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NUCLEAR SOLID CRUST ON ROTATING STRANGE QUARK STARS[1]

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ABSTRACT

We calculate in general relativity the thickness, mass, and moment of inertia of the nuclear solid crust that can exist on the surface of a rotating strange quark star suspended out of contact with the quark core by an electric dipole layer on the core surface and the centrifugal force. Aside from the interesting properties of such stars, a particular question of great import to the viability of the strange matter hypothesis is whether strange stars can undergo the observed phenomena of pulsar glitches. We find that the nuclear crust can have a moment of inertia sufficiently large that a fractional change can account for the magnitude of pulsar glitches, even giant glitches. However, before testing a detailed model of the coupling of the crust and quark core, not an easy problem, we are not able to draw the definite conclusion that strange stars can account for all phenomena associated with glitching such as the healing time and recurrence rate. The problem of understanding quakes on compact stars is, afterall, akin to predicting earthquakes. We study the particular sequence of stars, both rotating and stationary, that have the maximum possible crust density, the neutron drip density. The sequence has a minimum mass of about 0.015 $M_\odot$ or about 15 Jupiter masses. Stars near this limit have crusts of thickness tens to hundreds of kilometers and are small and dark and so could be hiding places of baryonic matter.

Subject headings: dense matter — elementary particles — stars: interiors

1. INTRODUCTION

The hypothesis (Bodmer 1971; Witten 1984) that strange quark matter may be the absolute ground state of the strong interaction is very difficult either to prove or disprove. On theoretical scale arguments, it is as plausible a ground state as the confined state of hadrons (Witten 1984; Farhi & Jaffe 1984; Glendenning 1990). It is understood that even if it is the ground state, little or no such matter would have survived the high-temperature era of the universe because of evaporation to hadrons and the universe would have evolved essentially independent of which is the true ground state (Alcock & Farhi 1985; Applegate & Hogan 1985; Madsen, Heiselberg, & Rissager 1986; Madsen & Olesen 1991; Madsen 1991). An up-to-date account of recent developments can be found in Glendenning (1990) and Madsen & Haensel (1991). If the hypothesis is true, then a separate class of compact stars could exist, called strange stars. They form a distinct and disconnected branch of compact stars and are not a part of the continuum of equilibrium configurations that include white dwarfs and neutron stars. In principle, both strange and neutron stars could exist. However, if strange stars exist, the Galaxy is likely to be contaminated by strange quark nuggets which would convert all neutron stars that they come into contact with to strange stars (Glendenning 1990; Madsen & Olesen 1991; Caldwell & Friedman 1991).

At the present time there appears to be only one crucial astrophysical test of the strange quark matter hypothesis, and that is whether strange quark stars can give rise to the observed phenomena of pulsar glitches. Glitches are sudden relatively small changes in the period of a pulsar, which otherwise increases very slowly with time due to the loss of rotational energy through radiation. They occur in various pulsars at intervals of days to months or years, and in some pulsars are small (Crab), and in others large (Vela) and infrequent ($\Delta\Omega/\Omega \sim 10^{-8}$ to 10^{-6}, respectively).

Glitches have been attributed variously to several causes related to the assumed structure of neutron stars. One such is the crust quake in which an oblate solid crust in its present shape slowly comes out of equilibrium with the forces acting on it as the period of rotation changes, and fractures when the built-up stress exceeds the sheer strength of the crust material (Ruderman 1969; Baym & Pines 1971). The period and rate of change of period slowly heal to the trend preceding the glitch as the coupling between crust and core reestablishes their corotation. The existence of glitches may have a decisive impact on the question of whether strange matter is the ground state. The reason is that *provided* the cores of the most massive of neutron stars have densities exceeding that required to convert hadrons to quark matter (of which strange matter is the lower state) and that such stars of sufficient mass can be made in natural processes, then almost certainly all other neutron stars have also been converted to strange stars (Glendenning 1990; Madsen & Olesen 1991; Caldwell & Friedman 1991) either as a result of being bathed themselves by a flux of stange nuggets estimated to be as high as 0.1 $(\mathrm{cm}^2\ \mathrm{s})^{-1}$ (Glendenning 1990) produced in strange star collisions in the Galaxy or formed in a supernova from a progenitor that already contains strange seeds, either acquired after formation of the progenitor star or as a result of the gaseous material out of which the progenitor star was formed already containing seeds of strange matter produced in the same way either within our Galaxy or in the neighboring cluster of galaxies (Caldwell & Friedman 1991). In any case the nugget will grow to consume the entire neutron star (Olinto 1987). Therefore since, under the above proviso, neutron stars have already been converted, strange stars must be capable of

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producing pulsar glitches; otherwise, the hypothesis of strange matter fails. Indeed, the claim has been made that it does fail on this account (Alpar 1987), but for many researchers the case is unconvincing since the properties of quark matter are not sufficiently well known (Alcock & Olinto 1988; Madsen & Hensel 1991). Of course, a bag model of quark matter is a gas and cannot support stress, but this is likely to be a poor description of quark matter at its lower range in density where asymptotic freedom is not necessarily established. We do not deal in this paper with the properties of strange matter itself but rather focus on the crust of hadronic matter that a strange star can support. Because the stange quark mass is greater than that of the up and down quarks, a strange star will possess a dipole layer on its surface due to the electrons it must contain for charge neutrality (Alcock, Farhi, & Olinto 1986). This electric dipole layer (and the centrifugal force) can suspend a nuclear crust on the surface of a rotating strange quark matter star out of contact with the quark core. Here we study the thickness, mass, and moment of inertia of the crust as a function of mass and rotational frequency of the star, since it is a possible site of the buildup of stress whose occasional release could be responsible for glitching or at least microglitching in hypothetical strange pulsars. Still another motivation concerns the cooling rate of strange stars, which is significantly altered by the presence of a crust (Pizzochero 1991).

Beyond the specific properties of strange stars, several of which are exotic, their existence or nonexistence carries information of a fundamental nature as to the true ground state of the strong interaction that no other experiment or fact has yet revealed (Witten 1984; Glendenning 1990; Madsen & Haensel 1991; Alcock 1991) so they are very interesting objects to study in their many facets.

2. DESCRIPTION OF THE CRUST

Because the strange quark mass is larger than that of the up and down quarks, equilibrium dense strange matter will contain an approximately equal mixture of all three, with a slight deficit of strange quarks. Since the Coulomb interaction is so much stronger than the gravitational, a star must be charge neutral to very high precision ($\sim 10^{-37}$ net charge per baryon). The net positive quark charge therefore must be balanced by electrons. As was discussed by Alcock et al. (1986), the electrons, because they are bound by the Coulomb force, extend several hundred fermis beyond the boundary of the strange star which itself has a surface thickness of the order of the strong interaction range, ~ 1 fm since this is the force that binds strange stars. The electric field at the surface is estimated to be $\sim 10^{17}$ V cm^{-1} and outwardly directed. It can therefore suspend a crust of charged material, which itself is overall neutral but in which the charges of opposite sign are displaced (as in the core). The crust is gravitationally bound to the core. By hypothesis, strange matter is absolutely stable, so neutrons will be dissolved into quark matter as they gravitate into the core. Therefore the maximum density of the crust is strictly limited by neutron drip. This density is about 4.3×10^{11} g cm^{-3}. Because this lies below the central density of the least massive stable neutron star by about three orders of magnitude, we anticipate that the minimum mass of strange stars with a crust of maximum density equal to the drip density will be less than the minimum mass of neutron stars.

We shall be interested in the nuclear crust of a strange star that has reached the final state of stellar evolution, namely cold catalyzed matter appropriate to the range of pressures found in the crust, no matter how the strange star acquired the crust, whether by accretion from the interstellar medium onto an initially, and possibly primordial, bare strange star, or whether during its creation in a supernova, the crust in this case being debris from the progenitor star.

Because we are not interested in the star structure on the scale of the thickness of the gap between core and crust, of the order of several hundred fermis (Alcock et al. 1986), the somewhat complicated situation just described can be very simply represented by the choice of equation of state. It should consist of two parts. (1) At densities below neutron drip it should be represented by the low-density equation of state of charge neutral nuclear matter. The most significant aspect of this density domain is that it consists of a Coulomb lattice of heavy ions immersed in an electron gas. The heavy ions become ever more neutron-rich as the neutron drip density is approached from below. The equation of state at subnuclear density down to very low density has been calculated by Baym, Pethick, & Sutherland (1971, hereafter BPS). Details of the structure of matter at densities below neutron drip can be found in this reference. To describe a strange star with the maximum possible crust density at its inner edge, we take the low-density equation of state for pressures below $P_{\rm drip}$. (2) At pressures above the neutron drip pressure in nuclear matter the equation of state corresponds to strange quark matter. We use the simplest form of the bag model equation of state for strange matter because as far as the relationship of energy density and pressure is concerned, it is accurate to within 4% of the more complicated form involving quark masses (Alcock et al. 1986). It is $P = (\epsilon - 4B)/3$. The edge of the strange star, if bare, occurs at $P = 0$, or equivalently, $\epsilon = 4B$. In the presence of a crust, the quark core will be slightly squeezed, and the pressure at the edge of the core will be small and equal to $P_{\rm drip}$. Correspondingly, the energy density at the edge of the core will be slightly larger than $4B$. The equation of state is illustrated in Figure 1 and represented by

$$P(\epsilon) = \begin{cases} P_{\rm BPS}(\epsilon) & \text{if} \quad P < P_{\rm drip} \\ \frac{1}{3}(\epsilon - 4B) & \text{if} \quad P \geq P_{\rm drip}\,. \end{cases} \tag{1}$$

