1.2-m telescope, and the intensified Reticon system on the 1.5-m Whipple Observatory telescope of the Smithsonian Astrophysical Observatory), but rapidly surveying large samples in most clusters requires 4-m-class telescopes. Since access to large telescopes is extremely competitive, the studies that have been done to date have been limited to modest numbers (3–6) of observations per star. Also acting to limit the number of observations per star is the rarity of spectroscopic binary stars in globular clusters, discussed in Sec. 2.1.3, which has forced the surveys to include large numbers of stars in order to get statistically significant results.

The complete characterization of the population of binaries in clusters is unfortunately many years off. What questions can and should be addressed first? A survey of a complete sample of globular-cluster stars for binaries can test two key predictions of the simulations of cluster dynamical evolution discussed in Secs. 3.3, 3.4, and 3.5: (1) that binaries should be more concentrated towards the center of the cluster than single stars and (2) that many binaries should be destroyed over a Hubble time in a typical dense cluster.

A simple test of the first prediction is to compare the radial distribution of binaries containing a giant with that of apparently single giant stars. A test of the second is that there should be fewer primordial binaries today in those clusters in which the binary hardening and destruction processes (described in Sec. 3.2) have been the most important. The time scale for these processes is approximately the time for another star to approach to within the binary's semimajor axis, a (Heggie 1980; Hills 1983; Hut 1984):

$$T_h = (2.5\times 10^9\text{yr})\left(\frac{10^4 M_\odot\,\text{pc}^{-3}}{\rho}\right)\left(\frac{1\ \text{AU}}{a}\right)\left(\frac{\langle v^2\rangle^{1/2}}{10\ \text{km s}}\right), \tag{1}$$

where ρ is the density of perturbing stars, and $\langle v^2\rangle^{1/2}$ is the rms velocity of these stars. This time is less than a Hubble time at the centers of many globular clusters for binaries with $a \gtrsim 1$ AU. Thus the models predict lower binary frequencies in those clusters where the binary destruction time T_h evaluated, say, at the center of the cluster is shorter. If clusters with present-day destruction times much longer than a Hubble time are included in the sample, such a study will also produce at least a lower limit on the initial frequency of binaries in globular clusters.

2.1.2 Implementing A Survey

The two questions that must be faced in implementing a search for binaries by radial velocities are (1) how to identify binaries with only a few observations per star and (2) how to turn the number of binaries discovered into the true binary frequency. The standard test for significant variability in a series of observations is the χ^2 statistic. Since velocity measurement uncertainties are usually determined from the data themselves, the proper statistic is actually the F (variance ratio) statistic, but, for samples with more than $\sim$200 degrees of freedom, the difference is slight. The problem with the χ^2 statistic is that it is sensitive to the assumption of normally distributed errors. Large errors can result from blunders in identifying the stars observed, but subtler effects may present a more serious problem.

For example, velocity errors due to miscentering the star on the slit (guiding errors) tend to be worse during times of good seeing, which come and go during a night. Another potential problem is the blending of stellar images in crowded cluster cores. Careful observational procedures can minimize some of these effects, as can estimating the uncertainties directly from the data, but adopting stringent limits for accepting the reality of variability is the wisest course. This, of course, reduces the number of binaries that are "detected."

A different sort of problem is real velocity variability that arises from causes other than orbital motion. Gunn and Griffin (1979, hereafter referred to as GG), Mayor et al. (1984), Lupton et al. (1987), and PLH all reported that the most luminous globular-cluster giants show more velocity variability than fainter stars, which is not the trend expected from binaries. The extensive data on M3 stars in PLH showed that the variability was largest within 0.5 mag of the giant-branch tip, though it was certainly present fainter than that and may have been present at a low level over the whole 1.75 mag range surveyed. This variability is attributed to motions in the atmospheres of the stars; the most variable are low-level photometric variables (PLH). Excluding the large-amplitude W Virginis and long-period photometric variables, the largest velocity ranges observed (always for stars within 0.5 mag of the tip), are about 8 km s^{-1}.

Thus a criterion that will identify binaries but reject "atmospheric" variables is a velocity range larger than 10 km s^{-1}. This restrictive limit could be reduced to 4 km s^{-1} (for measurement accuracies of about 1 km s^{-1}) for stars fainter than 0.5 mag below the tip of the giant branch. However, the most natural discovery criterion is a lower limit on the probability of obtaining a χ^2 larger than is observed. Another commonly used variability criterion, the ratio of the external to the internal uncertainties, is closely related to the χ^2 statistic. These criteria can be employed successfully for globular-cluster stars if the effect of atmospheric variability is included as an additional additive uncertainty (see, e.g., GG and PLH) and if velocity ranges greater than 10 km s^{-1} are required for the stars in the brightest 0.5 mag of the giant branch to be identified as binaries. Reasonable limits for the χ^2 probability are 0.01–0.001.

Once a criterion for identifying binaries has been established, the fraction of stars in a sample that satisfy the criterion is the discovery fraction. Any criterion will miss some binaries because of face-on orbits, long periods, low velocity amplitudes, or simple bad luck in the timing of the few observations. Each criterion has a corresponding discovery efficiency: the fraction of binaries that will, on average, be found. The efficiency depends on a complex way on the properties of the binaries and on the number and spacing of the observations, and is best determined by Monte Carlo simulations of the observations.

Figure 1 shows the results of such a simulation using the criterion that the velocity range be larger than 10 km s^{-1}. The sample consists of the 504 velocities given in GG and PLH for 110 M3 stars, which has an average time baseline of 10.6 yr. Binaries were chosen with periods between 0.1 and 100 yr and given a primary mass of 0.8 $M_\odot$. The four solid curves are the discovery efficiencies for four equal intervals of the logarithm of the mass ratio q in the range $0.33 > \log(q) > -1.0$. These curves are labeled by the mean mass ratio of the binaries discovered. The efficiencies were averaged in four bins per decade of period. The dashed lines show the bias introduced by the removal of binaries from this magnitude-limited sample by mass transfer resulting from the evolution of the primary into a red giant. This effect was determined by calculating the radius of every giant in the sample using a luminosity–radius relation constructed from the infrared photometry of Cohen et al. (1978). If the giant in a binary chosen for the simulation was larger than its Roche radius, then that binary was assumed to have been removed from the sample by truncation of the evolution of the giant or by coalescence through a common-envelope stage.

Figure 1(a) shows the discovery efficiency for binaries on circular orbits. Figure 1(b) is the same plot for a population of binaries with a "thermal" (see Hut 1985) distribution of eccentricities, e. This distribution is $f(e) = 2e$ and thus is weighted towards large eccentricities. The effect of mass transfer in elliptical orbits is approximated by eliminating systems whose giants overflow their Roche radii at periastron.

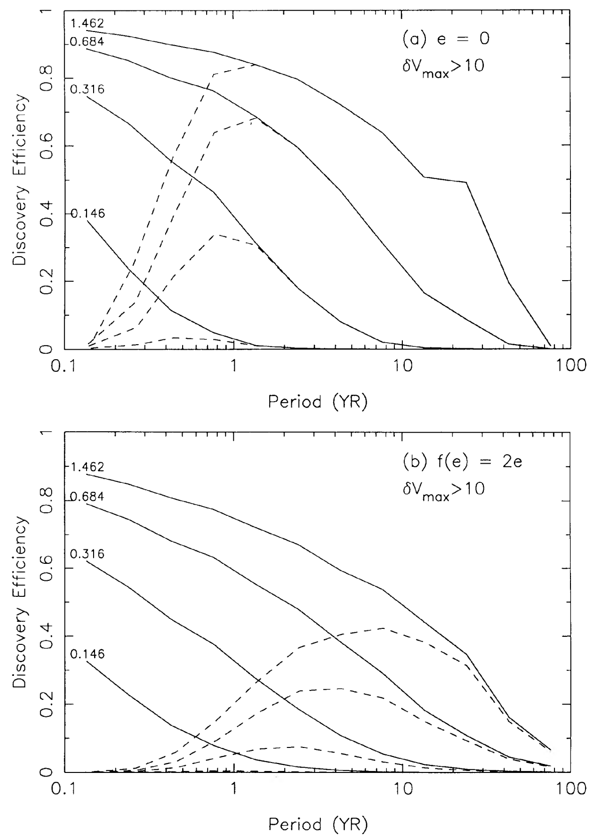

FIG. 1—Binary discovery efficiencies as a function of orbital period for the sample of 504 velocities from GG and PLH for 110 M3 stars. The discovery criterion is a velocity range larger than 10 km s^{-1}. The efficiencies were calculated from 1000 simulated samples in each of the 24 bins produced by dividing the interval between -1 and $+2$ in the logarithm of the binary period (in years) uniformly into 12 parts and the interval between -1 and $+0.33$ in the logarithm of the binary mass ratio into 4. The four solid curves are for the individual mass-ratio bins and are labeled with the mean ratio of the binaries discovered. The dashed lines show the effect on the discovery efficiency of the removal of binaries from the sample by the mass transfer resulting from the evolution of the primaries into red giants. (a) The discovery efficiencies for binaries with circular orbits. (b) The discovery efficiencies for binaries with a thermal distribution of eccentricities.

The results in Fig. 1 are typical of the best-observed samples of luminous globular-cluster stars. From these and similar diagrams for other clusters we conclude that:

(1) Binaries with elliptical orbits are generally somewhat harder to find than those with circular orbits, since they spend much of their orbit moving slowly near apastron.

(2) The biasing of the discovered binary orbits by the size of the giants is clearly significant, particularly when the orbits are elliptical. The bias can be reduced by observing fainter stars.

(3) The average discovery efficiency is high (25%–50%) for binaries with periods between about 0.2 and 10 years and with mass ratios larger than about 0.25. Binaries

outside these limits are unlikely to be discovered from observations of the brightest cluster giants.

Decreasing the size of the maximum-velocity range needed to "discover" a binary from 10 to 4 km s^{-1} (the criterion for fainter giants) typically adds 0.2 to the discovery efficiency. If measurement uncertainties are about 1 km s^{-1}, then using the χ^2 probability as the discovery criterion produces discovery efficiencies comparable to those for a 4 km s^{-1} velocity range, even for probabilities as low as 0.001.

There are several approaches for using discovery efficiencies to turn a discovery fraction into a binary frequency. One is simply to calculate the average efficiency over the range of periods and mass ratios for which the efficiency is, say, above 0.25. Assuming that all binaries discovered are probably within this range of periods and mass ratios and that there are no large variations of the binary frequency within the range, dividing the discovery fraction by the discovery efficiency produces an average binary frequency.

A more sophisticated, but also more model-dependent, procedure is to adopt period and mass-ratio distributions and to attempt to determine a multiplicative normalization. Simulations using the model binary population will yield the relationship between the discovery fraction and the normalization. The Abt and Levy (1976; corrected in Abt 1978, hereafter referred to as AL; see also Morbey and Griffin 1987) and Duquennoy and Mayor (1991, hereafter referred to as DM) log(period) distributions for field binaries are both slowly increasing with period over the range to which most globular-cluster studies are sensitive. However, the simulations discussed in Secs. 3.3 and 3.4 suggest that, at least in the denser globular clusters, the number of binaries will decrease with increasing period (wide binaries are more easily destroyed) and thus that using the field period distribution is perilous.

A problem for any approach to determining the binary frequency is the sensitivity of the discovery efficiency to the uncertain binary eccentricity distribution. Until the actual distribution is available, simulations must be made with both circular orbits and a thermal distribution of eccentricities. As more binary stars are discovered in globular clusters, efficiency simulations can be used to correct the observed period and eccentricity distributions to the true ones.

2.1.3 Current Results

Gunn and Griffin (GG) were the first to comment on the frequency of binary stars discovered by radial-velocity studies in globular clusters. They found no velocity variations among nonpulsating giants in M3 and in their then-unpublished data on giants in other clusters. This was in sharp contrast to their studies of Population I stars, and they argued that the binaries with separations between 0.3–10 AU (periods between 0.1 and 30 years) were much rarer in globular clusters than in the field Population I. This conclusion was criticized by Harris and McClure (1983), who pointed out that the globular-cluster giants had lower masses and larger radii than field Population I giants. Their simulations showed that the histogram of velocity differences for the GG data was compatible with the AL field binary frequency. They emphasized that firm conclusions were not permitted by the small published sample size (33 stars with multiple observations).

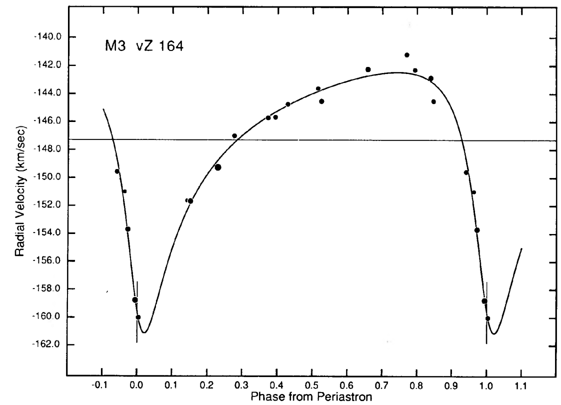

FIG. 2—Radial velocity versus phase for the binary star vZ 164 in M3. The solid circles are the observations, with the size proportional to the weight of the velocity. Typical measurement uncertainties are 1 km s^{-1}. The solid line is the fitted orbit, which has a period of 2657^d, an eccentricity of 0.58, a velocity amplitude (K) of 9.3 km s^{-1}, an argument of the periastron (ω) of 147°, and a mean velocity of -147.3 km s^{-1}. The Julian Date of the most recent periastron passage is 2448125.

The Harris and McClure paper prompted Latham and Pryor to undertake extensive new observations of the M3 giants. This resulted in the discovery of the first spectroscopic binary in a globular cluster, vZ 164 (Latham et al. 1985). Continuing observations have determined that the period of this system is 7.3 years. Figure 2 shows a plot of velocity versus orbital phase. The orbit, calculated by J. M. Fletcher (personal communication), is shown as a solid line. The data were taken with the Whipple Observatory 1.5-m telescope and the Multiple-Mirror Telescope (PLH; Latham, personal communication), the Dominion Astrophysical Observatory 1.2-m telescope (Hesser and McClure, personal communication), and the Canada–France–Hawaii telescope.

Pryor, Latham, and Hazen (PLH) used the new M3 observations of a sample of 110 stars to show that the binary frequency in that cluster was smaller than the AL frequency. Since only one binary was found, they followed Harris and McClure in comparing the histogram of velocity differences with those generated by simulated observations of a model binary population based on AL. They concluded that, if the binary population obeyed an AL period distribution, then the frequency of binaries with periods between 1 day and 100 years was less than 0.25 with about 95% confidence. For comparison, the AL frequency is about 0.50. (To be precise, this binary frequency is the fraction of the "systems" in the cluster that are binaries, where systems means both single stars and binaries treated as units.)

Alternatively, the small number of large velocity differences in the M3 data could be explained by altering the shape of the binary period distribution instead of its normalization. If we were to introduce a sharp cutoff in binary separation, all binaries with separations smaller than 1.5 AU would have to be eliminated (at 95% confidence). If the discovery efficiencies in Fig. 1 are used to convert the discovery fraction of 0.009 (1 star with a velocity range larger than 10 km s^{-1}) to a binary frequency, the frequency is 4% for periods between 0.2 and 20 years and mass ratios larger than 0.22. Of course, the uncertainty in this value is large.

The other important study of binary frequency in globular clusters during the early 1980s was that of Mayor et al. (1984). They used the CORAVEL photoelectric spectrometer to measure velocities, accurate to about 0.6 km s^{-1}, for a sample of 169 giants in 47 Tucanae. Of these, 64 stars that were cluster members on the basis of their velocities had 2 observations separated by more than 200 days. As in the M3 sample, the most luminous stars showed velocity variations. However, the 40 stars fainter than $V=12$ had no variations larger than 2 km s^{-1}. Using an analysis similar to that of Harris and McClure (1983), they concluded that there was less than a 2% probability that this sample of stars had the AL binary frequency. This sample was drawn from the region more than five core radii from the cluster center and thus the rejection of the AL binary frequency applies only to the outer part of the cluster.

In the mid-1980s, spectroscopic binaries were discovered in a number of globular clusters by the CORAVEL group (Mayor, personal communication) and by a group using the Dominion Astrophysical Observatory radial-velocity spectrometer on the Canada–France–Hawaii telescope (Pryor et al. 1987). Pryor et al. (1989a, hereafter referred to as PMHF) combined velocities from PLH, Mayor et al. (1984), and Lupton et al. (1987) with unpublished CFHT velocities to make the first estimate of the globular-cluster binary frequency that was based on the actual discovery of binaries.

The data consisted of 1253 velocities for 393 giants in the globular clusters 47 Tuc, M2, M3, M71, M13, and M12. The discovery criterion that the velocity range be larger than 10 km s^{-1} produced the six binary candidates in Table 1 with a reference of PMHF. Lupton et al. (1987) suggested that L782 in M13 was an undiscovered photometric variable, but it is at least as likely to be a binary. The resulting discovery fraction is 0.015, with a 95% confidence interval of (0.0056, 0.033). The confidence interval is based on the binomial statistics of how small (large) the discovery fraction could be and still have a 2.5% probability of seeing six or more (six or less) binaries.

The assumption of circular orbits and the AL period and mass-ratio distributions in order to evaluate the discovery efficiency for the PMHF data produces a binary frequency of 10% for periods between 1 day and 100 years, much less than the corresponding AL binary frequency of about 50%. A less model-dependent approach is to calculate an average discovery efficiency for the range of periods and mass ratios where the efficiency is significantly above zero. For binaries with periods between 0.2 and 20 years and mass ratios larger than 0.22, the average efficiency is 0.32 assuming circular orbits, and 0.12 assuming a thermal distribution of eccentricities. Thus the estimated frequency of binaries in that period and mass-ratio range is 5% or 12%, depending on the eccentricity distribution.

The AL study of field solar-type stars (as given in Abt 1978) yields a binary frequency of about 15% in this mass ratio and period range. The number of binaries with periods between 0.2 and 20 years in the DM sample of 164 nearby solar-type stars is 17. This number should be increased by at most 1 star for incompleteness and multiplied by about 0.72 to remove binaries with mass ratios below 0.22. Thus the estimate of the field binary frequency to be compared to that of the globular clusters is 8%. The DM binary frequency is more reliable than that of AL, *so the PMHF data now argue for at most a small deficiency of binary stars in globular clusters when compared to the field.*

PMHF also produced the first look at the radial distribution of binary stars in globular clusters. The six clusters all have half-mass relaxation times shorter than their ages and so the binaries should be centrally concentrated. The surprising result was that the binaries appeared to have the same distribution as the cluster light, and thus the same distribution as the giants and upper-main-sequence stars that produce the light. Though the sample consisted of only six binaries, a distribution for the binaries equal to that expected for a population of 1.8 $M_{\odot}$ stars (about twice the giant mass), calculated with thermal equilibrium models (King 1966; Da Costa and Freeman 1976; GG), could

TABLE 1
Some Probable Spectroscopic Binaries in Globular Clusters[a]

Cluster–Star	N_{obs}	Range (km s^{-1})	Period (yr)	Reference
NGC 5053–5	7	25	0.8	Pryor et al. 1991
NGC 5053–Q	7	13	?	Pryor et al. 1991
NGC 5053–967N	8	7	?	Pryor et al. 1991
M3–vZ164	19	19	7.3	Latham et al. 1985; PMHF
NGC 5466–227	13	8	0.6	Pryor et al. 1991
NGC 5466–187	7	6	?	Pryor et al. 1991
M13–L782	5	36	?	Lupton et al. 1987; PMHF
M71–I	27	14	0.95	PMHF; Pryor et al. (1992)
M71–71	18	22	3.7	PMHF; Pryor et al. (1992)
M71–CF8	6	13	?	CFHT
M71–81	6	8	?	CFHT
M2–AIV-17	8	15	?	PMHF
M2–P10	6	13	?	PMHF
M2–CR458	5	7	?	CFHT

[a]This table contains probable spectroscopic binaries for which C.P. had data as of the middle of 1991. N_{obs} is the number of velocities at distinct epochs and range is the difference between the largest and smallest velocities. Preliminary orbital periods are listed where they are known. M2–P10 is the anonymous tenth star in the list of Pryor et al. (1986). NGC 5053–967N is the more northerly of the bright double between stars 85 and Z in the finding chart of Sandage et al. (1977). A reference of "CFHT" indicates results from unpublished binary surveys by C.P. and his collaborators.

be rejected at the 98% confidence level. This result clearly deserves to be checked with larger samples. While stars inside and outside the cores of the clusters had similar numbers of velocity measurements, more subtle biases, due to crowding, against the discovery of binaries remain to be explored.

The comparison of the frequency of binary stars in different globular clusters is just beginning. Pryor et al. (1991) report preliminary results of a search for binary stars among a sample of 64 giants in the 2 low-density clusters NGC 5053 and NGC 5466. Binary destruction should have been unimportant in both. The strongest candidates are listed in Table 1. The variability of NGC 5053 stars G, 42, and 48 and NGC 5466 star 191 are less certain; observations of these stars are continuing. Note that a discovery rate of four to six stars is two to three times the average rate from studies of denser clusters where binary destruction should have been significant. Estimates of the discovery efficiency show that the hypothesis that the binary frequency in NGC 5053 and NGC 5466 is the same as that in the denser clusters can be rejected at the 85%–97% confidence level, depending on how many of the uncertain binaries are included.

Recently, Murphy et al. (1991) found that a number of objects near the center of the prototypical collapsed-core cluster M15 exhibit noticeable emission in the Ca II *H* and *K* lines. They interpreted that as evidence of induced chromospheric activity in cluster red giants located in hard, primordial binaries, since any magnetic fields in individual globular-cluster stars should have decayed long ago. Although they noted that an earlier study by Dupree et al. (1990) had detected Ca II emission cores in giants in other clusters, Murphy et al. (1991) preferred the binary explanation for the M15 observations.

In response to the Murphy et al. (1991) paper, Dupree and Whitney (1991) pointed out that the similarity of the M15 emission with that observed in luminous giants in other clusters is too strong to discount the conclusion that the M15 emission arises from the same source, namely, from hydrodynamically induced chromospheric activity in luminous giants. The large jitter amplitudes described above may, in fact, be closely related to this activity. Thus, Dupree and Whitney (1991) conclude that "... a simple detection of Ca II emission is not a unique signature of binaries," and they note that radial-velocity and/or emission-intensity variations would have to be observed before one could reasonably conclude that the M15 emission stars are actually binaries.

2.1.4 Future Prospects

The major obstacle to progress in this field is the 1.5% binary discovery fraction. Large samples of globular-cluster stars must be surveyed to produce a statistically reliable sample of binaries, particularly if comparisons between subsets of clusters or an estimate of the distribution of binary properties are to be made. Fortunately, a large reservoir of data exists in those stars whose velocities have been measured to study globular-cluster dynamics in the last 15 years. The recently developed fiber-fed spectrographs can obtain spectra for 20–30 such cluster stars simultaneously, making it possible to contemplate both larger and fainter samples than previously. These instruments are a great advance, but constraints on how close together the fibers can be placed make it difficult to measure many stars simultaneously in the dense centers of clusters.

For measuring velocities in cluster centers, Fabry–Perot narrow-band imaging in the vicinity of a strong absorption line is a promising technique that nicely complements the capabilities of the fiber-fed spectrographs. Using standard crowded-field stellar photometry packages with a sequence of images stepped in wavelength can produce short "spectra" and thus radial velocities for every star in the field. Data taken by Williams and Pryor with the Rutgers Fabry–Perot on the CTIO 4-m telescope for the clusters M15 and NGC 6397 have shown that nonphotometric conditions are not a serious obstacle and yield good velocities for about 100 stars in each cluster.

The next three to four years will produce much better values for the average binary frequency in both globular clusters and the field. The cluster samples should be large enough to decide whether there is an anticorrelation between frequency and average binary destruction time, which would argue for the destruction of binaries by dynamical processes. Currently about a dozen giants in binaries have been discovered in globular clusters. As they continue to be followed and new ones are found, we should get our first glimpse of the period and eccentricity distributions, though the bias from evolutionary effects will remain a problem.

An example of the kinds of projects that fiber-fed spectrographs are making possible is a search for binaries near the *base* of the giant branch in M71; it is being carried out by Barden et al. using first the Nessie and now the Hydra fiber feeds with the RC spectrograph on the Kitt Peak 4-m telescope. By observing fainter stars, this study is sensitive to shorter-period binaries than previous ones. It is currently following about 120 stars, many as faint as $V=17$, with a typical velocity accuracy of about 0.5 km s^{-1} (Barden et al. 1990).

2.2 Photometric Variables

2.2.1 Introduction

Certain types of stars can be unambiguously identified as binaries solely on the basis of their brightness variations, and it has long been appreciated that searches for such "photometrically variable binaries" might provide useful limits on the binary period distribution and frequency among main-sequence stars in globular clusters. Most attempts to identify photometric binaries in globulars have involved searches for short-period ($P \lesssim 5$ days) eclipsing binaries and cataclysmic variables for a number of practical reasons. First and foremost, the binary nature of these two classes of variable stars are indisputable (see Pringle and Wade 1985 for reviews of the properties of these systems).

Second, both classes of stars exhibit distinctive light variability that allows them to be readily discovered and studied with modern photometric instrumentation, even in rather crowded fields. Third, among main-sequence stars between spectral types F8 and K3, short-period eclipsing binaries are the most common sort of variable star (Kopal 1959; Webbink 1977; Ruciński 1985b). Stars in this spectral-class range are abundant in globular clusters. Similarly, the typical components of cataclysmic binaries—low-mass late-type dwarfs and white dwarfs—are also expected, on stellar evolutionary grounds, to be common in globular clusters. Thus, the raw materials required to form short-period eclipsing and cataclysmic binaries are in ample supply in globular clusters.

Good reviews of the early searches for photometric binaries in globular clusters can be found in Trimble (1980) and Webbink (1980). For the sake of continuity, some of the more important results discussed in these earlier papers will be briefly repeated in the following sections.

2.2.2 Eclipsing Binaries In Globular Clusters

Hogg (1973) noted that about a dozen relatively bright eclipsing binaries are located near globular clusters; only three were ever seriously considered possible cluster members (see Geyer and Vogt 1978 and Trimble 1977). Of these three, Geyer and Vogt (1978) concluded, on the basis of radial-velocity measurements, that the eclipsing binary associated with NGC 3201 and V78 near ω Centauri (NGC 5139) are probable nonmembers of the respective clusters. Alexander and Budding (1979) argued that the eclipsing system V3 located near the center of the metal-rich globular cluster M71 (NGC 6838; Sawyer 1953) is also unlikely to be a member of that cluster, as was subsequently confirmed spectroscopically by Liller and Tokarz (1981). Trimble (1977) independently concluded that none of the eclipsing binaries in Hogg's (1973) catalog of globular-cluster variables was a likely member of its respective cluster. Although Webbink (1980) noted that an additional 26 eclipsing binaries were located near globular clusters, he did not consider any of them to be likely cluster members. Thus, at the start of the 1980s, the best estimate of the number of eclipsing binaries in globular clusters was zero.

Because eclipsing binaries tend to be short-period systems, they are physically compact and are not likely to be found among the more luminous evolved cluster stars typically surveyed for variability. Thus, meaningful limits on the frequency of eclipsing binaries had to await systematic studies of relatively faint main-sequence globular-cluster stars. The first published search of that kind was reported by Trimble (1976). Based on her analysis of 35 Harvard survey plates of M55 (NGC 6809) obtained from 1935–1947, she was unable to find any eclipsing systems.

Soon afterwards, Niss et al. (1978) used the ESO 3.6-m telescope to obtain several deep plates over five nights to search for faint, short-period variables in ω Cen. They identified one certain eclipsing binary (NJL 5) and six additional candidates, all considerably fainter than the RR Lyrae variables in the cluster. Liller (1978) confirmed the variability of NJL 5, but concluded that it was too luminous to be a member of the cluster.

Jensen and Jørgensen (1985) and Margon and Cannon (1989) subsequently showed, on the basis of radial-velocity measurements, that NJL 5 is indeed a likely member of ω Cen; Liller's erroneous conclusion can be understood because NJL 5 is also a blue straggler and therefore more luminous than stars at the main-sequence turnoff of the cluster. Notice that the six other eclipsing-binary candidates identified in ω Cen by Niss et al. (1978) have not been studied in further detail, so their membership in the cluster remains unknown.

Irwin and Trimble (1984) analyzed three plates of M55 obtained on successive nights with the Anglo–Australian telescope. They identified eight faint variables as possible main-sequence eclipsing binaries; the variability of three of them was confirmed by Liller (private communication to Irwin and Trimble 1984) from an independent set of photographic plates obtained at CTIO.

Although photographic plates offer the large area needed to search large numbers of main-sequence stars for eclipsing binaries, their low quantum efficiency requires lengthy individual exposures, so few plates can be obtained on a given night. Furthermore, photographic photometry of faint stars in crowded fields is particularly difficult and imprecise. Most of the photographic surveys described above were therefore sensitive only to the small fraction of eclipsing binaries with relatively large amplitudes ($\gtrsim 0.5$ mag; Van't Veer 1975b; Ruciński 1974). Moreover, Webbink (1979) noted that most short-period binaries in globular clusters would be expected to be low-amplitude systems located on or near the main sequence, if the contact binaries were formed at the same time as the cluster (however, we note below that that need not be the case).

Most recent efforts to find faint variables in globulars have therefore employed CCD detectors. In one such study, Shara et al. (1988) obtained consecutive CCD exposures of fields in ω Cen and 47 Tuc (NGC 104) for 3.3 and 3.5 h, respectively. Although their photometry was suitable for detecting eclipsing variables with amplitudes $\gtrsim 0.2$ mag,[8] none was found out of the ~ 3500 main-sequence stars monitored in the two clusters. Given the relatively short duration of the observations, however, this study may have missed short-period detached binaries or even large-amplitude contact systems observed at unfavorable phases.

More recently, Mateo et al. (1990), using a long sequence of CCD exposures, discovered three eclipsing systems among the blue-straggler population of the metal-poor globular NGC 5466 (see Fig. 3). They argued that all three binaries are likely cluster members, a conclusion supported by more detailed analyses of their light curves (Kallrath et al. 1991). More recently, Mateo and Krzeminski (1990) have confirmed that the first two faint vari-

[8]Shara et al. (1988) claim that their results should have detected objects with whose light curves exhibit a standard deviation $\gtrsim 0.08$ mag. This translates to an *amplitude* limit of about 0.2 mag.

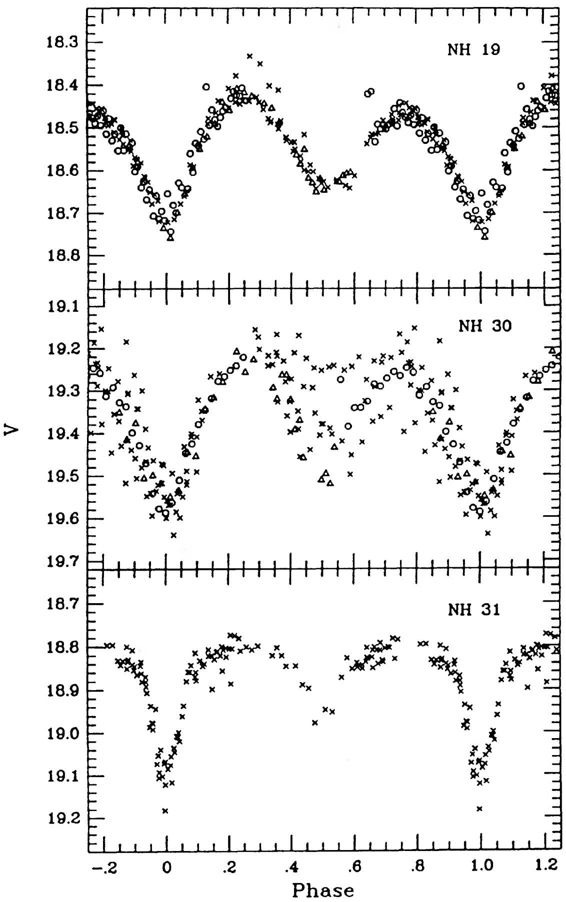

FIG. 3—V-band light curves of the three eclipsing binaries in NGC 5466. These results are based on measurements of over 100 individual CCD frames obtained over a 2-yr period. The scatter in the NH 30 light curve is presumably dominated by photometric errors in the crowded field near the center of the cluster. NH 19 and NH 30 exhibit the distinctive light curves of contact binary systems, while NH 31 is a detached binary. All three stars are blue stragglers.

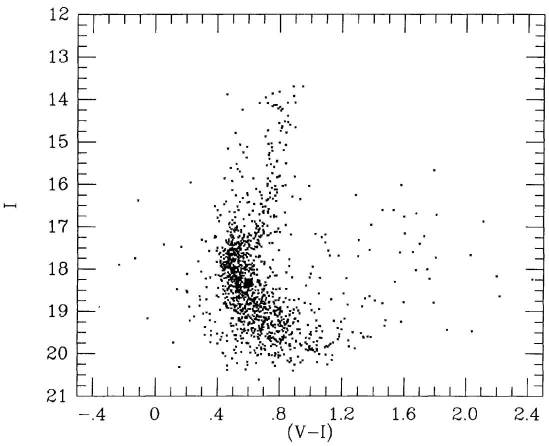

FIG. 4—An I versus $(V-I)$ CMD of the globular cluster NGC 6496. The point shown as a large solid square denotes the location of the newly discovered binary star identified in this cluster. Note that this binary is clearly located on the cluster main sequence, well below the turnoff point at $I \sim 17.5$. The zero points of the I and $(V-I)$ scales of this figure are approximate.

ables identified in M55 by Irwin and Trimble (1984) are in fact eclipsing binaries. Although their membership in the cluster remains to be definitively established, they are fainter than the cluster turnoff. If they are cluster members, they are almost certainly located on the main sequence.

Recent CCD observations at Las Campanas Observatory have uncovered further eclipsing binaries in a number of other southern globular clusters. Figure 4 illustrates that the binary found in one of these clusters—NGC 6496—is located on the main sequence of the cluster, well below the turnoff point. In a recent CCD photometric study of M71, Hodder et al. (1991) report the discovery of two probable binaries near the center of the cluster; one of them may be a short-period contact binary, while the other seems to be a longer-period binary of unknown type.

More recent CCD observations of M71 from Palomar Observatory have been used to identify additional binary candidates near the cluster center. The true nature of the variability of these stars remains to be definitively established. Table 2 lists the basic properties of all the known or suspected eclipsing binaries in globular clusters through August 1991. It is important to stress that, apart from NJL 5 in ω Cen, none of the binaries listed in Table 2 has been confirmed as a certain cluster member from radial-velocity or astrometric measurements.

All of the CCD studies described above have employed the same basic observational approach to find and identify eclipsing binaries: multiple CCD images are obtained of the cluster fields; photometry is performed on all the stars; the resulting lists of stellar positions and magnitudes are registered spatially, then converted to a common photometric system; stars exhibiting excessive variability compared to other stars of comparable brightness are identified as candidate variables. The real-time light curves of these candidates are then inspected to search for periodic variability consistent with short-period eclipsing-binary light curves. A χ^2 selection similar to that employed in searches for velocity variables (see Sec. 2.1) provides a useful means of generating a manageably short list of candidate variables.

This procedure is effective even in slightly nonphotometric conditions because of the large number of "local" photometric standards present in any globular-cluster field. However, instrumental effects or seeing fluctuations can readily produce "false" variables in the crowded fields typical of most globular clusters. This demands some care in the interpretation of the light curves of candidate variables. Most of the photometric searches for cluster binaries undertaken to date have involved observations of low-density or high-latitude clusters such as NGC 5466, M55, and M71, or of outer fields of rich clusters such as 47 Tuc and ω Cen.

