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ASTEROID FAMILIES:
RECENT RESULTS AND PRESENT SCENARIO

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Abstract. After several decades of frustrating results showing a generally poor agreement among different asteroid family classifications, recent studies based on high accuracy proper elements, as well as on objective statistical methods of cluster analysis have largely improved the situation. Now, a number of asteroid families have been recognized on the basis of different methods of cluster analysis, using asteroid proper elements data sets computed by means of different theories. For these reasons, they should be considered of very high reliability. Moreover, spectroscopic observations confirm in some cases these results, indicating surface compositions of the family members in agreement with a geochemically plausible parent body. However, in particular zones of the belt, like the Flora region, further efforts should be performed in order to establish the real consistence of the resulting clusterings of objects. In addition, the size distribution and the taxonomic types of some well established families seem to indicate particular features of the family sample when compared with the field objects. We recall that asteroid families, in the framework of asteroid collisional evolution, are of the highest importance for understanding the mechanisms of injection of fragments into the Earth-crossing zone through mean-motion and secular resonances and, as a consequence, for evaluating the impact rate on Earth of asteroidal objects.

Key words: Asteroid, asteroid families, proper elements, cluster analysis.

1. Introduction

Asteroid dynamical families have long been a subject of research for planetological science during the present century. Their discovery dates back to 1918, when Hirayama noticed the existence of big concentrations in the distribution of the asteroid orbital elements a, e and i. In some cases the observed condensations appeared too conspicuous to be due to chance. Hirayama devoted to this subject a long series of papers (Hirayama, 1918, 1919, 1920, 1923, 1928, 1933), and succeeded in evidencing a number of clusterings, which he called *families*. The best examples of these are the three well known Eos, Themis and Koronis families.

The basic and today generally accepted idea for explaining the existence of asteroid families is also due to Hirayama. According with this hypothesis, families were formed as the results of collisional events. They originated from the catastrophic disruption of single parent bodies, which were destroyed by high-velocity impacts with other objects of asteroidal size. In other words, the present family members should be collisional fragments which were ejected with comparatively low relative velocities, so that their orbital elements did not change too much with respect to each other and to the original parent body. This fact makes them still recognizable today.

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According with this scenario, the name "family" should be assigned only to groupings composed by members genetically related, rather than to any defined clustering of objects in the space of orbital elements. As a consequence, groupings that are recognizable only because they occupy particular zones of the belt which are isolated as a result of the presence of secular and mean motion resonances, should not be termed families. A well known example in this sense is given by the objects of the so-called Phocaea zone.

A very important point concerning asteroid dynamical families is that, as was already recognized by Hirayama, the time variations of the orbital osculating elements of asteroids due to the gravitational perturbations of the planets, mainly Jupiter, make these elements not suitable for the purposes of families identification. Instead, the *proper elements* a', e', i', defined as constants of integration of the differential equations giving the variation of the orbital elements in the framework of the secular perturbation theory, provide a set of elements which in principle could be considered constant. For this reason proper elements have been computed and analyzed by the authors who have studied the problem of asteroid family identification during the last decades.

Among them, and after the work of Hirayama, the most important studies have been published by Brouwer (1951), Arnold (1969), Lindblad and Southworth (1971), Carusi and Massaro (1978), Williams (1979, 1989), Kozai (1979). In addition, Van Houten *et al.* (1970) carried out a search for asteroid families using solely the data set provided by the Palomar Leiden Survey (PLS).

Of course, the different searches for asteroid families quoted above, differ largely in some important respects; in particular, different authors used various data sets of asteroid proper elements, different theories of proper elements computation, as well as different criteria of clustering identification. Moreover, various statistical tests of the results were applied by some authors, while they were not attempted at all in other studies. The general situation is shown in Table I. For a general review on this subject, see Valsecchi *et al.* (1989).

As can be seen in Table I, in spite of the noticeable effort produced by different researchers, the situation concerning asteroid families identification has been far from satisfactory until recently. In fact, a general agreement among the various proposed family lists has not long been achieved neither for what concerns the number nor the consistency of the families, with the only exception of the three major cases of Eos, Themis and Koronis.

On the other hand, the interest about asteroid families has been growing recently, mainly as a consequence of the improvements of our knowledge of the general process of collisional evolution of the asteroid belt (Chapman *et al.*, 1989; Davis *et al.*, 1989). In fact, it is evident that in this respect asteroid families have a crucial importance: their number can be considered as a significant constraint for the models aimed to describe the overall collisional evolution of asteroids, while the observed physical properties of their members can provide important information on the processes of catastrophic breakup of bodies of asteroidal size, as well as

TABLE I
Results of the asteroid families searches published until 1989. The general disagreement is evident for what concerns the number of recognized families. The results of the two most recent attempts of families identification (Zappalà *et al.*, 1990; Bendjoya *et al.*, 1991) are presented and discussed in the text

Author(s)	Numbered Asteroids	PLS Asteroids	Families found
Hirayama (1933)	1223	no	5
Brouwer (1951)	1563	no	28
Arnold (1969)	1735	no	37
Van Houten *et al.* (1970)	—	yes	14
Lindblad and Southworth (1971)	1735	yes	34
Carusi and Massaro (1978)	1861	yes	15
Williams (1979)	1796	yes	104
Kozai (1979)	2125	no	72
Williams (1989)	2065	yes	117

on the internal composition of asteroids. In this sense, the family members could give a unique possibility for "looking inside" asteroidal bodies, since their present exposed surfaces come from the interior of their parent bodies.

For these reasons, it is evident that the discrepancies among the different existing family lists are particularly frustrating, since they do not allow us to take profit of the potentially important physical information contained in the presently existing asteroid families. These discrepancies are due to various reasons. As already mentioned, they are basically a consequence of: (1) the different data bases considered by the different authors; (2) the different methods used to compute the orbital proper elements; (3) the different identification procedures used.

Among these causes, (1) is probably less important. While it is evident that in general the larger the data set, the better are the results, we can expect that the lack of a number of (mostly small) objects can mainly influence the ability to identify some families not easily recognizable and largely composed by small members. On the other hand, we do not think that the differences on the adopted data sets listed in Table I can explain the big discrepancies of the results. As an example, it is hard to believe that a difference of 61 objects between the Lindblad and Southworth (1971) and the Williams (1979) data sets can justify the big discrepancy on the number of families found by these authors (34 and 104, respectively).

More important appear to be the differences among both the adopted proper elements computation theories, and the methods used for identifying families out of the background in the space of proper elements. For what concerns the former problem, since it is often hard to quantify precisely the accuracies of the different theories of proper elements computation, as well as the influence that these can have on the overall results of asteroid families identification, some numerical and empirical approaches are probably more suitable. For instance, tests devoted to study, for selected asteroids chosen in different zones of the belt, the stability of the proper elements by means of numerical integrations spanning over a relatively large period time have been performed by Milani and Kneževič (1990). On the other hand, a good test could be also to apply a given method of families identification to a data set of asteroids whose proper elements are computed by means of different techniques. This can in principle allow us to infer what is the importance of the proper elements computation for the purposes of families identification. Such an exercise has been recently carried out (Zappalà *et al.*, 1992), and the results will be reviewed in Section 2.2.

