THE ASTROPHYSICAL JOURNAL, 401:371–377, 1992 December 10

FORCED RECONNECTION, CURRENT SHEETS, AND CORONAL HEATING

XIAOGANG WANG AND A. BHATTACHARJEE
Department of Applied Physics, 500 West 120th Street, Columbia University, New York, NY 10027
Received 1991 October 17; accepted 1992 June 16

ABSTRACT

The formation of current sheets in the solar corona is investigated by a simple model in which forced reconnection occurs due to the perturbation caused at the photospheric boundary by footpoint motion. The time dependence of the process is considered by means of an initial-value calculation. It is found that on the Alfvénic time scale, current sheets tend to develop with an amplitude that increases linearly with time. The effect of resistivity becomes important subsequently, and the reconnected flux at the separatrix increases quadratically with time. In the nonlinear phase, helicity-conserving islands support current sheets, and the rate of reconnection is given by a modified Sweet-Parker model. Implications for coronal heating are discussed.

Subject headings: MHD — Sun: magnetic fields — Sun: corona

1. INTRODUCTION

The problem of coronal heating is of central importance in solar physics. One of the most effective mechanisms for heating the solar corona is by the formation of current sheets. This mechanism has been proposed by Parker (1972) and has stimulated much research and some controversy. (See the recent reviews by Low 1990 and Browning 1991 for discussions of some of the relevant issues and extensive bibliographies.)

In a recent paper, we have considered the effect of a small but finite resistivity on the formation of current sheets in the corona (Bhattacharjee & Wang 1991). Of course, in the presence of resistivity the current density is not infinite since resistivity resolves the singularity in a boundary layer. We have given equilibrium solutions which are the "exterior" solutions of a boundary-layer problem and have shown that these solutions exhibit a tangential discontinuity (current sheet). The current sheet is formed as the result of the dynamical evolution of an initially smooth equilibrium to a neighboring singular equilibrium. We claim that current sheets must form on the separatrices of magnetic islands if the resistive dynamics governing this dynamical evolution is helicity-conserving. This claim is based on the work of Rosenbluth, Dagazian, & Rutherford (1973) and Waelbroeck (1989) who have considered the nonlinear evolution of kink-tearing modes in toroidal plasmas. In the corona, the motion of footpoints on the photospheric boundaries can deform a uniform magnetic field into a resistively unstable equilibrium. Now, it is known that linear resistive instabilities have somewhat different properties in coronal plasmas than they do in toroidal plasmas because the field lines are line-tied in the corona (Mok & van Hoven 1983; Otani & Strauss 1988; Strauss & Otani 1988). Despite these differences, we have shown that current sheets must form in coronal plasmas as they do in toroidal plasmas if the dynamics are helicity-conserving.

The analysis given in our recent paper is quasi-thermodynamical in that we have considered the relaxation of the coronal plasma from an equilibrium of high magnetic energy to a neighboring equilibrium of lower magnetic energy, keeping helicity fixed. The word "equilibrium" has been the subject of some controversy in the solar physics literature and calls for an explanation. We note that reconnection occurs in a narrow boundary layer (the "inner" region) in which inertial effects are indeed important. Away from the boundary layer, in the "exterior" region, the plasma obeys the equations of magnetostatics. Strictly speaking, the equilibrium solutions computed in our recent paper describe the structure of the solutions only in the "exterior" region. We emphasize the "exterior" region because that is what determines primarily the magnetic energy accessible for reconnection, and the inertial effects in the boundary layer are, in fact, caused by the basic tendency of the plasma to relax to a neighboring equilibrium of lower energy.

Magnetic reconnection can be either "free" or "forced." "Free" reconnection is caused by the spontaneous occurrence of an instability. "Forced" reconnection, on the other hand, is driven by changing the boundary conditions of a stable equilibrium. Since both free and forced reconnection processes which conserve helicity will produce current sheets (Waelbroeck 1989), the distinction between these processes should not be overemphasized. This is especially so for the solar corona in which the driving mechanism for both types of processes is the twisting motion of footpoints on the boundary. As explained in § 2 of this paper, which one of these processes actually dominates depends on the magnitude and the time dependence of the twist.