Of course, the frequency and precision of the observations bias the sorts of binary systems that can be detected as optical variables by these techniques. In Fig. 5 we have used the average observed Fourier components of contact binaries (Ruciński 1973) to construct characteristic light

TABLE 2
Probable Eclipsing Binaries in Globular Clusters

Cluster–Star	Period (days)	Type	N_{MS}	Reference
ω Cen	1.38	Algol (BS)	...	Niss et al. 1978
NGC 5466–NH 19[a]	0.342	W Ursae Majoris (BS)	...	Mateo et al. 1990
NGC 5466–NH 30	0.298	W UMa (BS)	...	Mateo et al. 1990
NGC 5466–NH 31	0.511	Algol (BS)	...	Mateo et al. 1990
M55–1	0.254	W UMa	...	LCO; Irwin and Trimble 1984
M55–2	0.245	W UMa	...	LCO; Irwin and Trimble 1984
M71–H3	~0.37?	W UMa	1500	Hodder et al. 1991
M71–H4	$\gtrsim$2?	Algol (?,BS)	1500	Hodder et al. 1991
M71–1	?	W UMa (?)	3700	Palomar
M71–2	~0.4	W UMa	3700	Palomar
M71–3	?	Algol (?)	3700	Palomar
M71–4	?	W UMa (?)	3700	Palomar
M71–5	?	?	3700	Palomar
NGC 6496–1	~0.3	W UMa	1100	LCO
NGC 6362–1	?	W UMa (?,BS)	...	LCO
NGC 4372–V4	0.242	W UMa	...	Kaluzny and Krzeminski 1992
NGC 4372–V16	0.308	W UMa	...	Kaluzny and Krzeminski 1992
NGC 4372–V22	0.415	W UMa (BS)	...	Kaluzny and Krzeminski 1992
NGC 4372–V30	0.406	Algol (BS)	...	Kaluzny and Krzeminski 1992
NGC 4372–V31	0.378	W UMa	...	Kaluzny and Krzeminski 1992
NGC 4372–V35	0.290	W UMa	...	Kaluzny and Krzeminski 1992

Notes: The NGC 5466, ω Cen, and NGC 6362 variables are also blue stragglers. N_{MS} refers to the number of main-sequence stars that were surveyed to find the binaries listed in the table. This quantity is undefined when fields were surveyed on the basis of existing information regarding the likely presence of eclipsing systems (e.g., M55) or for the work on the blue stragglers. "LCO" refers to results from ongoing work at Las Campanas Observatory; "Palomar" refers to results from ongoing work at Palomar Observatory. Stars with only question marks in the "Type" column are given half-weight in deriving the binary frequency as described in the text.

curves for two systems with orbital inclinations of 60 and 90 deg. Using a Monte Carlo procedure, these light curves were then "observed" in a sequence of 10-min "exposures" for a total of 5 h over two "nights" (4 h the first "night" and 1 h the following "night").

Figure 6 illustrates how the standard deviation of the observational light curves varies with period (between 0.25 and 1.0 days) for each inclination. The error bars in Fig. 6 represent the ± 1-sigma intervals derived from 1000 simulated observing "runs" at each period. It is apparent that for cases where the standard deviation for nonvariables is $\gtrsim 0.03$ mag (due to photometric errors), a 3-sigma detection of a binary with a full amplitude $\lesssim 0.2$ mag (corresponding to $i \lesssim 60°$) would generally be ambiguous; even some of the large-amplitude edge-on binaries with periods

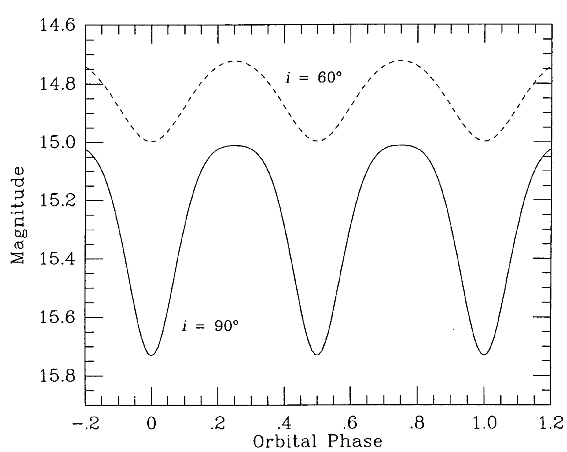

FIG. 5—Synthetic light curves for two hypothetical contact binaries with orbital inclinations of 60° (dashed line) and 90° (solid line). The curves represent binaries with equal-mass components and were constructed from the Fourier components given by Ruciński (1974) for typical contact binaries at these inclinations. These curves were used in the Monte Carlo simulations described in the text.

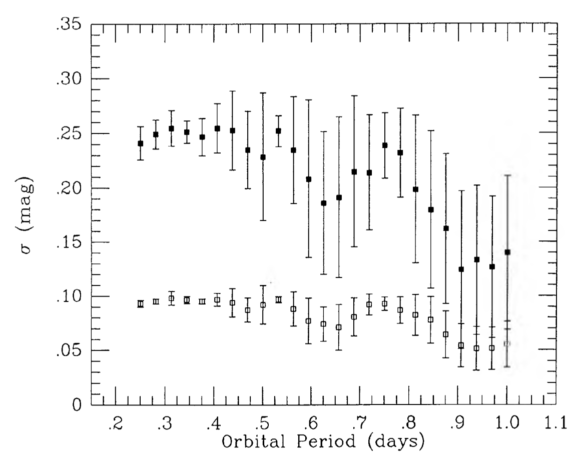

FIG. 6—The standard deviations of the "observations" of the stars represented by the light curves in Fig. 5 as a function of orbital period. The observations consist of a sequence of exposures spanning 5 h over two consecutive nights; each "exposure" was 10 min long. The initial phases were chosen at random and photometric errors of 0.03 mag per observation were assumed. One hundred simulations were run for each period. The solid squares represent observations of a contact binary with an inclination of 90°, the open squares represent observations of a contact binary with an inclination of 60°. Note that an observed standard deviation of 0.08 mag corresponds to a light-curve amplitude of 0.2 mag.

near one day might be missed owing to poor phase coverage.

From Fig. 6, we conclude that systems with orbital inclinations smaller than 60° would generally be missed by photometry of the precision and temporal coverage described above. Consequently, $\lesssim 50\%$ of all contact systems in globular clusters would be detectable in typical surveys, if there is no preferred orientation of the binary orbits. Longer-period, detached binaries are even more difficult to identify because (a) such systems are generally wider and therefore have a smaller inclination range in which eclipses can occur and (b) they vary less—if at all—between eclipses than do the highly distorted components of contact systems. Cluster binaries with periods in excess of about five days will therefore be very difficult to discover photometrically. This conclusion is consistent with the limited results available so far and summarized in Table 2.

Searches for short-period eclipsing binaries complement the other methods currently being used to discover binary systems in globular clusters (see Secs. 2.1 and 2.3). For example, detecting a binary sequence in a color–magnitude diagram requires excellent main-sequence photometry, effectively limiting this method to nearby, low-density clusters. Moreover, a binary sequence will only be visible if a significant number of binaries have mass ratios $\gtrsim 0.6$ (Romani and Weinberg 1991); in contrast, eclipsing contact binaries with mass ratios as low as 0.08 have (detectable) amplitudes $\gtrsim 0.2$ mag for inclinations $\gtrsim 70°$ (Ruciński 1974; Van't Veer 1975b).[9] Finally, short-period systems represent a population of binaries that cannot be identified by radial-velocity surveys of luminous red giants simply because these stars are too large to accommodate a binary system with $P \lesssim 30$ days. Multiobject spectroscopy of subgiant and main-sequence stars in clusters may ultimately provide the means to identify binaries in the 5–30-day period range, bridging the current gap separating systems detectable spectroscopically and photometrically.

Webbink (1976) and Vilhu (1982) have shown that the periods of eclipsing binaries can evolve significantly over the binary lifetimes owing to angular-momentum losses via gravitational radiation and—more importantly—via magnetized winds (see also Eggen and Iben 1989). The idea that the periods of short-period binaries evolve significantly is supported by the observed lack of binaries with orbital periods $\lesssim 2$ days in regions of active star formation (Mathieu et al. 1989) and among "unevolved" field binaries (Giuricin et al. 1984). In striking contrast, most contact binaries of later spectral types have periods considerably shorter than one day. Thus, short-period eclipsing systems in low-density globular clusters probably represent a primordial binary population in which the initial period distribution has been altered by angular-momentum-loss mechanisms rather than a population of binaries formed throughout the lifetimes of the clusters.

One important goal of searches for eclipsing variables in globular clusters is to estimate the binary frequencies in these stellar systems. The meager results reported in Table 2 suggest that the observed frequency of short-period main-sequence binaries is about 1 for every 950 ± 200 cluster dwarfs surveyed. We have assigned the uncertain main-sequence binary in M71 (M71-5) half-weight, and excluded binaries that are suspected to be blue stragglers (M71–H4, and NGC 6362-1) in deriving this result. In addition, we have excluded the null results of Shara et al. (1988) for 47 Tuc and ω Cen in estimating the cluster binary frequency; if those results are included, the observed binary frequency changes to 1 per 1500 ± 350 cluster dwarfs if we assume that 3500 main-sequence stars were uniformly monitored in the two clusters by Shara et al. (1988).

How does this compare to the frequency of short-period eclipsing binaries in the solar neighborhood? Duerbeck (1984) used the *General Catalog of Variable Stars* to estimate that the local density of *discovered* eclipsing binaries with periods <5 days and spectral types between A8 and K3 is 1.2×10^{-5} pc^{-3} [about 30% of Algol-type eclipsing binaries with periods less than 5 days occur in this spectral-type range (Herczeg 1984)]. With the luminosity function of local neighborhood stars given by Wielen et al. (1983), this implies that about 1 out of 830 main-sequence stars in this spectral-class range are eclipsing binaries with periods less than 5 days. If only F5 to late-G stars are considered, one out of 470 main-sequence stars is a short-period eclipsing binary.

Fleming et al. (1989), using an X-ray selected sample, deduced the frequency of contact binaries in the local neighborhood, and derived the local density of such binaries to be 8.5×10^{-5} pc^{-3}. However, this is an upper limit to the density of *optically observable* eclipsing contact binaries because (a) the X-ray selected sample is presumed to be insensitive to orbital inclination and (b) not all of the X-ray candidates are conclusively confirmed as binaries. If we assume that the critical inclination for detection of a field contact binary is 55° (Van't Veer 1975a), then only 43% the candidate binaries identified by Fleming et al. (1989) would have large enough photometric amplitudes for detection in an optical survey.

Thus, according to the X-ray data, about 1 out of 280 A8 to K3 main-sequence stars are *optically discoverable* contact binaries, while about one out of 160 F5 to late-G dwarfs are. Adding Algol eclipsing binaries with periods less than five days does not significantly change these results. Given the results derived from the Duerbeck (1984) and Fleming et al. (1989) samples, we conclude that about 1 out of 600 ± 250 A8 to K3 main-sequence stars in the local neighborhood is a short-period binary; if only F5 to late-G dwarfs are considered, the frequency is 1 out of 340 ± 150.

These results suggest that the binary frequency in globular clusters is between 30%–55% that of the solar neighborhood. Adopting a binary fraction in the solar neighborhood of 65% (Duquennoy and Mayor 1991), we conclude that the binary fraction in globular clusters is 20%–35%. Although this result is based on only three surveys of two clusters(!), it is nevertheless consistent with recent esti-

[9]In practice, these two methods can be applied simultaneously by simply alternating filters during long sequences of exposures of a single field.

mates, based on radial-velocity measurements (Carney 1983; Stryker et al. 1985; Latham et al. 1988, but see Abt and Willmarth 1987), of the binary frequencies for *field* halo stars. If the nondetection of eclipsing binaries in 47 Tuc and ω Cen by Shara et al. (1988) is considered, then the inferred cluster binary fraction is approximately 13%–22%. Of course, if we had adopted a lower binary fraction for the solar neighborhood (e.g., 25%; Abt 1987), our estimate of the binary fraction among main-sequence stars in globular clusters would be correspondingly lowered.

In deriving these estimates of the number of binaries in globular clusters, we have implicitly assumed the period distribution of cluster binaries is similar to that of binaries in the Galactic disk (Kraicheva et al. 1978; Duquennoy and Mayor 1991). To actually *derive* the initial period distribution of short-period binaries in globular clusters will require an excellent theoretical understanding of how their periods evolve within a cluster environment, along with better statistics from observations (i.e., more systems must be found!). For this purpose, well-defined *non*-detections of cluster binaries (e.g., Shara et al. 1988) are as important as positive detections. If sweeping assumptions regarding the period distribution of cluster binaries are to be avoided, efficient searches for cluster binaries with periods between 5–30 days will have to be developed.

2.2.3 Cataclysmic Variables

The binary nature of cataclysmic variables and novae (collectively referred to here as "cataclysmic variables," or CVs) is now well-established (Wade and Ward 1985), and their photometric and spectroscopic properties make them particularly distinctive. These characteristics suggest that CVs might be good tracers of the binary population in globular clusters. Although most globular-cluster CV candidates have revealed themselves through their X-ray properties, a number were first detected through their optical variability. Charles (1989) and Bailyn (1991) have recently reviewed the properties, statistics, and formation mechanisms for globular-cluster X-ray binaries and other CVs. In this section, we shall only mention some of the more recent efforts to identify CVs optically in globular clusters.

At least two novae have been observed very near the centers of (different) globular clusters (Hogg 1973). One of them—T Scorpii in M80 (NGC 6093)—was discovered over a century ago and was therefore the first binary of any sort found in a globular cluster. Margon et al. (1991) recently used *HST* to attempt to identify the M14 (NGC 6402) nova of 1938 in quiescence. Although they were able to resolve many new objects in the field surveyed from the ground by Shara et al. (1986b), no convincing candidate for the nova was identified. This is not necessarily surprising if the hibernation scenario for novae proposed by Shara et al. (1986a) and Livio (1991) is valid, but it does suggest that old novae may be quite difficult to identify in most globular clusters.

Of the numerous cataloged CVs located near globular clusters (Webbink 1980), only two have been seriously considered to be cluster members (Margon et al. 1981; Margon and Downes 1983). Recent spectroscopic studies suggest that the CV located near M30 (NGC 7099) is very likely not a cluster member (Machin et al. 1991), while the cataclysmic binary near M5 (NGC 5904) probably *is* a member of that cluster (Naylor et al. 1989). The long-term photometric properties of the M5 CV are also consistent with membership in the cluster (Shara et al. 1987).

Narrow-band Hα imaging has been used in systematic optical searches for CVs in globular clusters by Shara et al. (1985) and Grindlay et al. (1991). One possible candidate CV in NGC 6752 was reported in the latter study, but the star may instead be a late-type dwarf whose molecular bandhead mimicked strong Hα emission (Grindlay, private communication). Despite considerable effort, the number of known CVs in globular clusters remains very small. This may seem puzzling, as a naive extrapolation of the observed numbers of low-mass X-ray binaries (presumed to be accreting neutron-star binaries) suggests that CVs should be very common (Hertz and Grindlay 1983). More detailed considerations of the effects of the difference in mass between neutron stars and white dwarfs on their respective capture probabilities lead to much smaller predicted numbers of cataclysmic variables (Hut and Verbunt 1983; Verbunt and Meylan 1988). Thus, the difficulty in identifying large numbers of cataclysmic variables in globular clusters does not come as a surprise.

Unlike the short-period eclipsing systems described in the previous section, X-ray binaries and CVs are generally found in relatively concentrated clusters: only ω Cen and NGC 6712 possess such stars *and* have concentration parameters [$\equiv \log(r_{\text{tidal}}/r_{\text{core}}) \leqslant 1.6$]. Such a result is consistent with—and lends some support to—models in which CVs primarily form as a result of three-body encounters in dense cluster cores (Verbunt and Hut 1987; Charles 1989). They therefore provide important insights into the detailed dynamical interactions that form such binaries rather than about the primordial binary population of a cluster (although see Leonard 1989). The formation mechanisms of such binaries will be discussed further in Sec. 3 below.

2.2.4 Blue Stragglers

As noted above, some blue stragglers in globular clusters are known to be binaries through their photometric variability. A number of blue stragglers in ω Cen, NGC 5466, and NGC 5053 are also known to be pulsating "dwarf Cepheid" variables (Jørgenson and Hansen 1984; Mateo et al. 1992; Nemec et al. 1992; see Nemec and Mateo 1990 for a general review of these stars). If they are pulsating in the fundamental radial mode, their pulsational masses are approximately 70% larger than the turnoff masses of their respective parent clusters (see, e.g., Da Costa et al. 1986), consistent with the idea that they may be merged binaries (Eggen and Iben 1989; Mateo et al. 1990).

Alternatively, Leonard (1989) and Leonard and Fahlman (1991) suggest that blue stragglers result from the interactions of the components in relatively wide binaries. The important point to emphasize for the purposes of this review is that both mechanisms require the existence of a

TABLE 3
Photometric Binary Candidates (PBC)

Cluster	%PBC	r(core radii)	Reference
47 Tuc	<3	40	a
NGC 288	4	3	b
NGC 5053	3	2	c,d
M5	0	21	e
M4	<3	4	f
M13	<1.4	12	g
NGC 6752	<2	18	h
M30	<3	45	i

[a]Hesser et al. 1987.
[b]Bolte 1991a.
[c]Fahlman et al. 1991.
[d]Richer and Fahlman 1991.
[e]Richer and Fahlman 1987.
[f]Richer and Fahlman 1984.
[g]Richer and Fahlman 1986.
[h]Penney and Dickens 1986.
[i]Bolte 1987.

rich primordial binary population in order to form blue stragglers in low-density clusters.

Recently, blue stragglers have been identified in the cores of the highly centrally concentrated clusters NGC 6397 (Aurière et al. 1990) and 47 Tuc (NGC 104; Paresce et al. 1991; Guhathakurta et al. 1992). The "blueing" of the cores of some globular clusters with collapsed cores (Djorgovski et al. 1991) may also be evidence for rich, centrally concentrated populations of blue stragglers in such clusters. Although these discoveries suggest that blue stragglers may be ubiquitous components of globular clusters, they complicate the relatively simple relationship between those stars and binaries apparent from earlier studies of lower-density clusters. See also Sec. 2.3.5 for a discussion of the origin of blue stragglers.

2.3 Binaries In The Color–Magnitude Diagram

The study of cluster color–magnitude diagrams (CMDs) underlies much of our knowledge on stellar populations. Over the past few years, technological advances have allowed deep star counts to be obtained with high photometric accuracy, allowing the full two-dimensional distribution of the CMD to be exploited. Variations in metallicity, age dispersion, and unresolved binaries all contribute to the shape and breadth of the main sequence. Since a binary system combines the light from two stars, the object will appear on the CMD in a different location from the primary alone.

For example, if both main-sequence members of a binary system have the same mass and therefore the same color and luminosity, the CMD for a cluster with a population of binaries is simple: the binaries give rise to a sequence with identical color but brightened by 2.5 log 2 $=0.75$ mag because of the factor of 2 larger luminosity. On the other hand, if one member of the binary system always has a much smaller luminosity than the primary, the CMD appears to be only slightly wider than for single stars alone. The actual CMDs will lie between these limits.

There have been many observations that star clusters, both open and globular, indeed show a "second sequence" on the CMD, in the form of objects paralleling the main sequence but offset in magnitude by 0.75 mag at a constant color (e.g., Dabrowski and Beardsley 1977; McClure et al. 1981; Stetson and Harris 1988; Anthony-Twarog et al. 1990, to name a few).

2.3.1 The Second Sequence

The existence of a second sequence does not *necessarily* imply a surfeit of equal-mass binaries (although we will see below that there are cases where it does). For reasonable assumptions there is always a peak along the second sequence (Romani and Weinberg 1991, hereafter referred to as RW), although in practice the strength of the peak depends on the mass function, photometric accuracy, and distribution of mass ratios. This may be understood through the following simple example.

Suppose we have a cluster of single stars and arbitrarily pair them to form binary systems. At a particular value of the color $(B-V)_0$ there will be a peak in the distribution along the single-star main sequence, say at a magnitude $V=V_0$, since there will be many pairs with very discrepant luminosities. The pair made from two stars of identical type is offset from the main-sequence ridge line by $-2.5\log 2$ magnitudes in V. Making a pair with the given color $(B-V)_0$ and a magnitude V intermediate between V_0 and $V_0-2.5\log 2$ requires one star redder than $(B-V)_0$ and one bluer. Pairs with total color $(B-V)_0$ that are made of single stars within a small fixed interval of V_0, in general, have a distribution of their combined V that tends to peak near $V_0-2.5\log 2$; in other words, there are fewer pairs within the interval with $(B-V)=(B-V)_0$ where the components differ significantly from V_0.

RW considered a simple analytic main-sequence model which demonstrates these basic features. By choosing a linear relation for the main sequence on the CMD, $V=V_* +b(B-V)$ with $b\sim 5$ (Allen 1973), the binary distribution $F_2(B-V,V)$ becomes

$$F_2(B-V,V)$$
$$=\begin{cases} 2\Phi_2(M_1,M_2)\left\{\dfrac{1}{c}\dfrac{1+s^c}{1-s^{c-1}}\right\}\left\{\dfrac{s+1}{s}\right\} & \text{if} \quad 0<s<1 \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where $s\equiv L_2/L_1=10^{-0.4(V-M_1)}-1$, $c\equiv(b+1)/b$, and $\Phi_2(M_1,M_2)$ is the binary luminosity function. Notice that M_1 and s are implicitly functions of $(B-V)$ and V. The magnitude of the dimmer companion may be written $M_2 =M_1-2.5\log s$. For a smooth $\Phi_2(M_1,M_2)$, F_2 peaks at $s=0$ and 1 as discussed above. (Strictly speaking, F_2 diverges at $s=0$ and 1. However, the divergences are integrable and do not lead to an unphysical divergence in luminosity or number. Any physical distribution has a spread in stellar properties which ensures that the density in the CMD remains finite.)

As an example of how a CMD for a cluster containing binaries might look, consider the most straightforward case of negligible photometric errors, no metallicity or age dispersion among the cluster members, and equal-mass components. Here, the binary stars are recognized as a sequence running parallel to the main sequence and separated from it by 0.75 mag. Such a sequence (whose members we will term "photometric binary candidates," or PBCs) may have been seen in the globular cluster NGC 288 (Bolte 1991a), where 12 possible candidates (4% of all the stars) are seen in the parallel sequence in the magnitude range $V=19$–20. Spectra of nine of those candidates (two epochs separated by seven months) indicate that four show evidence at $>3\sigma$ for velocity variation in the range of 50–150 km s^{-1} (Margon et al. 1991).

Proof that this general technique does isolate cluster binaries has recently been provided by Bolte (1991b) who demonstrated that 10 of 17 PBCs in Praesepe are radial-velocity variables. That success rate is much higher than that of Griffin et al. (1988), who selected stars in the Hyades and found about 30% to be velocity variables. However, this photometric technique will always *underestimate* the true binary frequency in the field under study as binary systems with large mass ratios or with one component underluminous will not be discovered. That was clearly demonstrated in the work of Griffin et al. (1988) on the Hyades where numerous binary systems (recognized through radial-velocity variations) were found lying along the main sequence (see in particular their Fig. 2).

Globular clusters with very tightly delineated main sequences (because of high-precision observations in rather uncrowded fields) are tabulated in Table 3, together with an estimate of the binary frequency in the observed field whose location is indicated in units of the cluster core radius (Webbink 1986). The estimates are obtained without a detailed statistical analysis from the number of stars in a clearly defined parallel sequence in the CMD or simply from the number of stars approximately 0.75 mag brighter than the main sequence. No corrections for dynamical effects are included here, nor is any effort made to estimate the number of binaries that might have mass ratios significantly different from unity. Some fraction of the PBCs may be optical doubles (two stars lying along the same line of sight by chance), but in the cluster fields considered in Table 3 crowding is not a serious problem, so the number of such systems should be small. Thus the entries in the table correspond to only the obvious binary candidates in the fields observed.

While dynamical relaxation may have reduced the number of binaries seen at large core radii, and while binary–single and binary–binary interactions may have affected the primordial binary population and its distribution for most of the clusters in Table 3, the entries for NGC 288 and NGC 5053 are particularly interesting as those clusters possess long relaxation times (Webbink 1986) which may exceed the Hubble time in the fields actually observed. If that is so, then the approximately equal-mass primordial binary frequency (as judged solely from the PBCs actually seen) in those clusters is only a few percent of all the stars. Again, this result says nothing about binaries with either underluminous companions or with large mass ratios, however estimates of a few percent seem to be in line with general theoretical expectations (Murray, et al. 1991).

In more general situations where photometric errors are important in widening cluster main sequences, more sophisticated techniques (as discussed below) may be required to extract the number of binaries. The simulations shown in Fig. 7 (meant to model the CMD of NGC 5053 at two core radii from the cluster center) illustrate some of the problems. Panel 1 shows the CMD for a metal-poor globular cluster; the main sequence follows the 16 Gyr oxygen-enhanced isochrone for metal-poor stars (McClure et al. 1987, Table V) and the number of stars at each magnitude is taken from a power-law mass function with an exponent of 1.5 (Salpeter value=1.35), which is the measured value for NGC 5053 (Fahlman et al. 1991). The photometric errors are set to zero, there is no metallicity dispersion among the cluster stars, and no binaries are present.

Panel 2 shows the main sequence widened by the photometric errors (in both the magnitudes and the colors) found to be appropriate for a field at two core radii from the center of this cluster (Fahlman et al. 1991). The third CMD has the same fiducial sequence, with no photometric errors and 10% of the cluster tied up in binary stars. The masses of the primaries M_1 were chosen randomly from the mass function, and the secondary masses M_2 were generated from $dN/dq \propto q^{\beta}$ where $q=M_2/M_1$ ranged from 0.1 to 1.0. The diagrams in Fig. 7 use $\beta=0$, while simulations with $\beta=-2.5$ will be discussed shortly. Further discussion regarding the choice of q can be found in Leonard and Fahlman (1991).

The fourth panel of Fig. 7 widens the CMD of panel 3 with the appropriate errors. The important point is that about 9 of the cluster binaries brighter than $V=22$ could be recognized as PBCs. Finally, in CMD 5, 100% of the stars in the cluster are assumed to be binaries with mass ratios chosen as above while panel 6 again widens this CMD with the errors. Remarkably, it is now no longer obvious that there are PBCs in this CMD. While the main sequence is clearly wider than in panel 2, unless the photometric errors were very well understood, one might assume that there were no binary candidates at all in this field! An approach similar to that suggested by Romani and Weinberg (1991), which uses maximum-likelihood techniques, would be required to estimate the binary fraction properly here.

2.3.2 *Effects Of Crowding On Estimates Of Binary Frequency*

Assuming a binary mass ratio distribution function and a global cluster mass function, one can estimate or put limits on the fraction of binaries observed in a given field. The accuracy of the estimates will depend crucially on one's ability to define the systematic and random measurement errors for a given observation. In particular, one needs to know the observed distribution in $(B-V)$ and V for all true $(B-V)_0$ and V_0. Current CCD analysis pro-

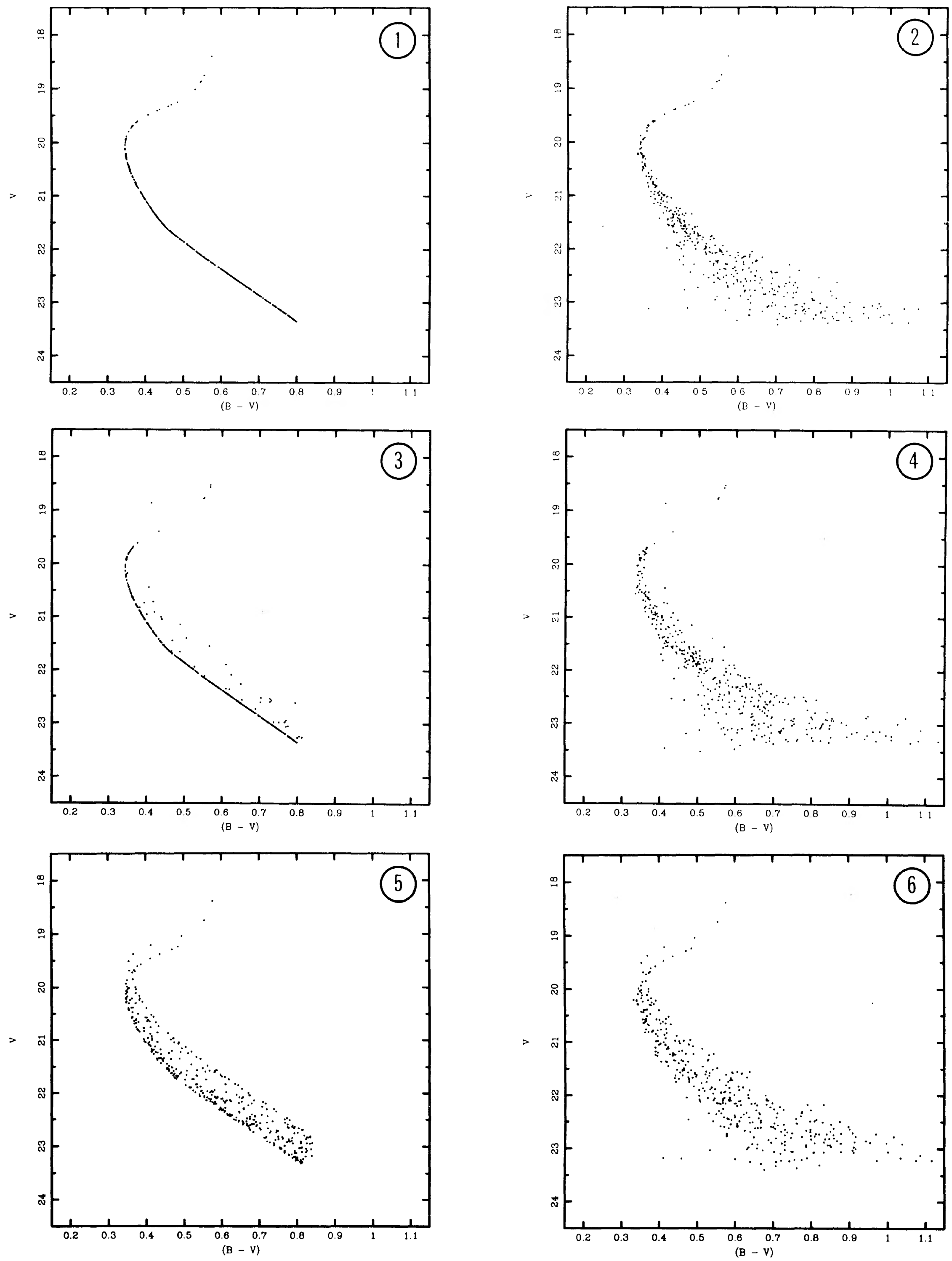

FIG. 7—A simulation of the CMD of the globular cluster NGC 5053 at 5 arcmin (two core radii) from the cluster center (no reddening corrections included). The six panels are as follows. (1) A metal-poor oxygen-enhanced isochrone for a 16 Gyr cluster. No errors in the photometry, no binaries, and no metallicity dispersion is included. (2) As in (1) except that Gaussian photometric errors with a dispersion $\sigma_M = 0.02e^{(M-22)}$ appropriate to this field are applied (Fahlman et al. 1991). Here M represents the apparent magnitudes B and V. (3) As in (1) except that 10% of the stars are binaries. (4) As in (3) except that the same errors as in panel (2) are applied. (5) As in (1) except that 100% of the stars are binaries. (6) As in (5) except that the photometric errors are applied.

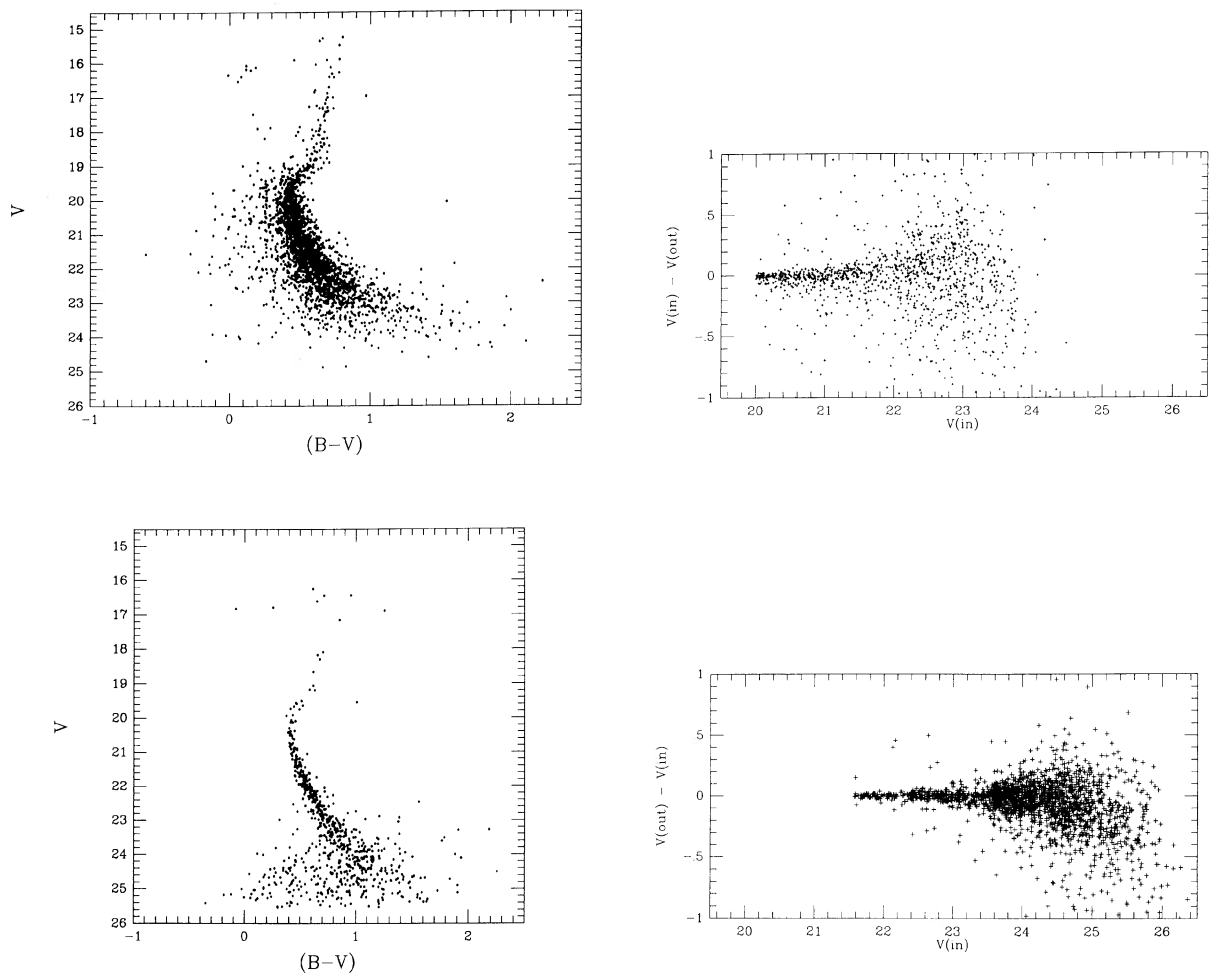

FIG. 8—The effect of photometric errors on the CMD of a real globular cluster NGC 5053, and thus on our ability to detect PBCs. The left-hand panels are the observed CMDs in the central field and at two core radii, while the right-hand panels indicate the error distributions in V for these two fields.

grams (e.g., DAOPHOT, DOPHOT) provide a way of estimating that distribution, although it is a time-consuming process.