On the other hand, the adopted identification method has a crucial importance. So far, most of the procedures applied by the various authors have been eminently visual. Moreover, in several cases no statistical test for the significance of the resulting clusterings found in the space of proper elements was performed. Instead, what we need is in principle a method which should satisfy some properties of *objectivity* and *reproducibility*. For this reason, some automated procedure managed by computer should be preferred, in order to avoid well known biases affecting the response of human eye in this kind of applications. In addition, the reliability of the resulting clusterings should be quantified on the basis of statistics, and taking into account the uncertainties of the adopted values of proper elements. In the next Section we present the results of two new searches for asteroid families which have been recently carried out (Zappalà *et al.*, 1990; Bendjoya *et al.*, 1991), taking into account the above requirements.

2. Families Reliability Tests

2.1. DIFFERENT METHODS APPLIED TO THE SAME DATA SET

Recently, two new independent methods of families identification have been applied to the same data set of asteroid proper elements (Zappalà *et al.*, 1990; Bendjoya *et al.*, 1991). This fact allows for the first time a direct comparison between different procedures, since the possible discrepancies in this case do not depend on anything but the methods themselves. Moreover, both methods are "objective", in the sense that they do not use any visual inspection of the data, but they largely employ mathematical algorithms executed by computer.

The data set used in both researches has been the 4.2 version of the list of asteroid proper elements computed by Milani and Kneževič. Such a sample, including about 4100 numbered asteroids, has been the largest ever used for families identification purposes. The proper elements of this data set have been computed by means of

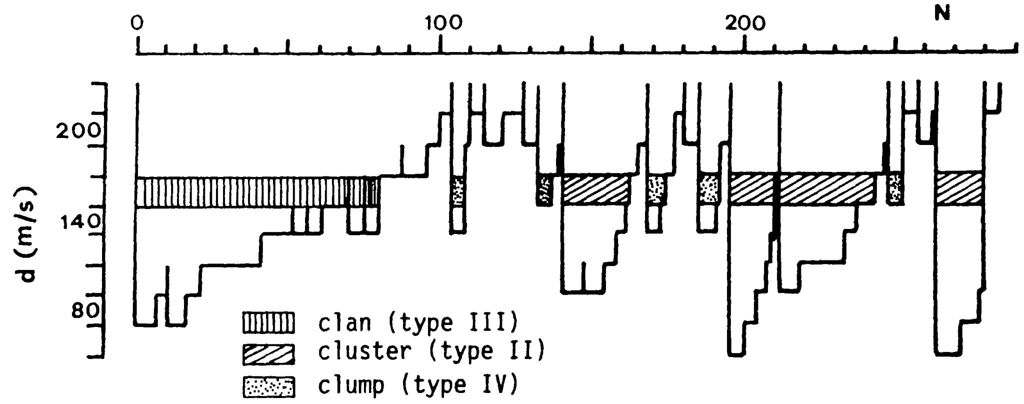

Fig. 1. An example of stalactite diagram referring to the sample of asteroids whose proper semi-major axis ranges between 2.5 and 2.825 AU (3/1 and 5/2 mean motion resonances with Jupiter). Examples of different nomenclatures for the resulting groupings are also indicated (see Section 4).

a second-order (in the planetary masses), fourth-degree (in the eccentricities and inclinations) secular perturbation theory, based on the Lie series technique. The accuracies of these proper elements depends in a complicated way on the location in the asteroid belt, and have been tested by means of numerical integrations of orbits (see Milani and Kneževič, 1990).

The first method is the so-called Hierarchical Clustering Method (HCM), and has been described by Zappalà *et al.* (1990). Basically, this method uses well known techniques of Multivariate Data Analysis in order to analyze the structures of the clusterings present in the space of proper elements. The basic idea is to introduce a definition of distance in this space; the mutual distances between the objects of the considered sample are then computed, and agglomerations are performed in such a way that it is possible to build a *dendrogram* containing all necessary information for families identification purposes. In particular, for any given value d of the distance, it is possible to derive immediately what are the existing clusterings composed by objects whose mutual distances are less than d. The results are expressed under the form of *stalactite diagrams*, an example of which is given in Figure 1. It has to be noticed that the adopted distance definition in the space of proper elements is connected with the assumption of a collisional origin for families, thus it exploits the known relationship (Gauss equations) between the ejection velocities of the fragments of a catastrophic impact and the corresponding differences of the orbital elements. With this assumption, the distances between the objects are actually expressed in m $\sec^{-1}$. In order to evidence the presence of families, one has to derive a confidence level of distance, for which no clusterings can be found having a non-negligible probability to be due to chance. Such a critical value of the distance can be found by generating, in the same portion of the proper elements space occupied by a given sample of real asteroids, a fictitious population of objects randomly chosen, but with the constraint to separately fit the a', e', and i' distributions of the real sample. In this way, the critical distance can be assumed to be the deepest level at which it is still possible to find sizeable agglomerations of of objects of the fictitious population (5 objects according to Zappalà *et al.*, 1990). In this way, the families present in the analyzed sample are evidenced, even if it is clear that a true collisional origin of these clusters cannot be assessed without a detailed analysis of the physical properties of their members (taxonomic types, observed mass distribution, etc.).

More recently, Bendjoya *et al.* (1991) have applied another procedure for asteroid families identification, namely the so-called Wavelet Analysis Method (WAM). The basic concept is that of *wavelet transform*, a mathematical tool which has been recently applied to the analysis of signals; its main advantage is that it is able to provide information on both the localization and the size of an event in a signal. If $s(x)$ is a one-dimension space signal in $L^2(\Re)$, the wavelet transform of this signal is a two dimension function $S(a,b)$, where a is a strictly positive scale-variable (size) and b a space-variable (localization). $S(a,b)$ is the decomposition of $s(x)$ onto a base of functions obtained from the dilatations and the translations of a unique function $\psi(x)$ of $L^2(\Re)$.

$$S(a,b) = a^{-\frac{1}{2}} \int_{-\infty}^{+\infty} s(x)\psi^*(\frac{x-b}{a})dx \tag{1}$$

$\psi(x)$ is called the analyzing wavelet. $a^{-\frac{1}{2}}$ is a $L^2(\Re)$ normalization factor and * denotes the complex conjugate.

