THE ASTROPHYSICAL JOURNAL, 396:364–368, 1992 September 1

MAXIMUM PRINCIPAL QUANTUM NUMBERS OF THE ATOMIC HYDROGEN IN THE SOLAR CHROMOSPHERE AND PHOTOSPHERE

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Received 1991 December 9; accepted 1992 March 6

ABSTRACT

The maximum principal quantum numbers n_m and the lowering of the continuum energy of hydrogen atoms in the solar chromosphere and photosphere are calculated. Several theories are discussed and the work focuses on two major mechanisms of the lowering of the continuum (the reduction of the ionization energy and the line merging). It was found that under the physical conditions existing in the solar chromosphere and photosphere the maximum principal quantum number varies, depending on the plasma temperature and density, from $n_m = 11$ to $n_m = 62$.

Subject headings: atomic processes — Sun: chromosphere

1. INTRODUCTION

The potential distribution in and around an atom immersed in a plasma is influenced by its bound electrons, free electrons, and free ions, and by bound electrons of other particles. Any atom exposed to these interactions requires less energy to be ionized (the so-called *reduction of the atomic ionization energy*) than the energy needed for ionization of the atom in vacuum. In addition, the interaction of the atom with other particles of the plasma causes the atomic energy levels to broaden increasingly toward states of higher excitation. The higher levels will eventually broaden enough to merge (the so-called *line merging*), before the series limit ($n \to \infty$; n is the atomic principal quantum number) is reached. If we denote the series limit (equal to the ionization energy of the ground-state atom) by E_0 then the last level below the atomic continuum (a result of either the reduction of the ionization energy or the line merging) will have energy E_m (with respect to the energy of the atomic ground state) equal to

$$E_m = E_0 - \Delta E_m , \quad (1)$$

where ΔE_m represents the lowering of the continuum energy by one of the two effects. The principal quantum number n corresponding to E_m can then be calculated in straightforward way.

The reduction of the ionization potential and the line merging are complex problems. All existing theories exclude external field and boundary effects and they assume that the gas contains only "free" particles (free electrons, ions and neutrals) and "bound" particles (the electrons and nuclei in the heavy particles). Even though such classification includes most of the plasma particles it is incomplete, because there can be a group of particles which are neither completely free nor completely bound, and which can be treated in terms of weak or strong pair correlations. (However, a satisfactory quantum-mechanical theory for the kinetics of particles with higher order correlations is not available). In addition, the electrons can be "free" for one phenomenon (for example, pressure) but not necessarily so for another (for example, conductivity). Given the additional difficulty in describing the electric potentials in vicinity of interacting particles, it is clear that the models of the reduction of the atomic ionization energy and of the line merging have an obvious uncertainty in their definitions and in their numerical results. Because of this, we discuss below several of such models most frequently used in studies of astrophysical and laboratory plasmas of low and moderate densities. (In dense plasmas, mechanisms other than those discussed here must be included; see Hummer & Mihalas 1988; Dappen, Anderson, & Mihalas 1987; Rogers 1986; Zimmerman & More 1980).

Knowledge of the maximum principal quantum number n_m of hydrogen atoms in the solar chromosphere and photosphere is important for studies of thermal properties (partition functions), and for optical diagnostics of the plasma. The range of the plasma conditions in the solar chromosphere and photosphere is as follows:

$$4000 \text{ K} \lesssim T \lesssim 8000 \text{ K} , \quad (2)$$

$$10^{11} \text{ cm}^{-3} \lesssim N_{\text{H}} \lesssim 10^{17} \text{ cm}^{-3} , \quad (3)$$

$$10^{10} \text{ cm}^{-3} \lesssim N_e \lesssim 10^{13} \text{ cm}^{-3} , \quad (4)$$