As a concrete illustration of the effect of photometric errors (mainly due to crowding) on the CMD of a real cluster (and hence on our ability to detect PBCs), we discuss the CMD of the very loose cluster NGC 5053. That cluster has a low central concentration ($c=0.75$, core radius $r_c=2.5$ arcmin) and consequently a long central relaxation time which Peterson and King (1975) estimate to be 5.5×10^9 yr. The cluster is thus dynamically young (although Fahlman et al. 1991 did observe mass segregation in NGC 5053) and the binaries we observe in it are probably primordial.

Figure 8 displays the CMD of this cluster in two regions, a 3.5×2.2-arcmin^2 field centered on the cluster and a similarly sized field located at 5 arcmin (two core radii) from the center. Associated with each CMD are the measurement errors as a function of magnitude. These were generated from Monte Carlo simulations, i.e., stars of known magnitude were added into each field and recovered, the difference between their input and recovered magnitude being an estimate of the measurement error. As can be seen directly the CMDs appear dramatically different. Without careful analysis, all information concerning PBCs is lost in the central regions, whereas about 3% of all the stars present in the outer field are PBCs. The similarity between the CMD in the outer field and the simulation in panel 4 of Fig. 7 should be noted as these diagrams have the same error distribution and number of stars.

Another source of error, as previously mentioned, results from physically unassociated, but unresolved, pairs which produce the same signature as a true pair. For any observed field, these optical doubles are a source of background noise. An estimate of the confusion is necessary to interpret the significance of a binary detection on the basis of the shape of the CMD. We will see below that it is more important to increase the resolution and thus lower the confusion than to improve the photometric accuracy below several hundredths of a magnitude. Lower accuracy can be traded for more sources (such as a larger field) but a confused field creates difficult systematic errors and a background that must be independently quantified.

TABLE 4
Number of CMD Data Points Required to Distinguish a Given Binary Fraction from $f_b=0$ with 95% Confidence

		Binary fraction f_b				
x	σ	0.01	0.03	0.1	0.3	1.0
0	0.01	1.5×10^4	1.6×10^3	1.4×10^2	16	1.5
	0.03	2.8×10^4	3.2×10^3	2.7×10^2	31	2.8
	0.05	4.2×10^4	4.7×10^3	4.0×10^2	46	4.2
0.5	0.01	3.3×10^4	3.7×10^3	3.2×10^2	37	3.3
	0.03	6.9×10^4	7.8×10^3	6.7×10^2	77	6.0
	0.05	1.0×10^5	1.2×10^4	1.0×10^3	1.1×10^2	10
1.0	0.01	6.9×10^4	7.8×10^3	6.7×10^2	77	7.0
	0.03	1.3×10^5	1.5×10^4	1.3×10^3	1.5×10^2	13
	0.05	1.9×10^5	2.1×10^4	1.8×10^3	2.1×10^2	19
1.35	0.01	9.3×10^4	1.1×10^4	9.0×10^2	1.0×10^2	9.4
	0.03	1.6×10^5	1.9×10^4	1.6×10^3	1.8×10^2	16
	0.05	2.3×10^5	2.6×10^4	2.2×10^3	2.6×10^2	23

2.3.3 Maximum-Likelihood Estimates

Given the limitations imposed by the photometric errors, how can we uncover the binary fraction? RW advocate investigating the binary population by statistical comparison of observed CMDs with theoretical models as they would appear if observed. In order to explore the power of this analysis of the CMD, we present some simple models under a variety of assumptions. Following RW, we assume a linear main sequence as in Sec. 2.3.1 above and a power-law mass-luminosity relationship $L\propto m^{3.5}$. At first, we will assume that members of binary systems are distributed in mass-like single stars. An alternative model will then be explored.

Beginning with the model binary sequence described in Eq. (2), we assume that the probability of observing an object has a smooth spread about the intrinsic values of $(B-V)$ and V with disperion $\sigma_{(B-V)}$ and σ_V. This idealized model yields a self-similar distribution for all $(B-V)$, so it is really only one-dimensional. To compare with a two-dimensional CMD, we bin in $(B-V)$ as a function of V and stack by matching values of the ridge line to form a one-dimensional profile. The total distribution on the CMD is then the sum of the single and binary distribution functions $F_{\rm tot}(V)=(1-f_b)F_{\rm MS}(B-V,V)+f_bF_2(B-V,V)$, where $F_{\rm MS}$ is the main-sequence distribution, F_2 is the pure binary distribution [cf. Eq. (2)], and f_b is the fraction of binary pairs in the distribution. Each model is uniquely specified by the measurement dispersion σ, the binary fraction f_b, and the exponent x of the power-law mass function $dN\propto M^{-x-1}\,dM$. We choose models with the three values $\sigma\equiv\sigma_{(B-V)}=\sigma_V=0.01$, 0.03, 0.05 and the four values $x=0.0$, 0.5, 1.0, 1.35. For uncrowded fields, $\sigma\approx0.03$ is representative of a good measurement at or below the turnoff. Under less favorable conditions and more crowded fields, σ may be larger by a factor of 2 or more.

Now imagine that we have two sets of objects drawn from two distributions of different f_b. The confidence with which we may discriminate two different distributions $F_{\rm tot}$ may be ascertained with the KS statistic; the cumulative distribution at fixed $(B-V)$ may be defined as $\mathscr{D}(V)=\int_V^{V_0}dV\,F_{\rm tot}(V)$. Alternatively, we may use the KS statistic to estimate the number of objects one would need to reject the hypothesis that the distributions are the same at a chosen confidence level. In particular, let us suppose we have a cluster sample with a given binary fraction f_b. How many pairs are needed to reject the hypothesis that $f_b=0$ at the 95% level? The answer for our grid of models is presented in Table 4.

For example, for a mass function $x=0.5$, one can distinguish a CMD with 10% binaries with $\lesssim1000$ stars for all values of σ considered. For a Salpeter mass function ($x=1.35$), one would require a factor of 2 more objects. It

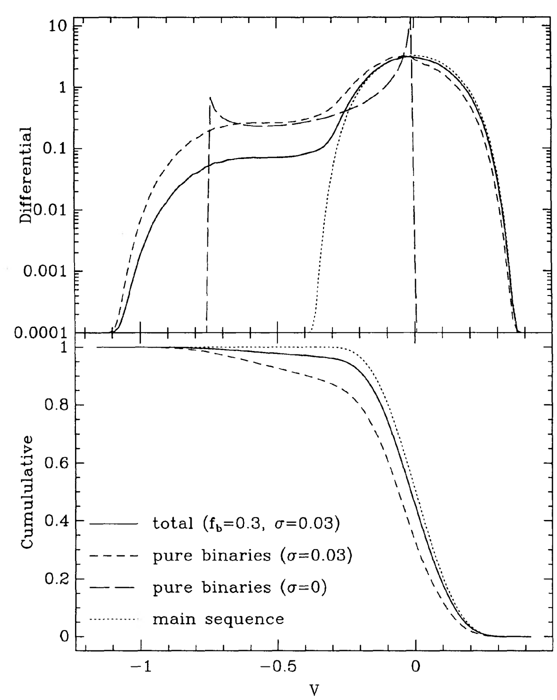

FIG. 9—Model with $x=0.5$, $f_b=0.3$, $\sigma=0.03$ (solid line). The upper panel shows the differential profiles, the relative number of sources in an interval dV, and the lower shows the cumulative distribution. The unsmoothed binary distribution (long dash), smoothed binary distribution (short dash), and smoothed main-sequence distribution (dotted) are shown for comparison.

TABLE 5
Number of CMD Data Points Required to Distinguish a Given Binary Fraction from f_b=0.05 with 95% Confidence

		Binary fraction f_b				
x	σ	0.01	0.03	0.1	0.3	1.0
0	0.01	9.1×10^2	3.6×10^3	5.4×10^2	23	1.6
	0.03	1.8×10^3	6.9×10^3	1.1×10^3	45	3.1
	0.05	2.6×10^3	1.0×10^4	1.6×10^3	67	4.6
0.5	0.01	2.1×10^3	8.1×10^3	1.2×10^3	53	3.7
	0.03	4.3×10^3	1.7×10^4	2.6×10^3	1.1×10^2	7.7
	0.05	6.4×10^3	2.5×10^4	3.9×10^3	1.7×10^2	11
1.0	0.01	4.3×10^3	1.7×10^4	2.6×10^3	1.1×10^2	7.7
	0.03	8.2×10^3	3.2×10^4	4.9×10^3	2.1×10^2	15
	0.05	1.2×10^4	4.5×10^4	7.0×10^3	3.0×10^2	21
1.35	0.01	5.8×10^3	2.3×10^4	3.5×10^3	1.5×10^2	10
	0.03	1.0×10^4	4.0×10^4	6.2×10^3	2.6×10^2	18
	0.05	1.4×10^4	5.6×10^4	8.6×10^3	3.7×10^2	25

is worth noting that for a given x, the number of required objects varies by ~ 3 as σ varies from 0.01 to 0.05. In performing such analyses of the CMD, there is a clear observational tradeoff in the number of stars required and photometric accuracy. Finally, at fixed σ and f_b the variation in number is a factor of ~ 6 between $x=0$ and $x=1.35$. As expected, the technique is much more sensitive at small x where the binary sequence is more pronounced.

The model with $x=0.5$, $f_b=0.3$, $\sigma=0.03$ is illustrated in Fig. 9. The upper panel shows the differential profiles, while the lower shows the cumulative distribution. The lower axis shows V magnitudes with the fiducial ridge line shifted to $(B-V)=V=0$. The unsmoothed binary distribution (long dash) has two cusps, at $V=0$ and $-2.5 \log 2$, and is shown along with the smoothed binary distribution (short dash). The dominant deviation between the single-star and pure-binary cumulative distributions occurs at $V \approx -0.3$. Notice that the deviation is due to the pedestal to the left of the main sequence, not the peak of the second sequence itself which is washed out by the dispersion. The smoothed distributions are much wider than σ owing to the slope of the main sequence; density from larger and smaller values of intrinsic $(B-V)$ are observed at $(B-V)=0$ causing the additional breadth. The spread about $V=0$ in the main-sequence distribution shows the extent of this effect. For $\sigma \gtrsim 0.05$, the binary sequence gets buried. This demonstrates that the efficiency of this approach will drop as confusion increases.

According to current estimates based on studied fields, the PBC fraction in globular clusters ranges from 0 to $\sim 5\%$ (see Table 3). Although the estimates may well vary even within a single cluster (e.g., as a function of radius), it is interesting to ask how many objects are needed to distinguish $f_b \equiv 0.05$ from neighboring values at the 95% level? As shown in Table 5, one would need $\sim 10^4$ objects to discriminate $f_b=0.03$ from $f_b=0.05$ even for $x=0$. Fortunately, data may soon be available using large-format CCD frames (2048×2048), so CMD analyses will be able to determine small values or differences in the binary fraction, as well as bounding f_b above zero with fewer observations. The variation in the number of objects required to discriminate the main sequence from a distribution with binary fraction f_b is illustrated in Fig. 10 (solid line) for the model $x=0.5$, $\sigma=0.03$. Also shown is the number required to differentiate a distribution with $f_b=0.05$ from other f_b. The drop in f_b with number is sharp; differences in f_b of ~ 0.1 are easily detected with $\lesssim 1000$ objects.

In summary, statistical analysis of presently observed CMDs may be used to detect binary fraction $f_b \gtrsim 0.05$ in relatively uncrowded fields. As both improvements in reduction techniques and the increasing size of CCD chips allow larger fields to be analyzed, the maximum-likelihood procedure may provide detailed quantitative information about the binary distribution in clusters.

2.3.4 *Testing Assumptions About The Mass-Ratio Distribution*

The distribution of the mass ratios in the binary components of clusters is poorly known. However, if one sees a

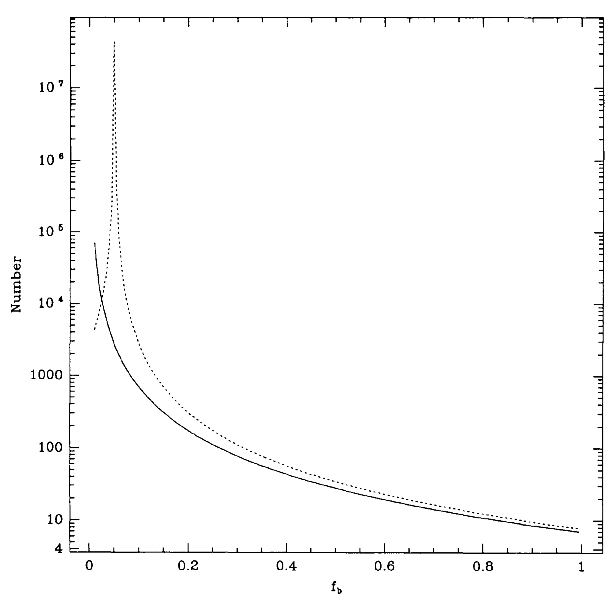

FIG. 10—Number of objects required on the CMD to distinguish a model with f_b=0 (solid line) and f_b=0.05 (dotted) line from models with other f_b. All models have x=0.5 and σ=0.03.

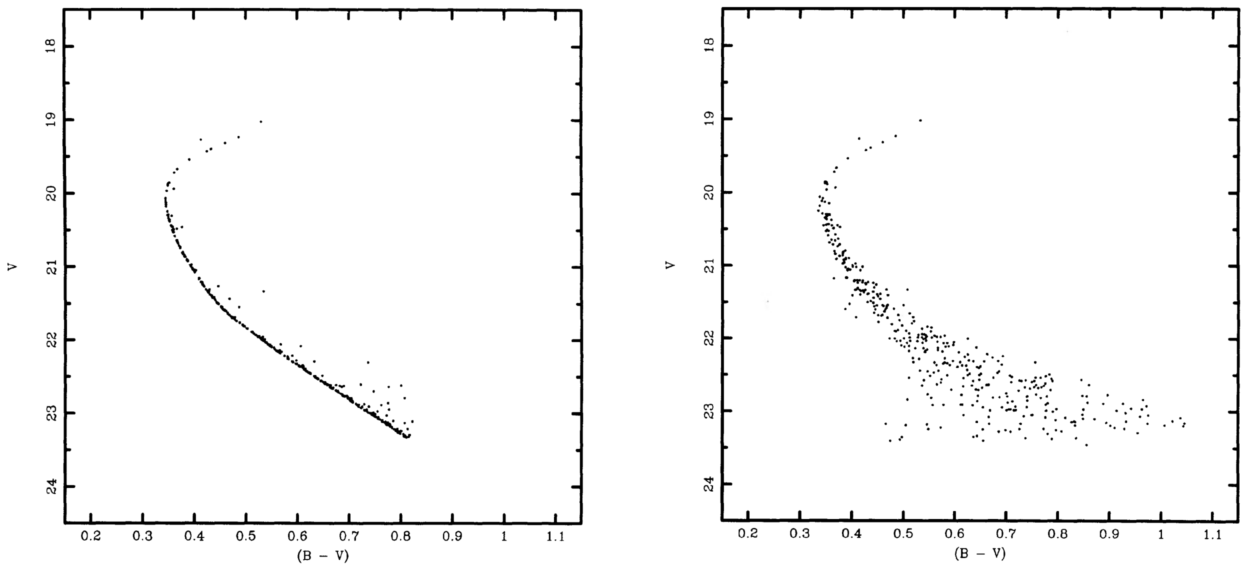

FIG. 11—Similar to Fig. 7 except that the binary mass ratios were determined as described in the text. In this case the cluster contains 100% binaries.

second sequence and can determine the mass function from the distribution along the ridge line, one can, in principle, test the hypothesis that the components of binary pairs tend toward equal mass.

To illustrate the sensitivity of PBC detection to the mass ratios of the binary components, we present a simulation similar to that shown in panels 5 and 6 of Fig. 7, except that now $\beta=-2.5$. With this choice of β, most binaries will contain at least one star of low mass. The result will thus be that PBCs will be difficult to detect as most will be found near the main sequence. Figure 11 illustrates a simulation of such a cluster which contains 100% binaries. The meanings of the panels are as in Fig. 7. Clearly it would be very difficult to assert that there are any PBCs in the cluster field by visual inspection alone.

The population of binaries in the solar neighborhood is relatively well-studied and provides at least a tentative model for clusters. Conversely, strong deviations from the local population would also be interesting, since that would indicate an alternative evolutionary history.

Abt and Levy (1976) studied the properties of binary systems among F3–G2 stars selected from the *Yale Bright Star Catalog* (Hoffleit 1982). This work was summarized by Abt (1983), but also see Morbey and Griffin (1987) for an alternate view. Abt and Levy identified two populations: a population with $a<100$ AU and one with $a>100$ AU. The components of the latter population, the wide binaries, are distributed like single stars whereas binaries with $a<100$ AU tend to have components of equal mass. For the F3–G2 dwarfs, Abt and Levy estimate the number distribution of secondaries to be $N_2 \propto m_2^{0.4}$.

The mass ratio $q=m_2/m_1$ $(m_2<m_1)$ has subsequently been considered by several authors. Halbwachs (1987) studied the distribution of q in both the visual binaries in the *Yale Bright Star Catalog* and a selected group of 205 spectroscopic binaries. He found that there was no evidence for a peak at $q=1$, but there was a possible maximum at $q\approx0.3$. No significant difference between the close and wide binaries was seen. Duquennoy and Mayor (1991) obtained quite similar results from a sample of 210 primaries selected from Gliese's catalog in the spectral range F7–G9; they suggested the following distribution for q:

$$\xi(q) \propto \exp\left[\frac{-(q-\mu)^2}{2\sigma_p^2}\right], \qquad (3)$$

where $\mu=0.23$ and $\sigma_p=0.42$. One of the inherent difficulties in the studies quoted above is the proper inclusion of selection effects.

A different tack was taken in the recent work of Eggleton et al. (1989). They incorporated all selection effects in the generation of a "synthetic" catalog, then compared the statistics of the synthetic and actual catalogs. Using the visual binaries in the *Yale Bright Star Catalog*, they also concluded that masses of components are correlated, although they did not estimate the actual profile.

Studies of the local population thus suggest that there is no reason to expect binary components to be distributed like single stars. Let us assume instead that the primaries are distributed like the single stars with the secondary masses distributed according to the $\xi(q)$ given by Duquennoy and Mayor (1991). For $x=0.5$, $\sigma=0.03$, and $f_b=0.3$, the binary distribution is quantitatively very similar to Fig. 9. Since there is little sign of the peak in the second sequence for x as small as 0.5 even for $f_b=1$, the observation of a pronounced second sequence suggests a population of binaries whose components are strongly weighted toward equal mass.

As an example, let us take a power-law distribution of mass ratio, $\xi(q)\propto q^\delta$; then half of the secondaries have $q>(1/2)^{1/1+\delta}$. A model with $x=0.5$, $\sigma=0.03$, $f_b=0.05$, and $\delta=0.5$, 1.0, 1.5, 3.0 may be distinguished from the $\delta=0$ model at 95% confidence with 1300, 590, 270, *and* 200 objects, respectively. The final model is illustrated in

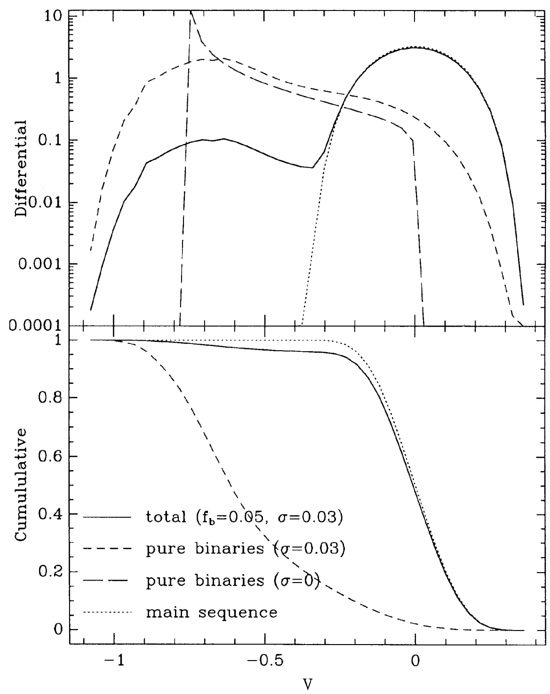

FIG. 12—Model with $x=0.5$, $f_b=0.3$, $\sigma=0.03$, $\delta=3$ shown as in Fig. 9.

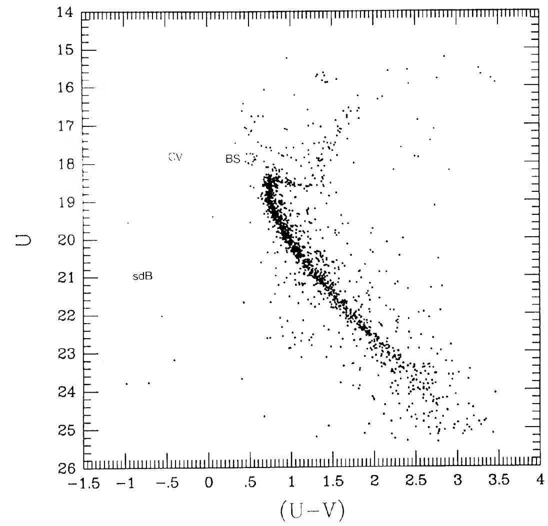

FIG. 13—The CMD in U, $(U-V)$ for the globular cluster M71, indicating the approximate locations of unusual stars thought to owe their peculiarity to binarity. Blue stragglers (BS), and B subdwarfs (sdB) are found in M71, while the location of a typical globular-cluster cataclysmic variable (CV) in quiescence is indicated.

Fig. 12 for comparison. It indicates that a population with preferentially equal-mass binaries should be easy to identify through analysis of the CMD.

2.3.5 Other Evidence For Binaries

Other features of globular-cluster CMDs may provide clues to the presence of binaries. In fact, any type of star not accounted for in standard evolutionary theory could be composed of binaries. That includes blue stragglers, subdwarf O and B stars, cataclysmic variables, dwarf Cepheids, extreme horizontal-branch stars, novae, and UV-bright objects. However, it is often unclear if the object really is or was a binary (e.g., blue stragglers and hot subdwarfs) or what the origin of the binary is (primordial or subsequently formed in the cluster). We list below three groups of objects which are believed to be binaries, provide a CMD for M71 in which an area containing some of these objects is isolated (Fig. 13), and produce a very crude estimate of the percentages of such objects in a few clusters.

(1) *Blue stragglers* are now known to be present in a wide variety of clusters ranging from those with very low-density cores [NGC 5053 (Nemec and Cohen 1989), NGC 5466 (Nemec and Harris 1987; Mateo et al. 1990), and NGC 7492 (Cote et al. 1991)] through to systems with quite dense stellar populations in their central regions [47 Tuc (Paresce et al. 1991), NGC 6397 (Aurière et al. 1990)]. The origin of blue stragglers remains controversial (Leonard 1989). At least three formation mechanisms that involve binary stars have been suggested: (a) The building up of low-mass companions in a binary system through mass transfer (McCrea 1964; Eggen and Iben 1989); (b) the merger of short-period binaries after a slow loss of orbital energy (Zinn and Searle 1976); and (c) the merger of stars during dynamical interactions involving binary systems (Hills and Day 1975; Hoffer 1983; Leonard and Fahlman 1991). Hydrodynamical studies of interacting stars have been presented by Benz and Hills (1987), Benz et al. (1989), Cleary and Monaghan (1990), McMillan et al. (1991), Goodman and Hernquist (1991), Davies et al. (1991), and Rasio and Shapiro (1991). Other possible mechanisms do not involve binaries at all (e.g., Wheeler 1979).

In the central regions of the loose clusters NGC 5053, NGC 5466, and M71 roughly 1% of all the stars measured down to $M_V=8$ are blue stragglers. In the least dense systems, where the formation of binaries by dynamical processes cannot be important, if the blue stragglers are in fact binaries, then those binaries must be primordial. For example, whereas up to 1600 physical collisions of stars are expected to have occurred in the core 47 Tuc over its lifetime, less than one has occurred in NGC 5466. Thus, any mechanism requiring the formation of blue stragglers via collisions of single stars is unlikely to be important in NGC 5466, but it may fully account for the population of blue stragglers in rich clusters such as NGC 6397 and 47 Tuc. The obvious warning is that blue stragglers are not necessarily always descendants of binary systems, especially those located in high-density clusters. The photometric variability of blue stragglers is discussed in Sec. 2.2.4.

(2) *Cataclysmic variables (CV)* are known to consist of a white-dwarf/red-dwarf pair and even in quiescence they occupy a unique region of the CMD. The CV M5V101 (Margon et al. 1981) in the globular cluster M5 appears in the $[U,(U-V)]$ CMD of Richer and Fahlman (1987) at

$M_U=3.5$ and $(U-V)_0=-0.8$, and this location is transferred to the CMD shown in Fig. 13 even though M71 is not known to contain any CVs. The number of CVs in globular clusters must be very small, as a hundred years of searching for variable stars in these systems has yielded fewer than half a dozen good candidates (Trimble 1980; Webbink 1980). See also Sec. 2.2.3 for a discussion of the photometric properties of CVs.

(3) *Hot subdwarf B stars (sdB)* seem to be ubiquitous in the central regions of globular clusters. Chan and Richer (1986) found 6 candidates brighter than $M_V=9.5$ in the core of M4 while Richer and Fahlman (1988) isolated 14 candidates among 5157 stars measured to $M_U=10$ in the central regions of M71. The brightest candidates in both of these clusters were confirmed spectroscopically as sdB stars by Drukier et al. (1989). In an unpublished $[(U, U-V)]$ CMD of the center of NGC 5053 Richer and Fahlman find about half a dozen sdB candidates among the 2087 stars measured to $M_U=8$. In more concentrated systems, even though no systematic survey yet exists, there is some evidence for their presence as can be noted in the CMD published by Bailyn et al. (1988) for ω Cen at two core radii. The very crude statistics that do exist thus seem to suggest that roughly 0.3% of all the stars measurable in the cores of globular clusters are of type sdB. One possible scenario for the origin of sdB stars is a merger of helium degenerates formed in close binary systems (Iben and Tutukov 1986). If the merged star has a mass less than about 0.5 $M_\odot$ it would not ignite its central helium and would simply evolve along a cooling track for a helium degenerate. Such a cooling sequence may have been seen in the CMD of M71 (Richer and Fahlman 1988).

2.3.6 Summary

With the current high precision of cluster photometry, the shape of the CMD can be used to investigate the properties of binaries in star clusters. With ~ 1000 objects and *a priori* knowledge of the mass function and measurement errors for a set of cluster observations, the binary fraction can be estimated or bracketed to within $\sim 10\%$. In addition, the relative strength of the second sequence may constrain the mass-ratio distribution of the binary components.

Strong second sequences are observed in both open and globular clusters. The strengths are not easily accounted for by mass functions with $x \geqslant 0$ if the masses of the secondaries are distributed like those of single stars or like those of spectroscopic binaries in the solar neighborhood. Since the cluster and field observational programs are rather different, it is certainly possible that the difference in population is caused by selection rather than Nature. On the other hand, if cluster binaries are markedly different from field binaries, comparison of the Galactic and globular-cluster populations may help determine the formation and evolution mechanisms.

Although the examples presented in this section are ideal, the approach is easily adapted to specific clusters (e.g., RW) and generalized to investigate specific binary models and different wave bands. Much can be learned by applying the statistical analysis to existing CMDs. In addition, with increased angular resolution, it should prove possible to measure the binary fraction as a function of distance from cluster centers and thus gain information on binary segregation and destruction processes.

With high-quality photometry it should be possible to measure the spread in the main-sequence ridge line due to fainter companions. That will allow stars fainter than the photometric limits of the study to be detected, "riding in on the coattails" of their brighter primary companions, thus providing information on the lowest-mass members of the cluster population. Finally, our results should be compared with the careful spectroscopic surveys of giants (e.g., Sec. 2.1) whose results suggest an incidence of binaries in the 10%–20% range. Those studies are sensitive to the bright, heavy post-main-sequence stars and thus probe closer to the cluster cores, in complement to CMD data.

Detailed analysis of CMDs will further improve our understanding of of binary frequencies in globular clusters. To date, most frequency estimates have been made using PBCs. Currently, the fraction of PBCs does not exceed a few percent of all the stars present; 3% or 4% seems to be an upper limit. That may still be significantly lower than the true binary frequency in globular clusters for several reasons.

First, PBCs seen to lie on a sequence parallel to and well separated from a cluster main sequence are expected to be approximately equal-mass systems. Binaries with large mass ratios or those with underluminous companions would not usually be included. Those systems would generally lie much nearer to the main sequence. Using the Hyades as an example, Griffin et al. (1988) find that only about a third of the cluster binaries are well separated from the main sequence. If these same statistics apply to globular clusters, the detection of 3% to 4% of the stars as PBCs suggests that more like 10% of the stars are actually binaries. Indeed, RW estimate a binary fraction of this order for M92.

Second, the cores are still by and large inaccessible to CMD analysis because of confusion; the fraction of binaries in the core of any cluster could be very much larger than that seen at many core radii where most of these statistics have been drawn.

2.4 X-Ray Binaries

Among the earliest discovered X-ray sources in our Galaxy a surprisingly large fraction is located in globular clusters. The globular clusters only contain a fraction of $\sim 10^{-4}$ of the mass of our Galaxy, but harbor 12 out of ~ 100 bright X-ray sources, with luminosities in excess of $\sim 10^{36}$ erg/s, associated with our Galaxy. Dim sources, at luminosities $L_x < 10^{35}$ erg/s, have been detected with *Einstein*, *EXOSAT*, and recently *ROSAT*, but only a handful of clusters has been observed to the level required to detect them. Efforts to detect those sources at optical and radio wavelengths have recently become successful. In what follows, we first discuss the bright sources and then the dim ones.

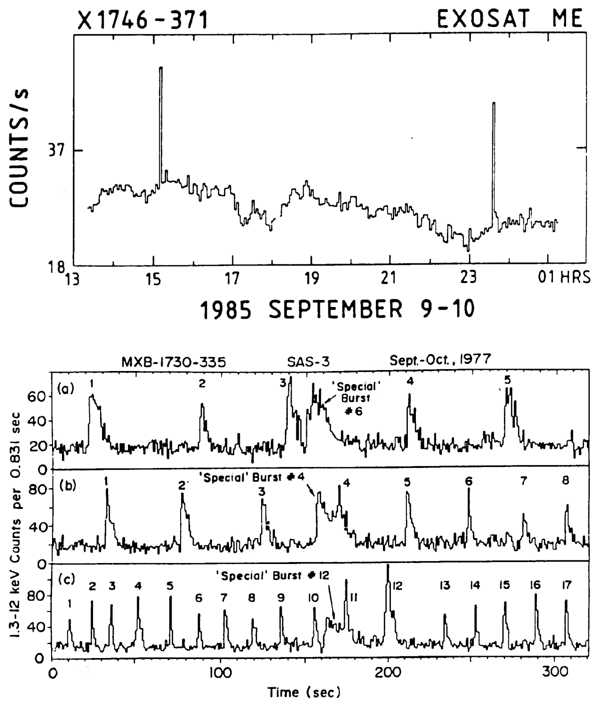

FIG. 14—Light curves for two X-ray sources in globular clusters. The top graph (from Parmar et al. 1989) shows the 1–10 keV count rate of the source in NGC 6441, plotted with a time resolution of 180 s, and features two X-ray bursts. Dips in the X-ray light curve occur around 13, 18, and 23 h; analogy with X-ray sources in the disk suggests that the distance between these dips, of about 5 h, may be an orbital period. The bottom graph (from Hoffman et al. 1978) shows the discovery graph of the rapid bursts from the source in Liller 1; most of the flux is emitted in rapid bursts. The "special bursts" observed occasionally are similar to the bursts seen in other sources (as for the source in NGC 6441 in the top graph).

2.4.1 Observations of the Bright Sources

The Fourth Catalog of X-ray Sources detected with the *Uhuru* satellite (Forman et al. 1978) contains six X-ray sources in globular clusters. One of them, in NGC 6440, dropped in luminosity by more than a factor 15, to a value below the detection threshold. Such a temporarily detectable source is called a transient. The other sources have all shown bursts of X rays, during which the X-ray flux first surges on a time scale of seconds by a factor up to 100, and subsequently subsides to the quiescent level on a time scale of tens of seconds.

The first of those bursts was observed with the *ANS* satellite, for the source in NGC 6624 (Grindlay et al. 1976). The *SAS-3* satellite discovered many more burst sources, both in globular clusters and in the Galactic disk; the observations are reviewed by Lewin and Joss (1983). Figure 14 shows an *EXOSAT* light curve of the source NGC 6441 with two bursts. SAS-3 also discovered a rather extraordinary source, located in the cluster Liller 1, and called the Rapid Burster (see below).

Two additional bright sources in globular clusters were detected by the *Hakucho* satellite during bursts; the permanent flux of those sources is thought to be below the *Uhuru* detection threshold (Makishima et al. 1981). Finally, the *ROSAT* sky survey added two bright X-ray sources in globular clusters. Because earlier surveys with the *HEAO-1* and *HEAO-2* satellites did not detect them (Hertz and Wood 1985; Hertz and Grindlay 1983), the new *ROSAT* sources must be variable; presumably they are transients (Predehl et al. 1991).

For permanently bright sources with $L_x > 10^{36}$ erg/s the survey of the Galactic globular clusters is virtually complete, in the sense that such sources would have been seen in any of the Galactic globular clusters. The bright sources currently known are listed in Table 6.

Interestingly, the luminosity distribution of the bright sources appears to show a lower cutoff at about 10^{36} erg/s. It has been suggested that sources whose time-averaged luminosity is less than this cutoff value become transients (White et al. 1984). The all-sky monitors on a number of newer satellites, such as *Ginga*, *Granat*, and *GRO* (BATSE), should do much to improve the time coverage of the transients and thus test this suggestion.

The Einstein satellite detected bright X-ray sources in globular clusters in M31 (Van Speybroeck et al. 1979; Long and Van Speybroeck 1983; Crampton et al. 1984; Trinchieri and Fabbiano 1991). The luminosities of these sources (as measured in a single observation with the Einstein satellite!) have a similar distribution as the luminosities of the low-mass X-ray binaries in our Galaxy, and are not correlated with the metallicity of the clusters (Verbunt et al. 1984).

2.4.2 Individual Systems

The rapid burster. Whereas other bursters have permanent X-ray radiation, on top of which the bursts arise, the so-called Rapid Burster in Liller 1 emits much of its flux in bursts. Most of them are very short bursts, occasionally interspersed with bursts similar to those seen in the other sources (Marshall et al. 1979, see Fig. 14). The Liller 1 source is a recurrent transient, being detectable for about one month roughly every half year (e.g., Kunieda et al. 1984b). The *Hakucho* satellite discovered that the first bursts of X-ray radiation when the source returns to detectability may last several hundred seconds; some observations with *Hakucho* and with *EXOSAT* found the source in a state in which the short bursts were replaced by continuous emission, making the source similar to the other burst sources (Kunieda et al. 1984a; Barr et al. 1987). As do some low-mass X-ray binaries in the Galactic disk, the Rapid Burster also shows quasiperiodic variations on time scale of 0.2–0.5 s; the properties of these variations are unique in their complexity, however, again setting the Rapid Burster apart from all other X-ray sources (Stella et al. 1988; Dotani et al. 1990a).