The wavelet transform acts like a sort of mathematical zoom and $S(a,b)$ can be seen as a representation of the a-sized details of $s(x)$ around the location b. As an analyzing wavelet, it can be shown that a good choice is given by the so-called mexican hat function (see Figure 2), given by:

$$\psi_\sigma(x) = (n - \frac{r^2}{\sigma^2})\exp(-\frac{r^2}{\sigma^2}) \tag{2}$$

n is the dimension of the space in which the signal is defined and r is the distance defined from the metric chosen in the considered space. For the application of wavelet analysis to the problem of asteroid families identification, each asteroid is represented by a point in the (a', e', i') space. The signal to be analyzed in this case is thus a set of Dirac functions $\delta(\mathbf{x} - \boldsymbol{\eta}_j)$, $\boldsymbol{\eta}_j$ being the position of the j^{th} point. In computer form, the wavelet transform is derived in a discrete way by computing the wavelet coefficients of the signal on each node of a frame superimposed to it. Let us consider a plane of data points over which a network is superimposed. The wavelet coefficient for the node $\mathbf{b}_i$ is obtained by substituting in Equation (1) $s(x)$ by the set of Dirac functions $\delta(\mathbf{x} - \boldsymbol{\eta}_j)$:

$$C(\sigma, \mathbf{b}_i) = \sum_{j=1}^{N} \psi\Big(\frac{\boldsymbol{\eta}_j - \mathbf{b}_i}{\sigma}\Big) \tag{3}$$

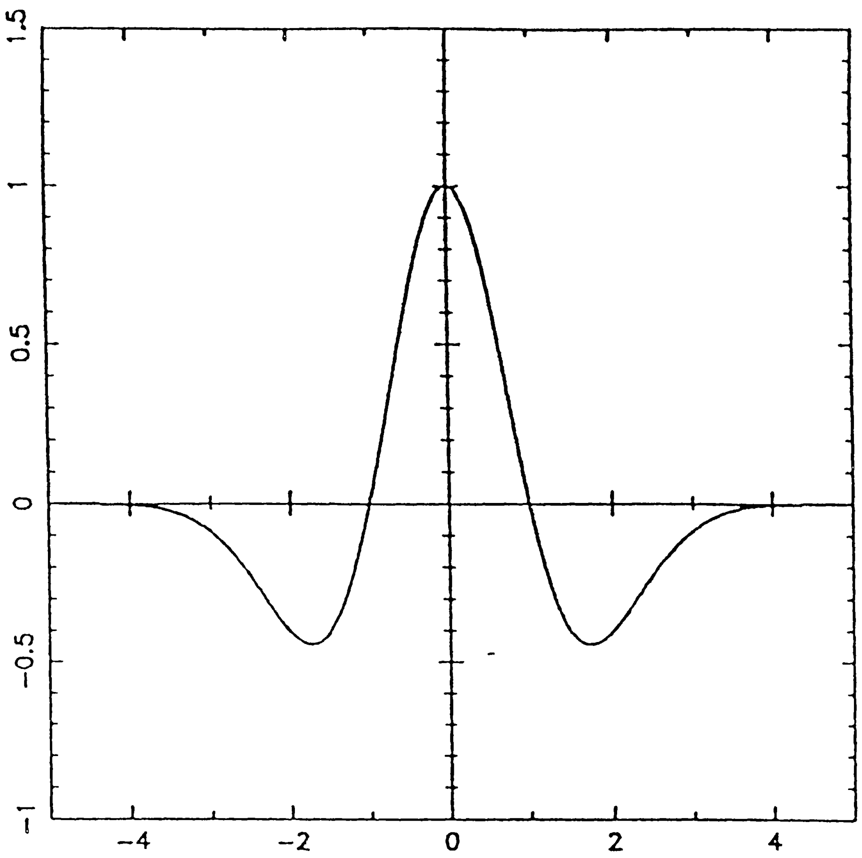

Fig. 2. Plot of the $1D$ mexican hat function. (Adapted from Bendjoya and Cellino, 1992).

where N is the number of data points. $C(\sigma, \mathbf{b}_i)$ is the wavelet coefficient at the node $\mathbf{b}_i$ for the scale σ. With the above choice of the mexican hat function as the analyzing wavelet, it follows that:

$$C(\sigma, \mathbf{b}_i) = \sum_{j=1}^{N} w_j$$

where

$$w_j = \left(2 - \frac{r_{ij}^2}{\sigma^2}\right) \exp\left(-\frac{r_{ij}^2}{2\sigma^2}\right) \tag{4}$$

r_{ij} being the distance between the i^{th} node $\mathbf{b}_i$ and the j^{th} point η_j. This distance is expressed on the basis of the chosen metric, which in the case of the Bendjoya *et al.* (1991) paper has been chosen to be identical to the metric used by Zappalà *et al.* (1990). It is easy to understand that a node centered on a uniformly populated area (see Figure 3, cases a and b) will have a wavelet coefficient close to zero, since the positive weights will be compensated by the negative ones, provided that the size of this area is big compared to the studied scale. On the opposite, for a node centered just on a structure (or on a hole) at a given scale, the wavelet coefficient

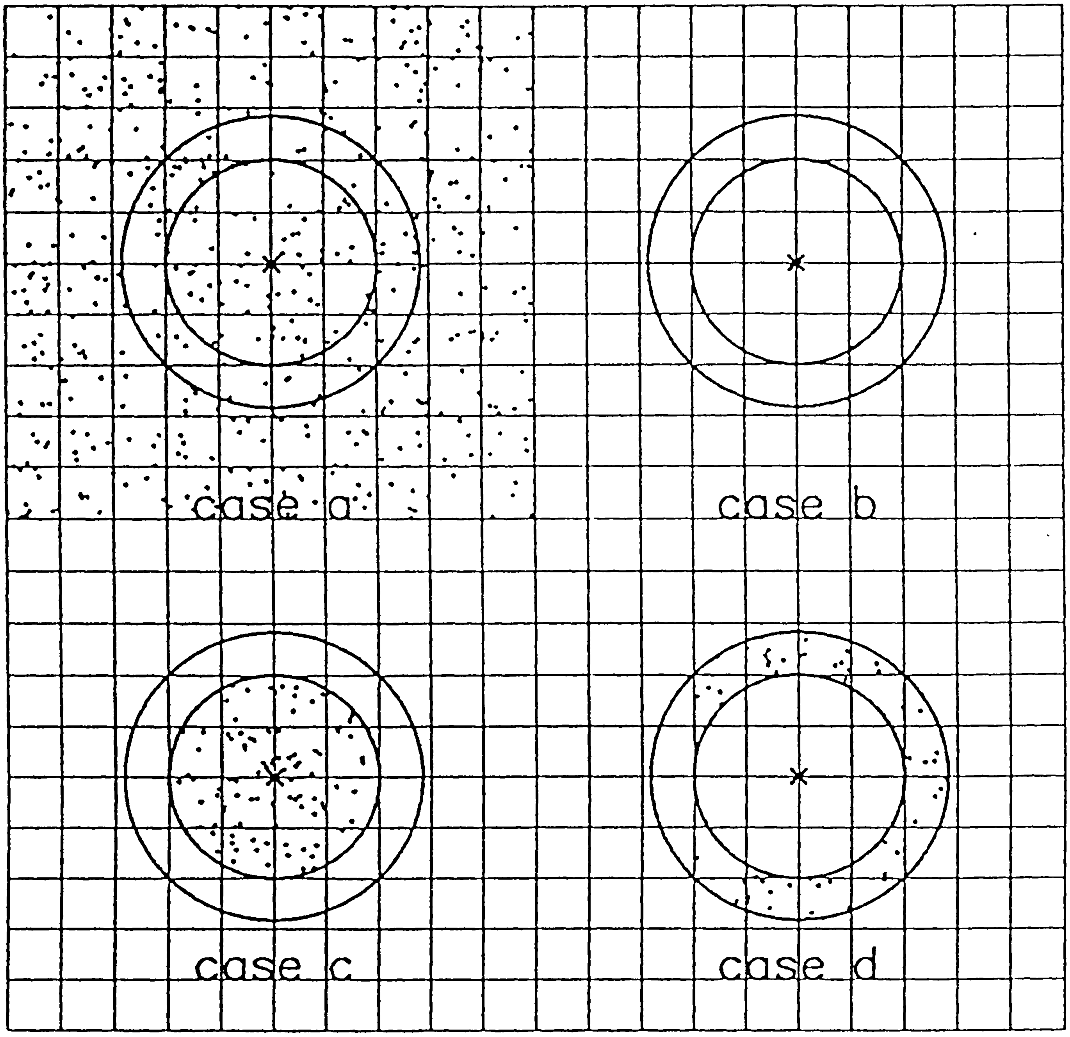

Fig. 3. Structure detection in the framework of Wavelet Cluster Analysis. (Adapted from Bendjoya and Cellino, 1992).