$$10^{-3} \lesssim x = N_e/N_{\text{H}} \lesssim \tfrac{1}{2} , \quad (5)$$

where T is the temperature, and N_{H} and N_e are densities of the hydrogen atoms and electrons, respectively. The fact that the ratio x in the chromospheric and photospheric plasma is not small allows one to consider the plasma electric charges as the dominant group of perturbers causing the reduction of the atomic ionization and the merging of the lines. In other words, the charged particle-atom interaction are more important for the lowering of the atomic continuum than atom-atom collisions. One should also add that the assumption of electric neutrality of the chromospheric and photospheric plasmas is a good one, and that the density of singly-ionized ions in the plasma is much higher than density of multiply-ionized ions. Therefore, we assume in our calculations that $N_e \approx N_+$ (N_+ is

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the density of singly ionized ions) so that the total density of charged particles in the plasmas is taken as $N \approx N_e + N_+ \approx 2N_e$.

2. REDUCTION OF IONIZATION ENERGY

2.1. *Debye-Huckel (DH) model*

This model is based on the assumption that the presence of electric charges around an atom weakens (because of the Debye-Huckel screening) the bond between any orbital electron and the rest of the atomic electrons and the nucleus. Using the Debye-Huckel potential, the reduction in the binding (ionization) energy of the atom immersed in plasma can be estimated as (see, for example, Griem 1964 and Weisheit 1975)

$$\Delta E_m \approx \frac{e^2}{r_D} , \quad (6)$$

where e is the elementary charge, and r_D is the Debye radius,

$$r_D = \left(\frac{kT}{4\pi e^2 N}\right)^{1/2} . \quad (7)$$

2.2. *Steward-Pyatt (SP) Model*

Steward & Pyatt (1966) formulated a model for the reduction of the ionization energy due to the presence of free electrons and ions using the finite-temperature Thomas-Fermi potential for the average electrostatic potential near nuclei of the plasma particles. They regarded bound electrons as part of the unperturbed ion, and the plasma electrons were decribed by nonrelativistic Fermi-Dirac statistics, while the ions were described by nonrelativisitic Maxwell-Boltzmann statistics. It was also assumed that the bound electrons do not contribute to the reduction of the ionization energy. The reduction of the ionization energy of an atom is given in the SP model as

$$\Delta E_m = \left[\left(\frac{6e^2}{r_D kT} + 1\right)^{2/3} - 1\right]\frac{kT}{4} . \quad (8)$$

2.3. *Ecker-Kroll (EK) Model*

Using the semiclassical approximation and combination of statistical and thermodynamic approaches Ecker & Kroll (1963) formulated a generalized Saha equation as a function of the Helmholtz free energy and, consequently, as a function of the chemical potential of the plasma. The electrostatic contribution to the chemical potential was then expressed in terms of the average electrostatic micropotentials (obtained from a solution of the Poisson equation) at a given particle. The final expression of the EK model for the reduction of the ionization energy of an atom in plasma in local thermal equilibrium was given as

$$\Delta E_m = \begin{cases} e^2/r_D & \text{when} \quad N \le N_{cr} \\ G_k e^2/d_e & \text{when} \quad N > N_{cr} \end{cases} , \quad (9)$$

where

$$d_e = (3/4\pi N)^{1/3} , \quad (10)$$

$$G_k = \frac{2.2(2N_{cr} e^2)^{1/2}}{N_{cr}^{1/3}(kT)^{1/2}} , \quad (11)$$

and the critical density N_{cr} is

$$N_{cr} = \frac{3}{4\pi}\left(\frac{kT}{e^2}\right)^3 . \quad (12)$$

2.4. *Duclos-Cambel (DC) Model*

Duclos & Cambel (1961) reexamined the earlier model of Ecker & Weizel (1958) using some properties of interactions of ions with small fluctuations of electric field in electrolytes. Their final expression for the reduction of the ionization energy was

$$\Delta E_m = \begin{cases} (1 + \sqrt{2})e^2/(2\sqrt{2}r_D) & \text{when } N \le N_{cr} \\ 6.45e^2N^{1/3} - \frac{1}{2}kT \ln (1.845e^2N^{1/3}/kT) - \frac{3}{2}kT & \\ & \text{when } N > N_{cr} . \end{cases} \quad (13)$$