An 8.5-h binary in M15. For two X-ray sources in globular clusters the orbital period has been determined. The first is the source in M15. It has been identified with the ultraviolet variable star AC 211 (Aurière et al. 1984) which was subsequently shown to have a radial-velocity period of about 9 h (Naylor et al. 1988). The orbital period was found to be present in optical photometry and X-ray

TABLE 6a
X-Ray Sources in Globular Clusters: Bright Sources

Cluster	X-ray Name		L_x (2–11 keV) erg/s UHURU	L_x (0.5–20 keV) erg/s HEAO-1	L_x (0.5–2.0 keV) erg/s *ROSAT*
NGC 1851	0512−40	B	$7.5-0.4\times10^{36}$	3.0×10^{36}	
NGC 6440	1745−20	T	$23.7-<0.7\times10^{36}$	$<0.1\times10^{36}$	
NGC 6441	1746−37	B	$28.5-14.2\times10^{36}$	14.6×10^{36}	
NGC 6624	1820−30	B R	$68.2-10.2\times10^{36}$	51.7×10^{36}	
NGC 6652	1836−33	T		$<0.4\times10^{36}$	0.8×10^{36}
NGC 6712	1850−09	B T RO	$40.0-<0.3\times10^{36}$	0.4×10^{36}	
NGC 7078	2127+12	B RO	$12.2-0.7\times10^{36}$	1.5×10^{36}	
Ter1	1732−30	B		$<1.4\times10^{36}$	
Ter2	1724−31	B	5.2×10^{36}	3.1×10^{36}	
Ter5	1745−25	B		$<0.26\times10^{36}$	
Ter6	1751−31	T		$<2.5\times10^{36}$	2.9×10^{36}
Lil1	1730−33	B R	$8.1-<0.02\times10^{36}$	12.6×10^{36}	

TABLE 6b
X-Ray Sources in Globular Clusters: Dim Sources

Cluster	X-ray Name	L_x (0.5–4.5 keV) erg/s *Einstein*
NGC 104	0021−72	6.9×10^{33}
NGC 1904	0522−24	7.3×10^{33}
NGC 5139	1324−47	0.5×10^{33}
NGC 5272	1340+28	4.0×10^{33}
NGC 5824	1501−33	20.1×10^{33}
NGC 6440	1746−20	1.8×10^{33}
NGC 6541	1804−43	2.1×10^{33}
NGC 6656	1833−24	0.2×10^{33}

In Table 6(a) the cluster name and the X-ray position are followed by three indicators of the nature of the source, viz. B if the source is a burster, T or R if the source is a transient or a recurrent transient, and R and/or O if the source has a probable radio and/or optical counterpart. The following columns give the X-ray luminosities as observed by *Uhuru* (maximum and minimum; after Bradt and McClintock 1983), *HEAO-1* (after Hertz and Wood 1984), and *ROSAT* (after Predehl et al. 1992). In Table 6(b) the X-ray luminosity as observed with the IPC detector of the *Einstein* satellite are given (from Hertz and Grindlay 1983). The X-ray luminosities are derived from the published fluxes using the distances to the clusters tabulated by Webbink (1985). All bright sources have been detected by *ROSAT* in its sky survey, but accurate fluxes are not yet available. In NGC 5139 and NGC 6656 more than one dim source has been detected; only the one in the core is listed.

photometry as well, and more accurately determined at 8.5 h (Ilovaisky et al. 1987; Hertz 1987). For a long time the M15 source was the only well-studied cluster X-ray source not known to be a burster. In 1988, an X-ray burst was finally detected with *Ginga* (Dotani et al. 1990b). The burst maximum is close to the Eddington limit for a neutron star, indicating that the flux observed is indeed a good fraction of the total flux emitted, and that we observe the neutron star directly, thus laying to rest earlier suggestions to the contrary. To fill its Roche lobe at the 8.5-h period, the donor must be slightly larger than a main-sequence star. Radio investigations of M15, which have uncovered a number of radio pulsars, also showed a source at the position of AC 211 (Kulkarni et al. 1990b; Johnston et al. 1991).

An 11-min binary in NGC 6624. A second orbital period was found in *EXOSAT* data of the X-ray source in NGC 6624, at 685 s (Stella et al. 1987). Such a period is reminiscent of the pulse periods of some X-ray pulsars, young neutron stars in orbit around O or B stars; such long pulse periods vary on time scales of decades (see, e.g., the plot in Nagase 1989). Reanalysis of the *Ariel V* and *Sas 3* data uncovered the 685-s period, and showed that it is remarkably stable, which indicates that it is an orbital period (Smale et al. 1987; Morgan et al. 1988). The short orbital period implies a helium-rich donor (Verbunt 1987). Ginga and *ROSAT* observations have extended the baseline on which the period can be followed, and show that the orbital period decreases on a time scale of about 10^7 yr (Tan et al. 1991; Van der Klis et al. 1992). This is rather surprising, as it implies that the donor star is nondegenerate, i.e., helium-burning. Analysis of Vela data led to the discovery of a 176-day period, on which the X-ray source in NGC 6624 changes its luminosity by a factor 2, rising in about one month, and then declining until the next rise (Priedhorsky and Terrell 1984). The X-ray source in NGC 6624 has also been detected in the radio (Johnston and Kulkarni 1992).

Radio and optical counterpart of the source in NGC 6712. One other bright X-ray source in a globular cluster has been observed in the radio, viz. the source in NGC 6712 (Lehto et al. 1990). That source is interesting in being located in a relatively low-density cluster, in contrast to all the other bright globular-cluster X-ray sources. A blue star, at $V\simeq21$, $(B-V)=0.2\pm0.5$ has been proposed as the optical counterpart (Nieto et al. 1990; Aurière and Koch-Miramond 1992). With the distance and reddening of NGC 6712, that implies an absolute magnitude $M_V\simeq5.6$, rather faint compared with the average low-mass X-ray binary, which has $M_V\sim1.0$ (Van Paradijs 1983).

2.4.3 *They Are Neutron Stars ...*

All bright sources in globular clusters that have been well-studied have shown X-ray bursts. Model calculations have shown that such bursts can arise if hydrogen is accreted at moderate rates, of order $10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$, onto a neutron star (Maraschi and Cavaliere 1977; Taam and Picklum 1979; for a review, see Lewin and Joss 1983; Van Paradijs and Lewin 1988). At these rates, the hydrogen fuses directly into helium, but helium fusion does not immediately follow. Instead, a layer of helium builds up until temperature and pressure reaches the value at which helium fusion may take place, causing the whole layer to fuse in an outburst. Although the models are not able to explain the complexity of many bursts, they do reproduce the time scales of the ordinary bursts, as well as the observed luminosities, reasonably well. The energy released per gram in helium fusion is of the order of 1% of the energy released by accretion onto a neutron star. The burst model therefore also explains the observation that the time-averaged energy emitted by the bursts is about 1% of the total energy emitted by the burst sources between the bursts.

2.4.4 *... But Are They Binaries?*

We now retrace the evidence that the x-ray sources in globular clusters are binaries.

(1) *X-ray luminosity.* The high X-ray luminosities can be explained by accretion of a fair amount of matter onto a compact object. For an accretion rate $\dot{M}$ onto an object of mass M and radius R we may estimate the resulting luminosity L and the effective temperature T_e of the emitted radiation, very roughly, with

$$L=\frac{GM\dot{M}}{R}\sim 4\pi R^2\sigma T_e^4. \tag{4}$$

Thus, for accretion onto a neutron star with a mass of $\sim 1.4\ M_{\odot}$ and a radius of some 10 km, we need $\dot{M}>10^{-10}$ $M_{\odot}\ \mathrm{yr}^{-1}$ to produce a luminosity $L>10^{36}\ \mathrm{erg\ s}^{-1}$. At these rates, the temperature of the emitting surface is indeed high enough for the bulk of the radiation to be emitted in the X-ray regime. As not all accreted mass need be efficient in producing X rays, the $\dot{M}$ derived is actually a lower limit to the real accretion rate. Such a large $\dot{M}$ cannot be obtained from the interstellar medium, but requires a donor star to accompany the neutron star. This is true *a fortiori* in globular clusters, which contain very little interstellar medium.

(2) *Position in the cluster.* If the X-ray sources in globular clusters are binaries containing neutron stars, mass segregation should cause them to be concentrated in the cluster cores. Accurate positions of the X-ray sources were obtained with the Einstein satellite, and show such a concentration (Grindlay et al. 1984). In Fig. 15 we show the distances to the cluster centers of the X-ray sources in globular clusters. In making this figure we use more accurately determined core radii and centers of several globular clusters (Chernoff and Djorgovski 1989; Shawl and White 1986), as well as better positions for some X-ray sources.

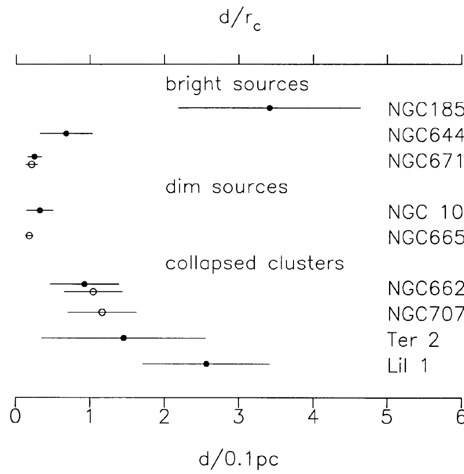

FIG. 15—The distances d between the center of the globular cluster and the X-ray source contained in it. Only those sources for which positions both of the center and of the X-ray source are known with an accuracy at the arcsecond level are indicated. For three bright and two dim sources in ordinary clusters, d is plotted in units of the core radius r_c. For four bright sources in post-core-collapse clusters, d is plotted in units of 0.1 pc. The "●" symbols indicate X-ray positions from *Einstein* HRI measurements (Hertz and Grindlay 1983); the "○" symbols give radio positions, from Johnston et al. (1992), Johnston and Kulkarni (1992), and Lehto et al. (1990). The cluster centers are from Shawl and White (1986), except for Ter 2 and Lil 1, for which we use centers given by Hertz and Grindlay (1985). The core radii are from Chernoff and Djorgovski (1989). To express d in parsecs, we use distances as tabulated by Webbink (1985). The identity of the radio sources with the X-ray sources is deemed probable, on the basis of the radio spectrum and the positional coincidence. The radio and optical positions of the source in NGC 7078 (M15) coincide, at a location which has a distance to the X-ray position of three times the $1-\sigma$ error of 1 arcsec given for the latter. As temporal variations at the orbital period are present both in the optical and the X-ray data, the identification of X-ray and optical source seems secure. We therefore only show the optical position in the graph.

Clearly, the bright X-ray sources are concentrated very strongly to the cluster centers, which suggests that they are more massive than the average cluster member. A single neutron star is already much more massive than the average cluster member. The single radio pulsars are also concentrated towards the cluster core. The concentration of X-ray sources in the core therefore provides no evidence that they are binaries.

(3) *Analogy with X-ray bursters in the Galactic disk.* Observationally, there is no evidence for any difference in X-ray properties between the X-ray bursters in globular clusters and those in the Galactic disk. In particular the luminosities are similar, and the X-ray spectra are similar, with $kT\sim 5$ keV. Orbital periods for the bursters in the Galactic disk have been determined with the advent of accurate CCD photometry and thanks to the ability of *EXOSAT* to observe these sources for several days on end without interruption (for a review, see Mason 1986). In analogy with the bursters in the Galactic disk, the X-ray bursters in globular clusters for which no direct evidence on binarity is available are also assumed to be binaries. For two X-ray sources in globular clusters, the orbital period has been measured; preliminary indication of an orbital

period may have been seen in the source in NGC 6441 (Parmar et al. 1989).

(4) *Could they be neutron stars accreting from massive disks*? Krolik (1984) argued that a close encounter between a neutron star and a main-sequence star may completely disrupt the latter, leaving a massive disk around the neutron star. This process has been brought back to our attention more recently by Phinney and Kulkarni (1992), who note that accretion from the disk onto the neutron star may spin it up, in which case the process is a very productive channel for the formation of single spun-up radio pulsars in globular clusters. During the accretion process the neutron star may be an X-ray source. Thus, we must also consider the possibility that (some of) the X-ray sources in globular clusters are accreting from a disk, without a companion. From the theoretical point of view, Johnston et al. (1992) estimate that the lifetimes of disk-fed systems will be too short to make it likely that any of the observed sources is such a short-lived system. For example, to make 100 spun-up radio pulsars in 47 Tuc requires 100 neutron stars fed by massive disks in the history of the cluster, which with lifetimes of 10^7 yr leads to a 10% chance of our seeing one right now. The crux of the argument, of course, lies in the lifetime of disk-fed systems, which is essentially unknown. Note however, that a lifetime of 10^9 yr for such systems would predict the presence of 10 bright X-ray sources in 47 Tuc, in clear contrast with observation. From the observational point of view, it is worth repeating that the X-ray sources in globular clusters are indistinguishable from the burster in the Galactic disk, from the point of view of luminosity distribution, temporal variability, and X-ray spectra. If they were neutron stars accreting from massive accretion disks, these similarities are accidental.

2.4.5 Lifetime of the Sources

The burst models discussed above indicate that bursts only occur at accretion rates of $\sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. The X-ray luminosities derived from these rates with use of Eq. (4) are of order 10^{37} erg s^{-1}, in good agreement with observations. Thus, the burst models indicate that the real accretion rates are not very different from those estimated by the rough Eq. (4) above.

We can use these models to estimate the lifetimes of such systems. A donor with a mass of 0.5 $M_\odot$, say, can keep up a transfer rate of $\sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ for a period of order 5×10^8 yr. This estimate could be wrong if our estimate of the accretion rate is very wrong, which as argued above is unlikely, or if the donor loses a large amount of mass without transferring it to the companion. There is no evidence for such large mass loss; nor is there an obvious reason why it would occur.

The second way to determine the lifetime of X-ray binaries is the measurement of their period derivatives. The number of systems for which such derivatives has been determined is limited, but one is the 11-min binary in NGC 6624, which has a period derivative of $\dot{P}_b/P \simeq -0.88 \times 10^{-7}\ \mathrm{yr}^{-1}$. This leads to an estimate of the lifetime of this system of about 10^7 yr.

Why are the estimates so different? The first assumes that the observed mass-transfer rate corresponds to a long-term average, while the second assumes that the currently observed period derivative equals a long-term average. According to evolutionary scenarios for binaries with helium-burning donors (Savonije et al. 1986), the period derivative currently observed is much higher than the long-term average, and the system is expected to spend most of its future life at a lower rate of mass transfer (hence lower luminosity), and a lower rate of change of orbital period. The currently obtained low estimate for the lifetime would then be accidental. The problem is of course that we are not sure we understand the long-term evolution of low-mass binaries.

2.4.6 Dim X-Ray Sources

The increased sensitivity of the Einstein satellite enabled it to discover a number of globular-cluster X-ray sources with much lower luminosities, of order 10^{33}–10^{34} erg s^{-1}. A single low-luminosity source was discovered in each of six clusters, while multiple sources were detected in two others: five in ω Cen and four in NGC 6656 (Hertz and Grindlay 1983). If one replaces the radius of the neutron star in Eq. (4) with the thousandfold larger radius of a white dwarf, one finds a luminosity 1000 times lower. Accordingly, Hertz and Grindlay argued that many dim X-ray sources are cataclysmic variables.

A possible problem with this interpretation is that no cataclysmic variables in the Galactic disks have such high X-ray luminosities, the reason being that, also according to Eq. (4), the temperature of the emitting area is low, causing the bulk of the flux to be emitted in the EUV. Verbunt et al. (1984) accordingly argued that the dim sources, at least those in the cluster cores, are in fact X-ray transients in their low state. This suggestion, already mentioned by Hertz and Grindlay as a possibility for the dim source in NGC 6440, gained in respectability when it was discovered that soft-X-ray transients in the Galactic disk can indeed have X-ray luminosities in the 10^{33} erg s^{-1} range (van Paradijs et al. 1987).

So far, the dim sources have eluded optical identification, although the position of the sources in ω Cen has been improved with *EXOSAT* observations (Verbunt et al. 1986). The accurate position of the source in 47 Tuc also did not provide an unambiguous optical counterpart (Aurière et al. 1989). As a result, some doubt has arisen as to whether the sources are really cataclysmic variables in the cluster (e.g., Margon and Bolte 1987).

If the sources in NGC 6656 (which is M22) and ω Cen outside the core are indeed binaries, it is somewhat difficult to understand how they could be so widely distributed in the cluster: one would expect binaries, whether cataclysmic variables or low-mass X-ray binaries, to be concentrated strongly towards the cluster core. One of the four sources in NGC 6656 is extended, and therefore not a binary (Hertz and Grindlay 1983). The study of the nature of the dim sources may be expected to receive a new impulse with the observations by the *ROSAT* satellite. As a first result, we mention that the radio source discovered by Johnston et al. (1992) coincides with the accurate position determined with *ROSAT* for the core source in NGC 6656 (Hasinger and Verbunt, private communication). This result is notable in establishing that radio observations may also help in determining accurate positions for the dim sources, as they have done for some bright sources.

2.4. The Nucleus of M33

So far, our review has mainly discussed observations of globular clusters around our own Galaxy. A great advantage of X-ray observations is that they can indicate the presence of binaries in globular clusters around other galaxies in the local group, such as M31, as discussed in Sec. 2.4.1.

Interestingly, there is one galaxy in the local group which has a core with a stellar density and velocity dispersion comparable to that of the densest globular cluster cores. This exception is the spiral galaxy M33. (The terminology becomes a bit confusing here, since we are addressing the inner structure of the nucleus, itself embedded in the "core" of the galaxy; we are thus addressing the existence of a probable core in the nucleus in the core!)

Recent observations of M33 by Kormendy and McClure (1991) give a central line-of-sight velocity dispersion of 22 km s^{-1}, only twice that in a typical globular cluster. Their observed upper limit on the core radius is $r_c \simeq 0.3$ pc. From these observations, Hernquist et al. (1991) argue that the true core radius is likely to be $r_c \lesssim 0.1$ pc.

In short, their arguments run as follows. From the virial theorem, or more specifically, the model of an isothermal sphere [Binney and Tremaine 1987, Eq. (4-123)], the local density $\rho(0.3\ \mathrm{pc}) = 2.0 \times 10^5\ M_\odot\ \mathrm{pc}^{-3}$. The corresponding local two-body relaxation time [Binney and Tremaine 1987, Eq. (8-71), and Spitzer 1987, Eq. (2.62) with $m = M_\odot$ and $\ln \Lambda = 10$] is $t_{\mathrm{relax}}(0.3\ \mathrm{pc}) = 9.4 \times 10^7$ yr. Since 0.3 pc is an upper limit to the core radius and since the central density is about three times that at the core radius, the central relaxation time is $t_{\mathrm{relax}}(0) \lesssim 3 \times 10^7$ yr. Such a short relaxation time implies that the system probably has already undergone core collapse.

With the observed velocity dispersion of 22 km s^{-1}, Hernquist et al. suggest that a collapsed core may have a true core radius $r_c \simeq 0.1$ pc. They offer three independent arguments for this value. First, simulations without primordial binaries show that cores oscillate after core collapse and that they spend most of their time near maximum expansion. They then typically contain 0.5% to 1% of the mass of a globular cluster. The corresponding core radius is $r_c \sim 0.1$ pc (Murphy et al. 1990). Second, a significant population of primordial binaries will cause core collapse to halt and reverse at relatively large core radii. Again the value is $r_c \sim 0.1$ pc (Goodman and Hut 1989; McMillan et al. 1990, 1991; Gao et al. 1991). Third, *HST* observations show that M15 has a core radius of $r_c \sim 0.1$ pc (Lauer et al. 1991).

Hernquist et al. conclude that the nucleus of M33 could have an efficiency of low-mass X-ray binary formation equal to that of all Galactic globular-cluster cores combined. This would suggest that about a dozen such binaries could be present. Their combined emission may explain the enigmatic, unresolved X-ray source in M33's nucleus, with its high X-ray luminosity of 10^{39} erg s^{-1} (Long et al. 1981).

2.5 Pulsars

2.5.1 Formation Mechanisms

In the Galaxy, the late stages of the evolution of a binary or triple star system initially containing at least one massive ($\gtrsim 8\ M_\odot$) star can involve accretion of matter from a companion onto a neutron star (for reviews, see Verbunt 1990 or van den Heuvel and Rappaport 1992). During the accretion phase, the neutron star shines as a bright X-ray source [referred to as a high-mass X-ray binary (HMXB) or a low-mass X-ray binary (LMXB), depending on whether the companion to the neutron star is more or less massive than 2 $M_\odot$].

The material accreting onto the neutron star carries the orbital angular momentum of the companion star. To accrete, most of this angular momentum must be removed, either in a wind from an accretion disk, or by viscous transport outward in the disk until tidal torques between the disk and companion restore it to the companion. The matter which eventually becomes attached to the neutron star's magnetic-field lines still carries enough angular momentum that accretion of only

$$\Delta M = 8 \times 10^{-3} P_{10}^{-4/3} I_{45}\, M_\odot \tag{5}$$

is enough to spin the neutron star (of moment of inertia $10^{45} I_{45}$ g cm^2) up to a very short period: $10 P_{10}$ ms. When accretion ceases, this rapidly rotating neutron star may be visible as a radio pulsar: a "recycled pulsar" (so called to distinguish it from "original manufacture pulsars," like the Crab pulsar, whose mass and spin were determined by the supernova explosions which created them).

2.5.2 Radio Searches

Motivated by the suggestions of evolutionary links between millisecond pulsars and LMXBs, and noting the high incidence of LMXBs in clusters, Hamilton et al. (1985) surveyed a dozen, nearby, dense globular clusters with the very large array (VLA). A point source was detected in the core of the cluster M28. Further observations showed this source to have the high polarization and steep spectrum characteristic of pulsars. Finally, in 1987 an international collaboration discovered a 3-ms pulsar (1821–1824) towards this cluster (Lyne et al. 1987).

Searches for cluster pulsars have now largely been completed to the sensitivity limits of existing instruments at Jodrell Bank, Arecibo, and Parkes. Integration times are long—$\gtrsim 2 \times 10^3$ s. The parameters of the binary pulsars discovered so far are presented in Table 7. A complete list of single, as well as binary pulsars in globular clusters may be found in Phinney (1992a). Given the faintness of the detected pulsars, it is not surprising that many of the discoveries have come from the 1000-ft Arecibo telescope. The cluster pulsars have luminosities similar to recycled

TABLE 7
Binary Radio Pulsars in Globular Clusters

Cluster	Pulsar	P_p (ms)	P_b (days)	e	$M_2^{\rm d}$ ($M_\odot$)	$P/(2\dot{P})$ (yr)	Reference
M15	2127+11C	30.5	0.33	0.68	1.3	$10^{8.3}$	e
Trz5	1744−24A	11.6	0.08	$<10^{-3}$	(0.09)	$>10^{9.3\,\rm a}$	f
47 Tuc	0021−72E[c]	3.5	2.22	<0.1	(0.16)		g
47 Tuc	0021−72J[c]	2.1	0.12	<0.03	(0.02)		g
M5	1516+02B	7.9	6.8[b]	0.13[b]	(0.2)		h
NGC 6539	1802−07A	23.1	2.62	0.22	(0.3)		i
M4	1620−26	11.1	191	0.025	(0.3)	$10^{8.3}$	j
NGC 6760	1908+00A	3.6	0.3[b]	0.0[b]			k
M53	1310+18	33.2	256	<0.01	(0.3)		l

Clusters are in order of decreasing central density.
[a]$\dot{P}$ believed (Phinney 1992) to be caused by the acceleration of the pulsar in its orbit about the dense cluster core.
[b]Provisional parameters: no phase-connected solution.
[c]The nine pulsars 0021−72D through M are only visible during moments of favorable scintillation, and have thus proved difficult to characterize and confirm.
[d]Mass of pulsar's companion (when in parentheses, tabulated value is $M_2 \sin i$, computed from the mass function assuming a pulsar mass of 1.4 $M_\odot$).
[e]Prince et al. 1991.
[f]Thorsett et al. 1991.
[g]Manchester et al. 1991.
[h]Wolszczan et al. 1989.
[i]D'Amico et al. 1990.
[j]McKenna et al. 1988.
[k]Anderson et al. 1990.
[l]Kulkarni et al. 1991.

pulsars in the Galactic disk. It is only the great distances to most clusters ($\gtrsim 7$ kpc) that makes their flux densities frustratingly low.

Besides the obvious luminosity selection effects, there are others which are well understood and can be modeled, such as those caused by data-sampling rate, dispersion measure, scattering by interstellar plasma, and Galactic synchrotron radiation. The last three are particularly important at meter wavelengths, the frequency of choice for cluster pulsar searches. Those selection effects certainly limit the sensitivity for clusters located in the bulge of the Galaxy—where, unfortunately, the richest clusters are located. Refractive scintillation in the interstellar medium can cause dramatic changes in the pulsars' flux on time scales of days to months, making evaluation of selection effects more difficult when, as is often the case, the discovery data are not accurately calibrated in flux. Pulsars may be preferentially discovered during refractive brightenings.

Because only recycled pulsars are found in globular clusters, it is important to note that standard pulse searches have less sensitivity to binary pulsars than to single ones. This occurs because a changing radial velocity smears the pulses in frequency, spreading the power in the Fourier transform of the pulse signal and making it harder to pick out of the noise (Johnston and Kulkarni 1991). To overcome this limitation, some groups have recently designed new search algorithms. Specifically, the approximation is made that the pulsar's radial acceleration is constant for the duration of observations. One then searches a variety of constant accelerations. The gain in signal strength is considerable for short-period binaries, and more than compensates for the reduced significance of peaks that results from the larger parameter space searched. This technique has been applied to clusters in which one pulsar has already been discovered, so the dispersion measure (DM) is known, and need not be searched for as well. The discovery of an 8-h binary in M15 (Anderson et al. 1990) by such a technique suggests that a number of compact binary pulsars remain to be discovered.

High-sensitivity radio synthesis imaging provides a powerful tool to probe the pulsar population in clusters. Imaging observations are unaffected by the many selection effects that plague pulse searches, including the reduced sensitivity to binary pulsars. The value of an imaging search is illustrated by the maps in Johnston et al. (1991), which show, e.g., that the cluster M92 cannot contain *any* pulsar, binary or otherwise, with 1.4 GHz flux density greater than 80 μJy—a limit challenging even for the giant Arecibo telescope. So far, only half a dozen clusters have been mapped with comparably high sensitivity. Observations of this kind provide our only observational constraint on the compact binaries which will be inaccessible to even the most sophisticated pulsar searches (Johnston and Kulkarni 1991). Indeed, the VLA data constrain the number of binaries not detected by ordinary pulse searches to be no more than twice the number detected (Kulkarni et al. 1990a).

3. THEORY

Globular clusters present us with a tantalizing problem. With $\sim 10^5$–10^6 member stars each, their dynamics is still too complex for detailed star-by-star simulation on present-day computers. However, much insight has been gained from approximate simulations, based on Fokker–Planck or conducting-gas-sphere models, as well as from direct N-body calculations with up to a few thousand particles.

In Sec. 3.1 we discuss the general features of globular-cluster evolution: core collapse, the nature of the energy sources that power the evolution after core collapse, and core oscillations. Since binaries play an important dynamical role in cluster evolution, we discuss in Sec. 3.2 how binaries are affected by the dense stellar environment of the inner regions of evolved globular clusters. The remaining three subsections review the various types of globular-cluster simulations which recently have been carried out: in Sec. 3.3 we discuss *N*-body simulations, in Sec. 3.4 Fokker–Planck simulations, and in Sec. 3.5 stochastic simulations.

3.1 Globular-Cluster Evolution

Globular clusters are in a stable equilibrium on a dynamical time scale, characterized by the time needed for a star to cross the half-mass radius, of order 10^6 yr. However, on a "thermal" time scale, in which individual stars can exchange significant amounts of energy, no cluster can achieve stability. At the half-mass radius, this two-body relaxation time scale lies in the range 10^8–10^{10} yr, while in a dense core of a centrally concentrated cluster it can take on much smaller values. At first, evaporation of stars from the cluster, past the tidal boundary imposed by the Galactic gravitational tidal field, causes the inner regions to shrink. At sufficiently high central densities, a gravothermal instability sets in locally, causing an increasingly rapid shrinking of the core.

This process of core collapse occurs on a time scale which is only a few times larger than the half-mass relaxation time. During and after collapse, infinite central density can be avoided only when some central energy source turns on. Several energy sources are possible, and they can be divided into three categories. Binaries can increase in binding energy, stars can undergo mass loss, or a black hole may form and subsequently swallow stars. All three processes result in direct or indirect heating of the cluster. For a short review of the field of globular-cluster evolution, see Elson et al. (1987). A general background for the dynamical evolution of globular clusters can be found in the excellent monograph by Spitzer (1987). In the following subsections we sketch the main arguments and results.

3.1.1 Core Collapse

Half a century ago, Ambartsumian (1938) and Spitzer (1940) independently predicted the inevitable decay of star clusters by evaporation. Two decades later, Antonov (1962) and Lynden-Bell and Wood (1968) showed how a star cluster undergoes internal core collapse, well before its final evaporation, because of the instabilities caused by the negative heat capacity of all self-gravitating systems. Also around that time, Hénon (1961, 1965) constructed the first cluster models exhibiting core collapse.

In the 1970s, star-cluster research received an enormous boost from the unexpected discovery of globular-cluster X-ray sources, which were explained as neutron stars that had been tidally captured into binary stars (Fabian et al. 1975; Press and Teukolsky 1977). On the theoretical frontier, a variety of numerical simulations began to sketch how star clusters evolve towards core collapse. These simulations were based on the Fokker–Planck approximation for the slow drift of stars in energy and angular-momentum space (cf. Spitzer 1975). Hénon (1975) was the first to extend a cluster-evolution model past core collapse. Soon afterwards, the first observational evidence for a collapsed core, which happened to be in M15, was presented (Newell and O'Neill 1978), although it was interpreted as a density cusp around a massive black hole. It took an additional 13 years, and the launch of the *Hubble Space Telescope*, to resolve the core of M15 (Lauer et al. 1991).

After a few years of quiet work by theoreticians trying to understand how to model a cluster after core collapse, the results of a number of different simulations were published around 1984; they were discussed and summarized in papers presented at the IAU Symposium 113 (Goodman and Hut 1984). Until 1990, nearly all simulations of star-cluster evolution were started with initial conditions in which all stars were single. Stars were allowed to form binaries either dynamically, through near-simultaneous close encounters of three stars leaving two of them bound, or tidally, through energy dissipation in the tidal bulges of one or both of the stars involved in a close encounter.

3.1.2 Post-Collapse Evolution of Globular Clusters

It was only after the early 1980s that simulations became sophisticated enough to be able to penetrate past the near-singular state of core collapse into the asymptotic regime of post-collapse evolution and eventual cluster evaporation (see Goodman and Hut 1985, and references therein). That decade saw a number of exciting theoretical discoveries. Gravothermal oscillations, first predicted on the basis of numerical simulations (Sugimoto and Bettwieser 1983; Bettwieser and Sugimoto 1984), were later also found in semianalytical studies (Goodman 1987). At about the same time, it was realized that close encounters and physical collisions between stars, previously invoked as possible explanations of cluster X-ray sources and blue stragglers (Fabian et al. 1975; Krolik 1983), could also have far-reaching dynamical consequences for the cluster as a whole (Goodman 1984; Ostriker 1985; Lee and Ostriker 1986; Lee 1987a,b).

Another important development was the observational discovery of many seeing-limited cores (cf. Djorgovski and King 1986), suggestive of remnants of core collapse, and quite well explained in terms of the results of multimass cluster simulations, with a judicious choice of stellar-mass function (Murphy and Cohn 1988; Murphy et al. 1990). Around the same-time, evidence was accumulating for the presence of a substantial population of primordial binaries in globular clusters. The present state of that evidence was discussed above in Sec. 2.

The evolution of a globular cluster after core collapse has only recently been studied intensively, and many aspects of our understanding of it remain uncertain and may change dramatically in the coming years. The *mean* behavior of the cluster after core collapse *is*, however, quite firmly established: the half-mass radius expands according to $r_h(t) \propto t^{2/3}$, where t is the time since core bounce, while the velocity dispersion drops according to $v \propto t^{-1/3}$. This relation may be derived from general principles, without any knowledge of the mechanism of energy generation in the core (cf. Hénon 1965, 1975), in a manner analogous to Eddington's (1926) prediction of the mass–luminosity relation for stars, which requires no precise knowledge of the nature of their internal energy generation.

The derivation goes as follows: (1) the half-mass relaxation time $t_{\rm hr}$ in a self-similar solution scales as $t_{\rm hr} \propto t$, the time since core bounce; (2) $t_{\rm hr} \propto N t_{\rm hc}$, where N is the number of stars in the cluster, $t_{\rm hc}$ is the crossing time at the half-mass radius, and we have neglected a factor log N; (3) if we neglect the slow change, due to escape, in mass and particle number, the virial theorem gives $t_{\rm hc} \propto r_h^{3/2}$; (4) combining these gives $t \propto r_h^{3/2}$ which leads to the results quoted above. In contrast, the rate of expansion of the core *does* depend on the details of the central engine (Cohn 1985; Ostriker 1985). Recently, semianalytical models for a variety of physical processes and approximations have been developed by Stodółkiewics and Giersz (1990) and Giersz (1990a,b).

Thus, regardless of the precise state of the core and of the physical processes going on there, after core collapse the cluster half-mass radius expands steadily. As it does so, the Galactic tidal field steadily removes the outermost stars, with the result that, eventually, the entire cluster is disrupted. The time scale for this process is longer than the core-collapse time, but only by a factor of a few.

3.1.3 The Central Energy Source

As mentioned above, several energy sources are possible in a cluster. One is binding energy extracted from binaries. Another is mass loss from the system by stellar evolution. Binaries can be formed by three-body dynamical capture or by two-body tidal capture. Enhanced mass loss can occur when stars collide and merge, forming heavier remnants with a much shorter lifetime under stellar evolution than original cluster stars. Finally, a black hole can form through repeated merging (for a recent review of these three mechanisms, see Goodman 1989).

The heating caused by each of these mechanisms occurs in quite different ways. Let us first look at binaries. A hard binary is defined as having a binding energy $\gg 1kT$, where $(3/2)kT$ is the mean stellar kinetic energy, a measure of the average thermal energy of a single field star. In the equal-mass case, a hard binary has an orbital velocity clearly exceeding the velocity dispersion of the system. When a hard binary encounters a single star, the three bodies tend to achieve equipartition, possibly after a temporary capture and/or exchange. In the process the binary will give energy to the escaping single star, and harden in the process. In this way, on average, the single star comes out of the scattering process with more energy than it went in with—hard binaries tend to heat their environment (cf. Heggie 1975; Hut 1983).

The second mechanism, mass loss, is a more indirect way of heating a star cluster. Let us consider the situation where a cluster loses a fraction ϵ of its mass, either through the escape of one or more stars, or through mass loss during stellar evolution (via a wind, or a helium flash, or the supernova explosion of a multiple-merger product). In all these cases, the mass loss will take place nearly instantaneously relative to the evolutionary time scale of the cluster. As a result, the kinetic energy will on average decrease by a factor ϵ (in case of winds or explosive mass loss), or less (in case of a slow diffusion of stars toward unbound orbits). However, the potential energy of a star cluster is quadratic in its mass, and a decrease in mass by a fraction ϵ will lead to a decrease in potential energy by a fraction 2ϵ. Therefore, the initial virial equilibrium, in which the potential energy was twice the kinetic energy of the cluster, cannot be maintained. After the mass loss, the potential energy will have decreased by much more than the kinetic energy. Therefore, effectively, the cluster will have been heated with respect to its new equilibrium configuration.