In our previous paper, we have considered the evolution of the coronal plasma from a smooth equilibrium to a neighboring singular equilibrium, but we did not deal explicitly with the time dependence of the process. In the present paper, we consider the time dependence of "forced" reconnection in a somewhat simplified, analytically tractable variant of Parker's model. We solve an initial-value problem similar to that of Hahm & Kulsrud (1985, hereafter HK) who considered a model problem suggested by Taylor (unpublished). Despite the differences between Taylor's model and Parker's model of the corona, the qualitative features of the two models have some similarities. In particular, we show that the coronal plasma evolves in phases: an ideal phase in which a current sheet tends to form with an amplitude that increases linearly with time t, and a width that shrinks as t^{-1}. Thus the current sheet is not a finite-time singularity. Resistivity intervenes before the singularity can be realized, and the ideal phase is followed by a reconnection phase. During the linear reconnection phase, the reconnected flux at the separatrix increases approximately quadratically with time. Following this linear reconnection phase is a nonlinear phase during which reconnection occurs at a rate that scales with resistivity in the same manner as the reconnection rate in the Sweet-Parker model. This turns out to be the helicity-conserving nonlinear phase discussed by Waelbroeck (1989), but omitted by HK. Though the current sheet formed in this phase is not strictly a δ-function, it is sufficiently singular that it causes intense coronal heating. We give estimates of the heating and compare the predictions of our analysis with recent numerical simulations.

2. TIME SCALES

In Parker's model of the solar corona (Parker 1972), the initial state is a straight uniform magnetic field

$$\boldsymbol{B} = B_0 \hat{z} , \tag{1}$$

contained between two horizontal planes at $z = \pm L$. (We use rectangular coordinates $[x, y, z]$.) The configuration (1) is a vacuum field and cannot undergo reconnection. It is meaningful to speak of reconnection, either free or forced, only after a current density is generated by the motion of footpoints. We assume that the motion of the footpoints generates a smooth magnetic field of the form

$$\boldsymbol{B} = B_0 \hat{z} + \nabla\chi \times \hat{z} , \tag{2}$$

considered by Parker (1972) and subsequently by others (van Ballegooijen 1985; Zweibel & Li 1987; Strauss & Otani 1988; Bhattacharjee & Wang 1991). If the configuration (2) is resistively unstable, and the instability is helicity-conserving, then a current sheet will form at a separatrix of equation (2) where $\nabla\chi = 0$ (Waelbroeck 1989; Bhattacharjee & Wang 1991). It is possible, however, that the free energy available for reconnection may not always be large enough to cause a helicity-conserving tearing instability. A current sheet can still form in such a configuration by the mechanism of forced reconnection which we treat here. For simplicity, in order to decouple free and forced reconnection, the quasi-static equilibria considered here are taken to be stable to tearing modes.

We consider the effect of continuous twisting motion of the footpoints on the configuration (2) in a characteristic time scale $\tau_0 \sim a/v_0$, where a is a typical length scale transverse to $\hat{z}$, and v_0 is a typical twisting velocity. The magnetic field between the two planes responds to the twisting motion on the boundaries on the Alfvén time scale $\tau_A = v_A/L \equiv \omega_A^{-1}$, where $v_A = B_0/(4\pi\rho)^{1/2}$ is the Alfvén speed and ρ is the density of the plasma. Compared with τ_A, the time scale of resistive diffusion $\tau_R = 4\pi a^2/\eta c^2$ is very slow in the solar corona, and we assume that

$$\tau_A \ll \tau_0 \ll \tau_R . \tag{3}$$

The inequality $\tau_A \ll \tau_0$ enables us to consider magnetic fields approximately in static equilibrium. The presence of a small but finite resistivity causes reconnection on a time scale τ_K which scales typically as a fractional power of η (e.g., as $\eta^{1/2}$ in the Sweet-Parker model). We have, therefore,

$$\tau_A \ll \tau_K \ll \tau_R . \tag{4}$$

Since τ_0 and τ_K are both bounded by τ_A and τ_R, we should consider the relative magnitudes of τ_0 and τ_K. If $\tau_K \ll \tau_0$, the reconnection rate is faster than the twisting rate, and the reconnection is driven mostly by the free energy already stored in the equilibrium—this is an instance of "free" reconnection. On the other hand, if $\tau_K \gg \tau_0$, the reconnection is "forced." In what follows, the regime of interest is

$$\tau_A \ll \tau_0 \ll \tau_K \ll \tau_R . \tag{5}$$

Typical parameters for the solar corona are (see, for instance, Withbroe & Noyes 1977; and more recently, Browning 1991): $\tau_A \sim 1$–10 s, $\tau_0 \sim 10^2$–10^3 s, $\tau_R \sim 10^9$–10^{10} s, and $\tau_K \approx (\tau_R \tau_A)^{1/2} \sim 10^4$–$10^5$ s. Inspection of the inequality (5) shows that it does correspond to many situations of interest.

2. QUASI-STATIC EQUILIBRIA

The incompressible resistive MHD equations are (in cgs units)

$$\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla\right)\boldsymbol{v} = \frac{\boldsymbol{J} \times \boldsymbol{B}}{c} - \nabla p , \tag{6}$$

$$\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} = \eta \boldsymbol{J} , \tag{7}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} , \tag{8}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} , \tag{9}$$

$$\nabla \cdot \boldsymbol{B} = 0 , \tag{10}$$

$$\nabla \cdot \boldsymbol{v} = 0 , \tag{11}$$

where the symbols have their usual meanings.