3. MERGING OF THE ENERGY LEVELS

3.1. *Inglis-Teller (IT) Model*

The main assumption of this model was that the merging of the broadened lines occurs at a principal quantum number, where the splitting of the Stark levels in the field of the perturbing (charged) particles is equal to about half the energy difference between two neighboring terms. The maximum principal quantum number n_m and the energy ΔE_m predicted by the Inglis-Teller (1939) theory are, respectively,

$$\log_{10} n_m = 3.10 - 0.13 \log_{10} N , \quad (14)$$

$$\Delta E_m = \frac{\text{Ry}}{n_m^2} = 3.15 \text{ Ry } a_0^{4/5} N_e^{4/15} , \quad (15)$$

where Ry is the Rydberg energy (13.6 eV), a_0 is the Bohr radius (5.29×10^{-9} cm), the electron density N_e is given in cm^{-3}, and where N (in cm^{-3}) is the total density of charged particles in the plasma; it was also assumed in the model that the plasma is electrically neutral in the neighborhood of the atom under consideration, which led to requirement

$$T < \frac{4.6 \times 10^5}{n_m} \text{ K} . \quad (16)$$

The main weakness of formula (14) is that the splitting of the Stark levels was simplified in the IT model by using an average splitting for all Stark components, and that a constant (mean) value of the plasma microfields in the plasma was used. Thus, the IT model neglects that fact that the distribution of the microfields depends on the plasma temperature and density. Despite this, the Inglis-Teller theory is still approximately valid (see discussion below). The theory was originally tested (successfully) against the data obtained by Mohler (1939) on the maximum principal quantum numbers observed for line series produced in a cesium discharge. (Mohler's measurements were made over a wide range of densities at temperature $kT = 0.43$ eV). The Inglis-Teller results show also good agreement with the experimental results of Boldt (1959) for line merging in nitrogen and oxygen over the temperature range from 10,500 to 13,000 K.

3.2. *Armstrong (AR) Model*

Armstrong (1964) proposed a line merging theory assuming that the intensity width of high levels determines the merging and that the merging is dominated by electron impact broadening; ion contribution was neglected. Furthermore, he assumed that it is more reasonable to use the electron impact width of the central Stark components than the quasi-static width for both ions and electrons (see Griem 1962). This is different from the width used by Inglis & Teller which was the width between outermost Stark components of a line. Therefore, merging may be predicted when the line cores are still identifiable if the Inglis-Teller Stark component width is used. Subsequently, Armstrong used the core profile as a major factor in impact broadening at large principal quantum numbers. Resulting formulae for the principal quantum number n_m and the energy ΔE_m are, respectively,

$$\log_{10} n_m = 3.213 + 0.071 \log_{10} kT - 0.143 \log_{10} N_e , \quad (17)$$

$$\Delta E_m = 4.707 \times 10^{-6}\left(\frac{N_e^2}{kT}\right)^{1/7} , \quad (18)$$

where N_e is in cm^{-3}, and kT and ΔE_m are in eV. This formula gives results which are close to the Inglis-Teller results and to the experimental data of Mohler (1939) and Boldt (1959). The criterion of validity of the impact approximation used by Armstrong was given by Baranger (1962) as

$$n < \left(\frac{3.8}{N_e \lambda_T^3}\right)^{1/6} , \quad (19)$$

where n is the principal quantum number of the perturbed state, and $\lambda_T = (h^2/2\pi m_e kT)^{1/2}$ is the thermal deBroglie wavelength. One should notice that even though the Inglis-Teller theory gives numerical results close to the results of the Armstrong electron-impact theory the former results do not show temperature dependence.