The third mechanism, heating by a black hole, is also an indirect process. The central hole will most likely capture stars on orbits which are confined predominantly to the central regions of the cluster. Therefore, they carry a relatively small fraction of the kinetic energy of the stellar population. Their capture again tends to increase the relative temperature of the remaining stellar population.

3.1.4 Core Oscillations

In some of the earliest post-core-collapse simulations, Sugimoto and Bettwieser (1983; Bettwieser and Sugimoto 1984) found chaotic fluctuations in the size of the core radius. They explained these as a consequence of the gravothermal instability, and therefore introduced the term "gravothermal oscillations" to describe them. In essence, the underlying physical mechanism can be simply described as follows. For a large number of stars in the system, the inner relaxation time scale is much larger than the half-mass relaxation time scale, which determines the overall rate of expansion. Therefore, the inner regions have the tendency to evolve on a time scale much smaller than the overall expansion time scale. As a result, the inner regions tend to get impatient, and a small fluctuation can trigger a local recollapse, followed by a local reexpansion. The larger the number of stars, the more the central and outer time scales are decoupled, and the more chaotic the oscillations become. A formal demonstration that the dynamical behavior of these oscillations is characterized by a low-dimensional chaotic attractor is given by Breeden et al. (1992; for a summary, see Cohn et al. 1991).

The gravothermal character of the core oscillations was confirmed explicitly by Goodman (1987), who performed a linear stability analysis of a new regular self-similar model for post-collapse evolution, and classified the different modes of behavior according to the type of linear instability they exhibit. He found that for a total number of stars $N < N_1$ his self-similarly expanding solution is linearly stable, while for $N_1 < N < N_2$ the solution is overstable, and for $N > N_2$ it is unstable. He estimates $N_1 \approx 7000$ and $N_2 \approx 40{,}000$. Although the instability for very large N has indeed the expected character of a gravothermal instability, we do not yet understand the nature of the overstability. It is presently somewhat unclear how this behavior is modified by the presence of complicating physical effects, such as a stellar mass spectrum or a substantial binary population.

Like gravothermal collapse, gravothermal oscillations now appear to be a ubiquitous phenomenon, at least in the models which treat the stars and all physical processes as continuous quantities. Inagaki (1986) and McMillan (1986, 1989) have expressed doubts as to whether the oscillations persist in real clusters, where the stars and the physical processes are discrete, and statistical fluctuations may be very large. This issue can probably only be resolved by direct N-body simulations of systems containing $>10^4$ stars.

3.2 Binary-Star Evolution

3.2.1 Physical Mechanisms

Binaries in globular clusters can evolve via a variety of mechanisms, closely coupled to the overall evolution of the cluster. They are: (1) mass segregation; (2) three-body interactions with other cluster stars; (3) binary–binary interactions; (4) recoil and ejection from the core and/or the cluster; (5) physical collisions and coalescence; (6) spiral-in of close systems; and (7) stellar and binary evolution. The final three items have not yet been included self-consistently in any dynamical simulation, although they are probably critical in determining the ultimate fate and appearance of cluster binaries.

Mass segregation occurs because the binaries, being heavier than average, tend to sink toward the cluster core, on a local relaxation time scale: $d \ln r/dt \sim t_r(r)^{-1}$, where t_r is the local relaxation time. In the denser inner regions, binaries can interact with single stars or other binaries. Binaries softer (i.e., less tightly bound) than $\sim 1kT$ tend to be quickly destroyed ("ionized") by such encounters (Heggie 1975; Mikkola 1983a,b; Goodman and Hut 1984), and are generally not included in cluster models. Harder binaries tend to harden (Heggie 1975; Hut 1983), shedding their excess energy as kinetic energy of the recoiling products, and thereby heating the cluster. The watershed, first determined by Hut (1983) for the equal-mass case, is only weakly dependent on mass ratio, and typically occurs when the relative velocity at infinity v_∞ is about $0.8v_c$. Here v_c is the relative velocity for which the field star f has just enough energy to ionize the binary (semimajor axis a, masses M_1, M_2):

$$v_c=\sqrt{\frac{GM_1M_2}{M_f}\frac{(M_1+M_2+M_f)}{(M_1+M_2)}\frac{1}{a}}\,. \qquad (6)$$

The (equal-mass) binary/single-star interaction cross section may be conveniently expressed as

$$\frac{d\sigma}{d\Delta}=\sqrt{\frac{\pi}{3}}\frac{63Gm_*a}{\langle v^2\rangle}\,\Delta^{-1/2}(1+\Delta)^{-4} \qquad (7)$$

(Spitzer 1987), where $a=Gm_*/E$ is the semimajor axis of a binary with binding energy $E>0$, and $\Delta=\delta E/E$ is the fractional change in binary binding energy resulting from the encounter. Each close ("resonant") three-body interaction [corresponding to the $(1+\Delta)^{-9/2}$ tail in Eq. (7) for $\Delta \gg 1$] produces, on average, a 40% increase in binary binding energy (Heggie 1975). When wider "flyby" encounters are considered too (the leading $\Delta^{-1/2}$ term, for $\Delta \ll 1$), the mean energy change per encounter drops to 20%. [Notice that this approximation does not have the correct asymptotic scaling for $\Delta \rightarrow 0$, since Eq. (7) can be integrated to yield a finite value for the total cross section. However, this shortcoming has only a minor effect on the overall energy-exchange cross section, as discussed in more detail by Hut et al. 1992.] As a binary hardens, its interactions become less frequent and more violent, so its *net* heating rate, averaged over many relaxation times, is constant, $\sim 0.3kTt_r^{-1}$.

Most binary encounters occur in or near the cluster core, where the density is highest. As encounters become more violent, the binary recoils to greater and greater distances from the center, redistributing to some extent the heating effect over the inner portions of the cluster. Nonetheless, the heating due to binary encounters can usefully be thought of as more or less localized to the cluster core and its immediate vicinity.

Binary–binary encounters have more possible outcomes than simple three-body interactions. In a flyby, each binary hardens in a manner similar to a three-body encounter with the center of mass of the other. In a resonance, however, the most likely result is the destruction of the wider binary involved, and the hardening of the other, with two single stars and a binary produced (Mikkola 1983a,b; Hut 1992). To some extent, this process can be modeled as a series of three-body interactions, although the true cross sections are different in detail. The second most likely outcome is the ejection of one star (usually a component of the wider binary), and the formation of a hierarchical triple system. In isolation, these systems have extremely long lifetimes; in a cluster, however, they are eventually perturbed into unstable configurations, so that the production of two single stars and a binary is once again the eventual outcome. Perhaps even more significant than their role in generating energy is the fact that binary–binary encounters act as an extremely efficient mechanism for destroying wide binary systems.

As a binary hardens and its encounters produce more and more recoil, it ultimately escapes from the cluster. Two processes which can avert this eventuality are physical collisions between stars, which become increasingly likely during resonant encounters as binary energies increase, and spiral-in of close binaries, due to gravitational radiation and/or magnetic braking.

When a stellar-mass spectrum is considered, and a choice of physical cluster parameters is (implicitly) made, the effects of stellar and binary evolution must also be taken into account. The reason for this is simple—after the initial evolutionary "burst," the time scales on which stars evolve, lose mass, and heat the cluster (Goodman 1984), can be similar to the stellar-dynamical time scales of the cluster itself. Binary evolution can affect the cluster dynamics (not to mention the appearance of the binaries themselves), e.g., by precipitating mergers or creating dwarf binaries from mass-transferring giants (cf. Taam and Lin 1992 and the discussion in Sec. 3.2.4 below). As simulations become more detailed, the inclusion of these effects will become more and more important, but no such self-consistent studies have yet been carried out.

3.2.2 Point Mass Dynamics

In globular clusters with core luminosity densities exceeding

$$\rho_V > 1.5\times 10^3\ L_{\odot V}\ \mathrm{pc}^{-3}(1\ \mathrm{AU}/a)\sigma_{10}\,, \qquad (8)$$

a primordial binary with semimajor axis greater than a will typically have encountered at least one other star intimately enough for an exchange reaction to have occurred, or at least for the binary's orbital parameters to have been changed significantly. In such clusters, therefore, the population of binaries with orbital periods longer than $P_b \sim (a/1\ \mathrm{AU})^{3/2}$ years will no longer be truly "primordial," but its stellar composition, and semimajor axis and eccentricity distributions will be determined in large part by past interactions with field stars and other binaries. This was realized in 1976 by Hills, who proposed that exchanging neutron stars into preexisting binaries might be a way to create X-ray binaries. This certainly occurs, but other processes described below probably dominate the rate of recycling of compact objects.

As discussed above, most calculations of binary evolution and heating by binaries in globular clusters have concentrated for simplicity on the case when all stars are identical point masses (Hills 1975; Hut and Bahcall 1983; Hut and Inagaki 1985; Hut et al. 1991). Hills and Fullerton (1980) were the first to make calculations of three-body encounters with mass ratios other than unity, but restricted their attention to zero impact parameter. The calculations revealed a number of intriguing features. These have recently been quantified by Sigurdsson (1991) and Sigurdsson and Phinney (1992), who computed complete differential cross sections for encounters of single stars with binaries (see Fig. 16). These covered a wide range of hardnesses, mass ratios, and initial eccentricities, and averaged (by Monte Carlo simulation) over phases and impact parameters. There are some similarities, but many important new features invisible in the equal-mass case.

The heaviest stars tend to be left in the final binary—as can be understood as a consequence of a tendency toward thermodynamic equipartition, in which the lightest star would have the highest velocity in the final state. This (most probable) exchange of a heavy field star M_f for a lighter star M_2 initially in a hard binary usually widens the binary by

$$a_f/a_i \simeq M_f/M_2\,. \qquad (9)$$

If the binary's binding energy were exactly preserved, Eq. (9) would be exact. In reality, the binding energy increases slightly (hard binaries get harder), so the orbit expansion is a little less than indicated by (SP3). After such an exchange, the binary is not only bigger, but also

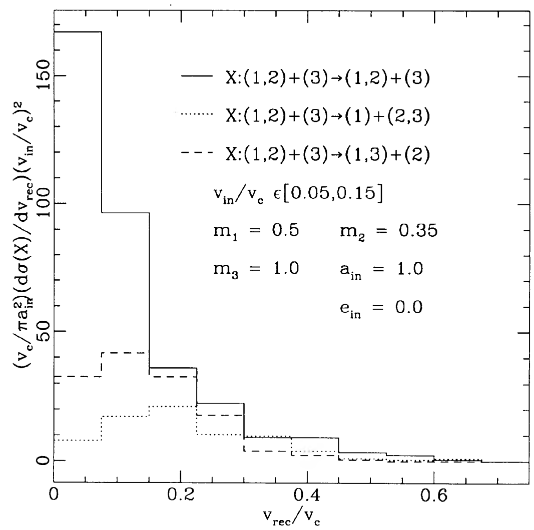

FIG. 16—Distribution of recoil velocities from flyby (solid line) and exchange (dashed and dotted lines) encounters of a field star of mass 1.0 (in arbitrary units) and hard binaries initially containing stars of mass 0.5 and 0.35. The field stars had velocity at infinity v between 0.05 and $0.15v_c$ [v_c defined by Eq. (8)]. The differential cross section for flyby encounters diverges at zero recoil (infinitely distant encounters), truncated in these simulations by the finite width of the beam of incident field stars. The cross sections for all exchanges and for flybys with $v_{rec}/v_c > 0.15$ have converged with the beam width used in this Monte Carlo simulation of 1000 encounters with random impact parameters with binaries of random orientations and orbital phases. Notice that the total cross section for exchange ejecting the heavier binary star (dotted line) is much less than that for ejecting the lightest star, but that the binary recoils with larger velocity in the rare cases when the heavier star is ejected. (From Sigurdsson 1991; Sigurdsson and Phinney, in preparation.)

heavier, so gravitational focusing is more effective. Its cross section for encounters is thus substantially larger than it was before the exchange. Consequently, a binary wide enough to be likely to have one exchange in some time period is likely to have several more encounters or exchanges in rapid succession after the first exchange. Since the heaviest stars tend to be left in the binary in such encounters, binaries are quite effective at soaking up heavy stars (e.g., neutron stars or heavy white dwarfs), even if none of these initially is in a binary. The tendency to exchange in the heaviest stars also has the consequence that the recoil speeds (and hence rate of ejection) of binaries involved in three-body exchanges are less than predicted by models in which all stars have equal masses, while the recoil speeds of the single (light) stars are higher.

3.2.3 Tidal Capture

Tidal capture occurs when a sufficiently close encounter between two unbound stars diverts enough orbital energy into the form of stellar oscillations that the pair becomes bound. Although this process is not as density-sensitive as dynamical ("three-body") binary formation, it is still strongly weighted towards the cluster core. Only in the densest globular clusters is the tidal binary-formation rate large enough to significantly influence the production or the appearance of a binary population. While tidal capture may have been responsible for the creation of some (but probably not all) of the blue stragglers found in globular clusters (see Secs. 2.2.5 and 2.3.5 above), it is in the context of neutron-star binaries that it has received most attention. Accordingly, in this section, we will confine our discussion to its role in the formation of bright X-ray sources and millisecond pulsars.

In globular clusters, the last "original manufacture pulsar" formed and faded more than ten billion years ago, and there are no HMXBs. However, LMXBs and recycled pulsars are observed in abundance. In fact, relative to the number of stars, the abundance of X-ray sources and recycled pulsars in globular clusters is between one and two orders of magnitude *higher* than it is in the Galactic disk (for a review, see Phinney and Kulkarni 1992). The explanation first proposed for the high abundance of X-ray sources was simply that the high density of stars in globular clusters made it not improbable that a neutron star could capture a passing star by raising a tide in it (Fabian et al. 1975; McMillan et al. 1987).

Since neutron stars are about twice as heavy as the heaviest luminous stars in a cluster, most of them should be found in the optical core of the cluster. The tidal capture rate is approximately

$$\mathscr{R}_{T2}=5\,\frac{\rho_V}{10^4 L_{\odot V}\ \mathrm{pc}^{-3}}\,\sigma_{10}^{-1.14}\,\frac{N_{\mathrm{ns}}}{700}\ \mathrm{per}\ 10^{10}\ \mathrm{yr}. \quad (10)$$

Here ρ_V is the central luminosity density of the cluster (derivable, e.g., from the data in Table 1 of Chernoff and Djorgovski 1989), $10\sigma_{10}$ km s^{-1} is the core velocity dispersion of turnoff-mass stars, and N_{ns} is the number of neutron stars in the cluster [$N_{\mathrm{ns}}=7\times 10^3 f L_V/10^{5.5} L_{\odot V}$ for a cluster of V-band luminosity L_V with an IMF of Salpeter slope between the 0.7 $M_\odot$ stars contributing the L_V and the >8 $M_\odot$ stars which contributed the neutron stars, of which a fraction f were retained in the cluster. Evidence that, with $f\sim 0.1$–0.3, this gives about the right number of neutron stars in M15 is provided by the accelerations of pulsars (Phinney 1992b)].

Globular clusters have central luminosity densities ranging from $>10^6\ L_{\odot V}\mathrm{pc}^{-3}$ to $<1 L_{\odot V}\mathrm{pc}^{-3}$. Tidal capture predicts that most of the X-ray sources should be found in the densest clusters, as is observed to be the case (Predehl et al. 1991). The observations are quite *inconsistent* with a model in which the incidence of X-ray sources is independent of, or negatively correlated with, cluster density (as would be expected if they were due only to evolution of primordial binaries, as in the Galaxy).

However, tidal encounters are not nearly as efficient at forming binaries as was commonly believed in the 1980s. If the Q's of the oscillations excited by a tidal encounter are small, the oscillations damp, rapidly heating the stellar envelope. Since the energy that must be dissipated by tides to circularize the orbit of the captured star is comparable to its gravitational binding energy, this heating can swell the star so much that subsequent passages are physical collisions which entirely disrupt it (McMillan et al. 1987; Ray et al. 1987).

On the other hand, if the Q's of the oscillations excited by the early passages are large, on subsequent encounters energy may be transferred from the tidal oscillations back to the orbit, unbinding the pair of stars again (Kochanek 1992). There is no consensus on the appropriate values of Q (these authors favor low Q), but whatever the value, it seems likely that the fraction of close encounters which lead to tidal capture rather than collision and disruption of the less dense star is likely to be much less than the 50%–66% assumed in early work.

Until about 1988, most astronomers (a) believed that the lifetimes of LMXBs and the globular-cluster X-ray sources (only two of which are actually known to be binary systems) were $\tau_X\sim 10^9$ yr, and (b) ignored the difficulties referred to in the previous two paragraphs. The long X-ray lifetimes were those given by the conventional theory of stable mass transfer driven by nuclear evolution and magnetic and gravitational radiation braking (see Verbunt 1990 for a review). The two-body tidal capture rates (9) with Salpeter IMFs give $\mathscr{R}_{T2}=8(f/0.1)$ per 10^9 yr for M15 and 47 Tuc, and $\mathscr{R}_{T2}=(0.1-0.3)(f/0.1)$ per 10^9 yr for the medium-density clusters M53, M4, and M13. With $\tau_X\sim 10^9$ yr, this gave an embarrassing excess of X-ray sources unless steeper IMFs were invoked (Verbunt and Meylan 1988).

Since 1988, however, a revolution in our understanding has occurred, and the problem has been reversed. For the X-ray sources themselves, there is a growing body of observational (e.g., Tan et al. 1991; Parmar et al. 1991; Hellier et al. 1990; Ryba and Taylor 1992) and theoretical (e.g., Podsiadlowski 1991) evidence that the lifetimes of many LMXBs are only $\sim 10^7$ yr. This increases the inferred birth rate by a factor of 10–100. Furthermore, recycled pulsars are supposed to be metamorphosed LMXBs, but the pulsars in low-density globular clusters have a birth rate much too high to be consistent with two-body tidal capture.

This high birth rate was first guessed from the pulsar luminosity function (Kulkarni et al. 1990a; for more up-to-date estimates, see Phinney and Kulkarni 1992; Johnston, et al. 1992) and can now be confirmed directly from measured pulsar ages (Phinney and Kulkarni 1992). In a high-density globular cluster like M15, where the pulsar birth rate (estimated from the observed timing ages) and X-ray binary birth rate (with a $\sim 10^7$-lifetime for typical cluster X-ray sources) are both inferred to be $\sim 3\times 10^{-8}$ yr^{-1}, two-body tidal capture alone may be adequate, provided the IMF is Salpeter-like (as suggested by dynamical models, see Phinney 1992b) and the fraction of neutron stars retained in the cluster $f\gtrsim 0.3$.

However, in medium density clusters with lower escape velocities, the retention fraction is likely to be lower, say $f\sim 0.1$, and even with a Salpeter IMF, two-body tidal capture seems to fail by a factor of several to produce the inferred birth rate of 1–3×10^{-9} yr^{-1} (Phinney and Kulkarni 1992). There are several lines of evidence which suggest that, in these lower-density clusters, most pulsars are formed by processes involving primordial binary stars. First, the retention fraction would depend less dramatically on escape velocity if some neutron stars formed in binary systems, since the center of mass would have a velocity considerably less than the kick imparted to the neutron star at birth (if the binary remains bound). Second, many of the cluster pulsars have binary companions in orbits which are difficult to understand in tidal-capture models (see Table 7, which lists the known binary pulsars in clusters; for a complete listing of all 29 pulsars in globular clusters, see Phinney 1992a).

If we neglect the difficulties in the physics of tidal capture, it would appear that tidal capture on giants could make wide binaries, at a rate 0.1–0.3 of that for capture on main-sequence stars (Verbunt and Meylan 1988; McMillan et al. 1990). However, even neglecting the difficulties of making binaries at all, we find it hard to imagine how two-body tidal capture can make *eccentric* binaries like 1516+02B in M5 ($P_b=7d$, $e=0.13$), 1802–07A in NGC 6539 ($P_b=2.6d$, $e=0.22$) or 1620–26 in M4 ($P_b=191d$, $e=0.025$). The clusters in which these binaries live have such low densities that it is improbable that passing stars will have induced eccentricities as large as those observed (Rappaport et al. 1989).

This difficulty can be resolved by interactions between neutron stars and primordial binary systems. About half of all binary–single encounters which lead to a collision leave the spectator star bound to the merged collision product, in an eccentric orbit (Sigurdsson 1991; Phinney and Kulkarni 1992). Third, as described below, primordial binaries have enormous tidal-capture cross sections, and a modest fraction ($\sim$5%–20%) of them can make the tidal-capture rate in moderate-density clusters comparable to the inferred large-pulsar birth rate there, if they have IMFs similar to those of the high-density clusters.

Consider two widely separated unbound stars of mass M and radius R. Then their total cross section for tidal capture of a field star of mass M_f and incident speed v_∞ is

$$\sigma_{T2} \simeq \frac{12\pi G(M+M_f)R}{v_\infty^2}. \tag{11}$$

Now suppose that the same two stars are in a hard binary of semimajor axis $a \gg R$, so the binary is mostly "empty space." A large fraction of the encounters where the field star approaches within $2a$ of the binary are resonant encounters, in which the three stars wander for a long time on chaotic orbits and repeatedly approach one another, like drunkards wandering in a boxing ring. Eventually two may hit each other, with cross section

$$\sigma_{T3} \simeq 5(a/R)^{1-\gamma}\sigma_{T2}, \tag{12}$$

as shown in Fig. 17. The exponent $1-\gamma \simeq 0.3$ when the stars all have masses comparable within a factor of 3 and $v_\infty < 0.5v_c$, but $1-\gamma \to 0$ if one star is much less massive than the others, or if $v_\infty \gtrsim v_c$. Since even main-sequence binaries can be hard with $a \sim 10^3 R$, this collision cross section can be $\gtrsim 50$ times that of two isolated stars. For more details, see Sigurdsson (1991) and Sigurdsson and Phinney, in preparation.

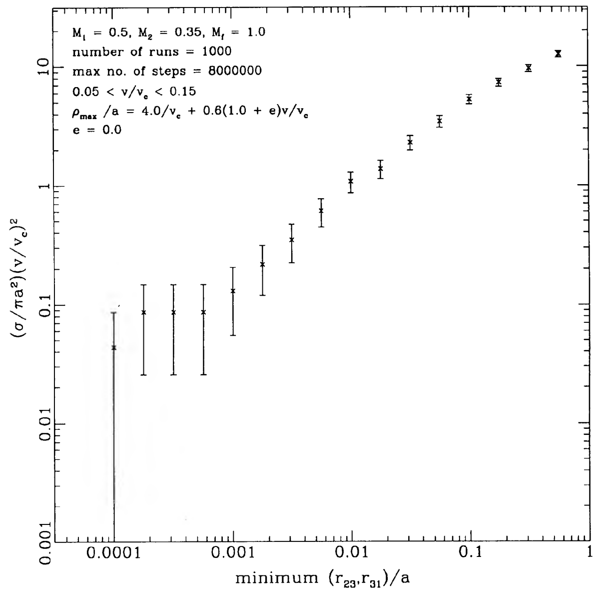

FIG. 17—Cross sections for close approaches of a field star of mass 1.0 (in arbitrary units) to either of two stars (of mass 0.5 and 0.35) initially in a circular binary of semimajor axis a. To determine the cross section, field stars were fired with velocity at infinity v between 0.05 and $0.15v_c$ [v_c defined by Eq. (8) at 1000 binaries with random orientations, phases, and impact parameters]. The cross section is cumulative (i.e., is for approach of the field star to less than the given distance from the other stars). Notice that the cross section for encounters within $10^{-2}a$ is roughly equal to the area of the binary times the gravitational focusing factor $(v_c/v)^2$. This large cross section for close encounters is due to the many passes in resonant encounters.

3.2.4 Stellar Evolution

Evolution of binaries in globular clusters, as elsewhere, may be driven by nuclear evolution of the component stars, in particular if expansion of a component star causes it to fill and overflow its Roche lobe. It may also be driven by loss of mass and/or angular momentum from the binary.

In a detached system, both stars in the binary are smaller than their Roche lobes. Tidal forces between the stars may cause the eccentricity of such a binary to decrease. In practice, such circularization is only efficient when at least one of the two stars has a radius of the same order as its Roche lobe. The reason for this is that the circularization time scale is proportional to $(a/R)^8$, where R is the radius of the star exerting the tidal force, and a the distance between both stars (Zahn 1977). Wide binaries may therefore only be expected to circularize as one of their components expands into a giant.

If the stars are close together, angular-momentum loss due to gravitational radiation may become a noticeable effect, and will cause the two stars to be driven together. The rate at which the distance between them decreases is proportional to a^{-4} (Peters 1964). In low-mass binaries, gravitational radiation will only affect the orbit within a Hubble time if the distance between the two stars is not more than a few solar radii. Mass loss from the binary may cause a larger loss of angular momentum. For example, a single star may lose an appreciable amount of angular momentum if its wind is forced by a magnetic field to corotate out to several stellar radii. This magnetic braking may be the cause of the loss of rotation velocity that late-type main-sequence stars experience as they age (Skumanich 1972; Soderblom et al. 1991).

If a companion of the star is able to keep it in corotation with the binary orbit, it effectively transfers the loss of angular momentum from the mass-losing star to the orbit. It is suspected that the amount of angular momentum lost depends mainly on the rotation rate of the star. Thus, by keeping the star in corotation, a companion will also help in boosting the loss of angular momentum. As a result, magnetic braking may cause close binaries to shrink. Because of our ignorance of the amount of loss of mass and of angular momentum of low-mass stars, the size of this effect is not known. It has been suggested that the process may be several orders of magnitude more efficient than gravitational radiation (e.g., Verbunt and Zwaan 1981).

The same processes leading to angular momentum loss still operate when one of the two stars fills its Roche lobe, so the binary is semidetached. The Roche lobe-filling star transfers matter to its companion, and this exchange of mass causes the distance between the stars to change. For example, if one assumes that the mass transfer conserves mass and angular momentum, i.e., that all mass lost by the donor is accreted by its companion, it is easy to show that the distance between the two stars will decrease (increase) if the mass-losing star is more (less) massive than its companion. As a result, mass transfer may run away if the donor is the more massive star, but may stabilize once mass transfer has caused the mass ratio to reverse.

If the donor is the less massive star, expansion of the orbit must cause the mass transfer to stop, unless loss of angular momentum reduces the distance between the two stars again, or unless the donor star itself is expanding. Thus, stable mass transfer in a low-mass binary may be driven by loss of angular momentum, in which case the mass-transfer rate $\dot{M}$ from the donor with mass M is given by the rate at which angular momentum J is lost: $\dot{M}/M \sim \dot{J}/J$. If gravitational radiation is the only mechanism for loss of angular momentum, the mass-transfer rate is of order $10^{-10}\ M_\odot$/yr for a main-sequence donor star. If the binary is small enough for a white-dwarf donor to fill its Roche lobe, the mass transfer is a very strong function of the white-dwarf mass. A relatively massive white-dwarf donor transfers mass very rapidly until its mass is $\leqslant 0.1\ M_\odot$, at which point the mass-transfer rate is low enough to be sustained for a longer period. It is suspected that magnetic braking is efficient in driving higher mass-transfer rates from main-sequence donor stars, but as noted above its actual efficiency is not clear.

Alternatively, stable mass transfer in a binary can be driven by expansion of the donor star into a giant, in which case the mass-transfer rate is set by the rate at which the donor expands, $\dot{M}/M \sim \dot{R}/R$. The expansion rate increases as the giant expands, and we therefore expect wider binaries to have higher mass-transfer rates. In short-period binaries loss of angular momentum may add to the mass-transfer rate. Some of the Algol systems known in globular clusters may be primordial binaries in which the originally more massive star is ascending the giant branch, and has started to transfer mass to its companion. If the mass-receiving star is a white dwarf or a neutron star, the binary will be a symbiotic variable or a low-mass X-ray binary.

Once the giant envelope has been transferred in its entirety, the giant core may cool to form a white dwarf. As most of the giant-envelope mass has been transferred rather than added to the core of the giant, the resulting white dwarf will have a low mass. Conservation of angular momentum will make the orbit considerably wider during the process of mass transfer from the giant. The binary in M4, which consists of a low-mass white dwarf and a neutron star in a wide orbit of low eccentricity, is the expected outcome of the evolution of a low-mass X-ray binary with a giant donor star. The evolution of a semidetached binary with a compact component has been reviewed by (for example) Verbunt 1990.

In a very close binary the situation may arise that both stars fill their Roche lobe, and form a contact binary. The evolution of contact binaries is a puzzle. Even though the component masses may differ appreciably, both stars seem to have the same surface temperature, which indicates efficient transport of energy between them. If magnetic braking were as efficient as has been surmised, contact binaries would merge rapidly, in contradiction to the observation that they exist in appreciable numbers. This may be taken as an indication that magnetic braking is not efficient in contact binaries. Contact binaries and the puzzle of their evolution have been reviewed by Ruciński (1985a) and Mochnacki (1985).

3.3 *N*-Body Simulations

At present, the conditions prevailing during the earliest stages of globular-cluster evolution are largely unknown. The substantial (and possibly fatal) dynamical effects associated with the almost immediate loss of the cluster's most massive stars (cf. Chernoff and Weinberg 1990) are generally not included in models of the long-term evolution of binary-rich clusters. Rather, a typical simulation starts with a cluster that has managed to survive the first few million years of stellar evolution. By that time, stellar mass-loss rates have been reduced to levels that are manageably small compared to other cluster time scales, and a full dynamical simulation of relaxation time scale phenomena can meaningfully be undertaken. The adopted initial conditions thus need only assume a mass spectrum extending up to $\sim$5–10 solar masses, along with a degenerate population, related to the "true" initial mass function and the escape probability of supernova remnants. It is generally also assumed that the cluster has reached a state of dynamical equilibrium by the time the simulation begins. The Plummer model (an $n=5$ polytrope) has become the "standard" initial distribution, largely for reasons of convenience and simplicity.

In fact, the numerical simulations that have been performed to data have started from even simpler initial conditions. Goodman and Hut (1989), McMillan, et al.

(1990, 1991; hereafter referred to as MHM), Gao et al. (1991, hereafter referred to as GGCM), and McMillan and Hut (1992, hereafter referred to as MH) all started their calculations with identical stars, some fraction f_B of which were selected randomly, and were "doubled" into binaries. Heggie and Aarseth (1992) have also performed some simulations with a mass spectrum, but without stellar or binary evolution. Thus, the only way in which binaries have so far been allowed to evolve in these self-consistent dynamical simulations has been through purely stellar-dynamical processes.

3.3.1. Direct Integration Methods

Conceptually at least, the most straightforward technique for following the evolution of a star cluster is the "brute force" approach of directly integrating the individual equations of motion of every star in the system. In other words, we simultaneously solve the $3N$ coupled differential equations:

$$\ddot{\mathbf{x}}_i = \sum_{j \neq i} \frac{-Gm_j(\mathbf{x}_j - \mathbf{x}_i)}{|\mathbf{x}_j - \mathbf{x}_i|^3} \quad (i=1,...,N), \qquad (13)$$

where the right-hand side of the equation is just the gravitational force on star i due to all other stars in the system. At any instant, knowing the positions and velocities of all stars, we can determine their accelerations from Eq. (13), and so advance the entire system by computing the next piece of each stellar trajectory.

In practice, however, there are many complications which limit the applicability of this approach to globular clusters. The two most serious stem from the long-range nature of the gravitational force and the enormous range of length and time scales that must be covered in a typical stellar-dynamical simulation. Gravity's infinite range means that every star interacts with every other star at every time step, so the computational cost of each force calculation scales as N. Thus, the cost of integrating N particles for one orbit (one crossing time) scales as N^2, and the cost of an integration to core collapse (a few relaxation times) scales as N^3, even if no account is taken of the more stringent accuracy requirements that must be applied to longer integrations. Gravity's singular behavior at small separations means that arbitrarily tightly bound binaries, with orbit periods $\ll 1/N$ crossing times, may form, slowing the calculation still further. For example, in a globular cluster, the total range in time scales might run from the orbital period of a tidal-capture binary—a few hours—to the half-mass relaxation time of the cluster—a few billion years. Add to this the facts that $O(100)$ time steps are required to follow a single binary orbit with reasonable accuracy and that overall cluster-evolution times can be hundreds of relaxation times, and the scope of the problem becomes evident.

Over the past three decades, the field of direct N-body simulation has advanced through a combination of algorithmic and technological improvements. The singularities associated with close stellar encounters are removed by two-, three-, and four-body regularization techniques (Kustaanheimo and Stiefel 1965; Aarseth and Zare 1974; Heggie 1974; Mikkola 1985). The wide range of time steps that remains even after regularization is efficiently handled by the use of individual or block particle time steps (Aarseth 1985; McMillan 1986; McMillan and Aarseth 1992), which effectively allow each stellar trajectory to evolve on its own natural time scale, instead of forcing an unnaturally small time step on most stars in the system. Problems relating to the calculation of long-range forces have been lessened—but not removed—by the introduction of neighbor schemes (Ahmad and Cohen 1973; Aarseth 1985), hybrid schemes (McMillan and Lightman 1984a) and, most recently, tree schemes (Barnes and Hut 1986; McMillan and Aarseth 1992). However, it is probably fair to say that the greatest improvements in the largest feasible value of N always have, and perhaps always will, come about through increases in computer speed, rather than by algorithm development.

Even with the use of the best available algorithms, as presently encapsulated in Aarseth's NBODY5 (Aarseth 1985) and its many descendants, the computational cost of collisional N-body simulations is high. For systems containing few hard binaries, it is still not currently feasible to follow systems with $N \gtrsim 10^4$ well beyond core collapse. Systems containing many binaries fare even worse, in large part because the algorithms necessary to regularize these systems do not lend themselves easily to vectorization or parallelization on supercomputers. The largest binary-rich systems that have been studied so far have contained only $\sim$2000–3000 members (McMillan et al. 1990, 1991; Heggie and Aarseth 1992), and even these studies were possible only because the binary population diminished rapidly in time as binary–binary encounters culled the herd.

Given these limitations, what can N-body simulations say about globular-cluster evolution? In fact, quite a lot. Calculations with as few as a few hundred particles produce core-collapse and evaporation time scales in reasonably good agreement with those found by "large-N" methods, such as those described in Sec. 3.4, and the ability of direct simulation techniques to follow in detail small-scale phenomena that are necessarily excluded from more approximate methods allows them to make detailed predictions about individual stellar encounters. Phenomena like stellar and binary evolution can be incorporated at little extra cost (so long as care is taken to introduce the correct time scaling into the description of these processes), and no simplifying assumptions about symmetry, isotropy, or equilibrium of the system need be made.

Small-N simulations do suffer from two inherent difficulties, however. The first lies in the inescapable fact that the value of N enters directly into the equilibrium structure of a post-collapse cluster, as first pointed out by Goodman (1984). The ratio of the core radius to the half-mass radius at the termination of the collapse increases as N is reduced. The second difficulty is related to the scale of stochastic fluctuations, which increases as N falls, making "thermodynamic" quantities like density and temperature difficult to define. In addition, it can easily be shown that a typical hard binary is ejected from a cluster with a binding energy of a few hundred kT. Since the energy of the entire cluster is $\sim NkT$, it follows that, in small systems, the energy of a single binary can dominate the cluster, while for large systems only the overall energetics of a binary population are important. In other respects, however, N-body simulations with small numbers of particles have proven themselves to be remarkably good indicators of the evolution of much larger systems.

3.3.2 Binary Formation

When we take a handful of stars and sprinkle them in a limited volume in space, the stars will change their distribution significantly during the first few crossing times. If the typical stellar velocities are significantly higher than are desirable for an equilibrium system, as dictated by the virial theorem, the whole system will initially expand. If the typical velocities are much lower, the system will begin to collapse, reach a minimum rms radius, and bounce back to a somewhat larger radius. In both cases, a fraction of the stars will escape. The remaining stars will at first show some large-scale motions, but soon violent relaxation will damp out these motions, on a time scale of order a few crossing times (the crossing time t_c can be defined in various ways, but in general is a measure of the time it takes for a typical star to cross the half-mass radius r_h of the system; r_h being defined as the radius which contains half of the total mass in stars).