For coronal loops, the length $L \sim 0.1$–5×10^{10} cm and the transverse dimension $a \sim 10^8$ cm (see, for instance, Browning 1991). Thus these loops are characterized by small values of the geometric parameter $\epsilon_0 \equiv a/L \ll 1$. Using ϵ_0 as an ordering parameter, it is possible to simplify considerably the resistive MHD equations. We consider force-free solutions with $\nabla p = 0$. In Parker's model, the total magnetic field can be written

$$\boldsymbol{B} = B_0 \hat{z} + \boldsymbol{b} , \tag{12}$$

where $|\boldsymbol{b}|/B_0 \sim \epsilon_0$. For twisting motions of the footpoints subject to the inequality (3), we have

$$\frac{v_0^2}{a^2} \sim \omega_0^2 \ll \omega_A^2 = \frac{v_A^2}{L^2} . \tag{13}$$

Hence, in the exterior region, the inertial term is much smaller than the Lorentz force term, and we can assume that the coronal plasma evolves through a sequence of quasi-static force-free equilibria,

$$\boldsymbol{J} = \alpha \boldsymbol{B} , \tag{14}$$

where α is a scalar function.

It is assumed that the photospheric footpoints at $z = \pm L$ are twisted with the velocity $\boldsymbol{v}_\pm = (\omega_0/K_0) \sin K_0 y \hat{x}$, where K_0 is a constant. This twisting motion generates the velocity field

$$\boldsymbol{v}_0 = \omega_0 \frac{\sin K_0 y}{K_0} \frac{z}{L} \hat{x} \tag{15}$$

in the coronal volume. Since $\omega_0 \tau_A \ll 1$, the coronal plasma responds quickly and generates the perturbed magnetic field $\boldsymbol{b}$, specified by Ohm's law,

$$\frac{\partial \boldsymbol{b}}{\partial t} - \nabla \times (\boldsymbol{v}_0 \times \boldsymbol{B}) = 0 . \quad (16)$$

With $\boldsymbol{v}_0$ given by equation (15), the configuration (2) is deformed with an equivalent helical wavenumber $k = k(t)$ along z, related to the frequency ω_0 by

$$\omega_0 = L\, dk/dt , \quad (17)$$

where $ka = 0(\epsilon_0)$. The magnetic field (2) changes to

$$\boldsymbol{B} \simeq B_0 \hat{z} + b_t \hat{x} + \nabla\psi \times \hat{z} , \quad (18)$$

where b_t and ψ obey the equations

$$\frac{\partial b_t}{\partial t} = \frac{\partial}{\partial z} v_0 B_0 , \quad (19)$$

and

$$\frac{\partial \psi}{\partial t} + \boldsymbol{v}_0 \cdot \nabla\psi = 0 , \quad (20)$$

with $\psi(\boldsymbol{r}, t = 0) = \chi(\boldsymbol{r})$. Integrating equations (19) and (20) with respect to time and assuming $k(0) = 0$, we get, respectively,

$$b_t = \frac{k}{K_0} B_0 \sin K_0 y , \quad (21)$$

and

$$\psi = \psi[x - x_*(t), y] , \quad (22)$$

where $x_*(t) = \int_0^t \boldsymbol{v}_0(t')dt' = z(\omega_0 t/K_0 L) \sin K_0 y$.

Equation (18) gives

$$\hat{z} \cdot (\nabla \times \boldsymbol{B}) \simeq -\nabla_\perp^2 \psi - kB_0 \cos K_0 y , \quad (23)$$

where $\nabla_\perp \simeq \hat{x}(\partial/\partial x') + \hat{y}(\partial/\partial y)$ with $x' \equiv x - x_*(t)$. From equations (14) and (23), we get

$$\frac{c}{4\pi}(-\nabla_\perp^2 \psi - kB_0 \cos K_0 y) \simeq \alpha B_0 , \quad (24)$$

where $\alpha = 0(\epsilon_0)$. Using the condition $\boldsymbol{B} \cdot \nabla\alpha = 0$ which gives

$$B_0 \frac{\partial \alpha}{\partial z} + \boldsymbol{b} \cdot \nabla_\perp \alpha \simeq 0 , \quad (25)$$

and the identity $\partial\psi/\partial z = -ky\, \partial\psi/\partial x$, we get

$$\nabla\psi \times \nabla\alpha \simeq 0 . \quad (26)$$

It follows from equation (26) that

$$\alpha = \alpha(\psi) , \quad (27)$$

if $\nabla\psi \neq 0$. If $\nabla\psi = 0$, where separatrices lie, current sheets can occur in violation of equation (27).