3.3. *Vidal (VI) Model*

Vidal (1966) studied the importance of the distributions of plasma microfields, focusing on improving the calculations of the intensity distributions in the region of the line merging. His method assumes the quasi-static profiles of the single lines taking into account only the linear Stark effect. Subsequently, the intensity distribution in the region of the merging is obtained by summing up the different line profiles.

The Vidal distributions of the microfields take into account the interaction of the perturbing particles, and the shape of the distributions, and therefore of the quasi-static profiles, depends on the ratio d_e/r_D (as before d_e is the mean distance between the perturbing particles and r_D is the plasma Debye radius). Several aspects of the line broadening were considered in the Vidal approach. First, the impact of the Doppler broadening is accounted, neglecting the microturbulent speed. Since for the higher series members the Doppler broadening main effect is smearing out the line center, the line center was smeared out, maintaining requirement of normalization, during Vidal's calculations; validiy of such procedure was verified by measurements (Vidal 1964, 1965) of the profiles of the higher members of the Balmer and Paschen series. Another issue studied in Vidal's work were the contribution of the quadratic Stark effect (see also discussion in Bethe & Salpeter 1957) and of the quadrupole interactions of the perturbers. It was found that both the effects can be neglected. (It turned out that numerical accuracy better than 2% is achieved in final calculations if the quadratic Stark effect is neglected as long as the electron density is smaller than $\sim 10^{17}$ cm^{-3}). A comparison of the Vidal (1966) predictions of the hydrogen line profiles with his measurements (Vidal 1964, 1965) of the profiles (at $N_e = 1.3 \times 10^{13}$ cm^{-3}) showed that for an accuracy of measurements of 3% the quasi-static theory is acceptable as long as the requirement (16) is met. In summary, one can say that the accuracy (confirmed by experimental data of Vidal 1966 and references within) of the Vidal approach in calculating N_e is better than 5% as long as the requirement (16) is met and the electron density is less than $\sim 10^{17}$ cm^{-3}. (The accuracy 5% in calculating N_e means an accuracy better than about half percent in calculating n_m if N_e is known).

For Balmer lines, the ratio of n_m resulting from the Vidal model to n_m obtained from the Inglis-Teller model can be given as

$$\frac{(n_m)_{VI}}{(n_m)_{IT}} \approx 1 + \frac{d_e}{6r_D} . \quad (20)$$

Applicability of the discussed models for the line merging is limited to plasmas fulfilling the requirements (16) and (19). These requirements can be rewritten, respectively, as

$$\lambda_1 < 1 \quad \text{and} \quad \lambda_2 < 1 , \quad (21)$$

where

$$\lambda_1 = 2 \times 10^{-6} n_m T , \quad (22)$$

$$\lambda_2 = 0.25 N_e \lambda_{T^3} n_m^6 , \quad (23)$$

where T is in kelvins, N_e in cm^{-3} and λ_T in centimeters. The N_e- and T-dependence of the parameters λ_1 and λ_2 is shown in Figure 1. As can be seen from there, the chromospheric and photospheric plasmas meet requirement (21).

4. RESULTS AND DISCUSSION

Values of ΔE_m obtained from different models at $T = 4000$ K and 8000 K are given in Figures 2 and 3, respectively. As can be seen from there, ΔE_m is a small fraction of the ionization energy E_0 of the ground-state hydrogen atom. The