After the stars settle down into a quasiequilibrium, further evolution takes place only on a thermal time scale t_r, the two-body relaxation time. Since $t_r(r)$ has a strong radial dependence, a convenient global measure is the half-mass relaxation time $t_{rh}=t_r(r_h)$. For large N, $t_{rh}\propto(N/\log N)t_c$. Neglecting the logarithmic factor, we see that a typical star has to wait for N/N_0 crossing times before it has forgotten its initial energy and angular momentum. Depending on the exact definition of t_c and t_h, N_0 is in the range $10<N_0<100$.

The earliest N-body calculations published were the ten-body simulations by von Hoerner (1960). These calculations had to be halted when the first binary was formed. Neither was the hardware fast enough, nor the software sophisticated enough to follow the long-term evolution of a perturbed binary in those days. Rapid improvement of both hardware and software allowed the treatment of dynamically formed binaries by the mid-1960s, but the maximum number of particles remained in the regime $N<100$ where relaxation effects could not be ignored on a crossing time. By the early 1970s, larger systems could be modeled, up to $N=500$ (for a review, see Aarseth and Lecar 1975). By that time a clear separation between relaxation time and crossing time could be observed. It was found that a typical star cluster underwent core collapse until a hard binary would appear in the center (sometimes more than one, but often just a single hard binary).

The energy released by one hard binary in a 500-body system is comparable to the kinetic energy of the whole system, by the time the binary reaches a binding energy of order $500kT$. With a hardness in this range, recoil in the next three-body reaction is likely to eject the binary from the system. As a consequence, the inner parts of the cluster will contract again, and produce a new binary. For larger N values, the relative perturbation due to the gravitational "burning" of one binary will be less, but the qualitative features are similar, as demonstrated by McMillan and Lightman (1984b). Even for N values in a typical globular-cluster range, $N\sim10^5$–10^6, at most a few binaries will be present in the core at any given time, in order to preserve a balance between energy loss in the outskirts of the cluster and energy production by the binaries (see Goodman 1984 for quantitative estimates).

The main shortcoming of N-body calculations so far has been the relatively small N values (up to $N=3000$, Makino 1989), some two orders lower than those of typical globular clusters. Fortunately, computer power is growing rapidly, and the 1990s may provide us with the first opportunity to model a globular cluster on a star-by-star basis. The hardware requirements of computing speeds in the Teraflop domain (Hut et al. 1988) may begin to become accessible in the coming years. An interesting approach to reaching such high speed has been pioneered in the GRAPE project (from "GRAvity PipE") at Tokyo University (Sugimoto et al. 1990), through the development of special-purpose hardware in the form of parallel Newtonian-force accelerators, in analogy to the idea of using floating-point accelerators to speed up workstations. For more information about the GRAPE hardware design, see Ito et al. (1990, 1991), and Fukushige et al. (1991). For software aspects, and choice of algorithms, see Makino et al. (1990) and Makino (1991a,b,c).

3.3.3 Primordial Binaries

As with other aspects of the initial cluster properties, there is little clear guidance on how to choose the parameters of a primordial binary population. Most simulations to date have simply assumed that the binaries are spatially distributed in the same way as the stars, with component masses (if unequal) chosen randomly from the stellar initial mass function. Binary fractions f_B ranging from $\lesssim3\%$ to $\sim20\%$ have been considered. Binary orbits are usually distributed uniformly in log(energy) across some range of interest which straddles the dynamically most interesting binaries, with $E\sim2$–$20kT$, and eccentricities are generally taken to be zero, or are thermally distributed.

MHM and MH have performed N-body simulations of small systems containing $\lesssim2000$ stars and containing $\sim5\%$–20% binaries. They started their calculations with a Plummer model of equal-mass stars, then doubled a fraction f_B of them into binaries, as described above. Binary energies were taken to be uniformly distributed in $\log(E)$, with $kT\lesssim E\lesssim20kT$. All binaries had zero initial eccentricity. MHM considered only isolated systems, MH included a Galactic tidal field. Only point-mass effects were considered in each series of runs and, in all cases, the simulations continued until the initial binary population (and possibly the cluster itself) was exhausted.

When primordial binaries are present in a cluster in any appreciable number, they rapidly come to dominate the dynamics, and continue to do so until all of the original binary population is destroyed. In the runs reported by MHM and MH, any primordial soft binaries (i.e., systems with $E \lesssim kT$) were assumed to have been destroyed already by single-star or binary–binary encounters. In the point-mass approximation, the only way in which a hard binary ($E > kT$) can be destroyed is by a collision with an even harder system. Because mass segregation concentrates binaries in the core, most binary destruction occurs within a few core radii of the cluster center.

So effective are binary–binary collisions at destroying binaries that relatively few of the systems in MHM's simulations actually reached the point where recoil ejected them from the cluster—only about 20% escaped, the rest were ultimately destroyed by encounters with harder systems. The binary-destruction time scales reported by MHM were as follows: 1/3 of all original binaries had been destroyed by the time of core collapse, $t_{\rm coll}$, 2/3 were gone after 3 $t_{\rm coll}$, 5/6 had disappeared after 10 $t_{\rm coll}$, and 9/10 after 30 $t_{\rm coll}$. By the end of the simulations ($\sim 30 t_{\rm coll}$), newly formed binaries had begun to dominate over primordial systems—the cluster had effectively reached the end of the "fossil-fuel-burning" epoch, and had rejoined the evolutionary track expected of a cluster with no primordial population.

As a result of mass segregation, the spatial distribution of binaries initially becomes significantly more centrally concentrated that that of the single stars—indeed, much of the initial collapse is driven by binary segregation. From the time of core collapse until the exhaustion of the binaries, MHM found that the binary half-mass radius lay within or close to the 25% radius of the stars. However, a significant fraction (several percent) of the binaries remained "parked" in the far halo until very late times. As the evolution proceeds, there is a tendency for the binary population to reexpand somewhat relative to the single stars. The core rapidly becomes binary dominated, with up to $\sim 50\%$ of the total core mass in the form of binaries. This fraction subsequently declines as binary heating supports the core, tending to maintain the total core mass while binaries are destroyed (see Fig. 4 of McMillan et al. 1991).

Binary heating results in a core significantly larger than would be expected for a cluster without primordial binaries—the core radius r_c at the point of collapse in MHM's simulations was about 10% of the half-mass radius r_h. Scaling arguments presented by Goodman and Hut (1989) imply that the corresponding fraction for a real globular, with $\sim 10^6$ stars, would be $r_c/r_h \sim 5\%$, about a factor of 2 higher than Goodman and Hut's own estimates.

In the absence of a tidal field, the response of the cluster as a whole to the heating is to expand, with the half-mass radius increasing by around a factor of 10 by the time all initial binaries are gone. In fact, this behavior owes little to the specifics of the binary population—roughly the same expansion occurs in a zero-binary case, where the internal heating comes entirely from dynamically formed systems ("three-body" binaries). The *overall* behavior of the cluster is determined essentially by the ability of the halo to transfer energy out of the inner regions, and is largely independent of the nature of the heat source.

When a Galactic tidal field is included, as in the work of MH, an extra time scale is introduced into the problem—namely, the time required for the cluster to dissolve—which is largely independent of the binary-destruction time. For the tidal parameters adopted by MH, the dissolution time is about 90 half-mass relaxation times, decreasing slightly with increasing f_B (to ~ 75 half-mass relaxation times for $f_B \sim 20\%$), while the binary-destruction time increases slowly with increasing binary fraction. MH find that there is a "watershed," at $f_B \sim 10\%$–15%, below which the binaries disappear before the cluster, and above which the binary mass fraction first falls, but eventually rises as cluster evaporation outstrips binary depletion. Thus, above this critical f_B, there is a *minimum* binary mass fraction (about 12% for $f_B = 20\%$). This might be used to place limits on the likely range of initial f_B corresponding to an observed present-day binary frequency—e.g., if the present-day binary fraction turns out to be 10%, an initial binary fraction of 20% may be ruled out.

The simulations carried out by MH span a wide range of parameters: their clusters have tidal radii $r_t = 4$, 8, and $12 r_h$, initial binary fractions $f_B = 5\%$, 10%, 15%, and 20%, and logarithmic initial binary energy distributions with $kT \leqslant E \leqslant 20kT$ and $kT \leqslant E \leqslant 400kT$. They find that, as f_B increases, the peak core binary mass fraction rises, from $\sim 25\%$ for $f_B = 5\%$, to $\sim 40\%$ for $f_B = 10\%$, to $\sim 50\%$ for $f_B = 20\%$. As the cluster evolves, and the core mass drops in all cases, the core binary mass fraction decreases for $f_B = 5\%$ and 10%, but remains roughly constant for $f_B = 20\%$. Figure 18 shows the evolution of (a) the total binary mass fraction and (b) the core binary mass fraction for a cluster with $N \sim 1200$, $r_t/r_h = 8$, and $f_B = 20\%$. MH speculate that, so long as the fraction of binaries in the most "active" range (2–20kT) remains the same, the overall energetics of the cluster are unlikely to change as the energy distribution is altered, although, of course, the observational appearance of the cluster may be markedly different.

3.4 Fokker–Planck Simulations

At the time of writing, direct N-body methods cannot be used to follow the dynamical evolution of a cluster of $N \geqslant 10^5$ stars over relaxation time scales. The computational burden of direct methods increases with N not only because of the larger number of interparticle forces that must be estimated at each time step, but also because of the increasing disparity between the half-mass relaxation and dynamical times: $t_{rh}/t_{dh} \approx N/26 \log N$ (Spitzer 1987). Fortunately, the large-N effects that make direct methods so expensive also tend to justify the use of statistical algorithms that are far less demanding computationally. Statistical methods represent most of the stars not individually but in groups sharing common characteristics. In this section, we shall concentrate on orbit-averaged Fokker–Planck approaches; some other statistical methods will be described in Sec. 3.5.

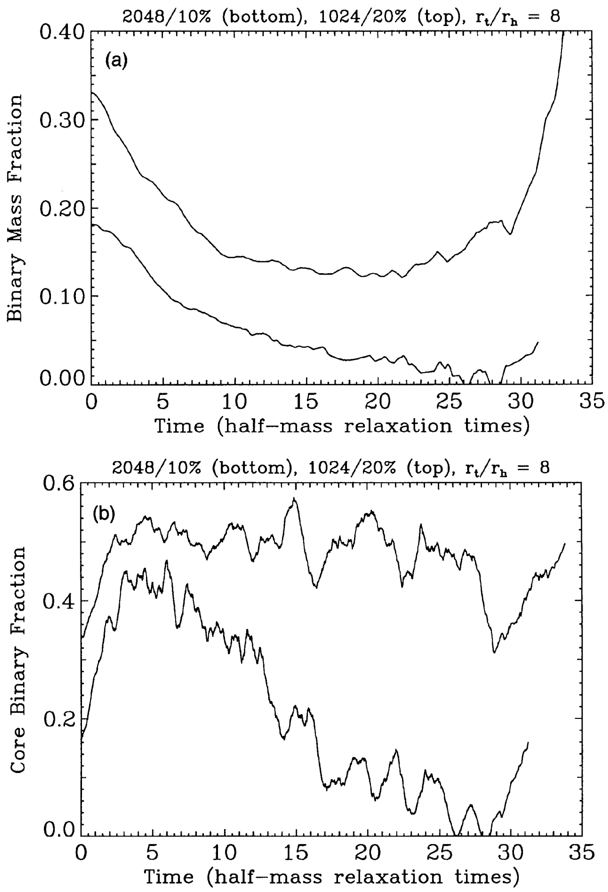

FIG. 18—Time evolution of (a) the total binary fraction and (b) the core binary fraction for two tidally limited clusters initially containing ~ 1200 and ~ 2200 stars, with $f_B = 20\%$ and $f_B = 10\%$, respectively. The ratio $r_t/r_h = 8$ in either case. The overall binary fraction for $f_B = 20\%$ reaches a minimum of about 12% at $t \sim 20$ initial half-mass relaxation times, while the core binary fraction stays close to $\sim 50\%$ right up until the dissolution of the entire cluster. For $f_B = 10\%$, both the overall binary fraction and the core binary fraction go to zero before the cluster itself is destroyed.

3.4.1 Fokker–Planck Formalism

When N is large, the cluster potential can be approximated by a smooth mean field, Φ, that varies little on time scales $\sim t_{dh}$, if one neglects rapidly varying external tidal fields and assumes that the cluster is many t_{dh} old, so any initial phase of violent relaxation has long since passed. Almost all theoretical studies in this subject have assumed spherically symmetric clusters. Individual stellar orbits in the mean field can be characterized by their energy per unit mass, $E_{\rm orb} = v^2/2 + \Phi$, and by their total angular momentum with respect to the cluster center, $J_{\rm orb} \equiv |\mathbf{r} \times \mathbf{v}|$; for most stars, these vary only on the long time scale t_{rh}. Since the time scale for phase mixing is $t_{dh} \ll t_{rh}$, the stars can be grouped according to their values of $E_{\rm orb}$ and $J_{\rm orb}$ without regard to orbital phase.

A distribution function $f = dN/d^3\mathbf{r}\, d^3\mathbf{v}$ is used to specify the density of stars in single-particle phase space and is written as a function of $E_{\rm orb}$, $J_{\rm orb}$, and time t with the presumption that $\partial f/\partial t \sim f/t_r$. If the stars are further distinguished by their mass or multiplicity, there will be a separate distribution function for each such group. For computational reasons, the dependence of f on $J_{\rm orb}$ is commonly ignored: this is found to be a good approximation for single stars in the inner parts of clusters not containing a black hole (Cohn 1980, 1985).

At large N, relatively weak two-body encounters dominate the relaxation process (cf. Binney and Tremaine 1987). Strong encounters, in which $\Delta E_{\rm orb}/E_{\rm orb} \sim 1$, do occur but have little effect on cluster evolution apart from an enhancement of the rate of escape of stars from the cluster (Hénon 1969; Goodman 1983). The evolution of $f(E_{\rm orb}, t)$ is therefore diffusive in character and can be described by the "isotropic orbit-averaged Fokker–Planck equation" (Hénon 1961; Cohn 1980)

$$\frac{\partial f}{\partial t} + \left\langle \frac{\partial \Phi}{\partial t} \right\rangle_E \frac{\partial f}{\partial E} = -\frac{1}{4\pi^2 p(E,t)} \frac{\partial}{\partial E} F_E ,$$
$$F_E = -D_E f - D_{EE} \frac{\partial f}{\partial E} . \qquad (14)$$

Here $4\pi^2 p(E_{\rm orb}, t)$ is the phase-space volume per unit $E_{\rm orb}$, and D_E and D_{EE} are diffusion coefficients computed by averaging the effects of weak encounters over this volume. The term involving $\langle \partial\Phi/\partial t \rangle_E$ represents the adiabatic change in $E_{\rm orb}$ caused by changes in the mean-field cluster potential.

Equation (14) and its generalization to the anisotropic case $f = f(E_{\rm orb}, J_{\rm orb}, t)$ have been applied very successfully to the evolution of clusters of single point-like stars up to core collapse. Comparison with direct N-body methods yields remarkable agreement up to the point when only ~ 30 stars remain in the core (cf. Spitzer 1987, and references therein). Extensions of the Fokker–Planck method to clusters containing binary stars have generally been less satisfactory, for several reasons.

(i) Each binary has an internal binding energy $E_{\rm int}$ between its components, in addition to its orbital energy with respect to the cluster potential. If the distribution of binaries over $E_{\rm orb}$ is explicitly modeled (this can sometimes be avoided), the number of independent variables and the computational difficulty of the calculations are significantly increased.

(ii) As described in Secs. 3.1.3 and 3.2, close ("superelastic") interactions between binaries and singles or binaries and binaries can generate orbital energy at the expense of internal energy. Because the important interactions yield $\Delta E_{\rm orb}/E_{\rm orb}$ and $\Delta E_{\rm int}/E_{\rm int}$ of order unity, however, they cannot be represented accurately as a diffusion in energy space but require the addition of nonlocal integral operators on the right-hand side of Eq. (14).

(iii) More fundamental than either of the above is the difficulty that no fully adequate theory of close-binary/single-star and binary–binary interactions exists. Such interactions are represented in the Fokker–Planck codes by differential cross sections, which are obtained separately by direct three- and four-body integrations. For point-like stars of equal mass, the binary–single cross sections are well-established; binary–binary cross sections are also available, if less securely known (Sec. 3.2). The situation for arbitrary unequal-mass ratios is much less well explored because of the larger number of parameters involved. Even worse, when tightly bound binaries interact with other stars, very close passages between stars can occur, so complicated hydrodynamic effects and outright stellar mergers, followed by uncertain stellar evolution, come into play. These last are potential problems for direct N-body methods as well.

3.4.2 "Three-Body" Binaries

Binaries can be formed by dissipationless three-body capture ("three-body binaries"), two-body tidal capture (Fabian et al. 1975), or as part of the star-formation process itself ("primordial binaries"). A few Fokker–Planck calculations have been made with three-body binaries as the central energy source (e.g., Cohn et al. 1989; Murphy et al. 1990; Lee et al. 1991), but the number of such binaries present in a post-collapse cluster is so small (Goodman 1984) that the binary population is not treated explicitly but is replaced by an energy-generation term depending on central conditions. Although they may have important dynamical consequences for the cluster, three-body binaries are not likely to be directly observable, and we shall not discuss them further here.

3.4.3 Tidal-Capture Binaries

Statler, Ostriker, and Cohn (1987; hereafter referred to as SOC) have used a modified form of Cohn's (1980) Fokker–Planck code to study the production of binaries by tidal capture in a cluster evolving up to and past core collapse. Their initial cluster contained $N=3\times10^5$ identical, unevolved stars of mass $M_*=0.7\ M_\odot$ and radius $R_*=0.57\ R_\odot$, and it had the structure of a Plummer model with half-mass radius $r_{h0}=1.47$ pc and central one-dimensional velocity dispersion $v_{c0}=11.6\ \mathrm{km\ s^{-1}}$. The cluster was considered to be isolated from external influences; in particular, the Galactic tidal field was neglected. All tidal-capture binaries were assumed by SOC to have a semimajor axis (separation between stellar centers) $a=2.5R_*$ after circularization.

SOC's initial model is rather compact compared to present-day globulars, but the principal dimensionless parameter determining the importance of tidal capture for cluster evolution is $\chi\equiv(v_h/v_*)^{1.8}/\ln(0.4N)$, where $v_h\equiv(0.4GM_{\rm cluster}/r_h)^{1/2}$ is the typical stellar velocity in the cluster, and $v_*\equiv(2GM_*/R_*)^{1/2}$ is the escape velocity from the surface of a star. The value of χ in SOC's initial conditions is typical of the more massive and centrally condensed present-day clusters (Illingworth 1976; Pryor et al. 1989b).

SOC concluded that tidal capture accelerates core collapse via the Spitzer (1969) mass-stratification instability, although core collapse occurred in their computations only slightly earlier than it does in parallel computations without tidal capture. Collapse was then arrested and reversed by binary–binary and binary–single superelastic encounters. After core collapse ($t>t_{cc}$), the cluster expanded in good agreement with the expected behavior $r_h(t)\propto(t-t_{cc})^{2/3}$, and interpolation in the figures from the paper shows that

$$t-t_{cc}\approx7t_{rh}(t) \tag{15}$$

at late times. The linear increase in t_{rh} with time, and even the numerical coefficient in Eq. (15), are expected to be almost independent of the central energy source, because the source must adjust its output to balance the outward transport of energy at $r\sim r_h$, which occurs by two-body relaxation.

This is well confirmed by comparison with the much earlier self-similar Fokker–Planck model of Hénon (1965), which assumed a point-like energy source of unspecified nature and implies a coefficient 7.2 in Eq. (15). By changing the efficiency of relaxation, a distribution of stellar masses may alter the coefficient but not the basic scaling. The Galactic tidal field, on the other hand, produces an entirely different scaling because it changes the structure of the outer parts of the cluster and greatly enhances the mass-loss rate [cf. Hénon (1961) and the review by Goodman (1989)].

Because the tidal binaries are extremely hard, in the stellar-dynamical sense, much of the energy released in superelastic encounters may be carried off by recoiling stars that escape the cluster entirely, although even then the ejected stars carry away negative potential energy and thereby indirectly "heat" the stellar distribution in the core (Sec. 3.1.3). SOC made calculations for several possible assumptions about the fraction of the recoil energy retained by the cluster. When all stars participating in superelastic encounters were assumed to be completely ejected ("heating by ejection only"), the mass-ejection rate was

$$\dot{M}_{\rm cluster}\approx-10^{-2}\frac{M_{\rm cluster}}{t_{rh}(t)} \tag{16}$$

in the post-collapse phase.

From the observational standpoint, the most interesting statistic may be the number of tidal-capture binaries present in the cluster ($N_{\rm tcb}$), which was substantial: from a peak of ≈1500 at core collapse, $N_{\rm tcb}$ very slowly declined through the post-collapse phase to ≈100 after $6\times10^3t_{rh}(0)$. Since the separation between components is very small, such binaries should in principle be detectable as eclipsing variables. Unfortunately, the binaries are predicted to be very strongly concentrated towards the cluster core, which is itself very small in SOC's calculations. As discussed in Sec. 2.2, observational searches have so far avoided the densest parts of the more centrally condensed clusters because of crowding problems, and it may be that these searches cannot yet be used to place interesting limits on the numbers of tidal-capture binaries in which both components lie on the main sequence. Theoretical calculations along the lines of SOC but with a distribution of stellar masses would be useful, because binaries with at least one relatively low-mass component are not expected to be so strongly concentrated towards the cluster center.

A number of effects not considered by SOC may conspire to reduce $N_{\rm tcb}$ either by reducing the rate of capture or by shortening the lifetimes of the binaries so formed.

Benz and Hills (1987) have shown that the cross section for stellar merging is about twice as large as that for tidal capture, and the ratio of cross sections may be even larger because of complications in the tidal-capture process not considered by these authors (cf. Sec. 2.5). Merged stars will probably evolve at the accelerated rate appropriate to their enhanced mass and shed all but $\sim 0.6\ M_\odot$ of this mass in their post-main-sequence evolution. The mass they discard will then be swept out of the cluster by whatever mechanism keeps globulars gas-free (Roberts 1988; Spergel 1991).

The net result is that merged stars contribute to $\dot{M}_{\rm cluster}$; this helps to expand the core and therefore reduces the rate of tidal captures required in the post-collapse phase. On the other hand, the lifetime of successful tidal captures may be reduced by angular-momentum loss, leading to merging: SOC have shown that the time scale for merging by gravitational radiation is somewhat longer than the time scale on which binaries are ejected, but other mechanisms, such as a magnetized wind, may be faster. Finally, hydrodynamic simulations of close encounters between tidal binaries and other cluster members indicate that mergers usually result (McMillan et al. 1991; Hernquist and Goodman 1991).

Thus the reduction in $N_{\rm tcb}$ is likely to be more than compensated by the production of mergers; although the observational signature of the latter is somewhat uncertain, it is likely that the more massive mergers would be classified as blue stragglers. For reasons discussed in Sec. 2.3.4, it is unlikely that all blue stragglers are the end products of stellar collisions or tidal capture. Nevertheless, it would be worthwhile to look for concentrations of blue stragglers in the denser cluster cores, such have already been discovered in NGC 6397 and 47 Tuc (Aurière et al. 1990; Paresce et al. 1991).

3.4.4 Primordial Binaries

Whereas tidal-capture binaries are expected to make up less than 1% of the cluster stars, the population of primordial binaries may well be $\sim 10\%$ (see Sec. 2). Tidal-capture binaries are always very hard, but primordial binaries have a broad range of internal binding energies, which increases the difficulty of modeling their effects on cluster evolution in a Fokker–Planck approach.

In recent work importantly influenced by the Monte Carlo simulations of Spitzer and Mathieu (1980), Gao et al. (1991, hereafter referred to as GGCM) have performed Fokker–Planck integrations of clusters containing primordial binaries. The distribution function was assumed to depend on t, E, and $E_{\rm int}$ only. For the binaries, the factored form

$$F(E,E_{\rm int},t)=f(E,t)g(E_{\rm int},t) \qquad (17)$$

was assumed, which makes the orbital energies independent of the internal energies. Binary–binary and binary–single interactions were computed in Monte Carlo fashion at each time step using cross sections from Mikkola (1983a,b; 1984a,b). No mechanisms for forming new binaries were included. Stars were given equal masses, and in the initial conditions (a Plummer model), 5%, 10%, or 20% of their number were twinned into binaries of twice the mass with internal energies uniformly distributed in the log. The calculations were continued for $\sim 100t_{rh}(0)$, $t_{rh}(0)$ being the initial half-mass relaxation time. No external tidal field was applied to the clusters, so r_h and t_{rh} expanded during the runs, and mass loss from the cluster was small because it occurred only by ejection, rather than by overflow of the tidal boundary. Because of this expansion, the age of the models in units of their *final* t_{rh} was typically only ~ 6.

GGCM's standard model contained 10% binaries by number and 18% by mass. During the first $10t_{rh}(0)$, mass segregation concentrated the binaries towards the core, and the core contracted. Energy generation by the binaries then stabilized the core, and there followed a period of some $40t_{rh}(0)$ during which r_c/r_h was almost constant at 1%–3%. During the final $40t_{rh}(0)$ of the computation, the core exhibited gravothermal oscillations, during which r_c/r_h varied over two to three orders of magnitude but was typically $\sim 2\%$ (and rarely much larger). These numbers for r_c/r_h are in good agreement with those estimated on analytic grounds by Goodman and Hut (1989). The time average of r_c/r_h is expected to depend very weakly on the total cluster population N for a given mass fraction in hard binaries. Gravothermal oscillations, on the other hand, are not expected in small-N systems such as have been studied by direct N-body methods, especially when the hard-binary fraction is as large as it is here; the oscillations may be stabilized even up to $N \sim 10^5$ by a distribution of stellar masses (Murphy et al. 1990).

About 70% of the initial binary population was depleted by the end of GGCM's standard case. This rate of depletion, which is roughly consistent with the results of McMillan et al.'s (1990) N-body simulations (Sec. 3.3), can be understood by an argument that can be extended to more general cluster models (see GGCM, Sec. 3.5): after collapse, the cluster is supported mainly by ejection, at a rate that is in good agreement with SOC's result (Sec. 3.4.3). GGCM find that about one unit mass of hard binaries is disrupted or ejected for every unit mass of stars ejected. Hence if f_0 is the initial mass fraction in hard binaries, then the binary population should disappear in $\sim 10^2 f_0$ relaxation times. Note, however, that the number of elapsed relaxation times should be determined by integrating t_{rh}^{-1} against time, since t_{rh} is time-dependent.

These arguments should also apply to the more realistic (but less often simulated) case of tidally limited clusters. Since a tidally limited post-collapse cluster has a life expectancy of only $\sim 20t_{rh}$ (cf. Hénon 1961; Lee and Ostriker 1987) or perhaps even $\sim 7t_{rh}$ in the presence of a distribution of stellar masses (Lee et al. 1991[10]), the cluster may dissolve before the hard-binary population is exhausted. GGCM estimate that this will be the case if $f_0 > 0.2$. This

[10]Unfortunately, these authors quote the ratio of the *age*, rather than the life expectancy, to t_{rh}, but the desired ratio can be extracted by analysis of their Fig. 6.

is in good agreement with the estimates of MH based on their N-body simulations [cf. Sec. 3.3; note that $f_0 = 2f_B/(1+f_B)$].

Close comparison does, however, reveal some shortcomings of the Fokker–Planck approach used by GGCM. In the N-body study, a large fraction of the binary population is found on highly eccentric orbits that spend little time in the core, as a consequence of recoil after close interactions; this effect cannot be represented by the factored distribution function (Sec. 3.4.4). Binaries do not so strongly dominate the mass of the core, and binary–single encounters contribute more importantly to the energy production rate than in GGCM's calculations, where binary–binary encounters produced ~ 5 times as much energy as binary–single ones. Fewer binaries are therefore lost for a given energy produced because binary–single encounters usually do not result in the disruption or ejection of the binary. This may be counterbalanced in the total binary disruption rate by the fact that the initial binary population in GGCM was harder on average than that in MH, and the softer binaries were the first to be destroyed in both studies.

One important lesson to be drawn from the Fokker–Planck studies is that to estimate the dynamical depletion of binaries from present-day core parameters can be hazardous because those parameters may have been very difficult in the past. Any cluster with $t_{rc} < 10^8$ yr and $r_c/r_h < 0.1$ may have undergone core collapse and may therefore have lost much of its original binary population.

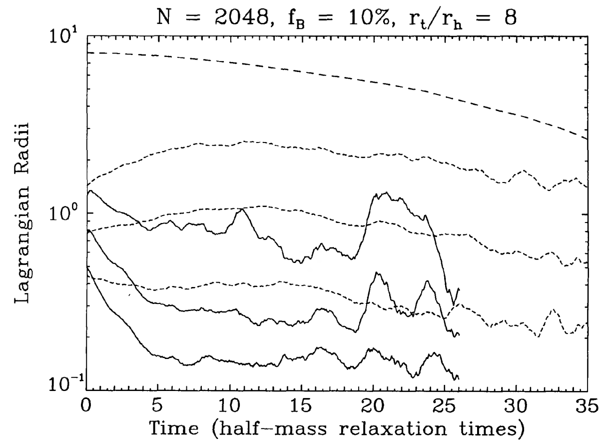

FIG. 19—Variation of the Lagrangian radii of single stars (dashed lines, 20%, 50%, and 80% of the total remaining mass) and binaries (solid lines, 25%, 50%, and 75% of the total) for a tidally limited cluster initially containing $\sim$2200 stars, with a binary fraction of 10%. The long dashed line at the top of the figure is the cluster tidal radius.

3.5 Stochastic Simulations

3.5.1 A Simple Model with Binary–Binary Scattering

At the present time, self-consistent dynamical models are still too simple for detailed comparison with cluster observations. N-body simulations offer only poor statistics, while Fokker–Planck and gas-sphere models must make sweeping assumptions about the nature of the cluster distribution function. In addition, neither includes any "real" physical stellar processes, even in the most rudimentary form.

Hut et al. (1992) have adopted a somewhat different approach, improving resolution of binary interactions at the expense of the underlying single-star distribution. Their motivation was to bridge the gap between N-body simulations (Sec. 3.3) and Fokker–Planck simulations (Sec. 3.4). Since the former lack the ability to model a sufficiently large number of particles, and the latter are not able to follow the individual large jumps in energy of the binaries, Hut et al. developed a stochastic treatment of these energy jumps against a frozen backdrop of a full cluster population of single stars. They consider the evolution of an extensive (10%) binary population in a 5×10^5 star tidally limited cluster, including all of the physical effects listed in Sec. 3.1 above, with the exception of the last (stellar evolution), but *neglecting* the dynamical evolution of the cluster potential. This apparently rather extreme approximation is supported by the fact that, in MH's runs (see Sec. 3.2), the Lagrangian radii of the single-star population do not in fact change much for an extended period following core collapse, as the cluster slowly evaporates (see Fig. 19).

The adopted cluster potential is the simplest possible crude approximation to a King model, consisting of a constant-density core, of radius r_c, lying within an isothermal inner halo, of radius r_h, surrounded by an outer halo where the density drops steeply to the tidal radius r_t. Thus, the density profile is

$$\begin{aligned} \rho &= \rho_c \qquad (r \leqslant r_c), \\ &= \rho_c (r/r_c)^{-2} \quad (r_c < r \leqslant r_h), \\ &= \rho_h (r/r_h)^{-4} \quad (r_h < r \leqslant r_t), \end{aligned} \tag{18}$$

where $\rho_h = \rho_c (r_h/r_c)^{-2}$. The cluster was assumed to consist of solar-type stars, with $r_h = 5$ pc, and $r_c{:}r_h{:}r_t = 1{:}50{:}500$, to represent a "typical" post-collapse cluster, such as M15.

Binaries, initially distributed as in MHM and MH, sink toward the core at a rate given by

$$\frac{1}{r_a}\frac{dr_a}{dt} = \frac{1}{t_r(r_{\text{eff}})}, \tag{19}$$

where r_a is the binary apocenter radius and the "effective radius" r_{eff} is obtained by an appropriate orbit average of the relaxation time t_r. Once in the core, they are allowed to experience interactions with single stars or other binaries, with both the rates and the outcomes of the interactions randomly selected from known cross sections. However, in addition to escape from the cluster and destruction by binary–binary scattering, provision is also made for binaries to be destroyed by actual stellar collisions during close encounters, and for the components to spiral together via the effects of gravitational radiation (adopted primarily as a well-defined, and easily implemented, orbit-decay mechanism—magnetic braking is likely to be at least as important). Figure 20 illustrates the various evolutionary processes by which binaries can evolve and be destroyed.

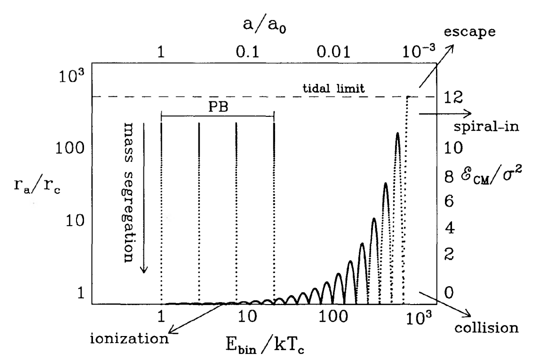

FIG. 20—Schematic evolution of binaries in globular clusters, in the plane defined by the apocenter radius r_a (also orbital energy, right scale) of their orbits in the cluster and their binding energy E_{bin} (also binary separation compared to that of a $1kT$ binary, top scale). Primordial binaries (PB) settle vertically to the cluster core, then harden in a series of steps via scattering, shown here schematically as mean 40% hops in E_{bin}. Several binary loss mechanisms are also indicated, as described in the text.

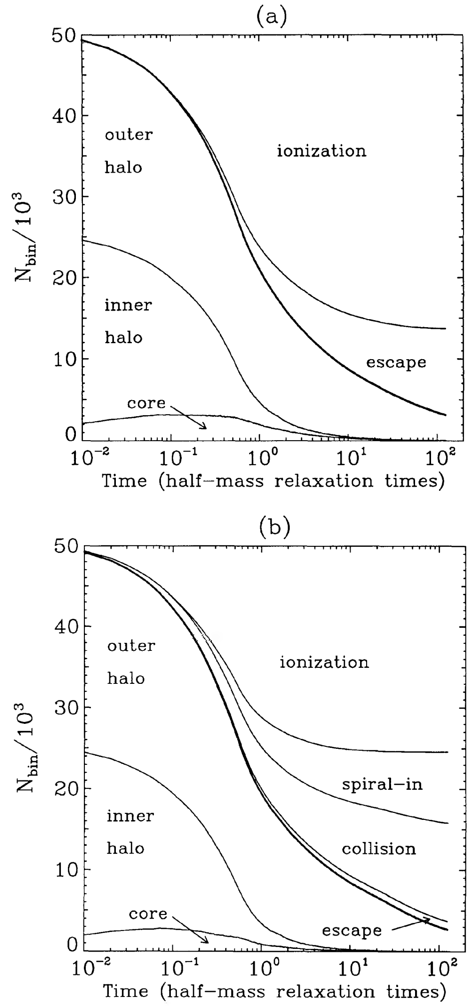

FIG. 21—Evolution of an initial 10% binary population in a model 5×10^5 star cluster. The thick full line separates surviving binaries (below) from systems destroyed or lost from the cluster (above). Thin lines further divide these regions, from bottom to top, indicating the number of binaries: (1) within the core; (2) within the the half-mass radius; (3) remaining in the cluster; (4) surviving, including escapers; (5) lost to collisions; (6) merged through gravitational-wave losses; and (7) disrupted in a scattering process. Not all of these channels exist for each simulation. (a) includes only stellar-dynamical processes. (b) allows the additional possibilities of physical stellar collision and spiral-in due to gravitational radiation.

