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ROTATIONAL PROPERTIES OF STRANGE STARS

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ABSTRACT

In this paper, we present results from an investigation of the rotational properties of strange stars, using models with a canonical value of the bag constant. The changes in structure resulting from uniform rotation have been calculated within the slow rotation regime and the minimum rotation periods consistent with stability to nonaxisymmetric perturbations have also been calculated. The minimum period is found to be set by the onset of instability in either the $m = 2$ or $m = 3$ mode. The first of these modes, which is probably inaccessible to standard neutron stars, may be the critical one for old strange stars spun up by accretion and this could be of importance in giving an observational test for distinguishing between strange stars and standard neutron stars.

Subject headings: dense matter — relativity — stars: rotation

1. INTRODUCTION

Following the suggestion by Witten (1984) that macroscopic strange quark matter might be absolutely stable and that neutron stars might rapidly convert into "strange stars" after formation, a number of authors have carried out investigations of the properties which these strange stars would have. (The term "strange quark matter" signifies a mixture of roughly equal numbers of up, down, and strange quarks together with a sufficient number of electrons to give electrical neutrality, and Witten conjectured that this might have lower energy per baryon than matter composed of ordinary nuclei even under conditions of low temperature and pressure.) From the studies by Witten (1984), Baym et al. (1985), Haensel, Zdunik, & Schaeffer (1986), and Alcock, Farhi, & Olinto (1986) a detailed picture has emerged of the structure and properties of nonrotating strange star models. The low-mass ones are essentially Newtonian objects with almost constant density and are held together mainly by the strong force rather than by gravity. (For these, the mass is roughly proportional to the cube of the radius, in sharp contrast with the situation for ordinary neutron stars where the mass is a decreasing function of radius.) However, for masses near to 1.4 $M_{\odot}$ (which seems to be "canonical" for those pulsars whose mass has been directly determined) the radii of the strange star models are comparable with those for conventional neutron stars and it is necessary to use the general relativistic equations for calculating their structure. The mechanisms by which neutron stars could convert into strange stars have been discussed by Alcock, Farhi, & Olinto (1986) and Olinto (1987).

Stimulated by the reported observation of a half-millisecond pulsar within the remnant of supernova SN 1987A (Kristian et al. 1989), interest later focused on the degree of rapid rotation which a strange star could support (Haensel & Zdunik 1989; Frieman & Olinto 1989; Glendenning 1989a, b; Lattimer et al. 1990). The subsequent withdrawal of the observational claim for the existence of this fast pulsar has now removed some of the urgent motivation for these studies but, nevertheless, the rotational properties of strange stars remain of interest if they are to be considered as serious models for pulsars. A persistent problem for the strange star picture (Alpar 1987; Caldwell & Friedman 1991) concerns the present lack of any model for explaining how strange stars could account for observed pulsar glitches. However, it is not clear how serious a difficulty this is (Alcock & Olinto 1988) and we take the view that strange star models are certainly worth further investigation.

In the present paper we concentrate on two questions: (1) what are the overall rotational properties of strange star models and to what extent do they differ from those of ordinary neutron stars? (2) If a strange star were to rotate fast enough for nonaxisymmetric instability modes to become excited, does the onset of these differ from the situation for neutron stars? In considering the first of these questions, we note that for all pulsars so far observed, the rotation may be considered as slow in the sense that rotational perturbations away from the structure of a nonrotating comparison model are always quite small. This allows rotational properties to be calculated by means of a perturbation method (Hartle 1967) which is much easier than solving the full equations (Friedman, Ipser, & Parker 1986) and gives results which can be conveniently scaled for different values of the rotational velocity. A sufficient requirement in order for the slow rotation approximation to be valid is that the ratio of centrifugal force to gravitational force should be small for all elements of the model. For a particle of unit mass moving with angular velocity Ω around a spherically symmetric nonrotating object of mass M_o and at distance r from its center, a general relativistic expression for the centrifugal force acting is given by

$$F_C = r\Omega^2 \frac{(1 - 3M_o/r)}{(1 - 2M_o/r - r^2\Omega^2)}, \tag{1}$$

(Abramowicz & Prasanna 1990, Abramowicz & Miller 1990; throughout this paper, we use units for which $c = G = 1$ except where otherwise stated.) For our slow rotation condition, we will require that the value given by equation (1) for an equatorial element of material (with r set equal to the radius of the model star R_o) should be small compared with the corresponding gravitational force (as given by $F_G = M_o/R_o^2$ both in New-