It is worth pointing out that considerable analytical simplification has been achieved here by an aspect-ratio (small ϵ_0) expansion. Without this small parameter, the problem of three-dimensional force-free fields (e.g. [14]) is not, in general, analytically tractable (except in the simple case α = constant). The question of the location of separatrices in a fully three-dimensional field is of great interest, but one that is not considered here. Here, for analytical simplicity, we consider a very long and thin coronal loop with a strong axial field which happens to be of considerable practical interest.

4. FORCED RECONNECTION IN PARKER'S MODEL

Since the twisting velocity (eq. [17]) is periodic along y, it is sufficient to consider reconnection in the domain $-a_0 \le y \le a_0 \equiv \pi/2K_0$. In the presence of even a small but finite amount of resistivity, islands will open up at the separatrix $y = 0$. We shall use a boundary-layer method which relies on the smallness of the island-width Δ (HK; Waelbroeck 1989). Near the separatrix, inertial and resistive effects are important. Away from the separatrix, where the island is not strongly perturbing, the plasma can be described by a linear quasi-static approximation. Considering only a single Fourier mode, we write

$$\psi(x, y) = \psi_0(y) + \tilde{\psi}(y) \exp(ik_0 x') , \quad (28)$$

where $k_0 x' = k_0(x - x_*) \equiv k_0 x - \omega t$,

$$\omega = \frac{2}{\pi} k_0 a_0 \omega_0 \frac{z}{L} \sin k_0 y , \quad (29)$$

and

$$\psi_0(y) \simeq \epsilon_0 B_0 y^2/2a_0 . \quad (30)$$

The problem separates into two regions: the inner region $|y| \sim \Delta \ll a_0$, and the outer region $\Delta \ll |y| \le a_0$. The outer region equation, obtained by linearizing equation (26), is

$$\tilde{\psi}'' - k_0^2 \tilde{\psi} = 0 . \quad (31)$$

Due to the motion of footpoints at the boundaries $z = \pm L$, the constant-ψ surfaces in the coronal volume are perturbed. With the footpoint displacement

$$\xi(x, y) = \xi(y) \exp\{i(k_0 x - \omega t)\} , \quad (32)$$

we have

$$\epsilon_0 B_0 y \xi(y) \simeq a_0 \tilde{\psi}(y) . \quad (33)$$

To be consistent with equation (15), we require $\xi(y) = -\xi(-y)$. If we now set $\xi(\pm a_0) = \pm \xi_0$, this model reduces to the model of forced reconnection proposed by Taylor (HK; Wang & Bhattacharjee 1992), with $\tilde{\psi}(\pm a_0) = \epsilon_0 B_0 \xi_0$. Equation (31) can then be solved following HK. The solution is

$$\tilde{\psi}(y) = \tilde{\psi}(0)\left[\cosh k_0 y \mp \frac{\sinh k_0 y}{\tanh k_0 a_0}\right] \pm \epsilon_0 B_0 \xi_0 \frac{\sinh k_0 y}{\sinh k_0 a_0} , \quad y \gtrless 0 . \quad (34)$$

Equation (34) describes two possible neighboring equilibria. If $\tilde{\psi}(0) = 0$, a current sheet occurs at $y = 0$, causing a finite jump in b_x given by

$$\Delta b_x = 2k_0 \tilde{\psi}(a_0) \frac{\cos k_0 x}{\sinh k_0 a_0} . \quad (35)$$

This equilibrium solution has the same topology as the initial unperturbed equilibrium. The other solution has $\tilde{\psi}(0) = \epsilon_0 B_0 \xi_0/\cosh k_0 a_0$ and is given by

$$\tilde{\psi}(y) = \epsilon_0 B_0 \xi_0 \frac{\cosh k_0 y}{\cosh k_0 a_0} . \quad (36)$$

This solution is smooth but has a topology different from that of the initial equilibrium in that it contains islands of width $(2a_0 \xi_0/\cosh k_0 a_0)^{1/2}$. In order to determine whether these equilibria are realized in practice, HK solve an initial-value problem. In the next section, we revisit their results.