Most binaries are destroyed by binary–binary interactions. In the point-mass approximation, the remaining binaries escape. With more realistic stars, the majority of the rest merge [as shown in Fig. 21, which compares the evolution of the binary population in the cluster in two cases—with only "point-mass" dynamics and (b) with the effects of physical collisions and spiral-in included]. At any instant, most of the remaining binaries are drifting in toward the center, before their first strong encounter. A typical binary spends most of its active life (after its first strong scattering event) in or near the cluster core. However, the few binaries that receive a recoil sufficient to place them outside r_h remain there long enough to make a significant contribution to the radial binary distribution—the "median" binary which has passed at least once through the core lies in or near the core itself, while the "time-averaged" such system resides at a much greater radius. This latter effect is strongly suppressed by collisions and spiral-in, both of which tend to lower the average distance of a binary from the cluster center.

Figure 22 shows the two-dimensional distribution of binaries in radius and binding energy after one half-mass relaxation time in the same simulation as Fig. 21(b) above. That time represents a typical stage in the drift-in of primordial binaries. At that epoch, there is about one recycled binary in the halo for every 20 core binaries; the ratio drops in later phases to about 1 in 40. Thus, we do not have to limit our search for recycled binaries to the core region, where they would be confined according to the simplest multimass King models. Instead, binaries should be present throughout the whole cluster, including the outer regions, which are easier to observe. Although the majority will reside in the core, there is likely to exist a significant halo population of recycled binaries, containing (for the above model) something in the range of 3%–5% of the total number of all recycled binaries. The numbers quoted here are, however, quite sensitive to the rather crude approximations adopted. For example, our calculation of angular-momentum loss by gravitational radiation was based on the assumption of orbits whose eccentricities were randomly thermally distributed. Had we instead assumed circular orbits, the overall effect of gravitational radiation would have been almost negligible.

Finally, the overall binary distribution at no time comes close to a multimass King model, implying that a dynamical model is imperative for any meaningful comparison of a theoretical binary distribution with observations.

Despite its approximations, the simple model just described provides considerable insight into the many processes governing cluster binary evolution. Stellar evolutionary effects, and their consequent modification of the binary population, were not modeled in the first series of runs.

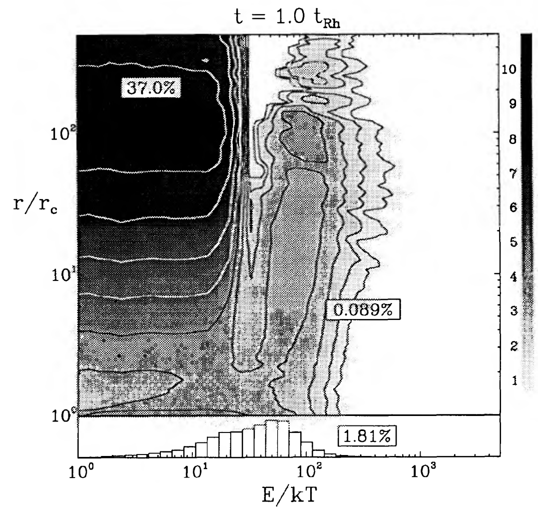

FIG. 22—Distribution of binaries in radius and binding energy, for a 10% population of primordial binaries, with collisions and spiral-in included [as in Fig. 21(b)], after one half-mass relaxation time. The upper portion of the plot shows the density ρ of halo binaries with respect to logarithmic binding energy $\ln E$ and the logarithmic radius $\ln r$, indicated by both grey level and contours. The normalization is intuitive to the coordinates used here, in the sense that equal areas of a given blackness denote an equal number of binaries, irrespective of the position in the figure, i.e., $\int\int \rho(E,r)\ d \ln E\ d \ln r$ is equal to the total number of binaries instantaneously in the halo. The grey scale is logarithmic, running from $\log \rho = -1.0$ (white) to $\log \rho = 4.5$ (black). The contour levels range from $\log \rho = -0.5$ to $\log \rho = 4.0$ in increments of 0.5, and are marked on the map on the right. The boxed percentages indicate the fraction of the initial binary population present in the form of "pristine" binaries (which have never passed through the core) and "processed" halo binaries (which have) are shown in the left- and right-hand insets. The lower panel shows the energy distribution of core binaries, drawn in the form of a histogram in $\log E$. The percentage of the initial binary population present in the core is also indicated.

With their inclusion, Monte Carlo simulations promise to be a valuable source of information on the appearance of binary-rich clusters.

3.5.2 A More Realistic Treatment of Binary/Single-Star Star Encounters

A more exact approach with a similar philosophy has been used by Sigurdsson and Phinney 1992 (see also Sigurdsson 1991) to study the evolution of a population of noninteracting test binaries in realistic clusters with realistic mass functions. The final mass function was computed from the initial mass function with the prescription for white-dwarf and neutron-star masses given in Chernoff and Weinberg 1990. The radii of the field stars were computed from a realistic mass–radius relation for each mass group (with appropriate fractions being given the radii of white dwarfs and neutron stars). Stars in the mass group containing giants were given radii from a probability distribution determined by $R(t)$ for the evolutionary sequences for giant stars from Fahlman et al. 1985.

The clusters have a statistically stationary background of stars in ten mass groups. The densities and velocity distribution of the stars of each group are determined by integrating the equations for the appropriate multimass King model. Into such a cluster are placed test "primordial" binary systems. The orbit of each such binary is integrated within the cluster potential; the orbit is determined by the Newtonian mean-field force, plus a dynamical-friction drag force (in the Chandrasekhar approximation, summed over the velocity distribution of all mass groups), plus a stochastic Fokker–Planck describing the small velocity kicks imparted by passing distant field stars (i.e., a random-walk implementation of the *diffusion* in velocity space), plus large-angle kicks determined by direct three-body integration as described below. Binary–binary interactions are not included.

If the binary were treated as a point mass, and the three-body integration turned off, the drag and the Fokker–Planck term would make the binary orbit perform a random walk. Averaged over long times, the positions of the binary would have a radial distribution appropriate to its mass in thermal equilibrium with the cluster. However, the model binaries, like real ones, sometimes have close encounters with field stars. These are treated as follows.

Each time step, the probability of collision with each type of field star is computed. If a random number is less than the sum of these probabilities, a close encounter is realized. The value of the random number determines the mass of the field star, and a second determines its velocity. Both have distributions computed for the position of the binary at the time of the encounter. The relative orbits of the three bodies are followed by direct integration until the encounter is resolved (i.e., all three stars are mutually unbound and on orbits which will not intersect in the future, as in ionization, or one star and a bound binary are mutually unbound and receding from each other, as in exchange or flyby). Physical collisions during the encounter are treated by adding the masses and momenta of the colliding stars, and treating them thereafter as a single star. If a binary still exists after the encounter is resolved, its center of mass velocity is transformed into the cluster frame, and the integration continues.

For a given binary, the critical field-star energy for ionization is nearly independent of the field-star mass. Thus if stars of all masses had equipartition of energy, stars of all mass groups would tend either to ionize or to harden a given binary. However, the velocities of light stars in the outer halo of a cluster are limited by escape (their dispersion is much less than the equipartition dispersion they have in the core). Furthermore, mass segregation means that the encounter rate of a binary in the halo is dominated by the lightest mass group, while in the core it is dominated by the heaviest group (neutron stars and white dwarfs, except in low-concentration clusters, where upper-main-sequence stars can dominate the encounters in the core). Thus, wide binaries which in a cluster core would be ionized can seem hard to, and be hardened by, the light stars in the halo before they reach the core.

In moderate-density clusters, only heavy binaries sink to the core and have interesting numbers of encounters. But in the densest clusters, the light binaries can be significant in the late (post-collapse) evolution. They do this by diffusing into the core and exchanging with a heavy star which makes them heavy enough to be equilibrium residents of the core. In the core they have many subsequent encounters, supplying heat and perhaps forming "exotic" objects in collisions.

Mass segregation has a dramatic effect on the types of stars and binaries involved in collisions. The central number density of a cluster increases more rapidly than linearly with the central V-band luminosity density, because the denser clusters have much higher central mass-to-light ratios. For multimass King models with central luminosity density $\rho_V(0) \gtrsim 3\times10^3 L_{\odot V}\,\mathrm{pc}^{-3}$, the total number density of stars in the core scales roughly as

$$n(0) \simeq 3\times10^4 \left(\frac{\rho_V(0)}{10^4 L_{\odot V}\,\mathrm{pc}^{-3}}\right)^{1.5} \mathrm{pc}^{-3}. \qquad (20)$$

(For more realistic Fokker–Planck models, the scaling is slightly less steep, since the outermost parts of the cluster never relax.) The core is mainly composed of heavy white dwarfs and neutron stars for $\rho_V(0)$ above the fiducial value of $10^4 L_{\odot V}\,\mathrm{pc}^{-3}$, and of main-sequence stars for $\rho_V(0)$ below that fiducial value. With a Salpeter IMF, in a dense cluster like M15, the fraction of objects in the core which are neutron stars is $\sim f/(1+f)$, where f is, as above, the fraction of neutron stars retained in the cluster.

Though the effects are complicated by ejection and destruction of binaries during encounters, mass segregation also has a major effect on the types of binaries involved in exchanges and collisions. Since these preferentially occur in the core, where the density is highest, the binaries involved in collisions tend to be the heaviest ones, which feel the strongest dynamical friction, and spend the largest fraction of their time in the high-density core. For example, with a binary IMF that picks stars independently from a Salpeter IMF extending down to 0.1 $M_\odot$ stars, the vast majority of all binaries in the cluster contain pairs of low-mass main-sequence stars. Yet, for given semimajor axis, equilibrium segregation of such primordial binaries would ensure that $\sim$52% of all binaries involved in exchanges and collisions contain at least one white dwarf, $\sim$81% have a primary more massive than 0.5 $M_\odot$, and $\sim$50% contain a primary more massive than 0.7 $M_\odot$.

Some of the main conclusions of the simulations are (see Figs. 23 and 24):

(1) In clusters with $\rho_V(0) \gtrsim 2\times10^4 L_{\odot V}$, most binaries which reach the core in a Hubble time (i.e., without being ionized) will have undergone more than one exchange encounter.

(2) Ejection of binaries from the cluster is initially much less important than in equal-mass simulations. This is because exchanges tend to eject the lightest star, leaving the heaviest stars in the binary. Thus the binary recoil velocity is lowered. Encounters involving very hard binaries in the densest clusters lead in steady state to $\sim$10%–30% of the hard binaries being found in the vicinity of the clusters' half-mass radii (Fig. 23), sinking back to the core whence they were ejected on a highly radial orbit. Such an ejection almost certainly explains why the tight binary pulsar 2127+11C is found some 2.7 pc (30 core radii) from the center of M15 (Phinney and Sigurdsson 1991).

(3) The repeated exchanges favoring retention of the heavy stars lead to the population of core binaries being dominated by heavy white dwarfs and neutron stars, even if none is initially in a binary at all (Fig. 24).

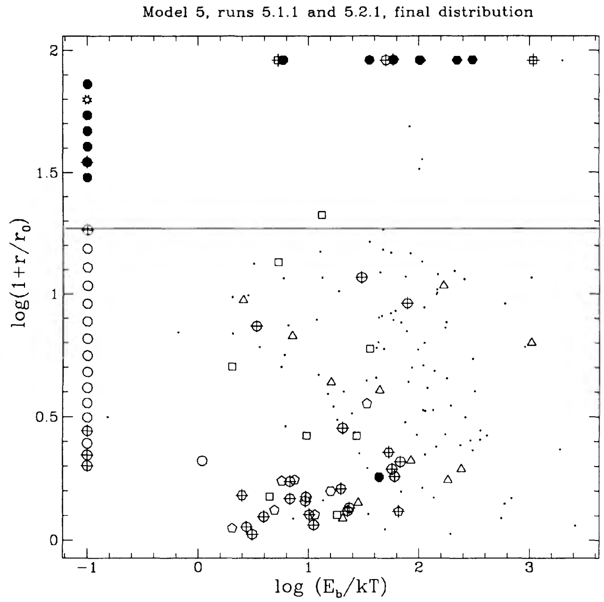

FIG. 23—Final distribution and fate after 5×10^9 yr of 187 primordial binaries in a simulated cluster resembling 47 Tuc (from Sigurdsson 1991; Sigurdsson and Phinney 1992). Test binaries were placed in a multimass King model cluster with *initial* mass-function slope $x=1$ (slightly flatter than Salpeter), and present masses in 8 groups ranging from 0.18 to 1.4 $M_\odot$. The model had core radius (for turnoff-mass stars) $r_0=0.35$ pc, half-light radius 17.6 pc, total mass 2×10^6 $M_\odot$, and a central core number density 3×10^5 pc^{-3}. The mean mass of objects in the core is 1.01 $M_\odot$, and the core line-of-sight velocity dispersion for the mean mass is 15 km s^{-1}. The 187 primordial binaries initially had their masses drawn independently from the same mass distribution as that of the surrounding cluster, except that $M_1>0.5$ $M_\odot$, $M_2>0.39$ $M_\odot$, and no binary was allowed to contain two neutron stars. Their initial semimajor axis distribution was uniform in log a for 0.005 AU $<a<$ 5 AU, and they initially had the same radial distribution as the heaviest (1.4 $M_\odot$) mass group. Except for the rows at left and top, each point shows the final position (radial distance from the cluster center) and internal binding energy (in units of $kT=\bar{m}\bar{\sigma}^2$ of the multimass King model) of each primordial binary. Binaries which were ionized are plotted in a vertical row on the left (unlike the bound binaries, they are *not* plotted at their radial positions at the end of the simulation). Notice that all but two of the initially soft binaries have been ionized. Binaries which were ejected from the cluster are plotted in a horizontal row at the top of the figure. The horizontal line indicates the half-mass radius. The fate of each binary is indicated by the symbol plotted. The small points represent binaries which had no close encounters. The $n+2$-sided open polygons are binaries which had n flyby encounters only. Crosses superposed on polygons represent binaries which exchanged stars with the field. Filled polygons are binaries in which one star collided and merged with a compact object (neutron star or white dwarf), while the third star remained bound. Notice that many of these binaries were subsequently ionized, and are shown at the left above the other ionizations. Reentrant star symbols indicate binaries in which a giant or subgiant collided with a main-sequence star (in the runs shown here, there were no collisions between two main-sequence stars). Notice that some binaries underwent a combination of the various encounters.

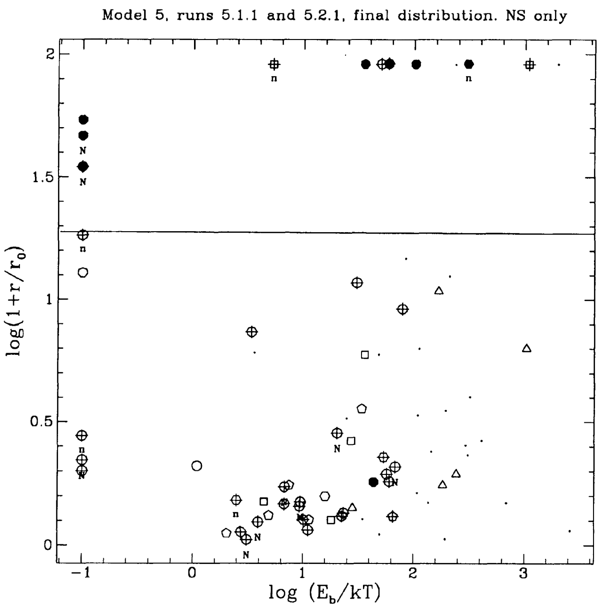

FIG. 24—Same as Fig. 23, except that only binaries containing neutron stars (or heavy white dwarfs in the same mass group—not distinguished in the simulations) are plotted. If the binary did *not* contain a neutron star initially, or if the binary was in collision where one of the stars colliding was a neutron star, a small "*n*" is plotted below the symbol for that binary. If the final binary contained two neutron stars, or was involved in a collision where one of the stars colliding *and* the third star were neutron stars, a small "*N*" is plotted below the symbol for the binary. Notice that no binary contained two neutron stars initially. Comparison with Fig. 23 shows that all but one of the binaries involved in exchanges ended up containing neutron stars, a consequence of mass segregation, gravitational focusing, and the predilection of three-body exchanges to leave in the binary the heaviest of the stars involved in the encounter, and thus widen the binary (see the text).

(4) Of all physical and tidal collisions in dense clusters, roughly $0.5f$ involve a neutron star: either directly (about 2/3 of the cases), or as a close bystander to the other two colliding stars, that it is likely to capture a significant fraction of the mass liberated in the disruption (1/3 of the cases).

(5) The other collisions are mainly white-dwarf/main-sequence collisions. Collisions between pairs of main-sequence stars are a minority.

(6) In the dense clusters, some 2% of all stellar collisions involving binaries are white-dwarf/white-dwarf encounters.

(7) In post-core-collapse clusters, most binaries will be ejected or destroyed (e.g., via gravitational radiation) in less than a Hubble time. Collision products (e.g., LMXBs, pulsars, CVs, blue stragglers, and the like) in these clusters are thus most likely formed by two-body processes.

(8) If the binary fraction $f_b \gtrsim 0.1$, the clusters with the highest rates of physical collisions and binary exchanges may be not the post-core-collapse clusters, but rather those with central luminosity densities near that of 47 Tuc. Such clusters are dense enough that (a) the core is starting to be dominated by compact remnants and (b) most primordial binaries reaching the core will undergo multiple exchanges (and, in a significant fraction, collisions), if as suggested by simulations (MHM, Gao et al. 1991, with equal-mass stars, and parametrized binary heating), a cluster can support itself against core collapse at this density for several billion years by "burning" its primordial binaries. Indeed, 47 Tuc has the largest known density of pulsars (Manchester et al. 1991) and blue stragglers (Paresce et al. 1991).

(9) The high-velocity giants discovered by Meylan et al. (1991) are perhaps most naturally interpreted as stars put into nearly radial orbits during a binary encounter. If they are a typical 2% of the stars in 47 Tuc, then almost every star in the cluster would have to be on such a radial orbit that the light profile could not be fit. One possible escape from this dilemma is that the high-velocity giants are stars put onto the giant branch *by their encounter* (e.g., by collision of a white dwarf and a main-sequence star). Then a large fraction of all giants, but not of all stars in 47 Tuc, would have to be recent products of binary collisions. One might expect that close inspection of the giants' spectra would reveal peculiarities, although the abundance patterns observed in cluster giants remain ambiguous.

4. SUMMARY AND OUTLOOK

In this section we summarize the recent results of each of the main observational and computational techniques. Note that nearly all the results mentioned have been obtained in the last few years, testifying to the enormous expansion of the field of study of binaries in globular clusters during this period. In the next few years, we expect the field to mature further, and to yield a comparable harvest of new results, theoretically as well as observationally. We offer specific discussions of this outlook in each subsection.

4.1 Radial-Velocity Variables

Radial-velocity measurements of globular-cluster stars have taken the first step of demonstrating the existence of spectroscopic binaries among cluster giants and producing an estimate of their frequency. According to these measurements, about 10% of globular-cluster stars are binaries with periods between 0.2 and 20 years and mass ratios larger than 0.22, with an uncertainty that is about a factor of 3. *This estimate argues for at most a small deficiency of binaries when compared to the frequency of similar binaries among the Population I field.* This change from the earlier conclusion of a significant deficiency of binaries in globular clusters is primarily due to changes in the estimated Population I binary frequency.

The next three to four years should produce more precise estimates of the binary frequency among cluster giants, the first measurement of the binary frequency for cluster main-sequence stars, and the first measurement of the binary frequency in clusters where the destruction of binaries by dynamical processes should have been unimportant. Those observations should allow us to test some of the predictions made by the models described in Sec. 3.

4.2 Photometric Variables

There are several obvious directions for future studies of photometrically variable binary stars in globular clusters. As emphasized in Sec. 2.2.2, the current constraints on cluster binaries imposed by these sorts of binaries rests on observations of eclipsing systems in only two clusters! Clearly, observations of a larger sample of clusters is needed. Although even main-sequence stars can be effectively monitored with modest-sized telescopes (~ 1 m), long observing sequences over many nights are needed to identify photometric binaries convincingly and study them effectively. Furthermore, even a "normal" frequency of eclipsing binaries implies that only 1 of 1000 main-sequence stars is likely to be variable. Thus, large numbers of cluster members must be surveyed. Fortunately, large CCDs and fast photometric reduction programs make such a survey practical.

The advent of multiobject spectrographs means that it should soon be possible to monitor hundreds and perhaps 1000 faint cluster members per night. Those observations will help bridge the period gap between current photometric (short-period binaries are most likely to eclipse) and spectroscopic techniques (radial-velocity studies of giants restrict searches to systems with periods of more than a month). Only in that way can we derive the true period distribution of binaries. However, it is at least encouraging that the frequencies derived from radial-velocity and photometric searches are similar.

With regard to CVs, it is clear that there may be a significant underabundance given the numbers of X-ray binaries and pulsars in globular clusters. Is that due to systematic differences in the photometric properties of CVs in globular clusters versus those observed in the solar neighborhood? Or are globulars somehow deficient in CVs? Future results from *ROSAT* and continued ground-based searches for globular-cluster CVs may help resolve the problems.

Recent discoveries of blue stragglers in the cores of dense globular clusters suggest that they are a common feature of massive stellar systems. However, the results also strongly suggest that more than one mechanism is likely to form blue stragglers in cluster environments. While the evidence that binarity is somehow associated with the formation of blue stragglers in low-density clusters is now very strong, it seems likely that direct stellar collisions or close encounters may be responsible for the presence of blue stragglers in dense clusters. Thus, those stars will be useful in studies of the binary frequency *and* dynamical interactions in globulars. However, it may be difficult to disentangle the mechanisms responsible for the formation of various sorts of stragglers. Further spectroscopic observations may shed some light on the matter, in the form of differences in chemical-abundance patterns between stragglers in clusters of different central concentrations. Photometric surveys of blue stragglers are also useful in directly estimating the masses of those that pulsate as dwarf Cepheids.

4.3 Binaries in the Color–Magnitude Diagram

In order to make accurate estimates of the binary frequencies in globular clusters from an analysis of their CMDs, the observer must (a) provide the cluster mass function, (b) define the systematic and measurement errors for a given observation, (c) estimate the binary mass ratio distribution, and (d) minimize the number of optical doubles associated with the data.

The cluster mass function should be derived in a global manner, so that corrections for dynamical effects do not dominate the results. That requires measuring a major fraction of the cluster from the core out to the tidal radius. An example of what can be done is presented in the study of NGC 5053 by Fahlman et al. (1991). The central density of that particular cluster is very low, so measuring faint stars right into the cluster core is feasible. With denser systems, higher spatial resolution will be required and here telescopes, such as the CFHT with the high-resolution camera, will be required to penetrate the crowded inner regions. The *Hubble Space Telescope* with corrected optics will be ideal for exploring the centers of dense clusters. Such data, supplemented with ground-based data obtained at the highest angular resolution possible (preferably with large-format CCDs), will provide excellent global-cluster mass functions in addition to the CMDs used in the analysis techniques discussed here. High-quality data will also eventually allow for a determination of the radial frequency of the binaries, which is an important check on the dynamical models.

Measurement errors can be determined accurately with Monte Carlo simulations, and that is routinely done by many observers. The systematic errors inherent in CCD studies are more difficult to account for (such examples might be calibration errors, variations in atmospheric extinction, flat-fielding errors) and are not often given enough consideration. A good example of careful attention to detail can be found in the study of M92 by Stetson and Harris (1988). Failure to account for the systematic errors inherent in cluster photometry can produce spuriously large estimates of the cluster binary frequency, as their effect is to widen cluster CMDs.

The binary mass-ratio distribution in globular clusters is basically unknown. A variety of models can be explored as was discussed in Sec. 2.3, but until the distribution is accurately determined, binary frequencies from cluster CMDs will remain uncertain. An interesting result which bears on this problem is the mass-ratio distribution in the halo, a distribution function that is still unknown, but one which could be measured observationally.

The most important improvement that can be made to the data used in constructing CMDs is better angular resolution. Data with low resolution yields numerous optical doubles, which appear in the CMD in the same manner as do real binaries, and constitute a background noise source that is difficult to quantify observationally or to model theoretically. The obvious solution lies in making observations under the best observing conditions possible, with instruments capable of giving the highest resolution. Here the

Hubble Space Telescope, as originally designed, would have been ideal; we will now have to wait for its refurbished optics to provide that kind of data. Ground-based observations at Cassegrain foci have generally not been made for a variety of reasons, such as the small size of the fields. With high resolution as the goal, those observations should be reconsidered, as should the use of telescopes equipped with adaptive optics, such as the high-resolution camera at the CFHT.

4.4 X-Ray Binaries

The discovery of bright X-ray binaries in globular clusters proved that close stellar encounters do occur there. The recent discovery of other binaries in globular clusters makes it likely that three-body processes may be more important in the formation of X-ray binaries than previously thought, i.e., that many X-ray binaries are formed when a neutron star is exchanged into a binary. Interestingly, all well-studied bright X-ray sources in globular clusters are neutron stars; none is a black hole. The bright X-ray sources are located in or very close to the cluster cores.

Study of the optical counterparts of X-ray binaries in globular clusters is hampered by the limited number of accurate positions, and by the difficulty of obtaining optical counterparts in crowded fields. The more sensitive observations obtained with *ROSAT* dramatically improve the positional accuracy of the faint sources. Radio detections of a number of sources have provided positions of subarcsecond accuracy. With the better spatial resolution now obtained with ground-based telescopes, such as the NTT, and with the *Hubble Space Telescope*, optical studies should help in elucidating the nature of the bright as well as of the faint X-ray sources. Those studied so far appear surprisingly faint in the optical.

The nature of the dim sources is uncertain. They may be soft-X-ray transients in quiescence, cataclysmic variables, or, especially those far from the cluster center, background X-ray sources not related to the cluster. The deep *ROSAT* observations will increase the number of dim ($L_x < 10^{34}$ erg s^{-1}) sources, and provide more accurate positions to enable optical followup. This may help to settle the questions about the nature of these dim sources. The luminosity function is further interesting in elucidating the relation between X-ray sources and millisecond radio pulsars, of which a large number has also been found in globular clusters.

4.5 Pulsars

The discovery in globular clusters of some 30 radio pulsars (9 in binaries with parameters determined) and 12 X-ray sources (2 in binaries with parameters determined) has taught us many things. About binaries in clusters, it has taught us that two-body tidal capture is not the only, or even the main, way in which neutron stars and white dwarfs capture companions and mass to accrete. Three- and four-body interactions involving primordial binaries probably dominate, especially in non-core-collapsed clusters. The pulsar 2127+11C, projected 2.7 pc from the center of M15, lies in a binary with binding energy $\sim$400 times the mean kinetic energy of stars in M15. This may be the first observed example of a binary ejected by recoil in an exchange interaction. As it sinks back to the core of M15, it will give its excess kinetic energy to the surrounding stars, doing its part to replenish the energy continually lost by the stars in M15's core.

The future is bright for observational tests of the simulations described in Sec. 3. The positions in the clusters of more pulsar binaries will be determined. Their radial distribution, periods, and companion masses, as well as those of the main-sequence binaries now becoming accessible to detection to surveys from multifiber spectrographs, will allow tests of the theory of relaxation, hardening, exchange, and ejection of binaries. Improved statistics will determine the fraction of high-velocity stars in cluster cores, providing a further check on the rate of encounters with binaries. The hardness of a cluster's binaries may even allow us to determine the past history of its central density. For example, we may learn whether 47 Tuc has just reached its current density, or whether it has been burning its binaries for billions of years, and is now on the verge of core collapse. Finally, we may eventually discover where are hidden the large expected number of white-dwarf analogs of the pulsar and X-ray binaries.

4.6 *N*-Body Simulations

The dynamics of binaries in the point-mass approximation has been extensively studied over the past two decades, especially in the idealized case where all stars are identical. The results of the extensive numerical simulations of Hills (1975), Hut (1983, 1984), and (most recently) Sigurdsson and Phinney (1992) provide not only a means of interpreting *N*-body experiments, but also an essential ingredient of both Fokker–Planck and Monte Carlo cluster simulations. While more exhaustive parameter-space mapping in the multimass case is highly desirable, and will become increasingly necessary as continuum simulations of clusters containing binaries become more sophisticated, it is fair to say that the purely stellar-dynamical aspects of binaries in clusters are fairly well understood.

By contrast, the problem of including realistic stellar-evolutionary and finite-size effects in studies of binary evolution is only now being addressed in detail. While *N*-body simulations might conceivably be extended, at little extra computational expense, to incorporate stellar collisions and evolution, this avenue is not open to Fokker–Planck simulations, which must characterize each stellar and binary species as a separate "fluid." When arbitrary mergers and consequent evolutionary changes are considered, the resulting Fokker–Planck description of the system is likely to be both impractically complex and of dubious validity.

Even in direct simulations, however, the absence of a complete theory of binary evolution limits the incorporation of "real" stellar physics into dynamical simulations, and restricts our ability to predict the appearance of binaries in real clusters. No simple (i.e., easily programmable) means of describing the behavior an interacting binary system over its entire lifetime presently exists. At present, the most convenient means of including important physical effects in simulations of all types is through the use of tabulated evolutionary "paradigms" based on more detailed calculations. The rules will undoubtedly become more complex with time, but the inclusion of real binary evolution is probably still some years away.

N-body simulations of clusters containing primordial binaries have clearly demonstrated that, *when binaries are initially present, even in small numbers, they rapidly come to dominate the evolution of the system, and continue to do so until they are all destroyed*. Thus, the structure of any cluster found to contain a substantial binary population should be expected to deviate markedly from that of a "single-star" system. Binaries are an extremely efficient means of destroying other binaries, and their superelastic interactions with other cluster members strongly influence their distribution within the cluster. The studies that have been carried out so far are very idealized, and offer little guidance on the evolution and appearance of cluster binaries, but these shortcomings are, at least in principle, quite straightforward to remedy.

Direct *N*-body calculations suffer from the computational expense of having to simultaneously integrate the *N* coupled equations of motion of all the stars in the system. Even with elaborate algorithms for minimizing the calculation of interparticle forces and treating close encounters between two or more stars in an efficient manner, it is still prohibitively expensive to follow a model cluster of more than $\sim$5000 stars past core collapse. When primordial binaries are included, things go from bad to worse—not only is the integration cost dominated by binary orbits, but these calculations are very difficult to vectorize or parallelize efficiently on a supercomputer, slowing the calculation still further. In a typical simulation (on a Cray Y-MP), adding a 10% population of primordial binaries increases the CPU time to core collapse by about a factor of 10, even though mass segregation speeds the collapse process itself by about a factor of 2.

The major advantage of *N*-body methods is that they make few sweeping assumptions about the nature of the stellar system under study, apart from those dictated by simple economics, and new physical processes are relatively easy to incorporate without major code reorganization. One rather dubious benefit of the high basic cost of this approach is that, once the price of admission is paid, even elaborate calculations of stellar collisions, mergers, evolution, and binary interactions do not add significantly to the total, and are therefore effectively free. Given the difficulties of studying multispecies models within the Fokker–Planck formalism, the increasing speed of computers, and continuing algorithmic advances, direct simulation techniques may yet become the method of choice for detailed simulations of large stellar systems.

4.7 Fokker–Planck Simulations

Despite their simplifying assumptions, Fokker–Planck methods have been very successfully applied to the dynamical evolution of globular clusters up to core collapse. Where comparison with *N*-body results has been possible, the agreement has generally been good. Although significant depletion of longer-period binaries is predicted, binaries appear to have little influence on this phase of cluster evolution, except insofar as they participate in dynamical relaxation like single stars of greater-than-average mass. One of the clearest theoretical predictions to emerge from these calculations is that, well before core collapse, the binary population should be strongly concentrated towards the cluster center, unless the core radius of the cluster at formation is so small that collapse occurs in less than a half-mass relaxation time. At present, the observational support for this concentration is weak or ambiguous at best, although that may be due in large part to the difficulty of finding binaries in crowded fields. There probably remains much to be learned by Fokker–Planck methods about binary mass segregation and disruption in precollapse clusters with realistic distributions of binary masses, mass ratios, and periods.

In the past decade, theoretical interest in cluster evolution has focused on the post-core-collapse phase. Here Fokker–Planck methods can probably be trusted to describe the evolution of the total cluster mass, half-mass radius, and perhaps—although there has not yet been much work on this—stellar mass distribution. It is clear that although core collapse is probably not catastrophic for the cluster, considerable evolution of cluster properties and stellar content occurs after collapse, especially when mass loss through the tidal radius is taken into account. These methods are less well suited to describing conditions in the cluster core, in or near which they predict most of the binaries to be found. Rough agreement has been found between Fokker–Planck and *N*-body simulations for the rate of destruction of binaries in the post-collapse phase. However, the binaries are less centrally concentrated in the *N*-body simulations because of recoil in close binary–binary and binary–single interactions, which are more accurately described by the *N*-body methods. Fokker–Planck calculations (and even more approximate conducting-gas-sphere models) have uncovered the fascinating phenomenon of gravothermal core oscillations, although these have yet to be confirmed by *N*-body studies, probably because of computational limits on *N*. The relevance of these oscillations to real clusters with a broad distribution of stellar masses remains uncertain.

It has become clear that the dynamical evolution of post-collapse clusters and their binary populations depends on more than point-mass Newtonian dynamics. Hydrodynamic interactions between stars are unambiguously predicted and probably significant for cluster evolution. Important work on these effects has been carried out by extensions of Fokker–Planck methods, but owing to the large number of physical parameters involved, the future probably lies with Monte Carlo and extended *N*-body techniques. Even with such techniques in hand, further progress in understanding post-collapse cluster evolution may be frustrated by our poor theoretical understanding of stellar mass-loss rates, angular-momentum loss rates in close binaries, and other issues in stellar structure and evolution. However, what is a nuisance for theory may be an opportunity for observation, as even our presently limited understanding of conditions in cluster cores would seem to predict the presence there of a variety of unusual stars and binary systems.

4.8 Stochastic Simulations

Simulations of multimass clusters containing binaries have shown that heavy white dwarfs and neutron stars are preferentially exchanged into primordial binaries which sink to the core of a dense cluster. These binaries will thus have lost all memory of their initial stellar composition. Their components will instead be representative of the heaviest stars encountered in the core of the cluster. Simulations incorporating a substantial binary population have clearly demonstrated the importance of binary–binary interactions, in good qualitative agreement with more detailed self-consistent simulations.

N-body integrations are detailed but expensive, while Fokker–Planck simulations are cheap, but of questionable validity in a multimass, binary-rich cluster. Stochastic methods provide one possible bridge between these two approaches, allowing realistic binary effects to be incorporated into otherwise smooth continuum models. The simple studies performed to date have limited predictive power, but they allow important effects to be discerned, and point the way toward more detailed simulations. In principle, the same physical effects that can be included directly into an *N*-body code can also be incorporated stochastically into a continuum model according to a Monte Carlo realization of the interactions of interest. The near future will probably see simulations of test binaries in evolving (Fokker–Planck) clusters. A more difficult step will be the marriage of a full component of directly integrated binaries to a Fokker–Planck background of single stars. This will allow self-consistent inclusion of binary–binary interactions, and the resulting nonconservation of even hard binaries with a self-consistent evolving cluster potential.