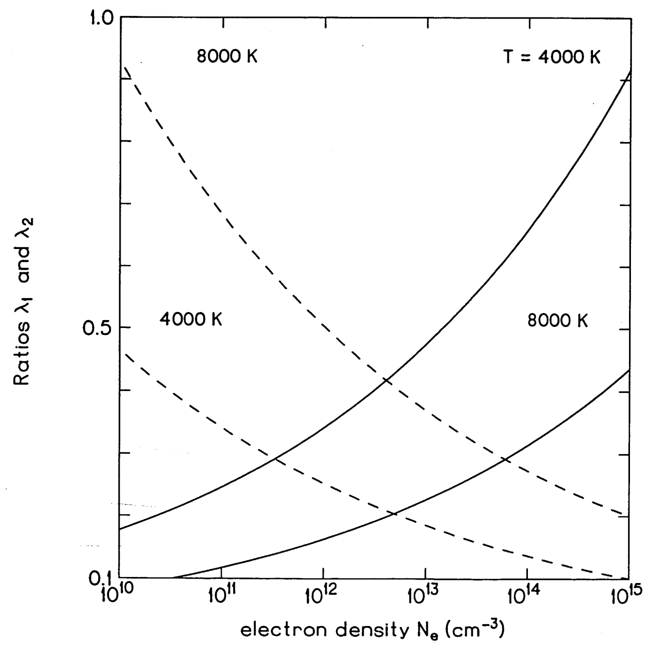

FIG. 1.—The ratios λ_1 (*solid curves*) and λ_2 (*dashed curves*) as functions of N_e for temperatures $T = 4000$ and 8000 K.

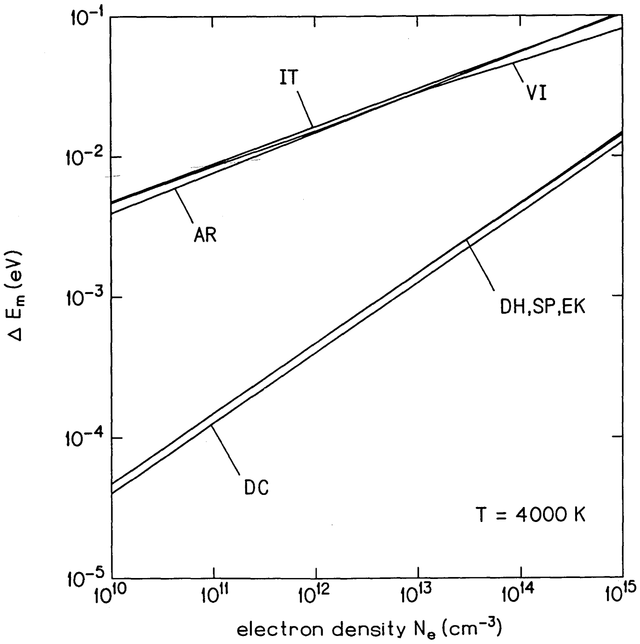

FIG. 2.—*The lowering* ΔE_m of the continuum of atomic hydrogen, as a function of electron density N_e at plasma temperature $T = 4000$ K. DH, SP, EK, and DC denote the Debye-Huckel, Steward-Pyatt, Ecker-Kroll, and Duclos-Cambel models for reduction of the ionization potential of the atom respectively. VI, AR and IT denote the results predicted by the line-merging theories of Vidal, Armstrong, & Inglis-Teller, respectively.