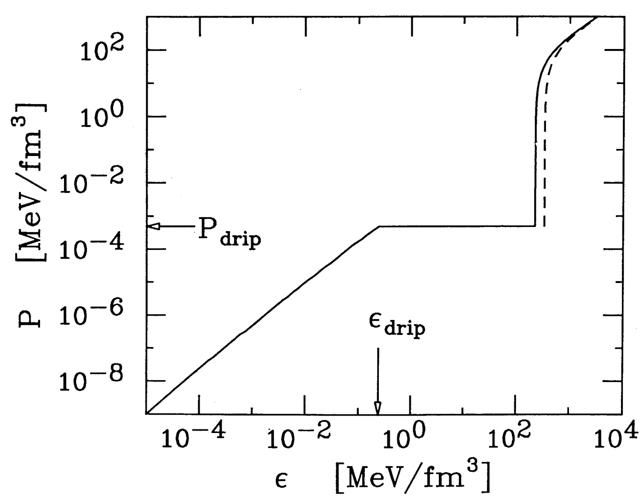

FIG. 1.—Equation of state of a strange star surrounded by a nuclear crust with inner density below neutron drip. The symbols $P_{\rm drip}$ and $\epsilon_{\rm drip}$ refer to drip pressure and drip energy density. The quark matter equation of state is calculated for $B^{1/4} = 145$ MeV (*solid line*) and 160 MeV (*dashed*).

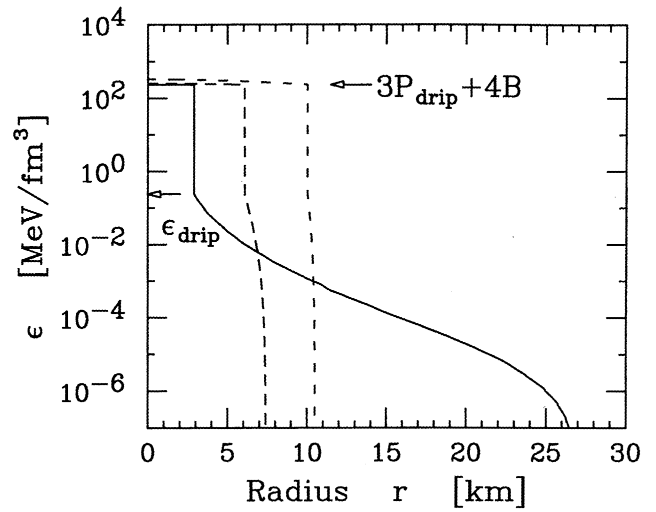

FIG. 2.—Energy density as a function of radial distance from the star's center for gravitational masses $M/M_{\odot} = 0.02$ (*solid line*), 0.20 (*dashed*), 1.00 (*dash-dotted*), and 1.50 (*dotted*). The bag constant is $B^{1/4} = 145$ MeV.

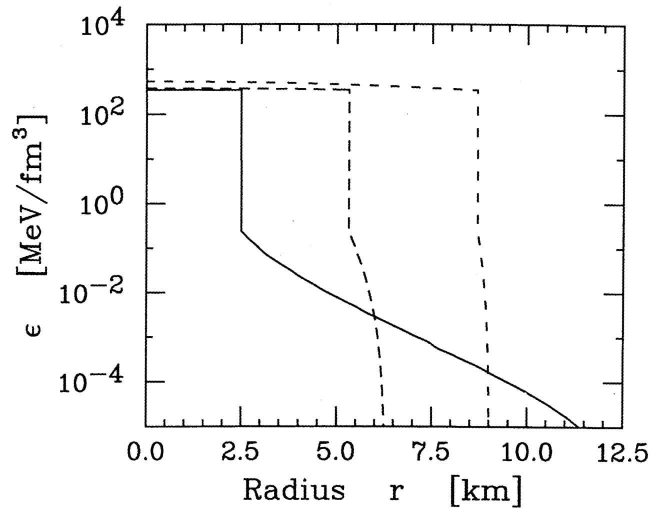

FIG. 4.—Same as Fig. 2, but for $B^{1/4} = 160$ MeV

Two different representative values for the bag constant for which three-flavor strange matter is stable have been chosen, $B^{1/4} = 145$ and 160 MeV. These values may serve to study the impact of the bag constant on the hadronic crust of strange stars. For massless strange quarks, these bag constants correspond to an equilibrium energy per baryon number of strange quark matter of about 830 and 915 MeV, respectively. For strange quarks of 100 MeV mass, they correspond to about 855 and 930 MeV, respectively (Farhi & Jaffe 1984). In other words, these choices represent strongly ($\sim$100 MeV) and weakly bound strange matter and in all cases correspond to strange quark matter being absolutely bound with respect to ^{56}Fe.

Pressure in a star is a continuous and monotonically decreasing function of the Schwarzschild radial coordinate, but naturally there is a discontinuity in energy density between strange quark matter and hadronic matter at the neutron drip pressure of hadronic matter. The energy discontinuity between hadronic matter at the dip pressure and strange quark matter at the same pressure is (cf. Fig. 1)

$$\Delta\epsilon \equiv (3P_{drip} + 4B) - \epsilon_{drip} , \qquad (2)$$

where ϵ_{drip} is the energy density of nuclear matter at the neutron drip point. The energy density of the star will suffer this discontinuity (across the several hundred fermi gap described above) at the radius where the pressure of the star falls from its central value to P_{drip}. The energy density and pressure profiles of several nonrotating sample stars are shown in Figures 2–5 for gravitational masses in the range $0.020 \leq M/M_{\odot} \leq 1.50$. The ends of this range correspond respectively to a very light star close to the lower mass limit and one closer to the upper mass limit. The sharp fall in the energy profile marks the boundary between the quark core and the nuclear crust.

By the hypothesis that strange matter is the absolute ground state, stable objects of strange matter from microscopic nuggets to stars could exist. However, the nuclear crust, suspended out of contact with the core by the electric dipole layer, is attached to the star by the gravitational interaction. For a

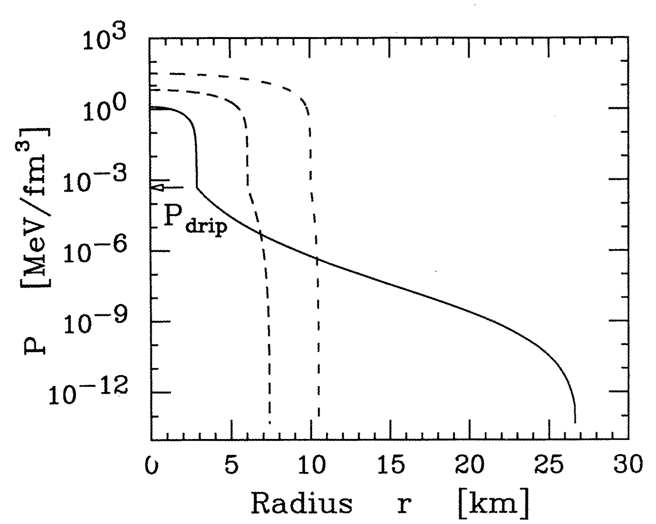

FIG. 3.—Pressure as a function of radial distance from the star's center for different gravitational mass (the labeling is the same as in Fig. 2). The bag constant is $B^{1/4} = 145$ MeV.

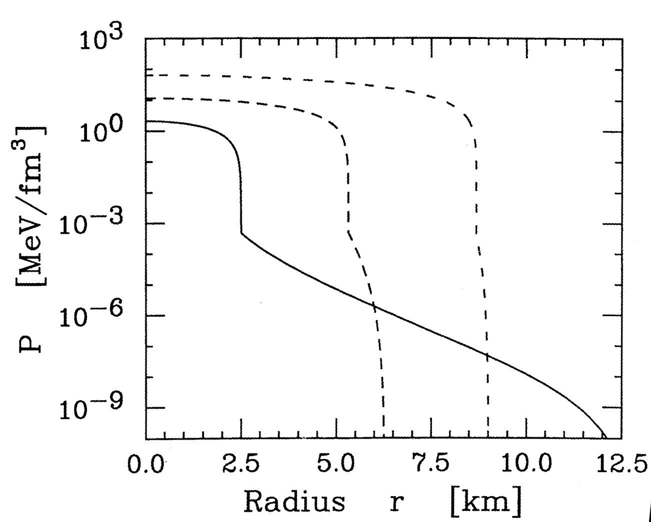

FIG. 5.—Same as Fig. 3, but for $B^{1/4} = 160$ MeV

given inner density of the crust, the maximum of which is the drip density, the crust will be thinner the more massive the strange quark core, and thicker the less massive it is. This can be understood as a consequence of the way the core radius scales with mass; for masses too small for gravity to play an important role, the relationship is $M = (4\pi/3)R^3\epsilon_0$ where $\epsilon_0 = 4B$ is the equilibrium density of strange matter. This relation is only somewhat modified near the most massive star in the sequence which gravity terminates. So since $R \propto M^{1/3}$, the (Newtonian) gravitational force acting on unit mass at the surface of the core is $M/R^2 \propto M^{1/3}$. The nuclear crust becomes gravitationally unstable ($dM/d\epsilon_c \leq 0$) at some minimum mass that depends on the inner density of the crust. When this density is the neutron drip density, we find a minimum mass star of $\sim 0.015\ M_\odot$, to be compared to $0.1\ M_\odot$ for neutron stars (BPS). So the thickening of the crust illustrated in Figures 2 and 4 for the $M/M_\odot = 0.02$ case, which is close to the lower mass limit for an inner crust density equal to the neutron drip density, illustrate the profiles very close to the lower end point of stable strange stars with nuclear crusts.