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tonian theory and in general relativity). Taking $M_o = 1.4\ M_\odot$ and $R_o = 11$ km (appropriate for strange matter with bag constant $B = (145\ \text{MeV})^4$ and vanishing mass for the strange quark—here the units used are ones for which $c = \hbar = 1$) then gives $F_C/F_G = 0.08$ for the fastest observed pulsars (with period $P = 1.6$ ms) indicating that a slow rotation approximation should give quite accurate results. (Note that for $P = 0.5$ ms, the ratio F_C/F_G obtained as above is 1.19, indicating that the slow rotation condition would be violated in that case as is already well known.) In a recent paper (Weber, Glendenning, & Weigel 1991), it has been shown that the slow rotation technique can also be used to obtain quite accurate results for some quantities even outside its strict range of validity. We note this point but will not pursue it here.

In discussing the second question, the instability modes which we will be concerned with are the Dedekind modes driven by gravitational radiation reaction and moderated by viscosity. The coefficient of bulk viscosity in strange matter is very large, and it has been suggested that this would lead to complete suppression of these instabilities for strange stars at all temperatures above 10^8 K (Sawyer 1989). We have now made detailed calculations (using a computer code constructed following the strategy of Lindblom and coworkers: Lindblom 1986; Cutler & Lindblom 1987; Ipser & Lindblom 1991) and find that the effect of bulk viscosity does not turn out to be sufficiently large to give the suppression predicted.

In this work, we consider models consisting only of strange matter and do not consider properties of a possible thin crust of ordinary material (for a discussion of this, see Alcock et al. 1985). We note here only that such a crust would be very thin and that, while it would greatly affect the surface properties and thermal behavior of the strange star, it would not greatly affect the properties which we are concerned with here. The plan of the paper is as follows. In § 2 we briefly review the properties of strange quark matter which are relevant for the present calculations; § 3 reviews Hartle's slow rotation technique (Hartle 1967) for calculating equilibrium models; § 4 describes the equilibrium properties of strange star models constructed in this way; §§ 5 and 6 are concerned with the onset of nonaxisymmetric instabilities, and § 7 contains discussion and conclusions.

2. EQUATION OF STATE OF STRANGE QUARK MATTER

Strange quark matter would consist of nearly equal numbers of up (u), down (d), and strange (s) quarks together with electrons (e) which give overall charge neutrality. For the conditions of interest here, the matter can be treated as cold since the temperature is always much smaller than the chemical potentials of the particles present. The three flavors of quarks and the electrons maintain chemical equilibrium via the weak interaction processes

$$d \to u + e + \bar{\nu}_e\ , \quad u + e \to d + \nu_e\ , \quad s \to u + e + \bar{\nu}_e\ ,$$
$$u + e \to s + \nu_e\ , \quad s + u \leftrightarrow d + u\ , \tag{2}$$

and the equilibrium composition is determined by the conditions of chemical equilibrium and charge neutrality. In the MIT bag model for confinement (Chodos et al. 1974) which we will use here, confinement is represented by having a false vacuum within the quark region which contributes a constant negative pressure $-B$, reducing the total pressure p of the medium below that associated with the individual quarks and electrons. The quantity B (the "bag constant") also appears as a positive contribution to the energy density ρ. There is a minimum value for the density of quark matter at which the pressure goes to zero. At densities very much greater than this, the quarks behave as free ultrarelativistic particles and the equation of state is soft ($p \to \rho/3$) but as ρ tends towards its minimum value, the effect of confinement forces becomes significant and the equation of state becomes much stiffer. We here include perturbative corrections to first order in the strong interaction coupling constant α_c and for the purposes of the equation of state we take zero mass of the strange quark ($m_s = 0$; in which case there are equal numbers of up, down, and strange quarks and no electrons). This gives