5. THE INITIAL-VALUE PROBLEM

We imagine that the boundary perturbation ξ, specified by equation (32), is set up slowly in time in accordance with equation (5). The plasma evolves through a sequence of quasi-static equilibria everywhere except the region near the separatrix where inertia and resistivity are important. These dynamical features can be described on the fast Alfvénic time scale by the ideal MHD equations and on a longer time scale by the resistive MHD equations. For very long and thin coronal loops of negligible beta with a strong axial magnetic field, it is appropriate to use the reduced MHD equations (Strauss 1976; HK). Using equations (7)–(9), it can be shown that the flux function ψ obeys

$$\frac{\partial \psi}{\partial t} + \boldsymbol{v} \cdot \nabla \psi = \frac{\eta c^2}{4\pi} \nabla_\perp^2 \psi \,, \tag{37}$$

where the velocity $\boldsymbol{v}$ can be obtained from a stream function ϕ, i.e.,

$$\boldsymbol{v} = \hat{z} \times \nabla \phi \,. \tag{38}$$

Note that the incompressibility condition (11) is satisfied identically. The curl of the momentum equation (6) yields a dynamical equation for ϕ, given by

$$\frac{\partial}{\partial t} \nabla_\perp^2 \phi + \hat{z} \cdot \nabla_\perp \phi \times \nabla_\perp \nabla_\perp^2 \phi = \frac{1}{4\pi\rho} \hat{z} \cdot \nabla_\perp \nabla_\perp^2 \psi \times \nabla_\perp \psi \,. \tag{39}$$

The velocity $\boldsymbol{v}_0$ specified by equation (15) is derivable from the stream function

$$\phi_0 = \frac{\omega_0 z}{K_0^2 L} \cos K_0 y_0 \,. \tag{40}$$

5.1. *The Linear Phase*

Following HK, for short times, we can linearize equations (37)–(39). Assuming perturbations of the form (28) and $|k_0^{-1} \partial/\partial_y| \gg 1$, and using the identity

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_0 \cdot \nabla\right) \exp(ikx') = 0 \,, \tag{41}$$

we can Laplace-transform and nondimensionalize equations (37)–(39) to obtain (HK)

$$\frac{d^2\Psi}{d\theta^2} = \epsilon\Omega(4\Psi + \theta U) \,, \tag{42}$$

$$\frac{d^2 U}{d\theta^2} = \theta\left(\Psi + \frac{\theta}{4} U\right) , \tag{43}$$

where

$$\Psi = \frac{k_0}{\epsilon_0 B_0} \int_0^\infty dt \exp(-pt)\tilde{\psi}(y, t) \,, \tag{44}$$

$$U = \frac{-4\epsilon k_0^2}{p} \int_0^\infty dt \exp(-pt)\tilde{\phi}(y, t) \,, \tag{45}$$

and

$$\theta = \frac{y}{\epsilon a_0} \,, \quad \epsilon^4 = \frac{p\tau_A^2}{4(k_0 a_0)^2 \tau_R} \,, \quad \Omega = \frac{\epsilon \tau_R p}{4} \,. \tag{46}$$

HK solve equations (42) and (43) to get

$$\Psi(0) = \frac{k_0^2 \xi_0}{p \sinh k_0 a_0} \left\{\frac{k_0}{\tanh k_0 a_0} + \frac{\mu}{2^{2/3}\epsilon a_0} I(\mu)\right\}^{-1} , \tag{47}$$

where

$$I(\mu) \equiv \int_{-\infty}^{+\infty} d\theta_1 \bar{Z}(\theta_1, \mu) \,, \tag{48}$$

$$\bar{Z} \equiv [(2/\mu)\Psi(0)]Z, \quad \theta_1 \equiv \sqrt{\theta}/2, \quad \mu \equiv 8\epsilon\Omega \,, \tag{49}$$

$$Z \equiv \frac{d^2\Psi}{d\theta^2} = \frac{\mu}{2}\left(\Psi + \frac{\theta}{4} U\right) . \tag{50}$$

In the limit $t \ll \omega_0^{-1} < \tau_1 \equiv \tau_A^{2/3}\tau_R^{1/3}$, HK show that the inverse Laplace-transform of equation (47) gives

$$\tilde{\psi}(0) \simeq \frac{2}{\pi} \frac{k_0^2 a_0^2 \epsilon_0 B_0 \xi_0}{\sinh k_0 a_0} \left(\frac{t}{\tau_K}\right)^2 , \tag{51}$$

where $\tau_K = (\tau_R \tau_A)^{1/2}$ is the characteristic time scale in the Sweet-Parker model of magnetic reconnection (Sweet 1958; Parker 1957). In this limit, the current sheet amplitude at the separatrix is given by

$$\tilde{J}(0) \simeq \frac{c}{2\pi^2} \frac{k_0^2 \epsilon_0 B_0 \xi_0}{\sinh k_0 a_0} \left(\frac{t}{\tau_A}\right) , \tag{52}$$

with a width that shrinks as t^{-1}. Hence the plasma tends to form a current sheet in the linear phase on the ideal time scale τ_A. However, such a sheet is not a finite-time singularity because it takes an infinite time to form a sheet of zero thickness. Furthermore, as indicated by equation (51), the effect of resistivity intervenes well before $t = \tau_1$.