A comparison of these figures for different bag constants B can be facilitated by noting the scaling laws that apply to strange stars (Witten 1984; Wang & Wang 1990). For a central energy density that is some fixed multiple of B, the mass and radius of bare strange stars corresponding to different assumptions about the bag constant scale as

$$M \propto 1/\sqrt{B}\,, \quad R \propto 1/\sqrt{B}\,. \tag{3}$$

The maximum mass strange star for given B corresponds to a central density of $19.2B$.

The mass-radius relationship for strange stars with a nuclear crust is shown in Figure 6, and since the crust is bound by the gravitational interaction, the relationship is qualitatively similar to the one for neutron stars, the radius being largest for the lightest and smallest for the heaviest in the sequence. Just as for neutron stars the relationship is not necessarily monotonic at intermediate masses. The radius of the strange quark core is also shown, and it follows the inevitable behavior of objects that are self-bound, namely the one mentioned above, $R \propto M^{1/3}$, which is only somewhat modified near the mass where gravity terminates the stable sequence.

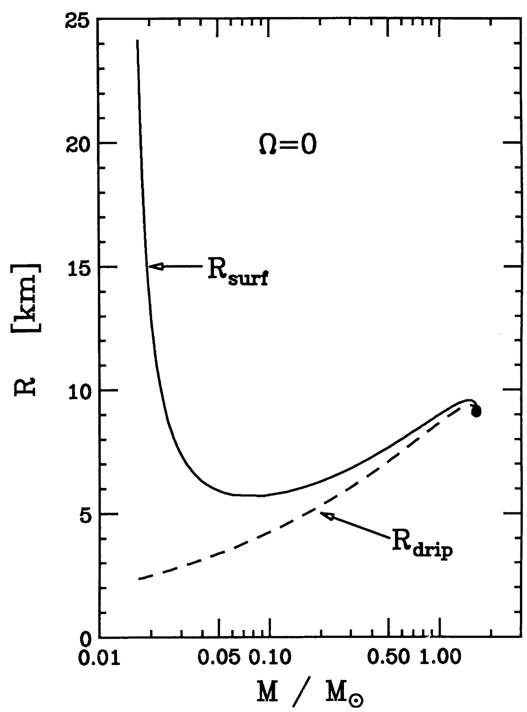

FIG. 6.—Radius as a function of mass of strange star with crust, and radius of the strange star core for inner crust density equal to the neutron drip, for nonrotating stars. Bag constant is $B^{1/4} = 160$ MeV. The solid dot refers to the limiting-mass model of the sequence.

3. PROPERTIES OF ROTATING STRANGE STARS WITH CRUST

3.1. *Outline of the Calculational Procedure*

We present in the following the properties of sequences of *rotating* strange star models possessing a nuclear crust which is suspended out of contact with the strange quark core by the electric dipole layer on the quark core surface. These models are constructed in the framework of Einstein's theory of general relativity by solving Hartle's perturbative stellar structure equations. Hartle's method was reexamined in Weber & Glendenning (1991, 1992) where it was found that it is a practical tool for constructing models of neutron stars down to rotational periods in the half-millisecond range, *when* the appropriate self-consistent condition is imposed so as to identify the Kepler frequencies (balance between gravity and centrifuge). We refer to these references for more details.

We shall assume that the crust and core rotate with the same angular velocity Ω. Since both are composed of charged particles, protons, electrons, and heavy ions in the case of the crust, quarks and electrons in the case of the core, we assume that they are coupled by the magnetic field of the pulsar.

No simple stability criteria are known for rotating star configurations in general relativity. An absolute upper limit on rotation is set by the Kepler frequency above which mass shedding would occur. This is the frequency at which the *maximum possible nuclear crust mass* can be supported by a strange star because the centrifugal force acting on the crust is at its maximum. Therefore, we construct entire sequences of models of strange stars that are rotating at their Kepler frequencies, $\Omega = \Omega_K$, at $\Omega_K/2$ and 0. The general relativistic value of Ω_K includes the important dragging effect of local inertial frames and of course the rotational deformation of the star and its mass increment due to the centrifugal decompression. For the maximum mass star of a sequence, the value of Ω_K is reduced considerably from the Newtonian Kepler frequency of a test particle circulating the corresponding nonrotating star, by about the factor 0.61–0.71 (Weber & Glendenning 1992), in agreement with an empirical formula deduced from numerical integration of Einstein's equations (Friedman, Ipser, & Parker 1989; Haensel & Zdunick 1989). The construction of rotating star models is considerably more complicated than is the case for the nonrotating Oppenheimer-Volkoff stars (e.g., Friedman, Ipser, & Parker 1986; Butterworth 1976; Butterworth & Ipser 1976).

3.2. *Hadronic Crust Mass*

The crust mass is of the order $10^{-5}\ M_\odot$ and so has little effect on the total gravitational mass of all but very light strange stars. These are, however, quite interesting because the stable strange quark core permits stable configurations of strange stars with crusts that are less than one-sixth of the mass of the lightest neutron star models, the least massive of which is $\sim 0.1\ M_\odot$ for the BPS equation of state. At some critical

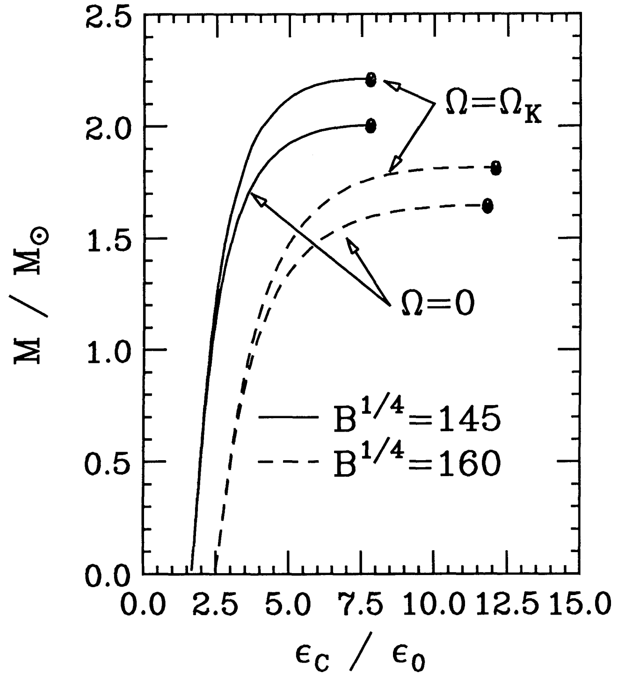

FIG. 7.—Gravitational masses of strange stars with hadronic crust as a function of central density (in units of normal nuclear matter density, $\epsilon_0 \approx 140$ MeV fm^{-3}) of nonrotating sequences ($\Omega = 0$) and sequences rotating at their Kepler frequencies ($\Omega = \Omega_K$). The solid dots refer to the limiting-mass model of each sequence.

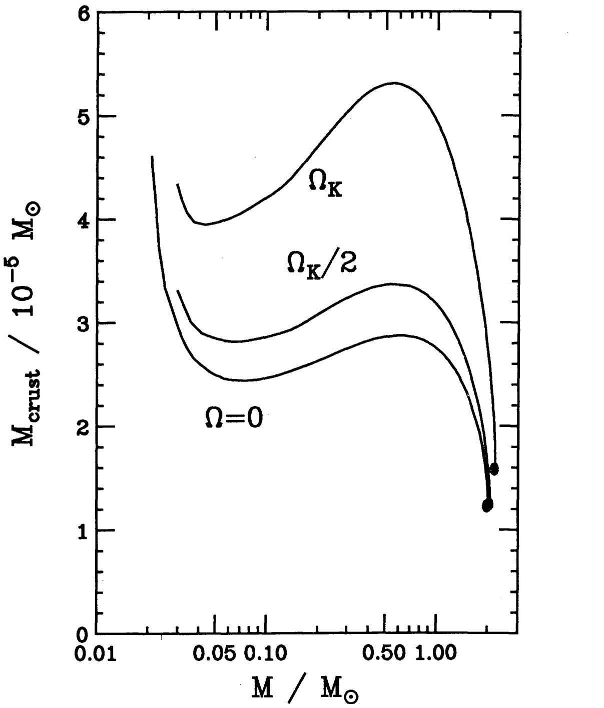

FIG. 8.—Mass of hadronic crust on a strange star as a function of total star mass for several rotation frequencies. Crust has maximum density equal to neutron drip. The solid dots refer to the limiting-mass model of each sequence. The bag constant is $B^{1/4} = 145$ MeV.

strange core mass, no crust is stable and the mass as a function of central density has a minimum. For more massive strange stars and for both bag constants we show the sequences corresponding to static stars and to stars rotating at their Kepler frequencies, Ω_K, in Figure 7. For the more massive stars, for which the Kepler frequency is greater, the mass that can be supported by a star of the same central density as a nonrotating one is larger for the rotating star by about 10%.

Next we turn our interest to the mass of the *hadronic crust* that can be supported by rotating strange stars by constructing sequences of models that rotate at their Kepler frequencies Ω_K, at $\Omega_K/2$, and at zero frequency. Figures 8 and 9 exhibit the crust mass as a function of total mass with frequency fixed at one of these values. As already noted, the gravitational force per unit mass at the surface of the core is $\propto M^{1/3}$ so that the light cores have small attraction and therefore have thick and massive crusts. At the opposite extreme, near the upper termination of the sequence, the crust is thin and therefore light. In between the behavior is not monotonic because of the depenence of the star and core radii on mass as shown in Figure 6.