(see Figure 3, cases c and d) will be strongly positive (or negative). In this way, a map of wavelet coefficients can be built, at different scales, in which all the positive coefficients are indicators of the presence of structures. Also in this case, there is the problem of identifying which clusterings cannot be due to chance. The solution is similar to that found in the case of the HCM: a quasi-random population is created, the wavelet analysis is applied to this fictitious sample, and a threshold is found for the wavelet coefficients, which gives the level of confidence for which the structures having larger coefficients can be accepted as "families". Finally, it has to be noticed the importance of the adopted scale for the family definition. As a matter of fact, at scales sufficiently large different structures merge together, while they disappear at scales too small (as en effect of "overzooming"). Thus, families are defined at the largest scale for which they do not grow appreciably, whereas they grow drastically at a scale slightly larger. In other words, the chosen scale is the largest for which a well-defined structure still exists, before merging with other

TABLE II

The families found by Zappalà *et al.* (1990) (HCM) and by Bendjoya *et al.* (1991) (WAM), and percentage of the common objects (with respect to the number of members found for each WAM family). For a definition of different zones of the asteroid belt, see also Table IV

Zone of the belt	HCM family	WAM family	Least-numbered asteroid		Intersection (%)
2	21a	–	525	Adelaide	–
2	21b	–	883	Matterania	–
2	22	–	763	Cupido	–
2	23	–	1047	Geisha	–
2	–	21	281	Lucretia	–
2	–	22	422	Berolina	–
2	–	23	700	Auravictrix	–
2	–	24	963	Iduberga	–
3	31	31a	4	Vesta	71
3	32	32	650	Amalasuntha	75
3	33	35	878	Mildred	85
3	34	33	1378	Leonce	100
3	35	31b	1933	Tinchen	50
3	–	34	142	Polana	–
4	41a	41a	15	Eunomia	96
4	42	43	110	Lydia	100
4	43	–	141	Lumen	–
4	44	41b	145	Adeona	100
4	45	44	170	Maria	90
4	46	45	668	Dora	100
4	47	46	847	Agnia	86
4	48	42	1272	Gefion	100
5	51	51	158	Koronis	76
6	61	61	221	Eos	85
7	71	71	24	Themis	51
7	72	–	137	Moelibea	–
7	73	72	490	Veritas	86

structures of the same region at larger scales.

As was mentioned at the beginning of this Section, the fact that both HCM and WAM have been applied to the same data set of asteroid proper elements, and also using the same definition of a metric in this space, allows us to perform a comparison particularly fruitful between their respective results. Since these two methods are mutually independent, it is clear that *a priori* a noticeable agreement between the results should be considered as a serious clue of the reality of the proposed families. In this sense, Table II shows that the situation appears to be particularly satisfactory. With the exception of the Flora region, the agreement is noticeably good for what concerns both the number of resulting families, and the identities of their members. In addition to the well known Eos, Themis and Koronis families, a dozen of other families are found by the two methods, and should be considered as highly reliable. Among them, we have to notice the existence of a small family having as the largest body the asteroid 4 Vesta. This large body is known to exhibit a peculiar spectrum dominated by the bands of basaltic material at 1 and 2 μm, a fact that makes Vesta also a promising possible parent body for a peculiar class of meteorites: the eucrites (see also Section 5). On the other hand, the apparent disagreement of the results in the zone of Flora is not surprising, due to both the particularly crowded background present in this zone, and the relatively high uncertainties in the proper elements of the asteroids belonging to this zone of the belt (see also Section 5).

2.2. SAME IDENTIFICATION METHOD APPLIED TO DIFFERENT PROPER ELEMENTS

Another interesting possibility to check the families reliability and to better understand what is the origin of the observed discrepancies among different identification attempts could be to apply a given identification method to data bases whose proper elements are computed by means of different procedures. In this respect, a study has been performed by Zappalà *et al.* (1992). These authors have selected the most recent data bases of asteroid proper elements, i.e. those of Williams (1989) and of Milani and Kneževič (1991, version 5.7), both based on high-order perturbation theories. Unfortunately they contain different samples of asteroids; the former refers to the first 2065 numbered objects with the addition of about 1200 PLS asteroids, the second contains 4258 numbered objects. In spite of the obvious problems deriving from the use of different samples, it can be interesting to take profit of this "disadvantage" by looking at the number of members belonging to the common families both in the region of their intersection (till asteroid 2065) and in the two other regions (PLS and high-numbered asteroids). The reliability of a given family can be improved if it is found to contain, in addition to a number of objects belonging to the intersection region of the two data sets, also a considerable amount of objects coming from both of the other regions.

The adopted families identification method has been the HCM (Zappalà *et al.*, 1990). We recall that by means of his analysis Williams (1989) found 117 families, while Zappalà *et al.* (1990) recognized 21 families only.

According with the Zappalà *et al.* (1992) analysis, this large discrepancy is mainly due to the method used for family identification rather than to the method of proper elements computation. In fact, considering groupings having at least 5 members, they have found 26 families using the Williams' data set against 20 families recognized when the elements of Milani and Kneževič have been considered;

TABLE III

A comparison between the families found by applying the Hierarchical Clustering Method to both the asteroid proper elements data set of Williams and the MK57 data set of Milani and Kneževič. An asterisk (*) indicates families not found in the Zappalà *et al.* (1990) paper, due to the smaller data set used

