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ON THE PHOTON DIFFUSION TIME SCALE FOR THE SUN

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ABSTRACT

It is pointed out that the common choice of 0.5–1.0 cm for the average step length of a photon diffusing through the Sun gives a diffusion time scale which is too short by an order of magnitude. Simple random walk theory with step lengths determined from a solar model gives the value of 0.090 cm for the average step length, which gives 1.7×10^5 yr for the diffision time scale of the present Sun.

Subject headings: diffusion — radiative transfer — Sun: interior

1. INTRODUCTION

An appreciation of the magnitude of stellar time scales is necessary for the understanding of stellar structure. The diffusion time scale of a star, the time it takes a photon to diffuse through a star, is helpful to appreciate the equation of radiative transfer. It is estimated (Shu 1982) by calculating the time of a random walk from the center to the surface with the step length given by an average mean free path of the photon. The value of the average step length for the present Sun, typically taken to be $l = 0.5$–1.0 cm (Shu 1982; Bahcall 1989; Hill 1977; Harwitt 1988; Bowers & Deeming 1984; Zirin 1988), is too large and gives a diffusion time scale in the range of 3×10^3 to 3×10^4 yr, which is much too short. We use simple random walk theory to establish the correct average step length for a photon diffusing through a star and apply it to the present Sun.

2. DIFFUSION TIME SCALE

The mean free path of a diffusing photon is

$$l = \frac{1}{\kappa\rho} , \tag{1}$$

where κ is the opacity and ρ is the density. To show that the usual choice for the average value is inappropriate, we plot the local mean free path in the present Sun in Figure 1. The model for the present Sun is from Guenther et al. (1992) who provide the density and opacity for every shell of the solar model.

It is clear from Figure 1 that the common choice of 0.5–1.0 cm for the average mean free path for the present Sun is inappropriate. The local mean free path remains under 0.1 cm for almost 50% of the solar radius and does not exceed 0.4 cm except near to the photosphere. All of the authors listed above have chosen an average step length which is greater than the local step length in shells extending over the inner 99% of the solar radius. While the estimates of the diffusion time scale are only order of magnitude ones, the reader takes them to be, we believe, within a factor of 2 of the correct time scale. They are not. The choice of the mean free path should be guided by a quantitative knowledge of the solar model and an understanding of the physical process. For example, the choice should be biased toward the center, where the photon spends more time since the mean free paths there are shortest. We use simple random walk theory to obtain the average mean free path for a photon diffusing through the Sun.

The random walk with unequal step lengths has been considered by Chandrasekhar (1943, 1954). The mean square displacement in a random walk of N steps of unequal size l_j ($j = 1, 2, \ldots, N$) is given by

$$3\langle r^2 \rangle = N\langle l^2 \rangle , \tag{2}$$

where

$$\langle l^2 \rangle = \frac{1}{N} \sum_{j=1}^{N} l_j^2 . \tag{3}$$

In the stellar model the step lengths within a shell are all taken to be equal. Let the number of steps of equal length l_j in the shell j be n_j and let the total number of steps to cross the star be

$$N = \sum_{j=1}^{S} n_j , \tag{4}$$

where S is the number of shells. Then, the mean square step length is given by

$$\langle l^2 \rangle = \frac{1}{N} \sum_{j=1}^{S} n_j l_j^2 . \tag{5}$$

Let f_i be the fractional radius of the ith shell, $r_i = f_i R$, where R is the radius of the star. Then applying equations (2), (3), and (5) to shells i and $i + 1$, we have

$$3f_i^2 R^2 = \sum_{j=1}^{i} n_j l_j^2 , \tag{6}$$

and

$$3f_{i+1}^2 R^2 = \left[\sum_{j=1}^{i} (n_j l_j^2) + n_{i+1} l_{i+1}^2 \right] . \tag{7}$$

Subtraction gives the number of steps in shell $i + 1$,

$$n_{i+1} = 3 \frac{(f_{i+1}^2 - f_i^2)R^2}{l_{i+1}^2} . \tag{8}$$

Since the total time for a photon to diffuse through the star is the sum of the times spent in each shell,

$$t_d = \sum_{i=1}^{S} n_{i+1} \left(\frac{l_{i+1}}{c} \right) = 3R^2 \sum_{i=1}^{S} \frac{(f_{i+1}^2 - f_i^2)}{l_{i+1} c} . \tag{9}$$