It should be noted that the crust mass is smaller for larger bag constants as can be seen by comparing Figures 8 and 9. Again this can be understood in terms of the gravitational force at the surface of the core, $\propto M^{1/3}$, while the mass itself scales as equation (3).

Figure 10 displays the dependence of the nuclear crust mass on Kepler period of stars in two sequences with different bag constants. The shortest period stars are the maximum limiting-mass models (because of their relatively small radii and the strong gravitational attraction). These are found to have periods in the range of $0.5 \lesssim P_K/\text{ms} \lesssim 0.6$ depending on the bag equation of state. The largest crust masses occur for star masses $\sim 0.7\ M_\odot$, and they lie in the range ~ 1–$5 \times 10^{-5}\ M_\odot$. It is striking that the crust mass depends only rather weakly on the Kepler period for most of the lighter stars.

The impact of rotation on M_{crust} for star frequencies $0 \leq \Omega \leq \Omega_K$ is shown in Figures 11 and 12 for a representative sample of star masses, M. Again, as in Figure 10, the larger

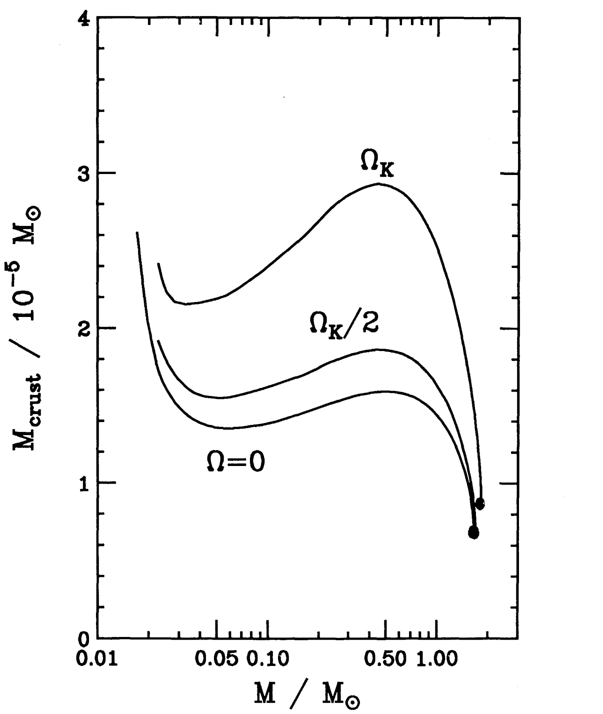

FIG. 9.—Same as Fig. 8, but for $B^{1/4} = 160$ MeV

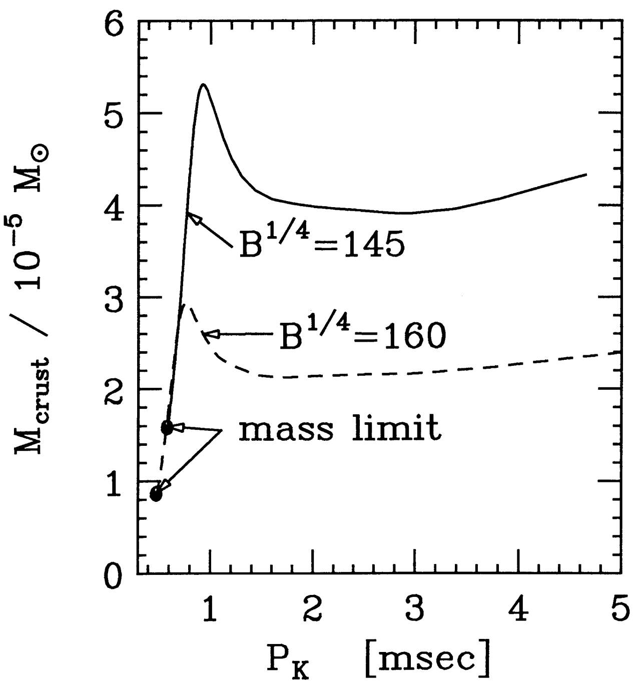

FIG. 10.—Mass of hadronic crust as a function of Kepler period, $P_K(\equiv 2\pi/\Omega_K)$. The most massive strange star models are those labeled mass limit; the lightest ones shown have masses of $M = 0.030\ M_\odot$ (for $B^{1/4} = 145$ MeV) and $M = 0.023\ M_\odot$ ($B^{1/4} = 160$ MeV).

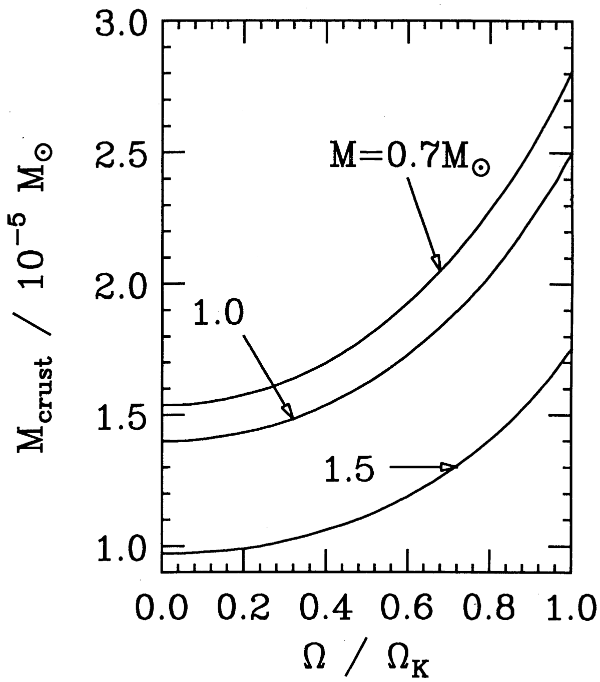

FIG. 12.—Same as Fig. 11, but for a $B^{1/4} = 160$ MeV

$B^{1/4}$, the smaller M_{crust}. Along the curves of these figures, the star's mass is kept constant, and therefore the crust mass is now a monotonically increasing function of the star's rotational frequency Ω. The constant mass curves also emphasize the dependence of $M_{crust}(\Omega)$ on M.

3.3. *Crust Thickness*

The crust thickness as a function of strange star mass is shown in Figures 13 and 14 for bag constants of $B^{1/4} = 145$ MeV and 160 MeV. As expected the crust thickness at the star's equator is larger than at its pole due to the centrifugal force. Of course the crust is thickest for the lighter stars because of the smaller gravitational force. The crust thickness

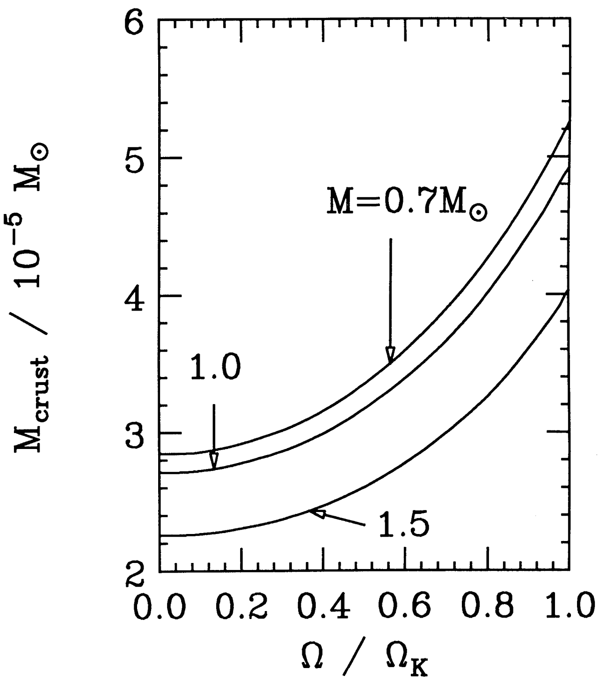

FIG. 11.—Hadronic crust mass as a function of rotational frequency Ω (in units of the Kepler frequency) for different star masses. The bag constant is $B^{1/4} = 145$ MeV.

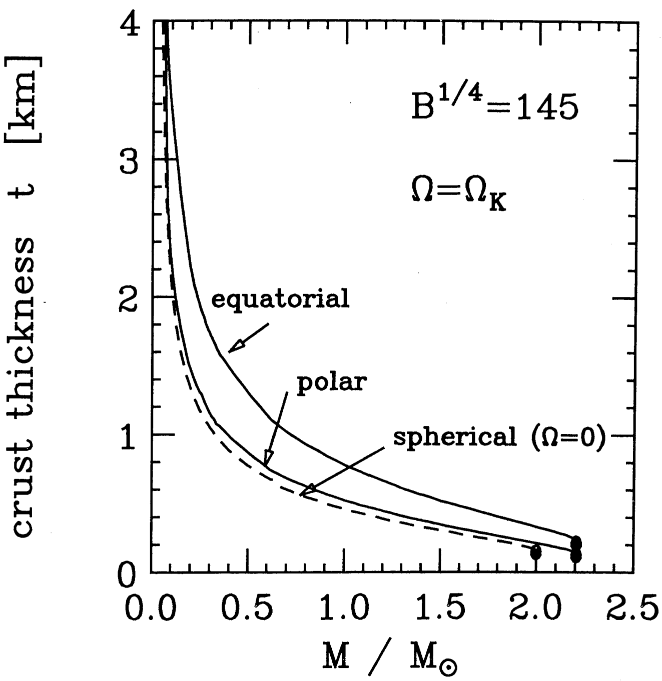

FIG. 13.—Thickness of crust at equator and pole as a function of star mass. The rotational frequency is Ω_K. Results for a nonrotating strange star sequence are labeled $\Omega = 0$. The bag constant is $B^{1/4} = 145$ MeV.