Family list Zappalà *et al.* (1990)]	Williams		MK 57		Intersection	remarks
	Num.	PLS	Num	H.Num.		
31 + 35	4	5	5	12	4	Vesta
32	5	3	9	32	5	F-type
33	4	29	2	21	2	
41	17	9	25	77	16	Eunomia
44	8	7	8	12	7	Adeona
45	14	2	17	21	13	Maria
46	7	5	6	14	6	
48	5	5	3	20	3	
51	54	29	54	101	54	Koronis
61	89	9	80	127	79	Eos
71	75	38	72	165	71	Themis
1183 (*)	3	10	2	12	1	
135 (*)	1	18	1	16	1	Hertha
808 (*)	4	7	3	4	3	

the "intersection clusterings" were found to be 16. The agreement was highly improved when only families containing at least 10 members were taken into account: from Williams' data set they obtained 15 clusterings, 14 of which represent also the complete sample recognized using the Milani and Kneževič data set. Table III shows in a more detailed way the results of the latter analysis. The columns list the family identification number (that adopted by Zappalà *et al.*, 1990 or, in the last three cases, the number of the least-numbered asteroid of the grouping), the members belonging to the Williams numbered and PLS objects, the members belonging to the Milani and Kneževič numbered asteroids (up to 2065 and beyond 2065), and the common objects present in both classifications. As already noticed the agreement is quite satisfactory, showing that the results are almost independent on the methods used for proper elements determination. In our opinion this result is important and encouraging for future studies on families, since it indicates that the most critical point is the choice of an objective statistical method for clustering identification. Moreover, that analysis shows that, even in the case of smaller permitted clusterings, less than 30 families can be identified using the Williams' data set. This small number has to be compared with the much larger amount of families found by Williams (117) using the same data set. The discrepancy can be easily explained if we admit that the Williams' criterion of family identification was too liberal or, alternatively, that the Zappalà *et al.* method was too conservative. However, the latter possibility seems in our opinion questionable, due to the reliability tests performed and to the previously discussed agreement with the results of Bendjoya *et al.* (1991). On the other hand, Williams used a quite subjective method, mainly based on a "visual" definition of clusterings. We suggest that he assigned an acceptable reliability also to groupings whose statistical significance was poor, often of the same level of the background groupings due to chance only. This argument can be understood better by looking at a typical *stalactite diagram* used by Zappalà *et al.* (1990) for their family representation. That of Figure 1 refers to their Zone 4 of the Main Belt, which is limited by the 1/3 and the 2/5 mean motion resonances with Jupiter. As already explained, the stalactite diagram shows, for different levels of the *distance*, the resulting clusterings. The width of a stalactite at each level is given by the number of objects belonging to the clustering at that distance level. The 160 m $\sec^{-1}$ level is the *quasi-random level*, i.e. the distance level under which no clusterings are found in the case of fictitious populations of quasi-randomly generated objects. It is easy to see that there are basically three kinds of stalactites. The so called type II is sharp and generally deep, and corresponds to families which are well defined and are found to have a large intersection of common objects both in the Williams and the Milani and Kneževič data bases. Type III is less sharp and its members grow gradually as long as the distance level increases; it follows that the level at which one "cuts" the stalactite in order to define the family membership is very crucial: a statistically significant grouping surely exists, but it is very hard to decide which are its real members. Numerical simulations performed by Bendjoya *et al.* (1992)(see the next Section for a more detailed presentation of the results) show that in such cases one has to make a choice between two possibilities: (a) to cut the stalactite at a low level, losing probably a significant amount of family members; (b) to be more liberal, cutting the stalactite at higher levels, taking into account that in this way a large percentage of interlopers can be included. This choice will probably remain a "philosophical" one, as long as observations can solve the compositional "puzzle" of the original parent body, allowing to eliminate objects whose physical and chemical features cannot be genetically related to the parent body. Finally, type IV stalactites are those that do not reach at present - or marginally reach - the rank of possible families on the basis of the Zappalà *et al.* (1990) statistical criterion. Most of them have been considered as real families by Williams (1989), and are therefore the main responsible for the large discrepancy found comparing Williams and Zappalà *et al.* classifications. Later in the text we will present a suggested new terminology for naming asteroid groupings identified in different ways (Farinella *et al.*, 1992), mainly based on the points addressed before.

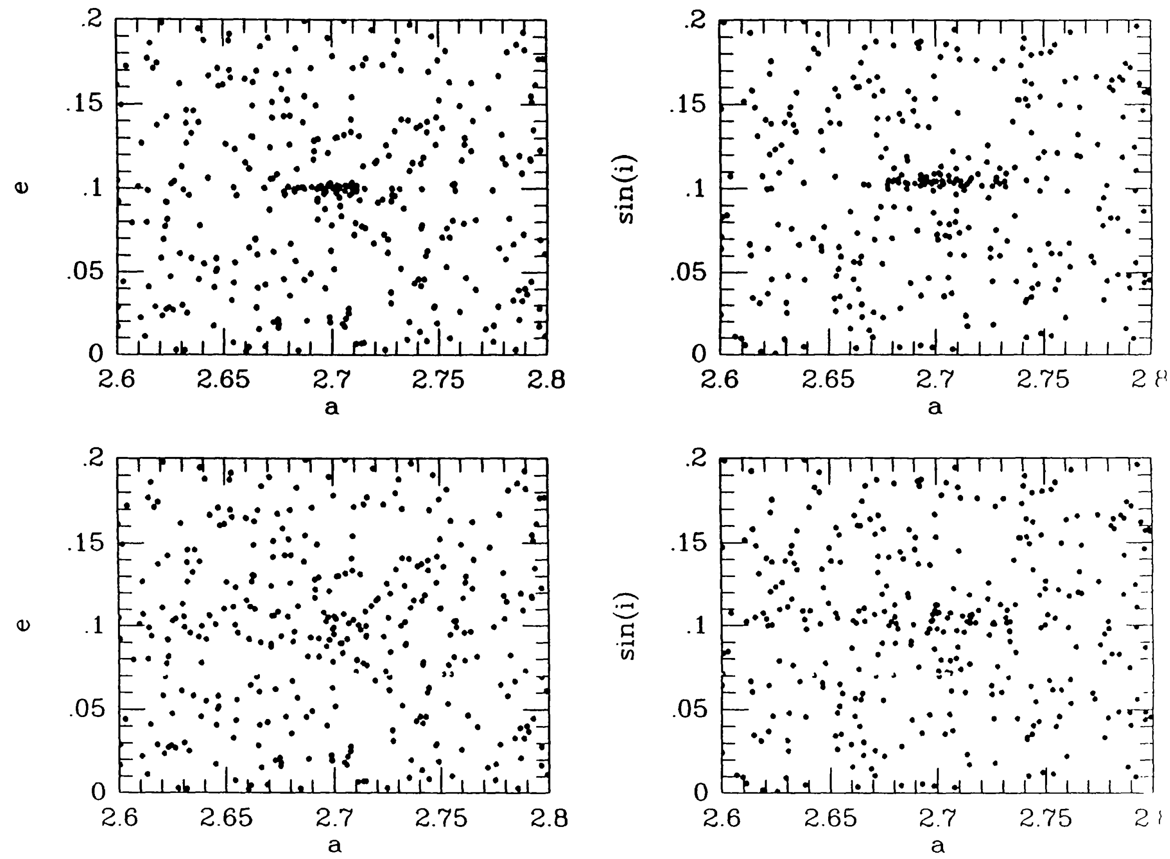

Fig. 4a. Projections on the a–e and a–$\sin i$ planes of the simulated family with a background of 300 objects, and with $V_{\min} = 50$ m sec^{-1} (top), and $V_{\min} = 150$ m sec^{-1} (bottom).