The authors are indebted to Roger Griffin for a thorough and meticulous reading of the manuscript. P.H. and S.M. acknowledge Grant No. AST900013P from the Pittsburgh Supercomputer Center. S.M. acknowledges the support of NASA Award No. NAGW-2559. J.G. was supported by grants from the Alfred P. Sloan Foundation and the David and Lucille Packard Foundation. M.M. acknowledges partial support from NASA through Grant No. HF-1007.01-90A awarded to the Space Telescope Science Institute which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA under Contract No. NAS5-26555, and from a Fullam/Dudley award from the Dudley Observatory. E.S.P. appreciates his support from the Alfred P. Sloan Foundation and NASA Grant No. NAGW-2394. C.P. is grateful to the time allocation committees of the CFHT, MMTO, KPNO, and CTIO for generous allocations of time to study binaries in globular clusters. He thanks his collaborators Taft Armandroff, Sam Barden, Murray Fletcher, Jim Hesser, Dave Latham, Robert McClure, Ed Olszewski, and Bob Schommer for making his research on binaries in globular clusters possible and for allowing him (and sometimes even encouraging him) to write a contribution to this review instead of analyzing our data. Jim Hesser and Robert McClure offered helpful comments on a draft of C.P.'s portion of this review. C.P. acknowledges a grant from the Rutgers University Research Council and Grant No. AST-9020685 from the National Science Foundation. H.B.R. would like to acknowledge the comments and suggestions of Peter Leonard and Greg Fahlman. H.B.R.'s research is supported by grants from the Natural Sciences and Engineering Research Council of Canada. F.V. is supported by the Netherlands Organization for Scientific Research (NWO) under Grant No. PGS 78-277. M.D.W. thanks IAS for its hospitality and acknowledges the support of NASA award No. NAGW-2224.

REFERENCES

Aarseth, S. J., and Lecar, M. 1975, ARAA, 13, 1
Aarseth, S. J. 1985, in Multiple Time Scales, ed. J. U. Brackbill and B. I. Cohen (New York, Academic), p. 377
Aarseth, S. J., and Zare, K. 1974, Celest. Mech., 10, 185
Abt, H. A. 1983, ARAA, 21, 343
Abt, H. A. 1978, in Protostars and Planets, ed. T. Gehrels (Tucson, University of Arizona Press), p. 323
Abt, H. A. 1987, ApJ, 317, 353
Abt, H. A., and Levy, S. G. 1976, ApJS, 30, 273
Abt, H. A., and Willmarth, D. W. 1987, ApJ, 317, 786
Ahmad, A., and Cohen, L. 1973, J. Comput. Phys., 12, 389
Alexander, M. E., and Budding, E. 1979, AaA, 73, 227
Allen, C. W. 1973, Astrophysical Quantities (London, Athlone)
Alpar, M. A., Cheng, A. F., Ruderman, M. A., and Shaham, J. 1982, Nature, 322, 153
Ambartsumian, V. A. 1938, Ann. Leningr. State Univ., No. 22., p. 19 [A translation appeared in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut, 1985 (Dordrecht, Reidel), p. 521]
Anderson, S. B., Gorham, P. W., Kulkarni, S. R., Prince, T. A., and Wolszczan, A. 1990, Nature, 346, 42
Anderson, S., Kulkarni, S., Prince, T., and Wolszczan, A. 1990 IAU Circ. No. 5013
Anthony-Twarog, B. J., Kaluzny, J., Shara, M. M., and Twarog, B. A. 1990, AJ, 99, 1504
Antonov, V. A. 1962, Vestnik Leningrad Univ., 7, 135 [A translation appeared in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut, 1985 (Dordrecht, Reidel), p. 525]
Aurière, M., and Koch-Miramond, L. 1992, IAU Circ. No. 5364
Aurière, M., Koch-Miramond, L., and Ortolani, S. 1989, AaA, 214, 113
Aurière, M., le Fèvre, O., and Terzan, A. 1984, AaA, 138, 415
Aurière, M., Ortolani, S., and Lauzeral, C. 1990, Nature, 344, 638

Bailyn, C. D., Grindlay, J. E., Cohn, H., and Lugger, P. M. 1986, in IAU Symp. No. 126, Globular-Cluster Systems in Galaxies eds. J. E. Grindlay and A. G. D. Philip (Kluwer, Dordrecht), p. 679

Barden, S., Armandroff, T., and Pryor, C. 1990, BAAS, 22, 1285

Barnes, J., and Hut, P. 1986, Nature, 324, 446

Barr, P., White, N. E., Haberl, F., Stella, L., Pollard, G., Gottwald, M., and Parmar, A. N. 1987, AaA, 176, 69

Benz, W., and Hills, J. G. 1987, ApJ, 323, 614

Benz, W., Hills, J. G., and Thielemann, F.-K. 1989, ApJ, 342, 986

Bettwieser, E., and Sugimoto, D. 1984, MNRAS, 208, 439

Binney, J., and Tremaine, S. 1987, Galactic Dynamics (Princeton, Princeton University Press)

Bolte, M. 1987, ApJ, 319, 760

Bolte, M. 1991a, ApJS, in press

Bolte, M. 1991b, ApJ, 376, 514

Bradt, H. V. D., and McClintock, J. G. 1983, ARA&A, 21, 13

Breeden, J. L., Packard, N. H., and Cohn, H. N. 1992, ApJ, to be submitted

Carney, B. W. 1983, AJ, 88, 623

Chan, E., and Richer, H. B. 1986, ApJ, 302, 257

Charles, P. A. 1989, in ESA-SP No. 296, X-ray Astronomy, eds. J. Hunt and B. Battrick, Vol. 1, p. 103

Chernoff, D. F., and Djorgovski, S. 1989, ApJ, 339, 904

Chernoff, D. F., and Weinberg, M. 1990, ApJ, 351, 121

Cleary, P. W., and Monaghan, J. J. 1990, ApJ, 349, 150

Cohen, J. G., Frogel, J. A., and Persson, S. E. 1978, ApJ, 222, 165

Cohn, H. N. 1980, ApJ, 242, 765

Cohn, H. N. 1985, in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel), p. 525

Cohn, H., Hut, P., and Wise, M. 1989, ApJ, 342, 814

Cohn, H. N., Lugger, P. M., Grabhorn, R. P., Breeden, J. L., Packard, N. H., Murphy, B. W., and Hut, P. 1991, in The Formation and Evolution of Star Clusters, ASP Conference Series, 13, ed. K. Janes (San Francisco, ASP), p. 381

Cote, P., Richer, H. B., and Fahlman, G. G. 1991, AJ, 102, 1358

Crampton, D., Cowley, A. P., Hutchings, J. B., Schade, D. J., and Van Speybroeck, L. P. 1984, ApJ, 284, 663

Dabrowski, J. P., and Beardsley, W. R. 1977, PASP, 89, 225

D'Amico, N. et al. 1990, IAU Circ. No. 5013

Da Costa, G. S., and Freeman, K. C. 1976, ApJ, 206, 128

Da Costa, G. S., Norris, J., and Villumsen, J. V. 1986, ApJ, 308, 743

Davies, M. B., Benz, W., and Hills, J. G. 1991, ApJ, 381, 449

Djorgovski, S., and King, I. R. 1986, ApJ, 305, L61

Djorgovski, S., Piotto, G., Phinney, E. S., and Chernoff, D. F. 1991, ApJ, 372, L41

Dotani, T. et al. 1990a, ApJ, 350, 395

Dotani, T. et al. 1990b, Nature, 347, 534

Drukier, G. A., Fahlman, G. G., and Richer, H. B. 1989, ApJ, 342, L27

Duerbeck, H. W. 1984, AaSS, 99, 363

Dupree, A. K., Harper, G. M., Hartmann, L., Jordan, C., Rodgers, A. W., and Smith, G. H. 1990, ApJ, 361, L9

Dupree, A. K., and Whitney, B. 1991, Nature, submitted

Duquennoy, A., and Mayor, M. 1991, AaA, 248, 485

Eddington, A. S. 1926, The Internal Constitution of the Stars (Cambridge, Cambridge University Press)

Elson, R., Hut, P., and Inagaki, S. 1987, ARAA, 25, 565

Eggen, O. J., and Iben, I. 1989, AJ, 97, 431

Eggleton, P. P., Fitchett, M. J., and Tout, C. A. 1989, ApJ, 345, 489

Fabian, A. C., Pringle, J. E., and Rees, M. J. 1975, MNRAS 172, 15P

Fahlman, G. G., Richer, H. B., and Nemec, J. M. 1991, ApJ, 380, 124

Fahlman, G. G., Richer, H. B., and VandenBerg, D. A. 1985, ApJS, 58, 225

Fleming, T. A., Gioia, I. M., and Maccacaro, T. 1989, AJ, 98, 692

Forman, W. et al. 1978, ApJS, 38, 357

Fukushige, T., Ito, T., Makino, J., Ebisuzaki, T., Sugimoto, D., and Umemura, M. 1991, PASJ, 43, 841

Gao, B., Goodman, J., Cohn, H., and Murphy, B. 1991, ApJ, 370, 567 (GGCM)

Geyer, E. H., and Vogt, N. 1978, AA, 67, 297

Giersz, M. 1990a,b, Nicolaus Copernicus Astronomical Center preprints 220, 221

Giuricin, G., Mardirossian, F., and Mezzetti, M. 1984, ApJS, 54, 421

Goodman, J. 1983, ApJ, 270, 700; erratum 1984, ApJ, 278, 893

Goodman, J. G. 1984, ApJ, 280, 298

Goodman, J. 1987, ApJ, 313, 576

Goodman, J. 1989, in Dynamics of Dense Stellar Systems, ed. D. Merritt (New York, Cambridge University Press), p. 183

Goodman, J., and Hernquist, L. 1991, ApJ, 378, 637

Goodman, J., and Hut, P. (eds.) 1985, IAU Symp. No. 113, Dynamics of Star Clusters (Dordrecht, Reidel)

Goodman, J., and Hut, P. 1989, Nature, 339, 40

Griffin, R. F., Gunn, J. E., Zimmerman, B. A., and Griffin, R. E. M. 1988, AJ, 96, 172

Grindlay, J., Gursky, H., Schnopper, H., Parsignault, D. R., Heise, J., Brinkman, A. C., and Schrijver, J. 1976, ApJ, 205, L127

Grindlay, J. E., Cool, A. M., and Bailyn, C. D. 1991, in The Formation and Evolution of Star Clusters, ed. K. Janes (San Francisco, ASP), p. 396

Grindlay, J. E., Hertz, P., Steiner, J. E., Murray, S. S., and Lightman, A. P. 1984, ApJ, 282, L13

Guharthakurta, P., Yanny, B., Schneider, D. P. and Bahcall, J. N. 1992, AJ, 104, in press

Gunn, J. E., and Griffin, R. F. 1979, AJ, 84, 752

Halbwachs, J. L. 1986, AA, 168, 161

Halbwachs, J. L. 1987, AA, 183, 234

Hamilton, T. T., Helfand, D. J., and Becker, R. H. 1985, AJ, 90, 606

Harris, H. C., and McClure, R. D. 1983, ApJ, 265, L77

Heggie, D. C. 1974, Celest. Mech., 10, 217

Heggie, D. C. 1975, MNRAS, 173, 729

Heggie, D. C. 1980, in Globular Clusters, NATO Advanced Study Institute, ed. D. Hanes and B. Madore (Cambridge, Cambridge University Press), p. 281

Heggie, D. C., and Aarseth, S. J. 1992, MNRAS, 257, 513

Hellier, C., Mason, K. O., Smale, A. P., and Kilkenny, D. 1990, MNRAS, 244, 39P

Hénon, M. 1961, Ann. d'Astrophys., 24, 369

Hénon, M. 1965, Ann. d'Astrophys., 28, 62

Hénon, M. 1969, AA, 2, 151

Hénon, M. 1975, in IAU Symp. No. 69, Dynamics of Stellar Systems, ed. A. Haili (Dordrecht, Reidel), p. 133

Herczeg, T. 1982, in Landolt Börnstein, Vol. 2b, p. 408

Hernquist, L., and Goodman, J. 1991, ApJ, 378, 637

Hernquist, L., Hut, P., and Kormendy, J. 1991, Nature, 354, 376

Hertz, P. 1987, ApJ, 315, L119

Hertz, P., and Grindlay, J. E. 1983, ApJ, 275, 105
Hertz, P., and Grindlay, J. E. 1985, ApJ, 298, 95
Hertz, P., and Wood, K. S. 1985, ApJ, 290, 171
Hesser, J. E., Harris, W. E., VandenBerg, D. A., Allwright, J. W. B., Shott, P., and Stetson, P. B. 1987, PASP, 99, 739
Hills, J. G. 1975, AJ, 80, 809
Hills, J. G. 1976, MNRAS, 175, 1P
Hills, J. G. 1983, AJ, 88, 1269
Hills, J. C., and Fullerton, L. W. 1980, AJ, 85, 1281
Hills, J. G., and Day, C. A. 1975, Astrophys. Lett., 17, 87
Hodder, P. J. C., Nemec, J. M., Richer, H. B., and Fahlman, G. G. 1992, AJ, in press
Hoffer, J. B. 1983, AJ, 88, 1420
Hoffleit, D. 1982, The Bright Star Catalog (New Haven, Yale University Press)
Hoffman, J. A., Marshall, H. L., and Lewin, W. H. G. 1978, Nature, 271, 630
Hogg, H. S. 1973, Publ. D. D. O., 3, No. 6
Hut, P. 1983, ApJ, 272, L29
Hut, P. 1984, ApJS, 55, 301
Hut, P. 1985, in IAU Symp. No. 113, Dynamic of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel), p. 231
Hut, P. 1992, in X-Ray Binaries and Recycled Pulsars, ed. E. P. J. van den Heuvel and S. A. Rappaport (Dordrecht, Kluwer), p. 317
Hut, P., and Inagaki, S. 1985, ApJ, 298, 502
Hut, P., and Bahcall, J. N. 1983, ApJ, 268, 319
Hut, P., Murphy, B. W., and Verbunt, F. 1991, A&A, 241, 137
Hut, P., Makino, J., and McMillan, S. L. W. 1988, Nature, 336, 31
Hut, P., McMillan, S. L. W., and Romani, R. W. 1992, ApJ, to appear
Hut, P., and Verbunt, F. 1983, Nature, 301, 587
Iben, I., Jr., and Tutukov, A. V. 1986, ApJ, 311, 742
Inagaki, S. 1986, PASJ, 38, 853
Illingworth, G. 1976, ApJ, 204, 73
Illovaisky, S. A., Aurière, M., Chevalier, C., Koch-Miramond, L., Cordoni, J.-P., and Angebault, L. P. 1987, AaA, 179, L1
Irwin, M. J., and Trimble, V. 1984, AJ, 89, 83
Ito, T., Makino, J., Ebisuzaki, T., and Sugimoto, D. 1990, Comput. Phys. Commun., 60, 187
Ito, T., Ebisuzaki, T., Makino, J., and Sugimoto, D. 1991, PASJ, 43, 547
Jensen, K. S., and Jørgensen, H. E. 1985, AaAS, 60, 229
Jørgensen, H. E., and Hansen, L. 1984, AA, 133, 165
Johnston, H. M., and Kulkarni, S. R. 1991 ApJ, 368, 504
Johnston, H. M., and Kulkarni, S. R. 1992, ApJ, in press
Johnston, H. M., Kulkarni, S. R., and Goss, W. M. 1991, ApJ, 382, L89
Johnston, H. M., Kulkarni, S. R., and Phinney, E. S. 1992, in X-ray Binaries and the Formation of Binary and Millisecond Pulsars, ed. E. P. J. van den Heuvel and S. A. Rappaport (Kluwer, Dordrecht)
Kallrath, J., Milone, E. F., and Stagg, C. R. 1991, preprint
Kaluzny, J., and Shara, M. M. 1987, ApJ, 314, 585
Kaluzny, J., and Krzeminski, W. 1992, preprint
King, I. R. 1966, AJ, 71, 64
Kochanek, C. S. 1992, ApJ, 385, 604
Kopal, Z. 1959, Close Binary Systems (London, Chapman and Hill), p. 525
Kormendy, J., and McClure, R. D. 1992, ApJ, to be published
Kraicheva, Z. T., Popova, E. I., Tutukov, A. V., and Yungel'son, L. R. 1978, Astron. Zh., 55, 1176; English translation in Sov. Astrophys., 22, 670
Krolik, J. H. 1983, Nature, 305, 506
Krolik, J. H. 1984, ApJ, 282, 452
Kulkarni, S. R., Anderson, S. B., Prince, T. A., and Wolszczan, A. 1991, Nature, 349, 47
Kulkarni, S. R., Narayan, R., and Romani, R. W. 1990a, ApJ, 356, 174
Kulkarni, S. R., Goss, W. M., Wolszczan, A., and Middleditch, J. 1990b, ApJ, 363, L5
Kunieda, H. et al. 1984a, PASJ, 36, 215
Kunieda, H. et al. 1984b, PASJ, 36, 807
Kustaanheimo, P., and Stiefel, E. L. 1965, J. Reine Angew. Math., 218, 204
Latham, D., Hazen-Liller, M., and Pryor, C. 1985, in IAU Colloq. No. 88, Stellar Radial Velocities, ed. Philip and Latham, p. 269
Latham, D. W., Mazeh, T., Carney, B. W., McCrosky, R. E., Stefanik, R. P., and Davis, R. I. 1988, AJ, 96, 567
Lauer, T. R. et al. 1991, ApJ, 369, L45
Lee, H. M. 1987a, ApJ, 319, 772
Lee, H. M. 1987b, ApJ, 319, 801
Lee, H. M., Fahlman, G. C., and Richer, H. B. 1991, ApJ, 366, 455
Lee, H. M., and Ostriker, J. P. 1986, ApJ, 310, 176
Lee, H. M., and Ostriker, J. P. 1987, ApJ, 322, 123
Lehto, H. J., Machin, G., McHardy, I., and Callanan, P. 1990, Nature, 347, 49
Leonard, P. J. T. 1989, AJ, 98, 217
Leonard, P. J. T., and Fahlman, G. G. 1991, AJ, 102, 994
Lewin, W. H. G., and Joss, P. C. 1983, in Accretion-Driven Stellar X-Ray Sources, ed. W. H. G. Lewin and E. P. J. van den Heuvel, p. 41
Liller, M. H. 1978, IBVS, No. 1527
Liller, M. H., and Tokarz, S. P. 1981, AJ, 86, 669
Livio, M. 1991, in The Physics of Classical Novae, ed. A. Sassatella and R. Viotti (Berlin, Springer), p. 342
Long, K. S., D'Odorico, S., Charles, P. A., and Dopita, M. A. 1981, ApJ 246, L61
Long, K. S., and Van Speybroeck, L. P. 1983, in Accretion-Driven Stellar X-Ray Sources, ed. W. H. G. Lewin and E. P. J. van den Heuvel, p. 117
Lupton, R., Gunn, J. E., and Griffin, R. F. 1987 AJ, 93, 1114
Lyne, A. G., Brinklow, A., Middleditch, J., Kulkarni, S. R., Backer, D. C. and Trevor, T. R. 1987, Nature, 328, 399
Lynden-Bell, D., and Wood, R. 1968, MNRAS, 138, 495
Machin, G. et al. 1991, MN, 250, 602
Makino, J. 1989, in Dynamics of Dense Stellar Systems, ed. D. Merritt (Cambridge, Cambridge University Press), p. 201
Makino, J., Ito, T., and Ebisuzaki, T. 1990, PASJ, 42, 717
Makino, J. 1991a, ApJ, 369, 200
Makino, J. 1991b, PASJ, 43, 621
Makino, J. 1991c, PASJ, 43, 859
Makishima, K. et al. 1981, ApJ, 247, L23
Manchester, R. N., Lyne, A. G., Robinson, C., Damico, N. and Bailes, M. 1991, Nature, 352, 219
Maraschi, L., and Cavaliere, A. 1977, Highlights Astron., 4, Part I, p. 127
Margon, B., Anderson, S. F., Downes, R. A., Bohlin, R. C., and Jakobsen, P. 1991, ApJ, 369, L71
Margon, B., and Bolte, M. 1987, ApJ, 321, L61
Margon, B., and Cannon, R. 1989, The Observatory, 109, 82
Margon, B., and Downes, R. A. 1983, ApJ, 274, L31
Margon, B., Downes, R. A., and Gunn, J. E. 1981, ApJ, 247, L89
Margon, B., Wilcots, E., and Bolte, M. 1991, BAAS, 23, No. 2
Marshall, H. L., Ulmer, M. P., Hoffman, J. A., Doty, J., and

Lewin, W. H. G. 1979, ApJ, 227, 555
Mason, K. O. 1986, in The Physics of Accretion onto Compact Objects, ed. K. O. Mason, M. G. Watson, and N. E. White (Berlin, Springer), p. 29
Mateo, M., Harris, H. C., Nemec, J. M., and Olszewski, E. W. 1990, AJ, 100, 469
Mateo, M., and Krzeminski, W. 1990, BAAS, 22, 1284
Mateo, M., Nemec, J., Harris, H., C., and Olszewski, E. W. 1992, in preparation
Mathieu, R. D., Walter, F. M., and Myers, P. C. 1989, AJ, 98, 987
Mayor, M. et al. 1984, AA, 134, 118
McClure, R. D., Twarog, B. A., and Forrester, W. T. 1981, ApJ, 243, 841
McClure, R. D., VandenBerg, D. A., Bell, R. A., Hesser, J. E., and Stetson, P. B. 1987, AJ, 93, 1144
McCrea, W. H. 1964, MNRAS, 128, 147
McKenna, J., and Lyne, A. G. 1988, Nature, 336, 226
McMillan, S. L. W. 1986, ApJ, 307, 126
McMillan, S. L. W. 1989, in Dynamics of Dense Stellar Systems, ed. D. Merritt (Cambridge, Cambridge University Press), p. 207
McMillan, S. L. W., and Lightman, A. P. 1984a, ApJ, 283, 801
McMillan, S. L. W., and Lightman, A. P. 1984b, ApJ, 283, 813
McMillan, S. L. W., McDermott, P. N., and Taam, R. E. 1987, ApJ, 318, 261
McMillan, S. L. W., Hut, P., and Makino, J. 1990, ApJ, 362, 522 (MHM)
McMillan, S. L. W., Hut, P., and Makino, J. 1991, ApJ, 372, 111
McMillan, S. L. W., Cranmer, S. R., Shorter, S. A., and Hernquist, L., 1991, in The Formation and Evolution of Star Clusters, ed. K. Janes (San Francisco, ASP), p. 418
McMillan, S. L. W., and Aarseth, S. J. 1992, ApJS, submitted
McMillan, S. L. W., and Hut, P. 1992, in preparation (MH)
Meylan, G., Dubath, P., and Mayor, M. 1991, ApJ, 383, 587
Mikkola, S. 1983a, MNRAS, 203, 1107
Mikkola, S. 1983b, MNRAS, 205, 733
Mikkola, S. 1984a, MNRAS, 207, 115
Mikkola, S. 1984b, MNRAS, 208, 75
Mikkola, S. 1985, MNRAS, 215, 171
Mochnacki, S. W. 1985, in Interacting Binaries, ed. P. P. Eggleton and J. E. Pringle (Dordrecht, Reidel), p. 51
Morbey, C. L., and Griffin, R. F. 1987, ApJ, 317, 343
Morgan, E. H., Remillard, R. A., and Garcia, M. R. 1988, ApJ, 324, 851
Murphy, B. W., and Cohn, H. N. 1988, MNRAS, 232, 835
Murphy, B. W., Cohn, H. N., and Hut, P. 1990, MNRAS, 245, 335
Murphy, B. W., Rutten, R. G. M., Callanan, P. J., Seitzer, P., Charles, P. A., Cohn, H. N., and Lugger, P. M. 1991, Nature, 351, 130
Murray, S. D., Clarke, C. J., and Pringle, J. E. 1991, preprint
Nagase, F. 1989, PASJ, 41, 1
Naylor, T., Charles, P. A., Drew, J. E., and Hassall, B. J. M. 1988, MNRAS, 233, 285
Naylor, J. et al. 1989, MN, 241, 25
Nemec, J. M., and Cohen, J. G. 1989, ApJ, 336, 780
Nemec, J. M., and Harris, H. C. 1987, ApJ, 316, 172
Nemec, J., and Mateo, M. 1990, in Confrontation Between Stellar Pulsation and Evolution. ed. C. Cacciari and G. Clementini (ASP, San Francisco), p. 64
Nemec, J., Mateo, M., Burke, M., Richer, H., Fahlman, G., and Olszewski, E. W. 1992, in preparation
Newell, B., and O'Neil, E. J. 1978, ApJS, 37, 27
Nieto, J. L. et al. 1990, AaA, 239, 155
Niss, B., Jørgensen, H. E., and Lautsen, S. 1978, AaAS, 32, 387
Ostriker, J. P. 1985, in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel), p. 347
Paresce, F., Shara, M., Meylan, G., Baxter, D., and Greenfield, P. 1991, Nature, 352, 297
Parmar, A., Smale, A. P., Verbunt, F., and Corbet, R. H. D. 1991, ApJ, 366, 253
Parmar, A. N., Stella, L., and Giommi, P. 1989, AaA, 222, 96
Penny, A. J., and Dickens, R. J. 1986, MNRAS, 220, 845
Peters, P. C. 1964, Phys. Rev. 136, 1224
Peterson, C. J., and King, I. R. 1975, ApJ, 80, 427
Phinney, E. S. 1992a, Philos. Trans. R. Soc. Lond. A, in press
Phinney, E. S. 1992b, MNRAS, in press
Phinney, E. S., and Sigurdsson, S. 1991, Nature, 349, 220
Phinney, E. S., and Kulkarni, S. R. 1992, Nature, in press
Podsiadlowski, P. 1991, Nature, 350, 136
Predehl, P., Hasinger, G., and Verbunt, F. 1991, AaA, 246, L21
Press, W. H., and Teukolsky, S. A. 1977, ApJ, 213, 183
Priedhorsky, W., and Terrell, J. 1984, ApJ, 284, L17
Pringle, J. E., and Wade, R. A. (eds.) 1985, Interacting Binary Stars (Cambridge, Cambridge University Press)
Prince, T. A., Anderson, S. B., Kulkarni, S. R., and Wolszczan, A. 1991, ApJ, 374, L41
Pryor, C., Hesser, J. E., McClure, R. D., Krismer, M., and Fletcher, J. M. 1992, BAAS, 24, 775
Pryor, C., Latham, D., and Hazen, M. 1988, AJ, 96, 123
Pryor, C., McClure, R. D., Fletcher, J. M., Hartwick, F. D. A., and Kormendy, J. 1986, AJ, 91, 546
Pryor, C., McClure, R. D., Fletcher, J. M., and Hesser, J. E. 1989a, in Dynamics of Dense Stellar Systems, ed. D. Merritt (Cambridge, Cambridge University Press), p. 175
Pryor, C., McClure, R. D., Fletcher, J. M., and Hesser, J. E. 1989b, AJ, 98, 596
Pryor, C., McClure, R. D., Hesser, J. E., and Fletcher, J. M. 1987, BAAS, 19, 676
Pryor, C., Schommer, R. A., and Olszewski, E. W. 1991, in The Formation and Evolution of Star Clusters, ed. Janes (San Francisco, ASP), p. 439
Rappaport, S., Putney, A., and Verbunt, F. 1989, ApJ, 345, 210
Rasio, F. A., Shapiro, S. L., and Teukolsky, S. A. 1989, in Dynamics of Dense Stellar Systems, ed. D. Merritt (Cambridge, Cambridge University Press), p. 121
Ray, A., Kembhavi, A. K., and Antia, H. M. 1987, A&A, 184, 164
Richer, H. B., and Fahlman, G. G. 1984, ApJ, 277, 227
Richer, H. B., and Fahlman, G. G. 1986, ApJ, 304, 273
Richer, H. B., and Fahlman, G. G. 1987, ApJ, 316, 189
Richer, H. B., and Fahlman, G. G. 1988, ApJ, 325, 218
Richer, H. B., and Fahlman, G. G. 1991, unpublished
Roberts, M. S. 1988, in IAU Symp. No. 126, Globular Cluster Systems in Galaxies, ed. J. E. Grindlay and A. G. Davis Philip (Dordrecht, Reidel), p. 411
Romani, R. W., and Weinberg, M. D. 1991, ApJ, 372, 487 (RW)
Ruciński, S. M. 1973, Acta Astron., 23, 79
Ruciński, S. M. 1974, Acta Astron., 24, 119
Ruciński, S. M. 1985a, in Interacting Binaries, ed. P. P. Eggleton and J. E. Pringle (Dordrecht, Reidel), p. 13
Ruciński, S. M. 1985b, in Interacting Binary Stars, ed. J. E. Pringle and R. A. Wade (Cambridge, Cambridge University Press), p. 85
Ryba, Taylor 1992, ApJ, 380, 557
Sandage, A., Katem, B., and Johnson, H. L. 1977, AJ, 82, 389

Savonije, G. J., de Kool, M., and van den Heuvel, E. P. J. 1986, AaA, 155, 51
Sawyer, H. B. 1953, JRASC, 47, 229
Shara, M. M., Kaluzny, J., Potter, M., and Moffat, A. F. J. 1988, ApJ, 328, 594
Shara, M. M., Moffat, A. F. J., and Hanes, D. A. 1985, in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel), p. 103
Shara, M. M., Livio, M., Moffat, A. F. J., and Orio, M. 1986b, ApJ, 311, 163
Shara, M. M., Moffat, A. F. J., and Potter, M. 1987, AJ, 94, 357
Shara, M. M., Moffat, A. F. J., Potter, M., Hogg, H. S., and Wehlau, A. 1986a, ApJ, 311, 796
Shawl, S. J., and White, R. E. 1986, AJ, 91, 312
Sigurdsson, S. 1991, Dynamics of Neutron Stars and Binaries in Globular Clusters, Caltech Ph.D. thesis
Sigurdsson, S., and Phinney, E. S., 1992, in preparation
Skumanich, A. 1972, ApJ, 171, 565
Smale, A. P, Mason, K. O., and Mukai, K. 1987, MNRAS, 225, 7P
Soderblom, D. R., Duncan, D. K., and Johnson, D. R. H. 1991, ApJ, 375, 722
Spergel, D. N. 1991, Nature, 352, 221
Spitzer, L. 1940, MNRAS, 100, 396
Spitzer, L. 1969, ApJ, 158, L139
Spitzer, L. 1975, in IAU Symp. No. 69, Dynamics of Stellar Systems, ed. A. Haili (Dordrecht, Reidel), p. 3
Spitzer, L. 1987, Dynamical Evolution of Globular Clusters (Princeton, Princeton University Press)
Spitzer, L., and Mathieu, R. D. 1980, ApJ, 241, 618
Statler, T. S., Ostriker, J. P., and Cohn, H. N. 1987, ApJ, 316, 626
Stella, L., Haberl, F., Lewin, W. H. G., Parmar, A. N., van Paradijs, J., and White, N. E. 1988, ApJ, 324, 379
Stella, L., Priedhorsky, W., and White, N. E. 1987, ApJ, 312, L17
Stetson, P. B., and Harris, W. E. 1988, AJ, 96, 909
Stodółkiewics, J. S., and Giersz, M. 1990, Nicolaus Copernicus Astronomical Center, preprint p. 218
Stryker, L. L., Hesser, J. E., Hill, G., Garlick, G. S., and O'Keefe, L. M. 1985, PASP, 97, 247
Sugimoto, D., and Bettwieser, E. 1983, MNRAS, 204, 19
Sugimoto, D., Chikada, Y., Makino, J., Ito, T., Ebisuzaki, T., and Umemura, M. 1990, Nature, 345, 33
Taam, R. E., and Lin, D. N. C. 1992, preprint
Taam, R. E., and Picklum, R. E. 1979, ApJ, 233, 327
Tan, J. et al. 1991, ApJ, 374, 291
Thorsett, S. E., and Nice, D. J. 1991, Nature, 353, 731
Trimble, V. L. 1976, BAAS, 8, 443
Trimble, V. L. 1977, MN, 178, 335
Trimble, V. L. 1980, in Star Clusters, ed. J. Hesser (Dordrecht, Reidel), p. 259
Trinchieri, G., and Fabbiano, G. 1991, ApJ, 382, 82
Van den Heuvel, E. P. J., and Rappaport, S. A., (eds.) 1992, X-ray Binaries and the Formation of Binary and Milliscecond Pulsars (Kluwer, Dordrecht)
Van der Klis, M. et al. 1992, AaA, submitted
Van Paradijs, J. 1983, in Accretion-Driven Stellar X-Ray Sources, ed. W. H. G. Lewin and E. P. J. van den Heuvel, p. 189
Van Paradijs, J., and Lewin, W. H. G. 1988, Adv. Space Res., 8, No. 2-3, p. 461
Van Paradijs, J., Verbunt, F., Shafer, R. A., and Arnaud, K. A. 1987, AaA, 182, 47
Van Speybroeck, L., Epstein, A., Forman, W., Giacconi, R., Jones, C., Liller, W., and Smarr, L. 1979, ApJ, 234, L45
Van't Veer, F. 1975a, AaA, 40, 167
Van't Veer, F. 1975b, AaA, 44, 437
Verbunt, F. 1987, ApJ, 312, L23
Verbunt, F. 1990, in Neutron Stars and their Birth Events, ed. W. Kundt (Kluwer, Dordrecht), p. 179
Verbunt, F., van Paradijs, J., and Elson, R. 1984, MNRAS, 210, 899
Verbunt, F., and Hut, P. 1987, in The Origin and Evolution of Neutron Stars, ed. D. J. Helfand and J. H. Huang (Dordrecht, Reidel), p. 187
Verbunt, F., and Meylan, G. 1988, A&A, 203, 297
Verbunt, F., Shafer, R. A., Jansen, F., Arnaud, K. A., and van Paradijs, J. 1986, AaA, 168, 169
Verbunt, F., and Zwaan, C. 1981, AaA, 100, L7
Vilhu, O. 1982, AaA, 109, 17
von Hoerner, S. 1960, Z. Astrophys., 50, 184
Wade, R. A., and Ward, M. J. 1985, in Interacting Binary Stars, ed. J. E. Pringle and R. A. Wade (Cambridge, Cambridge University Press), p. 129
Webbink, R. F. 1976, ApJ, 209, 829
Webbink, R. F. 1977, ApJ, 215, 851
Webbink, R. F. 1979, ApJ, 227, 178
Webbink, R. F. 1980, in IAU Symp. No. 88, Close Binary Stars: Observations and Interpretation, ed. M. J. Plavec, D. M. Popper, and R. K. Ulrich (Reidel, Dordrecht), p. 561
Webbink, R. F. 1985, in IAU Symp. No. 113, Dynamics of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel), p. 541
Wheeler, J. C. 1979, ApJ, 234, 569
White, N. E., Kaluzienski, J. L. and Swank, J. H. 1984, in High Energy Transients in Astrophysics, ed. S. E. Woosley (New York, AIP), p. 31
Wielen, R., Jahreiss, H., and Kruger, R. 1983, in The Nearby Stars and the Luminosity Function, ed. A. G. D. Philip and A. R. Upgren (Schenectady, Davis), p. 163
Wolszczan, A., Anderson, S. B., Kulkarni, S. R., and Prince, T. A. 1989, IAU Circ. No. 4880
Zahn, J.-P. 1977, AaA, 57, 383 (erratum in 1978, AaA, 67, 162)
Zinn, R., and Searle, L. 1976, AJ, 209, 734