For $t \gg \tau_1$ but $t \sim \tau_2 \equiv \tau_R^{3/5}\tau_A^{2/5}$, HK calculate an extension of the linear phase using the so-called "constant-ψ" approximation. However, we show in the next section that for most cases of physical interest, nonlinear effects play a significant role for times much shorter than τ_2 (Waelbroeck 1989; Wang & Bhattacharjee 1992). This generally makes the "constant-ψ" linear phase academic.

5.2. *The Nonlinear Phase*

A necessary criterion for the validity of linear theory is that the island width w be much smaller than the reconnection layer width Δ. An upper bound for Δ is the current sheet width in the linear phase,

$$\Delta = \frac{2k_0 B_0 \epsilon_0 \xi_0}{\sinh k_0 a_0} \Big/ \frac{4\pi}{c} \tilde{J}(0) = \frac{\pi}{2k_0} \frac{\tau_A}{t} \,. \tag{53}$$

The island-width is given by

$$w = 2\left(\frac{a_0}{\epsilon_0 B_0}\right)^{1/2} \tilde{\psi}^{1/2}(0) = 2k_0 a_0 \left(\frac{2}{\pi} \frac{\xi_0 a_0}{\sinh k_0 a_0}\right)^{1/2} \frac{t}{\tau_k} \,. \tag{54}$$

Then the ratio

$$\frac{w}{\Delta} \doteq \frac{4}{\pi}\left(\frac{2}{\pi}\frac{\xi_0 a_0}{\sinh k_0 a_0}\right)^{1/2} k_0^2 a_0 \left(\frac{t}{\tau_c}\right)^2 , \quad (55)$$

where $\tau_c \equiv \tau_A^{3/4}\tau_R^{1/4} \ll \tau_1$, increases quadratically with time. For most cases of physical interest $k_0 a_0 \sim 1$, whence

$$\frac{w}{\Delta} \sim \left(\frac{\xi_0}{a_0}\right)^{1/2}\left(\frac{t}{\tau_c}\right)^2 . \quad (56)$$

To make estimates, we take $\xi_0/a_0 \sim 10^{-2}$. With $\tau_c \sim 10^2\tau_A < \omega_0^{-1} < \tau_1 \sim 10^3\tau_A$, we see that w can exceed Δ for $t \lesssim \tau_1$ ($\ll \tau_2$). Hence, nonlinear effects play an essential role for $t > \tau_1$. For $t \sim \tau_k$ ($\ll \tau_2$), the plasma enters the Sweet-Parker reconnection phase.

In our previous paper (Bhattacharjee & Wang 1991) we have demonstrated the formation of a current sheet on the time scale τ_k from the requirement of helicity conservation in the nonlinear regime. (Needless to say, helicity conservation holds in the linear regime as well, but it is the persistence of this conservation law in the nonlinear regime which distinguishes dynamics that produce a current sheet from dynamics that do not.) We shall not repeat the demonstration here; the integral equation that describes the exterior region has been discussed in detail elsewhere (Rosenbluth et al. 1973; Waelbroeck 1989; Bhattacharjee & Wang 1991). Instead, we consider here the interior region where resistivity is important. In the Appendix, we show, using a slightly modified form of the original Sweet-Parker (SP) theory (Sweet 1958; Parker 1957; Park, Monticello, & White 1984; Waelbroeck 1989; Wang & Bhattacharjee 1992), that

$$\tilde{\psi}_{SP}(0, t) \simeq \sqrt{2}\epsilon_0 B_0 a_0 \left(\frac{k_0 \xi_0}{\sinh k_0 a_0}\right)^{3/2} \frac{t}{\tau_k} . \quad (57)$$

The amplitude of the current sheet in this nonlinear phase is

$$\tilde{J}_{SP}(0) \simeq \frac{\sqrt{2}c}{4\pi}\frac{\epsilon_0 B_0}{a_0}\left(\frac{\tau_R}{\tau_A}\right)^{1/2}\left[\frac{k_0 \xi_0}{\sinh k_0 a_0}\right]^{3/2} . \quad (58)$$

The width of this sheet, from equation (A5), is seen to be

$$\Delta_{SP} \simeq \sqrt{2}\left[\frac{\tau_A}{\tau_R}\frac{\sinh k_0 a_0}{k_0 \xi_0}\right]^{1/2} a_0 . \quad (59)$$

Combining equations (52) and (58), we conclude that the amplitude of the current sheet increases at first linearly with t till it reaches the valve (58) in the nonlinear Sweet-Parker phase. Subsequently, the current sheet passes on to the Rutherford regime in which it decays on the time scale of resistive diffusion (Rutherford 1973; HK 1985; Wang & Bhattacharjee 1992).