TABLE IV

Number densities for the different zones of the asteroid belt and for the three cases analyzed in the simulations

	Density
Zone 2 ($2.065 \leq a < 2.300$ AU)	0.16
Zone 3 ($2.300 \leq a < 2.501$ AU)	0.08
Zone 4 ($2.501 \leq a < 2.825$ AU)	0.04
Zone 5 ($2.825 \leq a < 2.958$ AU)	0.03
Zone 6 ($2.958 \leq a < 3.030$ AU)	0.05
Zone 7 ($3.030 \leq a < 3.278$ AU)	0.04
Simulations Background 300	0.04
Simulations Background 600	0.08
Simulations Background 900	0.12

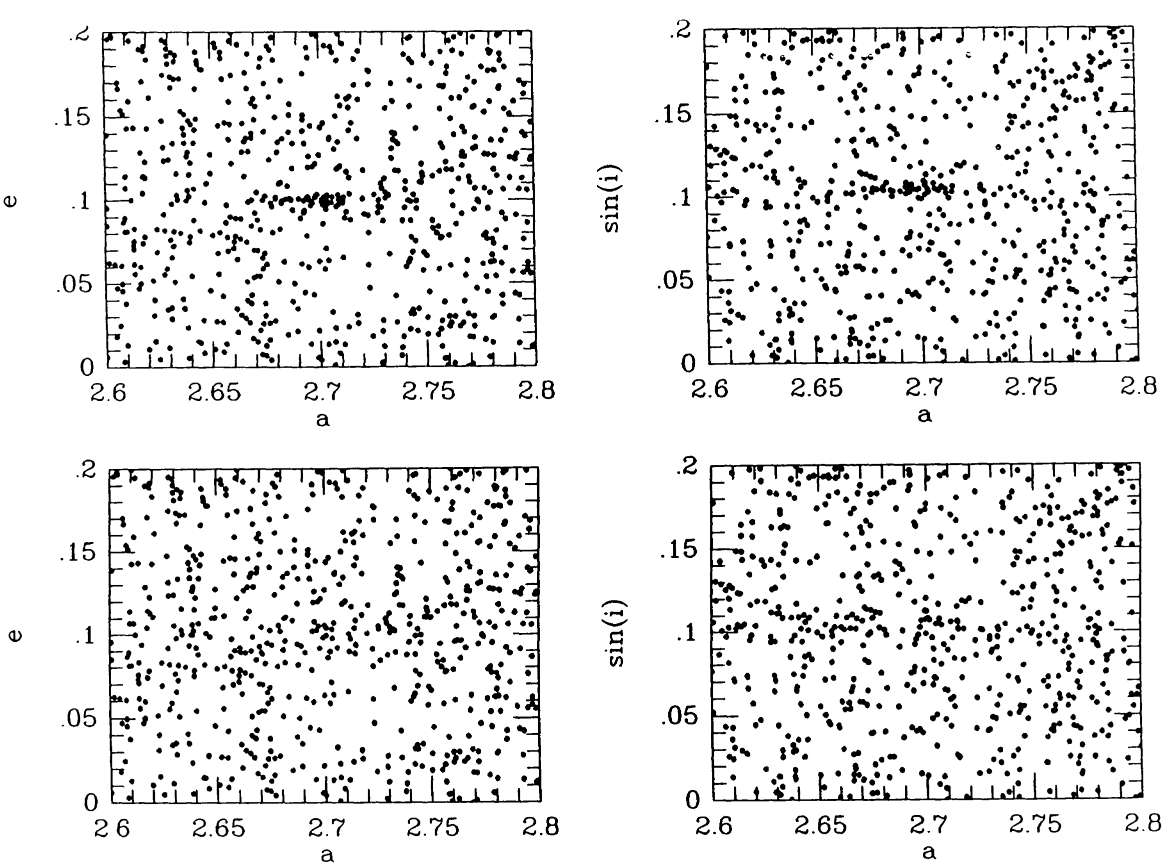

Fig. 4b. Projections on the a–e and a–$\sin i$ planes of the simulated family with a background of 600 objects, and with $V_{\min} = 50$ m sec^{-1} (top), and $V_{\min} = 150$ m sec^{-1} (bottom).

3. Simulated Families

In order to check the efficiency of the two most recent methods of families identification (Zappalà *et al.*, 1990, Bendjoya *et al.*, 1991), Bendjoya *et al.* (1992) have applied them to a family scenario which is *a priori* known. Admitting that asteroid families are generated in a collisional event, a sample of simulated families was artificially built, by assigning to each family member a typical ejection velocity according with a power-law distribution, justified by laboratory fragmentation experiments. The number dN of fragments having a mean ejection velocity between v and $v{+}dv$ is given by:

$$\begin{array}{ll} dN = 0 & \text{if } V < V_{\min} \\ dN = CV^{-\alpha} dV & \text{if } V > V_{\min} \end{array}$$

where $V_{\min}$ is the minimum ejection velocity and C is a constant. The exponent α has been taken equal to 3.25 in order to follow the results coming from laboratory experiments, (see, e.g., Fujiwara and Tsukamoto, 1980) while the diameter of the parent body has been chosen equal to 100 km. A family of 50 members has been simulated for $V_{\min}$ equal to 50, 100 and 150 m sec^{-1}, and then it has been plunged in a background of 300, 600 and 900 fictitious asteroids. 9 cases have been obtained, corresponding to different densities and to different dispersions

TABLE V
Comparison of the results of simulated families identification by means of Hierarchical Clustering Method (HCM) and Wavelet Analysis Method (WAM)

Background		$V_{\min}$ (m sec^{-1})					
		50		100		150	
		$N_{\text{Fam.}}$	$N_{\text{Int.}}$	$N_{\text{Fam.}}$	$N_{\text{Int.}}$	$N_{\text{Fam.}}$	$N_{\text{Int.}}$
300	HCM	50	2	46	1	45	5
	WAM	50	1	47	1	41	3
600	HCM	50	3	46	4	36	11
	WAM	49	4	46	4	40	9
900	HCM	50	2	46	2	29	4
	WAM	49	4	46	7	36	11

of the family members; Figures 4a and 4b show some of these cases in the *a*-*e* and in the *a*-sin*(i)* planes. Table IV gives the number densities for each case, which can be compared with the real number densities of the different zones of the asteroid belt (as defined by Zappalà *et al.*, 1990). It appears that, except for zone 2 which represents the so called "Flora region", most of the real families have to be compared with the simulations corresponding to backgrounds of 300 and 600 asteroids. The results are quite encouraging. In the case of a background of 300 objects the family was always recognized (with at least 41 members), for every ejection velocity and by both methods. The number of interlopers never exceeded 5 objects. Similar results have been also obtained when a background of 600 asteroids was considered. However, in this case the number of interlopers increased till 11 for the highest ejection velocity (150 m sec^{-1}). The cases related to a background of 900 objects show an increase of this trend (however, at least 29 family members have been detected by both methods), pointing out the problems connected with the number of interlopers already evidenced for type III groupings. In particular, in a simulation obtained using a very high minimum ejection velocity (200 m sec^{-1}) and a background of 900 asteroids, the method of Zappalà *et al.* (1990) recognized only 8 asteroids to be part of the family, with the addition of only 2 interlopers, while the method of Bendjoya *et al.* (1991) was able to identify 29 family objects but with the addition of 27 interlopers. On the other hand, we have to notice that the quite "conservative" result obtained by applying the method of Zappalà *et al.* (1990) can be in principle improved, in the sense of being able to recognize a much larger family, if one decides to scale with the density of the background the critical distance level for cutting the stalactites; further studies on this subject are in progress, even if they cannot really overcome the "interlopers" problem connected with type III clusterings. Table V presents in details the results obtained by Bendjoya *et al.* (1992), evidencing for each case the number of "real" family members as well as the number of the interlopers.