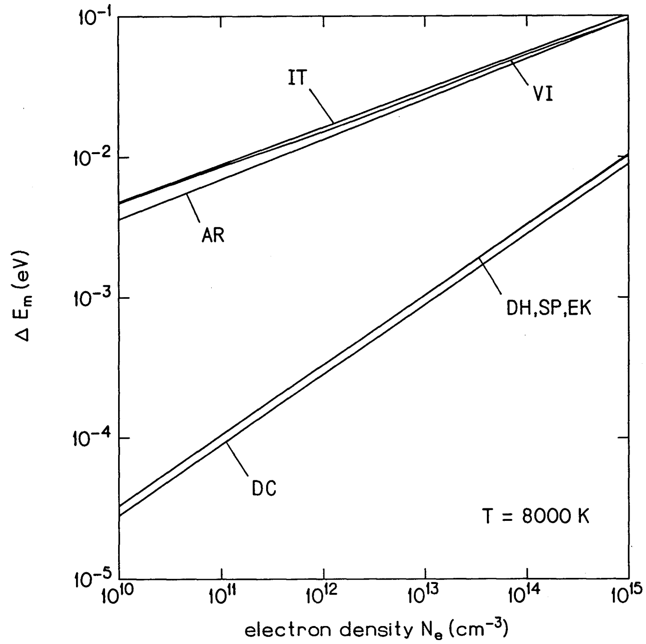

FIG. 3.—The lowering ΔE_m of the continuum of atomic hydrogen, as a function of electron density N_e at plasma temperature $T = 8000$ K. DH, SP, EK, and DC denote the Debye-Huckel, Steward-Pyatt, Ecker-Kroll, and Duclos-Cambel models for reduction of the ionization potential of the atom, respectively. VI, AR, and IT denote the results predicted by the line-merging theories of Vidal, Armstrong, and Inglis-Teller, respectively.

N_e-dependence of ΔE_m is stronger than the T-dependence (Typically, ΔE_m decreases by about factor 0.7 as T increases from 4000 to 8000 K).

The N_e-dependence of the quantum number n_m in hydrogen atoms is shown, for $T = 4000$ K and 8000 K, in Figures 4 and 5, respectively. All the three models of the line merging give results close to each other. It seems, however, that the Vidal (VI) model may be the most appropriate when the merging of the hydrogen Balmer lines is used to determine the chromospheric and photospheric electron density; then, the accuracy of determining N_e should be better than 5%.

The results given in Figures 2–5 indicate that the EK model may be the most appropriate, in the considered range of plasma density and temperature, for calculating the reduction of the atomic ionization energy. This conclusion agrees with the results of a study by Ecker & Kroll (1963) (who made comparison of their model with earlier models by Unsold [1948], Abe [1959], Vedenov & Larkin [1959], Brunner [1960], & Green [1961]) and with an analysis of Sweeney (1978). It was also shown in the latter study that the SP model is thermodynamically inconsistent with the Maxwell identities for thermodynamic functions, and it was concluded that such inconsistencies can produce significant errors. One should also add that there is a common agreement (see Hummer & Mihalas 1988) that neglecting of the nonideal plasma effects in the models of reduction of the ionization energies is well justified in such plasma.

One should mention that the present results, evaluated for uniform plasma in local thermal equilibrium will also be valid in nonequilibrium models of the chromospheric and photospheric plasmas. This is because the hydrogen energy levels in the region of the line merging (and above) are usually in thermal equilibrium with the free electron continuum. Also, a higher state will not exist in a hydrogen atom if the ionization frequency of the state is so high that the life time of the state is less than its orbital period ($\tau_n = 1.52 \times 10^{-16} n^3$ s). In addition, Trubnikov & Yavlinskii (1965) calculated the Debye radius at which there are no higher bound states in atomic hydrogen in

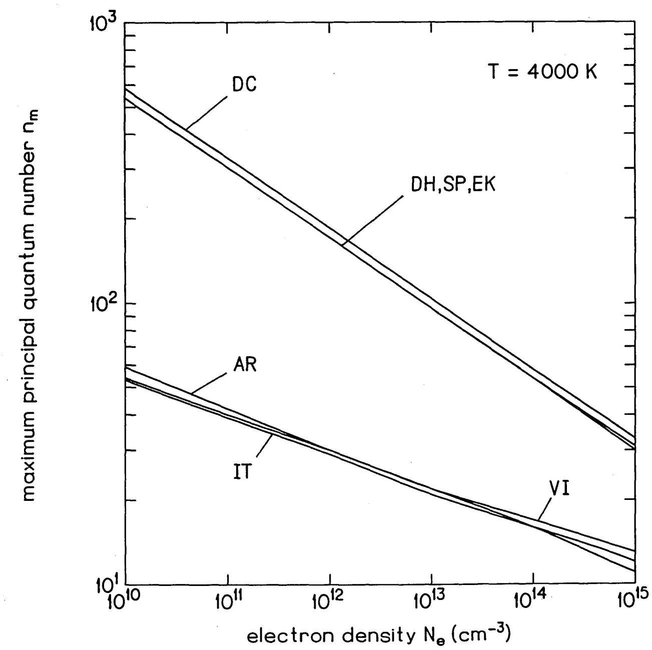

FIG. 4.—The maximum quantum number n_m of the hydrogen atoms in the chromospheric and photospheric plasma as predicted by the models discussed in the text ($T = 4000$ K). Meaning of the symbols is the same as in Fig. 2.