These results are qualitatively in accord with numerical simulations. That forced reconnection causes current sheets has been seen in reduced MHD simulations (Park et al. 1984; Biskamp 1986). In the context of the solar corona, relevant numerical simulations have been carried out by Strauss (1990, 1991) and Mikic and coworkers (1990). For the problem of forced reconnection discussed in this paper, we cite a recent paper by Strauss (1991). The initial conditions for Strauss's simulation are roughly similar to ours. We draw the reader's attention to Figure 3 of Strauss's paper which shows that in the initial phase the peak value of the current density grows linearly with t. The departure from the linear behavior for intermediate times is not predicted by our analysis. A difference between the simulation and the analysis lies in the time dependence of the footpoint motion. In the analysis, it is assumed that $\omega_0 \tau_A \ll 1$, i.e., the characteristic time scale for the twisting is much longer than the Alfvén time scale. However, in the simulation it appears that $\omega_0 \tau_A \lesssim 1$, which suggests that the twisting is built up faster than the present calculation allows for. This may account for the departure from linear growth observed in the simulation for intermediate times.

We remark that in Strauss (1991), no instability is seen, and according to the nomenclature introduced in § 1, this is indeed a case of forced reconnection. The earlier work of Strauss & Otani (1988), by contrast, followed the evolution of an unstable kink-tearing mode, and is an example of free reconnection. In the presence of a freely reconnecting instability, the current sheet develops exponentially on the short time scale. In forced reconnection, by contrast, the current sheet grows out of the stable MHD continuum at an algebraic rate on the short time scale. On the nonlinear Sweet-Parker time scale τ_k, both types of reconnection processes lead to flux destruction at an algebraic rate. The current sheet that is formed on this nonlinear time scale eventually smooths out on the time scale τ_R of resistive diffusion.

6. CORONAL HEATING

It is estimated that the energy loss from the solar corona into the chromosphere at the two magnetized ends in the Parker model is of the order of 10^2 W m^{-2} for quiet regions and 10^3–10^4 W m^{-2} for active regions (Withbroe & Noyes 1977). Clearly, a heating mechanism is necessary to maintain thermal equilibrium for which the power flux out of the ends must be balanced by the internal heating power. If we assume that there are no current sheets and that the current is smoothly distributed, then the ohmic heating power can be estimated as

$$P_0 \sim \eta J_0^2 AL \sim \frac{b^2\tau_R}{4\pi} a^2 L \sim 10^{10} - 10^{11} \text{ W} , \quad (60)$$

where $\tau_R \sim 10^9$–10^{10} s, $a \sim 10^8$ cm, $L \sim 10^{10}$ cm, and the transverse field $b \sim 3C^2(a/L)B_0$ (Berger 1992) with $B_0 \sim 10^3$ gauss and the winding number $C \sim 1$. Equation (60) corresponds to an energy flux $P_0/A \approx 10^{-1}$–10^{-2} W m^{-2} which is much less than is required for thermal equilibrium.

On the other hand, in the presence of a current sheet, the current density $J \approx J_0 a_0/\Delta$ where Δ is the sheet thickness caused by forced reconnection. It is important to know how widespread the current sheets are in the coronal volume so that we can apportion correctly their contribution to coronal heating. This has been done by Parker (1989) and more recently by Berger (1991) who conclude from kinematic considerations that most of the current in the presence of random twists of the footpoints must reside in current sheets. The heating power due to these sheets can be estimated as

$$P \sim \eta J^2 \Delta a_0 L \sim \left(\frac{a_0}{\Delta}\right) P_0 \sim \left(\frac{\tau_R}{\tau_0}\frac{\xi_0}{a_0}\right)^{1/2} P_0 \approx 10^4 P_0 , \quad (61)$$

where we have used equation (59) and the estimate $\xi_0/a_0 \sim 10^{-2}$. Equation (61) gives a flux (10^2–10^3) W m^{-2} which appears to be adequate for quiet regions, but not quite enough for active regions. Larger values of flux should be possible for larger values of either ξ_0/a_0, or the winding number or both, but we do not pursue this point here because it lies beyond the domain of validity of our calculation.

7. CONCLUSIONS

In this paper, we consider the evolution in time of a coronal loop in which reconnection is forced by the twisting motion of footpoints. This study complements our previous work (Bhattacharjee & Wang 1991) in which we show that helicity-conserving reconnection processes in the corona, be they free or forced, produce current sheets at separatrices. Our previous work considered the relaxation of the plasma through a sequence of neighboring equilibria; here we consider the time evolution of forced reconnection explicitly. In a simple model, we show that current sheets tend to form on the Alfvén time scale with an amplitude that increases linearly with time. Due to the presence of a small but finite resistivity, these current sheets open up to form islands which evolve subsequently into a reconnection phase with a rate of reconnection approximately consistent with the Sweet-Parker model. The heating caused by these sheets in the linear and nonlinear phases can be substantial.