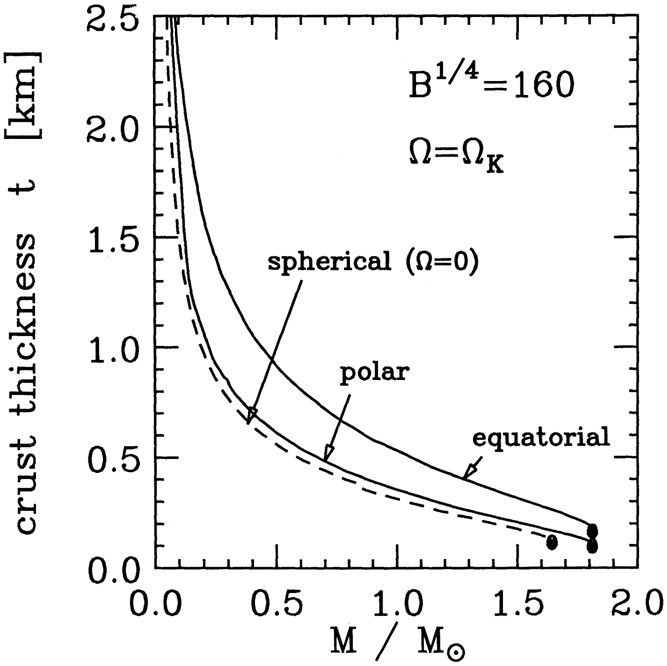

FIG. 14.—Same as Fig. 13, but for $B^{1/4} = 160$ MeV

of a nonrotating strange star is exhibited for the purpose of comparison. It is interesting that even the polar thickness increases with rotational frequency when comparing stars of the same mass. This is because (1) the centrifugal force opposes gravity, so that a rotating star of the same mass as a nonrotating one is relatively decompressed; (2) the lower density rotating star also has its mass redistributed, becoming oblate; (3) the gravitational force of the mass in the sphere of polar radius, R_p, acting on unit mass at R_p is less than at the surface of a spherical nonrotating star.

$$F \sim \frac{[(4/3)\pi R_p^3]\epsilon_<}{R_p^2} \propto (R_p \epsilon_<) < R\epsilon \,, \qquad (4)$$

where $\epsilon_<$ and ϵ are average densities for the decompressed and nonrotating stars respectively; (4) the remainder of the mass contained in the oblate star outside the sphere of polar radius is at greater distance to the pole compared to the same mass distributed over the nonrotating star and so experiences a smaller gravitational attraction. So in brief, the nuclear crust on a rotating strange star experiences a smaller gravitational attraction to the core, even at the pole and so is thicker everywhere when compared to a nonrotating star of the same core mass (essentially the total mass).

The two Figures 15 and 16 display the crust thickness at the equator and the pole of strange stars of constant masses as a function of rotational frequency. One sees that t is a monotonically increasing function of Ω. The impact of rotation on t is the smaller the more massive the strange star model of a sequence. The nuclear crust thickness on very light strange stars can be hundreds of kilometers thick (Fig. 6), but for the range of masses shown in Figures 15 and 16, it is less than 1 km at the Kepler frequency.

3.4. *Moment of Inertia*

3.4.1. *Expression for the Moment of Inertia in General Relativity*

The following line element is introduced for the calculation of the expression of the moment of inertia, I (cf. Hartle 1967; Hartle & Thorne 1968) ($G = c = 1$):

$$\begin{aligned} ds^2 &= g_{\alpha\beta}\, dx^\alpha dx^\beta \\ &= -e^{2\nu(r,\Omega)} dt^2 + e^{2\lambda(r,\Omega)} dr^2 + e^{2\mu(r,\Omega)} d\theta^2 \\ &\quad + e^{2\psi(r,\Omega)} [d\phi - \omega(r, \Omega) dt]^2 \,, \end{aligned} \qquad (5)$$

where the coordinates will be referred to in the order $x^\alpha \equiv t, r, \theta, \phi (\alpha = 0, 1, 2, 3)$. In this line element, ω is the angular velocity of the local inertial frame and depends on the radial coordinate

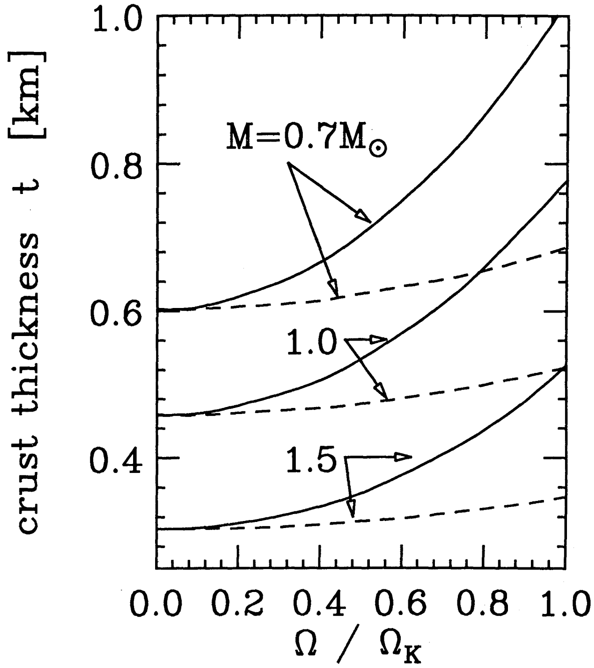

FIG. 15.—Crust thickness as a function of rotational frequency, Ω, for several star masses. The solid and dashed lines refer to the thickness at the star's equator and pole. The bag constant is $B^{1/4} = 145$ MeV.

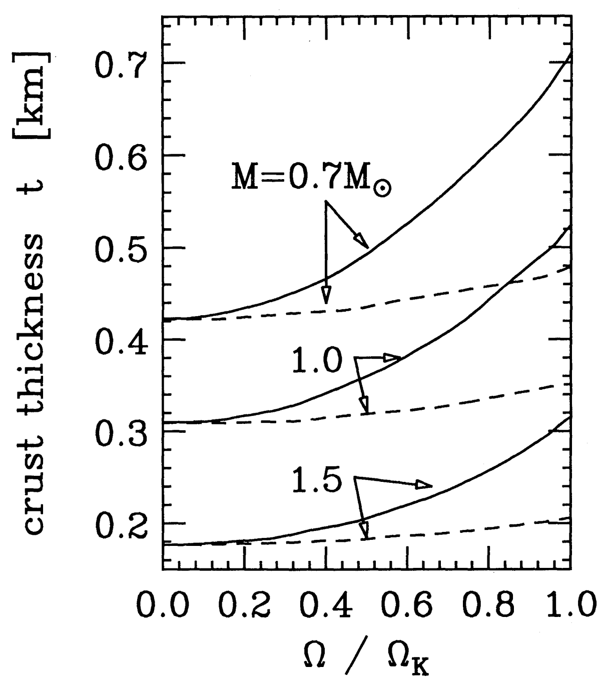

FIG. 16.—Same as Fig. 15, but for $B^{1/4} = 160$ MeV

r. It is related to the star's rotational frequency Ω, which is a constant throughout the fluid for uniform rotation. We recall that it is the difference between these frequencies, $\bar{\omega} \equiv \Omega - \omega$, with which the fluid inside the star moves (dragging effect of local inertial frames; Hartle 1967).

The metric functions in the line element of equation (5) have the form

$$e^{2\nu(r,\Omega)} = e^{2\Phi(r)}\{1 + 2[h_0(r, \Omega) + h_2(r, \Omega)P_2(\cos\theta)]\} , \quad (6)$$

$$e^{2\psi(r,\Omega)} = r^2 \sin^2\theta\{1 + 2[v_2(r, \Omega) - h_2(r, \Omega)]P_2(\cos\theta)\} , \quad (7)$$

$$e^{2\mu(r,\Omega)} = r^2\{1 + 2[v_2(r, \Omega) - h_2(r, \Omega)]P_2(\cos\theta)\} , \quad (8)$$

$$e^{2\lambda(r,\Omega)} = \left[1 + \frac{2}{r}\frac{m_0(r, \Omega) + m_2(r, \Omega)P_2(\cos\theta)}{1 - 2m(r)/r}\right] \times \left[1 - \frac{2m(r)}{r}\right]^{-1} . \quad (9)$$

The quantity $\Phi(r)$ in equation (6) denotes the metric function of a spherically symmetric object and $m(r)$ the mass within r for the corresponding spherical star, and P_2 is the Legendre polynomial of order 2. The perturbation functions m_0, m_2, h_0, h_2, and v_2, which vanish for a spherical star, are to be calculated from Einstein's field equations and are given as solutions of Hartle's stellar structure equations (Hartle 1967; Hartle & Thorne 1968) which are here implemented with the self-consistency condition as discussed by Weber & Glendenning (1991, 1992) and Weber, Glendenning, & Weigel (1991).