When all the step lengths are equal, this formula reduces to the one quoted by Shu (1982). Hence the single, constant step length required to give the diffusion time is

$$\frac{1}{\bar{l}} = \sum_{i=1}^{S} \frac{(f_{i+1}^2 - f_i^2)}{l_{i+1}} . \tag{10}$$

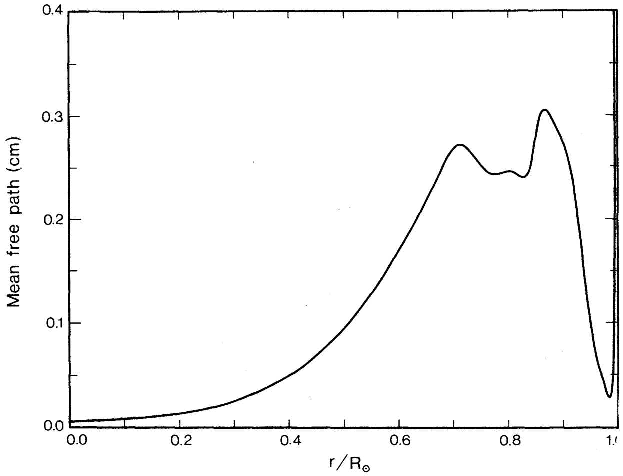

FIG. 1.—Local mean free path in present Sun vs. normalized radial distance for the Guenther et al. (1992) standard solar model.

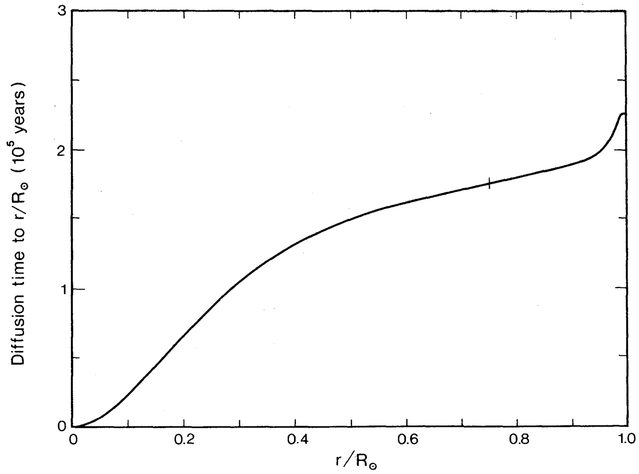

FIG. 2.—Diffusion time for a photon from center to radius r for the Guenther et al. (1992) standard solar model. The inner edge of the convection zone is marked with a slash.

3. DISCUSSION

The time for a photon to diffuse from the center of the Sun to radius r is plotted in Figure 2 as if the whole Sun were radiative. In the solar convective zone, however, the energy transfer is by convection. Since the crossing time of the convective zone by convecting elements is very much shorter than the time for the photon, starting from the center, to reach the inner edge of the convective zone by diffusion or to cross the zone by diffusion, the diffusion time scale of the Sun should be taken as the time for a diffusing photon to reach the inner edge of the solar convective zone. The average step length to reach the inner edge of the solar convective zone is $\bar{l} = 9.0 \times 10^{-2}$ cm, giving a diffusion time scale for the Sun of $t_d = 1.7 \times 10^5$ yr. The average velocity of the photon between the center and the inner edge of the convective zone is $v = 0.75 R_\odot / t_d = 0.97$ cm s^{-1}. The total number of steps taken by the diffusing photon, N, is found from equations (4) and (8) to be 1.0×10^{25}. The average step length of the photon equals the local mean free path at $r/R_\odot = 0.49$.

We conclude that the average step length of the diffusing photon and the diffusion time scale for the Sun are $\bar{l} = 9.0 \times 10^{-2}$ cm and $t_d = 1.7 \times 10^5$ yr.

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