$$p = \tfrac{1}{3}(\rho - 4B) \quad \text{for } \rho \geq 4B\ . \tag{3}$$

A more extensive description of quark matter can be found in the articles by Farhi & Jaffe (1984), Haensel et al. (1986), and Alcock et al. (1986). Some previous authors (e.g., Frieman & Olinto 1989; Haensel et al. 1986) have used equations of state including finite masses for the strange quark. We here restricted attention to the case $m_s \to 0$ as far as the equation of state (eq. [3]) is concerned in order to simplify the presentation but take $m_s = 100$ MeV when calculating the bulk viscosity (see below).

In the following sections, we will use equation (3) as our microphysical model for the interior of strange stars, taking $B = 10^{14}$ g cm^{-3} [which is closely equivalent to $(145\ \text{MeV})^4$ or 56 MeV fm^{-3}]. The baryon number density n_B and the relativistic adiabatic index Γ are given by

$$n_B = \left[\frac{4(1 - 2\alpha_c/\pi)^{1/3}}{9\pi^{2/3}\hbar}(\rho - B)\right]^{3/4}\ , \tag{4}$$

$$\Gamma = \frac{n_B}{p}\frac{dp}{dn_B} = \frac{4}{3}\frac{(\rho - B)}{(\rho - 4B)}\ . \tag{5}$$

At very high densities, Γ approaches the critical value 4/3 but it increases dramatically at lower densities, due to confinement forces, and tends to infinity as $\rho \to 4B$. In § 4 below, we give values of the pressure averaged adiabatic index $\bar{\Gamma}$ for the strange star models described, in order to indicate the effective stiffness of the material.

For the studies of the onset of nonaxisymmetric instabilities which we will be discussing in § 6, it is necessary to consider the ways in which oscillations can be damped. Two of the relevant mechanisms are shear and bulk viscosity. An approximate expression for the shear viscosity of strange matter (to lowest order in α_c) is given by (Haensel & Jerzak 1989)

$$\eta = 7.0 \times 10^{15}\left(\frac{\alpha_c}{0.1}\right)^{-3/2}\left(\frac{T}{10^{10}\ \text{K}}\right)^{-2}\left(\frac{n_B}{n_{B0}}\right)^{5/3}\ \text{g cm}^{-1}\ \text{s}^{-1}\ , \tag{6}$$

where n_{B0} is the nuclear matter density ($n_{B0} = 1.7 \times 10^{38}$ cm^{-3}). By comparison with numerical computations of the full expression for η presented by the same authors, it is found that expression (6) consistently underestimates η by a factor of about 2.5. In our calculations we have used expression (6) multiplied by this factor and have taken $\alpha_c = 0.15$. For strange matter η has the same temperature dependence as for ordinary neutron star matter ($\propto T^{-2}$) but is typically larger for equivalent density and temperature.

Bulk viscosity in strange matter arises as a consequence of the slowness of weak interactions in changing the concentrations of the u, d, and s quarks as matter is successively compressed and rarefied. Calculations of bulk viscosity have been carried out by Sawyer (1989) and a convenient analytic fit to his numerical results is given by

$$\zeta = 1.20 \times 10^{27}\left(\frac{n_B}{n_{B0}} - 0.18\right)\left(\frac{T}{10^{10}\ \mathrm{K}}\right)^2\left[\left(\frac{\sigma_o}{10^4\ \mathrm{rad\ s^{-1}}}\right)^2 + 4.9\left(\frac{n_B}{n_{B0}}\right)^2\left(\frac{T}{10^{10}\ \mathrm{K}}\right)^4\right]^{-1} \mathrm{g\ cm^{-1}\ s^{-1}}\,, \quad (7)$$

where σ_o is the oscillation frequency as measured in the local mean rest frame of an element of stellar fluid. This expression (which is appropriate for $m_s = 100$ MeV) follows the one given by Cutler, Lindblom, & Splinter (1990), The temperature dependence of ζ is quite different from that for neutron star matter (for which $\zeta \propto T^6$), and the values of ζ are very much larger in the case of strange matter. It does not necessarily follow, however, that the effects of bulk viscosity in strange matter will be overwhelming. These effects depend both on the value of ζ and on the degree of compressibility of the material and strange matter can be extremely stiff at the densities of interest particularly when ρ is near to the minimum value ($4B$).