We are interested in the moment of inertia of azimuthally symmetric, uniformly rotating, relativistic stars in equilibrium. Such rotating bodies are symmetric about the axis of rotation and therefore will not radiate gravitational waves. Under these restrictions, the expression for the moment of inertia is given by (Hartle 1973)

$$I(\mathscr{R}, \Omega) = \frac{1}{\Omega}\int_{\mathscr{R}} dr\, d\theta\, d\phi\, \mathscr{T}_0^3 \sqrt{-g} . \quad (10)$$

In the above equation, $\mathscr{R}$ denotes an axially symmetric region in the interior of a body where all matter is rotating with the same angular velocity Ω. The quantities g and $\mathscr{T}$ refer to the determinant of the metric tensor [$g \equiv \det(g_{\alpha\beta})$] and the energy momentum density tensor $\{\mathscr{T} \equiv \mathscr{T}[\epsilon, P(\epsilon)]\}$. For the metric of equation (5) one finds

$$\sqrt{-g(r, \Omega)} = e^{\lambda(r,\Omega)}e^{\mu(r,\Omega)}e^{\nu(r,\Omega)}e^{\psi(r,\Omega)} . \quad (11)$$

From the expression for the energy momentum density tensor, given by

$$\mathscr{T}_\alpha^\beta = -(\epsilon + P)u_\alpha u^\beta + P\,\delta_\alpha^\beta , \quad (12)$$

one readily obtains

$$\mathscr{T}_3^0 = -(\epsilon + P)u_3 u^0 . \quad (13)$$

The quantity $u^\alpha = (u^0, 0, 0, u^3)$ denotes the fluid's four-velocity. The condition of uniform rotation, which is assumed throughout this work, is expressed by $u^3 = \Omega u^0$ (Hartle 1967). From the normalization condition $u_\alpha u^\alpha = -1$ one obtains

$$u^0 = (-g_{00} - 2g_{03}\Omega - g_{33}\Omega^2)^{-1/2}$$
$$= (e^{2\nu(r,\Omega)} - \bar{\omega}(r, \Omega)^2 e^{2\psi(r,\Omega)})^{-1/2} \quad (14)$$
$$\approx e^{-\Phi(r)}\{1 - [h_0(r, \Omega) + h_2(r, \Omega)P_2(\cos\theta)] + \tfrac{1}{2}e^{-2\Phi(r)}r^2 \sin^2\theta\, \bar{\omega}(r, \Omega)^2\} . \quad (15)$$

The covariant component u_3 is given by

$$u_3 = g_{3\alpha}u^\alpha = g_{30}u^0 + g_{33}u^3$$
$$= \bar{\omega}(r, \Omega)e^{2\psi(r,\Omega)}u^0 . \quad (16)$$

By means of equation (16), the velocity u_3 in equation (13) can be eliminated in favor of the metric functions (and therefore the perturbation functions, which are the solutions of Hartle's stellar structure equations) of equations (6)–(9). One arrives at

$$\mathscr{T}_3^0(r, \Omega) = -(\epsilon + P)\bar{\omega}(r, \Omega)e^{2\psi(r,\Omega)}(u^0)^2$$
$$= -(\epsilon + P)\bar{\omega}(r, \Omega)e^{2\psi(r,\Omega)}[e^{2\nu(r,\Omega)} - \bar{\omega}(r, \Omega)^2 e^{2\psi(r,\Omega)}]^{-1}$$
$$\approx -(\epsilon + P)\bar{\omega}(r, \Omega)e^{2\psi(r,\Omega)} \quad (17)$$
$$\times [\![e^{-\Phi(r)}\{1 - [h_0(r, \Omega)P_2(\cos\theta)] + \tfrac{1}{2}e^{-2\Phi(r)}r^2 \sin^2\theta\, \bar{\omega}(r, \Omega)^2\}]\!]^2 . \quad (18)$$

Using the expressions derived for $(-g)^{1/2}$ and $\mathscr{T}_3^0$ of equations (11) and (17), one finds from equation (10) for the moment of inertia

$$I = 4\pi\int_0^{\pi/2} d\theta \int_0^{R(\theta)} dr\, \frac{e^{\lambda(r,\Omega)}e^{\mu(r,\Omega)}e^{\nu(r,\Omega)}e^{\psi(r,\Omega)}[\epsilon + P(\epsilon)]}{e^{2\nu(r,\Omega)-2\psi(r,\Omega)} - \bar{\omega}(r, \Omega)^2}\, \frac{\bar{\omega}(r, \Omega)}{\Omega} . \quad (19)$$

This expression is valid through quadrupole deformation of the star due to rotation and is the one we use to evaluate the moment of inertia of rapidly rotating, axially symmetric, relativistic stars.

If the rotational deformation were ignored (i.e., spherical star) it would become

$$I_{\rm sph} = \frac{8\pi}{3}\int_0^R dr\, r^4 \frac{\epsilon + P(\epsilon)}{\sqrt{1 - 2m(r)/r}}\, \frac{\bar{\omega}(r, \Omega)}{\Omega}\, e^{-\Phi(r)} . \quad (20)$$

This last well-known expression evaluated with $\bar{\omega}/\Omega \to 1$ is appropriate for a slowly rotating star for which frame dragging and rotational deformation are negligible. As we shall see later it is fairly accurate up to $\Omega \approx \Omega_K/2$.

3.4.2. *Results for the Moment of Inertia*

The moment of inertia of the hadronic crust, $I_{\rm crust}$, that can be carried by a strange star as a function of star mass for a sample of rotational frequencies, $\Omega = \Omega_K$, $\Omega_K/2$, and 0 are shown in Figures 17 and 18 as a function of M. Because of the relatively small crust mass of the limiting-mass models of each sequence (see Figs. 8 and 9), the ratio $I_{\rm crust}/I_{\rm total}$ is smallest for them (Figs. 17 and 18, *solid dots*). The less massive the strange star the larger its radius (Fig. 6) therefore the larger both $I_{\rm crust}$ as well as $I_{\rm total}$, the dependence on M being such that their ratio $I_{\rm crust}/I_{\rm total}$ is a monotonically decreasing function of M. We find that there is only a slight difference between $I_{\rm crust}$ for $\Omega = 0$ and $\Omega = \Omega_K/2$. This confirms that equation (20) is a good approximation for stars rotating below $\Omega_K/2$, while for $\Omega > \Omega_K/2$ the full expression of equation (19) would need to be used except for very light stars.

Absolute values of the crust's moment of inertia as a function of gravitational star mass and of crust mass are shown in Figures 19 and 20 and Figures 21 and 22, respectively, for three rotational frequencies. They are qualitatively similar to the behavior of the crust mass exhibited in Figures 8 and 9. The discussion of these previous figures applies to the present case as well and will therefore not be repeated. The corresponding

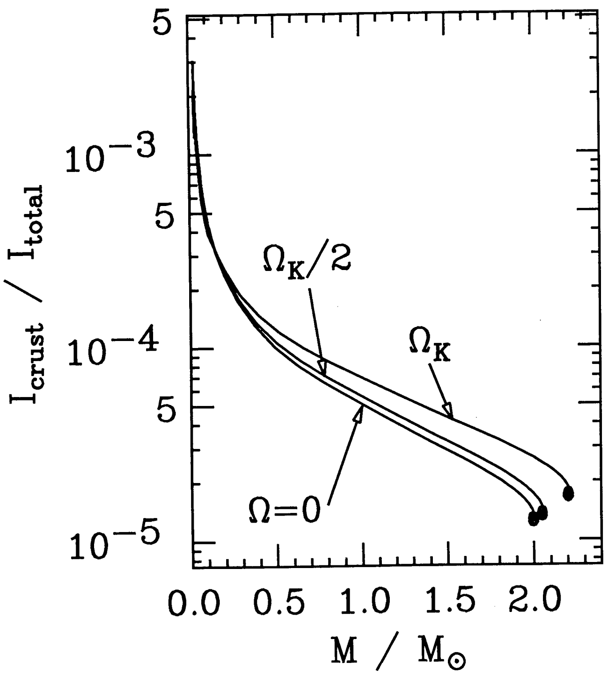

FIG. 17.—The ratio I_{crust}/I_{total} as a function of star mass. Rotational frequencies are shown as a fraction of the Kepler frequency. The solid dots refer to the limiting-mass models. The bag constant is $B^{1/4} = 145$ MeV.

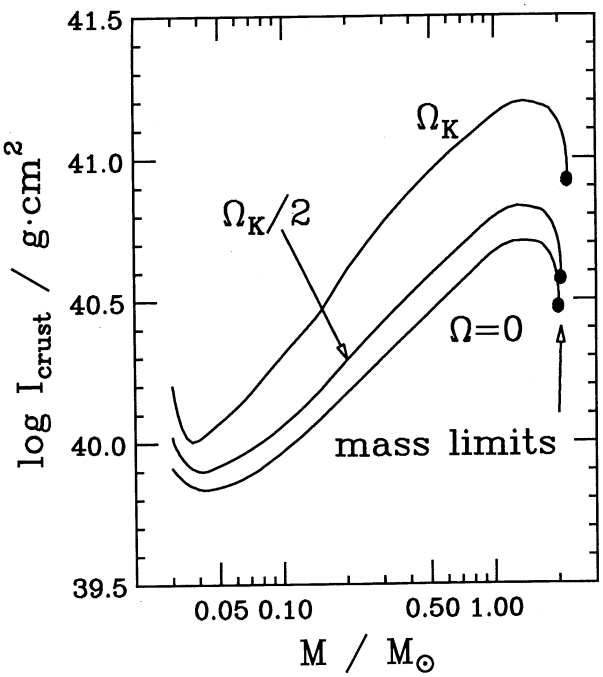

FIG. 19.—The moment of inertia of the hadronic crust as a function of star mass for three rotational frequencies. The solid dots refer to the limiting-mass models. The bag constant is $B^{1/4} = 145$ MeV.

masses related to the curves of these two figures can be inferred from Figures 8 and 9.