That work permits to believe *a posteriori* in the reliability of the previous analyses performed by Zappalà *et al.* (1990) and Bendjoya *et al.* (1991) on real families, since, as already noticed, the number densities of the zones of the real asteroid belt are mostly comparable with those of the easier cases of the simulations (50 objects in a background of 300 asteroids). For the most difficult cases (such as for the Flora region, where, on the other hand, the two classifications have shown a very poor agreement) the study permits to point out the need for making a distinction between different kind of clusterings and for having a new nomenclature.

4. A New Nomenclature for Asteroid Family Identification

From the results outlined in Sections 2 and 3, it seems that confusion and ambiguities about family classification arise mainly as a consequence of using the single word "family" for naming asteroid groupings identified in different ways and subjected to different significance tests. This is particularly true for groupings identified "by eye"(i.e., through visual inspection of the asteroid distribution) when they are compared with groupings found by almost automated clustering algorithms. Farinella *et al.* (1992) suggested a "family scheme" by recognizing four types of groupings in proper elements space, distinguished both by the techniques used for their identification and by the adopted tests of statistical significance. Figure 5 reports this scheme. Farinella *et al.* (1992) call the type IV families, introduced in Section 2, **clumps**, and they can be defined as groupings recognized by visual inspection but not subjected to rigorous statistical tests. Groupings which have shown to be statistically significant are divided in two categories: **clusters** (or type II families) and **clans** (or type III families). Clusters are groupings for which an unequivocal membership definition is possible through a clearcut separation from the "random" background and from other groupings; clans are statistically significant also, but lie within a background so dense and/or are so close to each other that they cannot be separated in a clearcut manner. Obviously, new data could in the future allow clumps to become clusters or clans, as well as clans can become clusters or vice versa. Finally, Farinella *et al.* (1992) proposed to call **families** only those clusters or clans for which some physical evidence indicates that their members have a genetic relationship (and not just a dynamical one, like the asteroids of the Phocaea region). This evidence can be based on collisional physics (e.g., mass and velocity distributions should be in agreement with the results of laboratory experiments or at least physically reasonable); or it could be based on taxonomy, from which a given family shows to be cosmochemically plausible.

5. Open Problems and Future Work

In the framework outlined before, it is hard to insert the grouping(s) found in the region of Flora (zone 2 in Zappalà *et al.*, 1990). If we accept the idea that the region

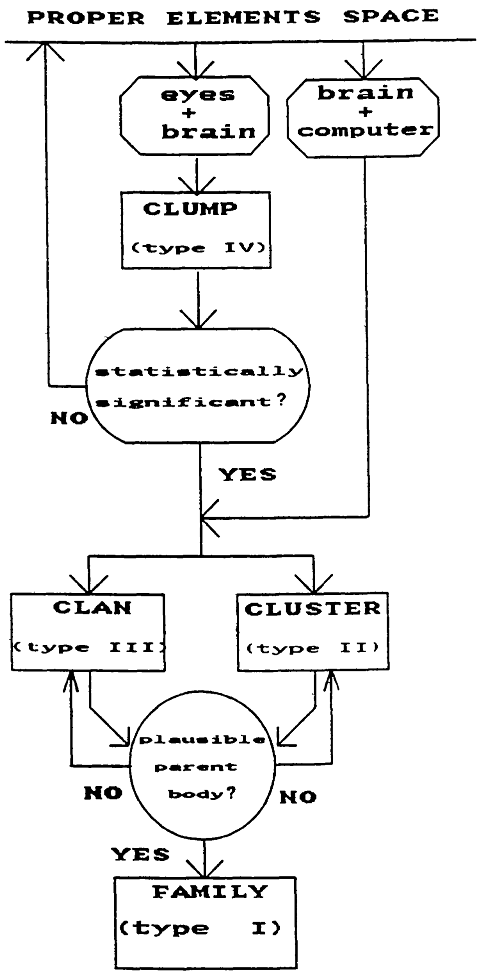

Fig. 5. Scheme of the proposed new nomenclature for asteroid groupings. (Adapted from Farinella *et al.*, 1992).

of Flora is actually a region very highly populated with respect to the other regions of the belt (after having taken into account the obvious problems of observational selection effects) and therefore dominated by a very dense background, it follows that we have to admit only the presence of several clumps, but not that of clusters, clans or families. On the other hand, if we believe that the background cannot be very different from that of the adjacent zone 3, a very large clan can be recognized. The large dispersion of its members can be possibly due to the presence of strong secular and mean-motion resonances. Further studies should be performed on this problem, mainly aimed to improve the stability of the proper elements, which, even using high-order perturbation theories, seems to be not yet completely satisfactory.

A support to the hypothesis of a big clan is given by the analysis of the size distribution of the asteroid belt. Cellino *et al.* (1991) have shown, through an analysis of the IRAS data, that families have a slope of the power-law describing their size distribution much steeper than that of field asteroids. The same feature is present when all the asteroids belonging to the zone of Flora are taken into account.