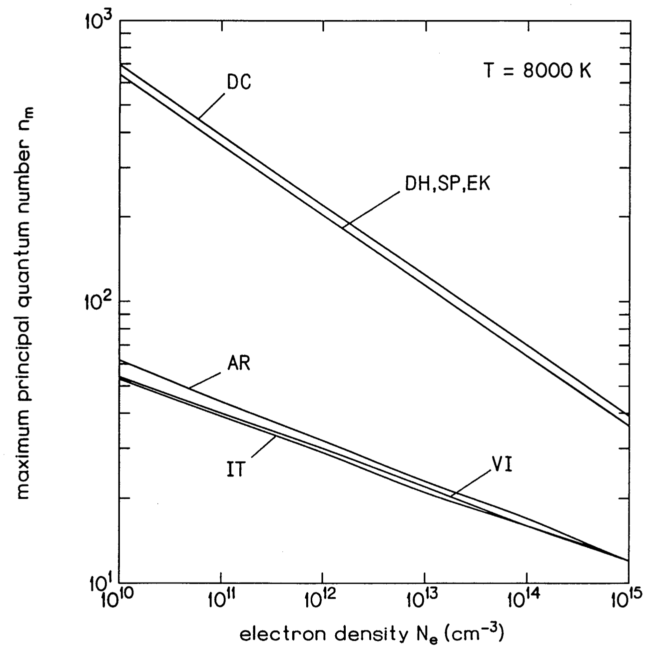

FIG. 5.—The maximum quantum number n_m of the hydrogen atoms in the chromospheric and photospheric plasma as predicted by the models discussed in the text ($T = 8000$ K). Meaning of the symbols is the same as in Fig. 2.

presence of the Debye-Huckel potential; such states do not exist if

$$r_D < 0.84a_0 = 0.44 \text{ Å} . \quad (24)$$

It should be noted that in the chromospheric and photospheric plasma the atomic level $(E_m)_{lm}$ at which the lines merge is always below the highest level $(E_m)_{red}$ resulting from the reduction of the atomic ionization energy. It should be emphasized, however, that not the former but rather the latter level must be taken as the highest level in calculations of the statistical sums of the atomic internal partition functions. This is because the atom excited to a level between $(E_m)_{lm}$ and $(E_m)_{red}$ is still stable; that is, it still requires some perturbation in order to reach the free electron continuum. This differs from the "status" of the levels between $(E_m)_{red}$ and E_0 (ionization limit in vacuum), because the atom excited to these levels is truly unstable. Thus, knowledge of the level $(E_m)_{lm}$ is of importance for the spectral studies of the plasma, but not for determination of the plasma equilibrium partition functions.

Ending, one should mention an important issue resulting from the fact that the lowering of the continuum would be different for different species of the plasma, while physically there can be just one plasma continuum. This yet-unsolved problem is of no consequence for spectroscopy, but it requires careful examination in studies of thermodynamic models of plasmas.

We thank Eugene Avrett, Werner Däppen, and Hans Griem for valuable comments.

W. H. S. appreciates the support of Sallie Baliunas during the work. This work was supported by the National Science Foundation through a grant for the Institute for Theoretical Atomic and Molecular Physics at Harvard University and Smithsonian Astrophysical Observatory, and a grant from the Scholarly Studies Program and Langley-Abbot fund of the Smithsonian Institution.

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