The following two Figures 23 and 24, show the absolute values of the moment of inertia of the nuclear crust as a function of the star's moment of inertia for three different rotational frequencies.

4. DISCUSSION AND SUMMARY

Strange quark matter stars, if they exist, have the remarkable property of possessing a very strong electric dipole field on their surfaces (Alcock et al. 1986) which can support a nuclear crust out of contact with the quark core and with an inner

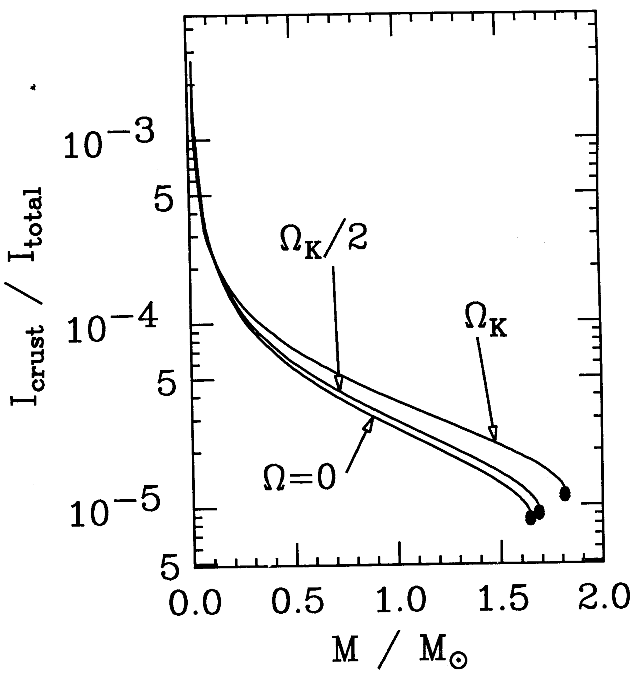

FIG. 18.—Same as Fig. 17, but for $B^{1/4} = 160$ MeV

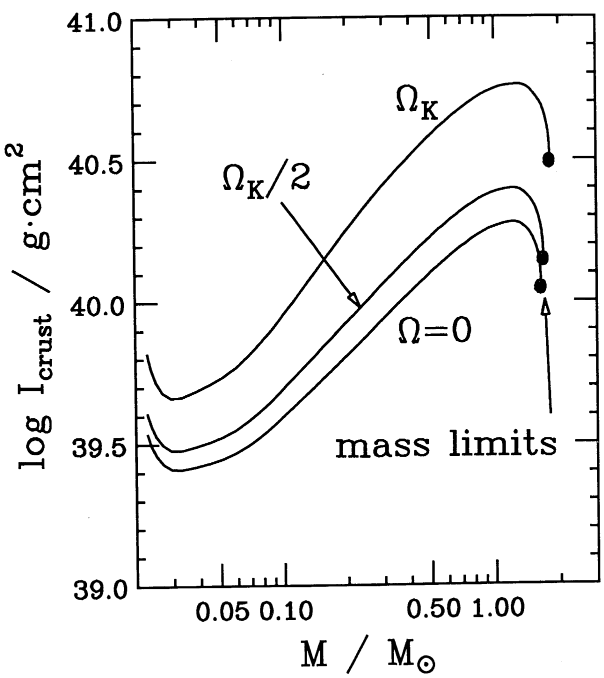

FIG. 20.—Same as Fig. 19, but for $B^{1/4} = 160$ MeV

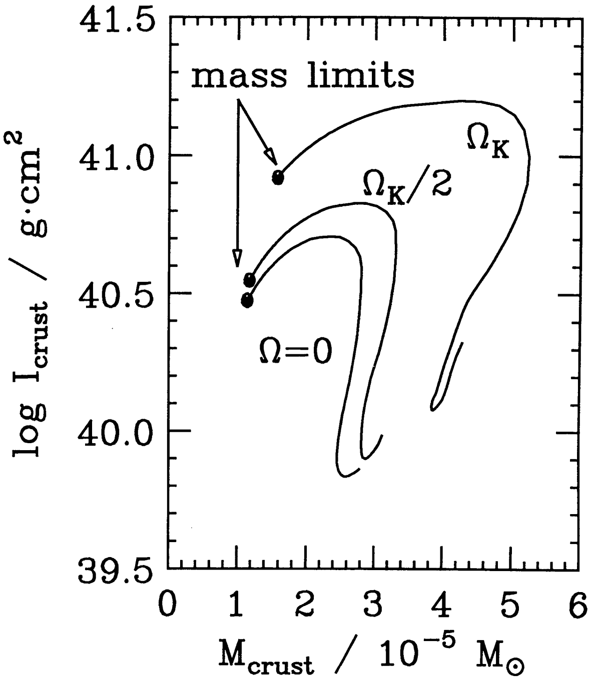

FIG. 21.—The moment of inertia of the hadronic crust vs. its mass for three rotational frequencies. The solid dots refer to the limiting-mass models. The bag constant is $B^{1/4} = 145$ MeV.

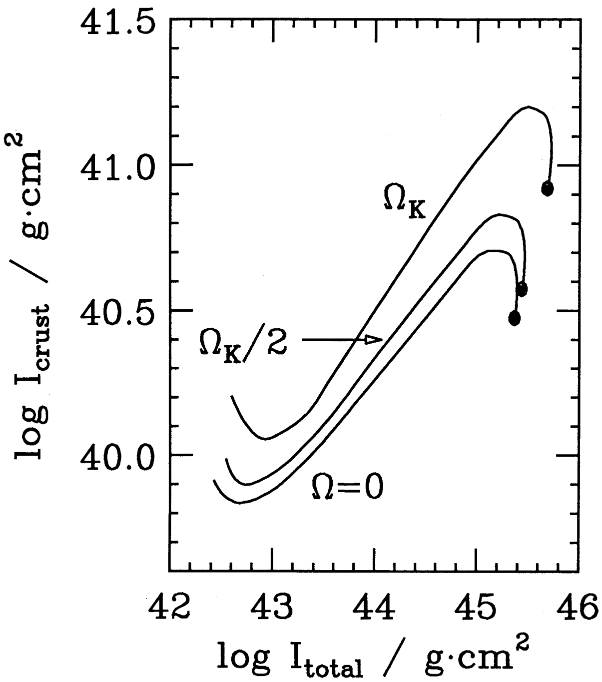

FIG. 23.—The moment of inertia of the hadronic crust vs. total moment of inertia for three rotational frequencies. The solid dots refer to the limiting-mass models. The bag constant is $B^{1/4} = 145$ MeV.

density as high as the neutron drip density. This is higher than the central density of white dwarfs. In the course of time, bare strange stars are likely to accrete nuclei, perhaps in proportion to the cosmic abundances in the interstellar medium, developing ever thicker crusts. While most of this material swept up by a strange star will consist of the lightest elements, in the presence of the gravitational field of the core, such material will undergo thermonuclear burning. For a core of a solar mass and radius of a few kilometers, the gravitational field is enormous compared to that found in normal stars that may be more massive but are also much much larger. The burning processes may therefore resemble those considered in the evolution of ordinary stars as they burn from hydrogen through to iron cores except for at least two major differences.

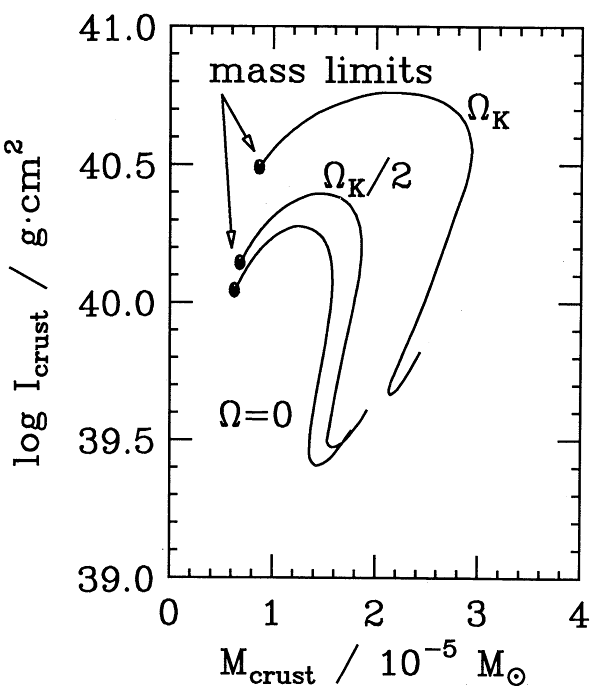

FIG. 22.—Same as Fig. 21, but for $B^{1/4} = 160$ MeV

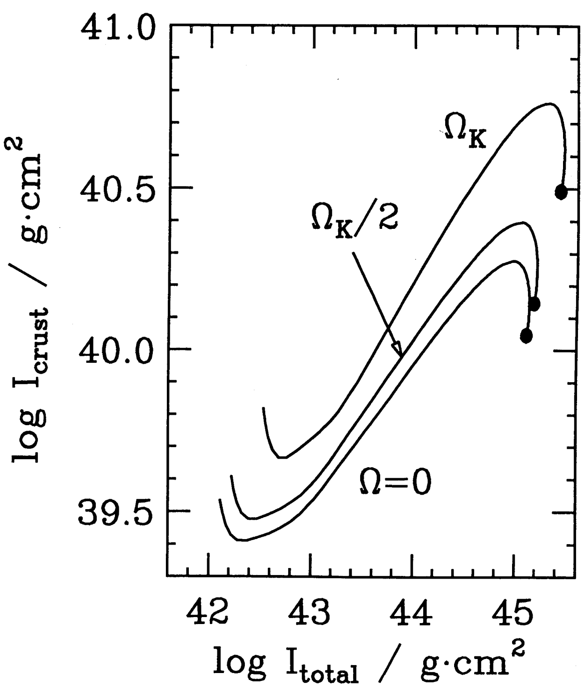

FIG. 24.—Same as Fig. 23, but for $B^{1/4} = 160$ MeV

As already remarked, the highest possible inner density of the crust is the neutron drip density, much higher than the density ever found in ordinary stars and even several orders of magnitude higher than that of white dwarfs. So the burning proceeds even further to exceedingly neutron rich nuclei. Second, the time scale for burning may be very fast by comparison, except it is likely to be limited by the accretion rate. There are obviously many interesting aspects of such a scenario to be investigated. They are not the subject of the paper but are mentioned so as to round out the discussion and suggest areas of study.