This research was supported by the National Science Foundation under grant No. ATM 91-00513 and PHY 89-04035. Most of the research was completed when both authors were visiting the Institute of Theoretical Physics (ITP), the University of California, Santa Barbara in fall, 1991. Both authors would like to express their gratitude to the members and staff of the ITP for their hospitality, and to the participating members of the program on Topological Fluid Mechanics for stimulating discussions.

APPENDIX

In this Appendix, we give a derivation for the reconnected flux $\tilde{\psi}_{SP}(0)$ in the nonlinear regime (eq. [57]), using a slightly modified version of the Sweet-Parker model.

Near the separatrix, since $\boldsymbol{v} \cdot \nabla\psi \simeq 0$, we have

$$\frac{\partial \tilde{\psi}_{SP}}{\partial t} = \frac{\eta c^2}{4\pi} \tilde{\psi}''_{SP} \approx \frac{\eta c^2}{4\pi} \frac{\Delta \tilde{b}_x}{\Delta} , \tag{A1}$$

where Δ is the width of the current sheet and $\Delta \tilde{b}_x$ is given by

$$\Delta \tilde{b}_x \approx \frac{2k_0 \epsilon_0 B_0 \xi_0}{\sinh k_0 a_0} . \tag{A2}$$

By mass conservation, for incompressible plasmas,

$$v_y a_0 \simeq v_x \Delta , \tag{A3}$$

where v_y is the flow entering the reconnection region, and v_x is the exiting flow. Also, in the inner limit of the external region

$$\frac{\partial \tilde{\psi}_{SP}}{\partial t} = \frac{1}{2} v_y \Delta \tilde{b}_x . \tag{A4}$$

From equations (A1)–(A4), we get

$$\Delta = (\eta c^2 a / 2\pi v_x)^{1/2} . \tag{A5}$$

The velocity v_x is obtained from energy conservation. This gives

$$\frac{1}{2} \rho v_x^2 = \frac{1}{8\pi} \left(\frac{\Delta \tilde{b}_x}{2}\right)^2 , \tag{A6}$$

$$v_x = \frac{1}{2} V_A \left(\frac{\Delta \tilde{b}_x}{B_0}\right)^2 . \tag{A7}$$

From equations (A1), (A3), and (A5), we get

$$\frac{\partial \tilde{\psi}_{SP}}{\partial t} \simeq \sqrt{2} \epsilon_0 B_0 a_0 \left[\frac{k_0 \xi_0}{\sinh k_0 a_0}\right]^{3/2} . \tag{A8}$$

Making use of

$$\eta \tilde{J}_{SP} = \frac{1}{c} \frac{\partial \tilde{\psi}_{SP}}{\partial t} \tag{A9}$$

and integrating equation (A8) with respect to time, we obtain equations (58) and (57), respectively.

REFERENCES

Berger, M. A. 1991, A&A, 252, 369
———. 1992, private communication
Bhattacharjee, A., & Wang, X. 1991, ApJ, 372, 321
Biskamp, D. 1986, Phys. Fluids, 29, 1520
Browning, P. K. 1991, Plasma Phys. Controlled Fusion, 33, 539
Hahm, T. S., & Kulsrud, R. M. 1985, Phys. Fluids, 28, 2412 (HK)
Low, B. C. 1990, ARA&A, 28, 491
Mikić, Z., Schnack, D. D., & van Hoven, G. 1989, ApJ, 338, 1148
Mok, Y., & van Hoven, G. 1982, Phys. Fluids, 25, 636
Otani, N. F., & Strauss, H. R. 1988, ApJ, 325, 468
Park, W., Monticello, D. A., & White, R. B. 1984, Phys. Fluids, 27, 137
Parker, E. N. 1957, J. Geophys. Res., 62, 509
———. 1972, ApJ, 174, 499
———. 1989, Geophys. Astrophys. Fluid Dyn., 45, 159
Rosenbluth, M. N., Dagazian, R. Y., & Rutherford, P. H. 1973, Phys. Fluids, 16, 1984
Rutherford, P. H. 1973, Phys. Fluids, 16, 1903
Strauss, H. R. 1976, Phys. Fluids, 19, 134
———. 1991, ApJ, 381, 508
Strauss, H. R., & Otani, N. F. 1988, ApJ, 326, 418
Sweet, P. A. 1958, Electromagnetic Phenomena in Cosmical Physics (NY: Cambridge Univ. Press), 123
van Ballegooijen, A. A. 1985, ApJ, 298, 421
Waelbroeck, F. L. 1989, Phys. Fluids B, 1, 2372
Wang, X., & Bhattacharjee, A. 1992, Phys. Fluids B, 4, 1795
Withbroe, G. L., & Noyes, R. W. 1977, ARA&A, 15, 363
Zweibel, E., & Li, M.-S. 1987, ApJ, 312, 423