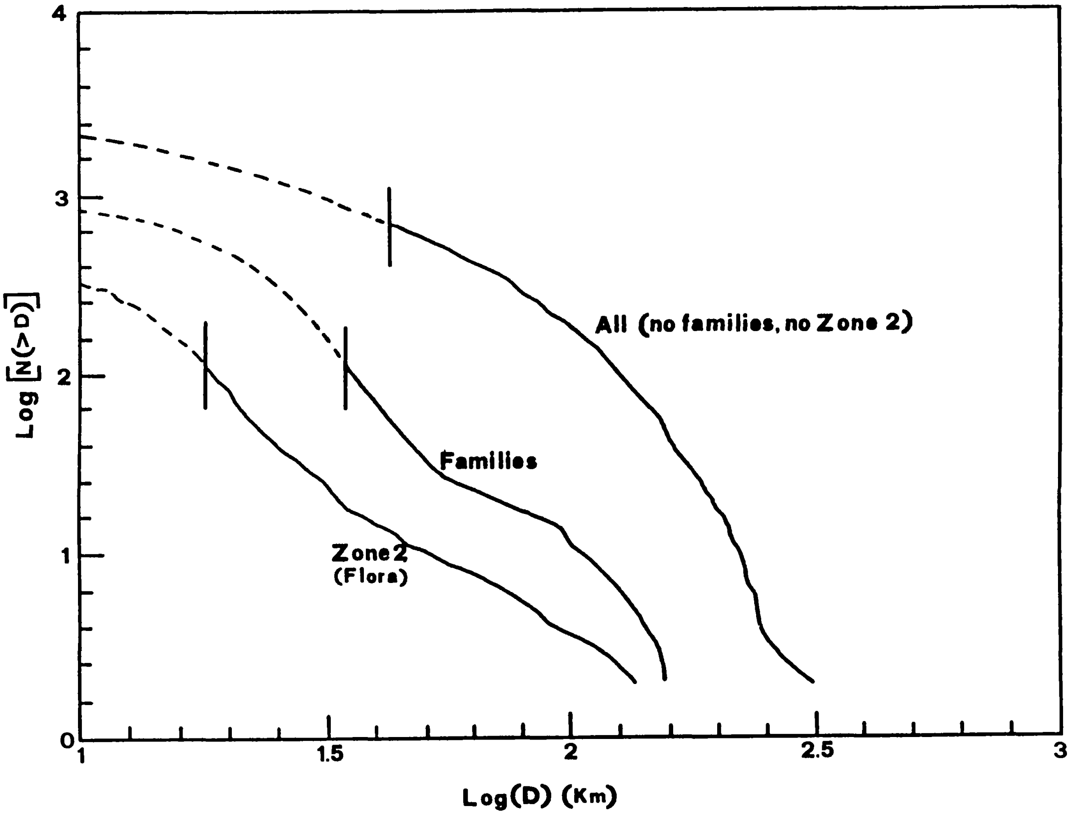

Fig. 6. The cumulative size distributions of family asteroids, Flora region objects, and asteroids not belonging to either of the above samples.

Figure 6 shows the distributions of non-family, family and zone 2 asteroids. One can conclude that the similarity of the distributions of family and Flora region asteroids plays in favor of the idea of a single very large clan in zone 2. However, it remains still to investigate why the families show such a different size distribution. Is this fact due to the relatively young age of the families, i.e. to the fact that families still represent the outcome of a single impact and not a fully relaxed collisional population ? Simulations performed for testing the dispersion of family members due to their subsequent collisional evolution can partially solve this problem (Davis, private communication).

Another interesting question about families is the following: could the impact event which has originated a family modify the taxonomic type of the fragments with respect to the parent body? The reply is surely "yes" if we consider a differentiated parent body, since the fragmentation should expose fragments coming from the mantle and from the core; on the other hand, the reply should be "no" for an homogeneous parent body. However, it is interesting to notice that two of the most reliable families, i.e., Koronis and Eos, show members whose taxonomic types are well defined and similar (and therefore consistent with an homogeneous parent body), but significantly different from the "normal" types of the zones of the belt they belong to. In particular, Eos family members have a very strange tax-

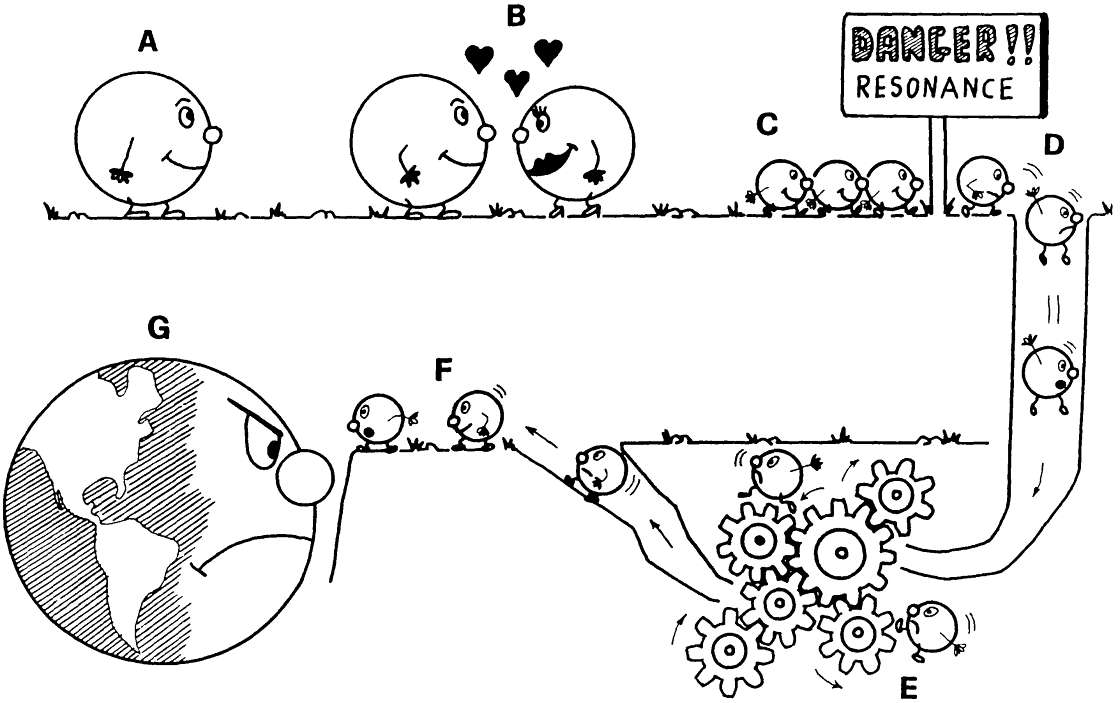

Fig. 7. The geological implications of the asteroidal families are described in this drawing. During the solar system lifetime a main belt asteroid (A) is expected to have a "catastrophic" encounter with another object (B). As a consequence of this collisional event, several fragments can be produced forming an asteroid family (C). However, if the catastrophic impact takes place not too far from some mean-motion or secular resonance, some fragments can be injected into them (D) and their orbital evolution can be controlled by a complex mechanism (E) able to increase the eccentricity well enough to transfer the fragments into the inner-planets region (F). It follows that highly energetic impacts with the Earth have to be considered as relatively frequent events (G).

onomic type, which is defined as K type in Tholen's classification (1984). Should we believe that the Eos parent body, the only one object of relatively large size to be disrupted in the Eos region (zone 6 in Zappalà *et al.* (1990)), was a "unique" asteroid in the belt, or will we think about a modification of a common taxonomic type (maybe S) after the catastrophic fragmentation which originated the family? Moreover, we have also to recall that the S-type associated to the objects of the zone of Flora seem significantly different from the S-type of the other regions (Dermott *et al.*, 1985). Again, one could connect this fact with the relatively young age of some families.

Finally, let us spend some words about the family of Vesta. Very recent observations of some asteroids belonging to the Vesta's family defined by Zappalà *et al.* (1990) have shown spectra which allow to classify these objects as belonging to a taxonomic type which seems intermediate between S and V (Tholen, private communication), and R and V (Binzel, private communication), and to call them "vestoids". Can the Vesta's family be the source of the Earth Approaching Asteroids of class V and, more generally, the source of the eucrite meteorites ? Preliminary simulations performed by Rosa *et al.* (1992) seem to show that a not negligible percentage of fragments coming from a crater-like impact on Vesta could reach the 3/1 resonance with Jupiter, known to represent one of the most efficient sources of injection of objects in the inner planets region (Wisdom, 1985).

As a final remark let us say that the introduction of automated clustering algorithms for families identification seems finally to have solved most of the ambiguities and confusions related to this old but fundamental problem. This should permit to start further and more detailed physical studies of the families in the general framework of the asteroid collisional evolution.

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