Our paper has focused very specifically on a particular class of strange stars, those for which the crust has the maximum possible density, the neutron drip density, higher than which neutrons would simply gravitate into the core and be converted to quark matter. Matter at the densities of the crust is a Coulomb lattice of iron and nickel nuclei all the way from the inner edge to the surface of the star. The sequence of stars has a minimum mass, including core and crust, which is ~ 0.015–$0.017\ M_{\odot}$ or about 15–17 Jupiter masses, and a maximum mass which is ~ 1.6–$2\ M_{\odot}$ for non-rotating stars, and ~ 1.8–$2.2\ M_{\odot}$ for those that are rotating at the Kepler frequency, depending on the bag constant. In each case above, the larger of the pair of numbers refers to the lower value of the bag constant and therefore to more deeply bound strange quark matter. The above quoted minimum mass is smaller than that of neutron star sequences, about $0.1\ M_{\odot}$ (Baym et al. 1971). Of course there are other sequences with a lower density crust for which the minimum mass star of the sequence may be even lower. We have not investigated these but conjecture that down to some minimum inner crust density they too will have a minimum mass below which a strange star with crust is not gravitationally stable. For crusts with densities in the white dwarf domain and strange quark cores of low mass, the stability will be essentially that of white dwarfs. All such stable low-mass objects may be of considerable importance since they may be difficult to detect and therefore may effectively hide baryonic matter. A gravitational lensing method of detection like that proposed for brown dwarfs may be feasible (Alcock et al. 1992). In fact, these objects, which have larger masses but smaller radii than Jupiters, may nonetheless be indistinguishable from them by gravitational lensing detection.

We have computed a number of properties of the nuclear solid crust that can be supported out of contact with a strange star as a function of mass of the star and of its rotational frequency. Roughly speaking, the mass and moment of inertia of the crust increases by about a factor 2–3 between zero frequency and the Kepler frequency. Between the same frequencies they vary by factors of about 5 and 15, respectively, through the sequence of stars from lightest to most massive.

Of interest to the subject of cooling of strange stars, the crust thickness for the strange star at the lower mass limit is hundreds of kilometers, decreases very rapidly with mass to ~ 12 km for very light strange stars of mass $\sim 0.02\ M_{\odot}$ (Fig. 6) and is a fraction of a kilometer for the star at the maximum mass. Its mass lies within a factor 5 of $10^{-5}\ M_{\odot}$ depending on core mass and rotation frequency and is about a factor of 2 larger for the smaller bag constant.

Perhaps of considerable relevance to the question of whether strange stars can exhibit glitches in rotation frequency, we find for the moment of inertia that $I_{\rm crust}/I$ varies between 10^{-3} for the lightest stars and $\sim 10^{-5}$ at the upper mass limit and differs by about a factor 2–3 depending on the bag constant. If the angular momentum of the pulsar is conserved in the quake then the relative frequency change and moment of inertia change are equal,

$$\frac{\Delta\Omega}{\Omega}=\frac{|\Delta I|}{I_0}>\frac{|\Delta I|}{I}\equiv\frac{fI_{\rm crust}}{I}\sim(10^{-5}\ \text{to}\ 10^{-3})f,\quad(0<f<1)\,, \tag{21}$$

where I_0 is the moment of inertia of that part of the star whose frequency is changed in the quake. It might be that of the crust only, or some fraction or all of the star, depending on how strongly the crust and core are coupled. The factor f in the above relation represents the fraction of the crustal moment of inertia that is altered in the quake, $|\Delta I|=fI_{\rm crust}$. Since observed glitches have relative frequency changes $\Delta\Omega/\Omega$ of 10^{-9} to 10^{-6}, a change in the crustal moment of inertia by less than one-tenth would cause a giant glitch even in the least favorable case. Of course that is not the whole story. There remain the questions of whether there can be a sufficient build-up of stress and also of the recoupling of crust and core which involves the healing of the pulsar period. This is probably a very complicated process that does not simply involve the recoupling of two homogeneous substances. The variation of the pressure through the core can be as large as five orders of magnitude (cf. Fig. 3) over which the properties of quark matter may vary considerably. The pressure variation of the crust from inner edge to the surface of the star is even greater.

We note that whereas in the crust quake model of neutron stars, the solid crust and an assumed almost purely *neutron* core are supposed to be weakly coupled by the magnetic field, in strange stars all particles (quarks and electrons) carry electric charge, so the crust and core or parts thereof should be rather strongly coupled. It is also worth noting that glitch phenomena in their magnitude, frequency of occurrence, and healing time, vary greatly from one pulsar to another. Indeed the rate of glitching of PSR 1737−30 is nearly an order of magnitude greater than any other known pulsar (McKenna & Lyne 1990). The possibility of highly individualistic behavior appears possible for crust quakes on strange stars because of the variation of the thickness and mass of the crust with the mass and frequency of the star, not to mention the temperature dependence of the shear modulus, the magnetic coupling between the crust and core with a possibly varying structural nature of the quark core with depth as well as the varying nature of the nuclear crystalline crust with height above the core. Of course many of these remarks pertain also to neutron stars.

The recoupling of crust and strange quark core is likely to have a long time scale, not unlike that expected for neutron stars (days to years; Manchester & Taylor 1977). Strange stars with crusts are likely to have a relatively short time scale coupling mechanism in addition, namely the abrasion of part of the inner surface of the crust by the core following a quake. Recall that the crust is suspended above the core by a few hundred fermis by the strong electric dipole layer on the core. If the relative change in the moment of inertia is attributed, for the sake of rough order of magnitude estimate, to a change in the radius of the star, then the relation $\Delta I/I = 2\Delta R/R$ holds. For the above quoted range of relative frequency glitches, we have $\Delta R \sim (10^{-3}$ to 1) cm. So we expect a quake in the crust to cause the crust to momentarily come into physical contact with the core, a contact or bouncing that is unlikely to be symmetrical, and in any case that will transfer matter and therefore angular momentum to the core. However, this abrasion of parts of the inner surface of the crust will be a momentary perturbation. The abraded material will be dissolved into the quark core. So superposed on the frequency change caused by the crust quake we suggest that there may be a smaller series of glitches associated with this transient mechanism.

Alpar (1987) argues against the existence of strange stars because of observed pulsar glitches. His argument is twofold. One has to do with the assumption that strange quark stars have no internal structural differentiation that could give rise to pulsar glitches so that a bare strange star could not glitch. Our opinion on this is that we simply do not know enough about quark matter to make such a ruling and have alluded to at least some of the other uncertainties above. Second, Alpar argues that even a strange star with a nuclear crust cannot glitch. This is based on assuming the near equality of $I_{\rm crust}/I$ and $\Delta\dot{\Omega}/\dot{\Omega}$ with the first ratio $\sim 10^{-5}$ by computation, and the second $\sim 10^{-3}$ to 10^{-2} by observation, yielding an apparent contradiction. However, using the same notation as in equation (21) we may write

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} \equiv \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega}\frac{|\Delta I|}{I_0} = \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega}\frac{f I_{\rm crust}}{I_0} > (10^{-1} \text{ to } 10)f \quad (22)$$

yielding a small f as before, $f < (10^{-4}$ to $10^{-1})$. (See also Baym et al. 1969, eq. [5] and Manchester & Taylor 1977, 193.) We have used measured values of the ratio $(\Delta\Omega/\Omega)/(\Delta\dot{\Omega}/\dot{\Omega}) \sim 10^{-6}$ to 10^{-4} for the Crab and Vela pulsars, respectively. (See Manchester & Taylor 1977, table on 118.) So the observed range of the fractional change in $\dot{\Omega}$ is consistent with the crust having the small moment of inertia calculated and the quake involving only a small fraction, f, of that, just as in equation (21). Nevertheless, without undertaking a study of whether the nuclear solid crust on strange stars could sustain a sufficient buildup of stress before cracking to account for such a sudden change in relative moment of inertia, or whether the healing time and intervals between glitches can be understood, we cannot say definitely that strange stars with a nuclear solid crust can account for any complete set of glitch observations for a particular pulsar. However, we have laid part of the ground work for such an assessment. And we have shown that quite plausible fractional changes in the crustal moment of inertia can account for the magnitudes of glitches. Nevertheless, we should keep in mind that the problem of quakes or rotational timing glitches on compact stars, *whether* neutron stars or strange stars, does not appear to be simpler in nature than the prediction of earthquakes, a science with a notable dearth of successes!

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Note added in manuscript.—The referee drew our attention to a thesis by P. F. C. Romanelli (1986, Massachusetts Institute of Technology), in which the author computes some static configurations of a nuclear crust on a strange core of all stable masses of the core, including those with very thick crusts and small cores.

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