3. EQUATIONS OF STRUCTURE

In this section, we give a brief introduction to Hartle's technique for constructing general relativistic models of stars in slow uniform rotation, which we will then use in § 4 to calculate the structure of slowly rotating strange stars. More details of the method can be found in the papers by Hartle (1967), Hartle & Thorne (1968), Chandrasekhar & Miller (1974), and Miller (1977).

For nonrotating spherical stars, the relativistic structure equations are the Tolman-Oppenheimer-Volkoff (TOV) equations which, for any given value of the central density ρ_c, can be solved to give the total radius, R_o, and gravitational mass, M_o, of the configuration. The structure equations for a slowly rotating relativistic star are derived by expanding the fluid and field equations about the nonrotating solutions, in powers of the angular velocity Ω, retaining only first and second-order terms. At first order in Ω, the only change from the nonrotating solution is the appearance of a term ω which represents dragging of the inertial frames. The centrifugal deformations enter at second order in Ω.

For slow rotation, the stationary axisymmetric line element

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}(d\phi - \omega\, dt)^2 + e^{2\mu_2}\, dr^2 + e^{2\mu_3}\, d\theta^2\,, \quad (8)$$

can be written (correct to order Ω^2) as

$$ds^2 = -e^{2\nu_o}[1 + 2h_o(r) + 2h_2(r)P_2(\cos\theta)]dt^2 + e^{2\lambda_o}\left\{1 + \frac{e^{2\lambda_o}}{r}[2m_o(r) + 2m_2(r)P_2(\cos\theta)]\right\}dr^2 + r^2[1 + 2k_2(r)P_2(\cos\theta)]\{d\theta^2 + \sin^2\theta[d\phi - \omega(r)dt]^2\}\,, \quad (9)$$

where $P_2(\cos\theta)$ is the second order Legendre polynomial, ν_o and λ_o are metric functions for a nonrotating, spherical configuration having the same central density as the rotating one and the functions h_o, m_o, h_2, m_2, and k_2 are all of order Ω^2. Only spherical and quadrupole deformations are generated at this order.

For calulating h_o, m_o, h_2, m_2, and k_2, the field equations are solved together with the equation of hydrodynamic equilibrium. For a perfect fluid (which we assume in this section) the hydrodynamic equilibrium equation can be written in the form

$$\nu + \tfrac{1}{2}\log(1 - v^2) + \mathscr{P} = \text{constant}\,, \quad (10)$$

where $v \equiv e^{\psi-\nu}(\Omega - \omega)$ is the fluid velocity as measured by a local zero-angular-momentum observer and the function $\mathscr{P}(r, \theta)$ is defined by the relation $(\rho + p)d\mathscr{P} = dp$. We write $\mathscr{P}(r, \theta)$ to second-order in Ω as $\mathscr{P}(r, \theta) = \mathscr{P}_o(r) + \delta\mathscr{P}_o(r) + \delta\mathscr{P}_2(r)P_2(\cos\theta)$.

The quantities describing the rotational perturbations are all proportional to either Ω or Ω^2 and, for any fixed value of ρ_c, equilibrium configurations for different values of Ω can be obtained by scaling the results from a single calculation. The field equations together with the hydrodynamic equilibrium condition give rise to a set of ordinary differential equations which are integrated out from $r = 0$ to the stellar surface (where $p = 0$). The requirement that the metric functions should join continuously to the known analytic exterior solutions then enables one to calculate globally defined quantities (the perturbed mass M, the total angular momentum J, and the quadrupole moment Q) as well as completing the calculation for the frame dragging and for the shapes and mean radii of the isobaric surfaces.

Another quantity of interest for our discussion is the Keplerian angular velocity of a free test particle moving on a circular geodesic orbit in the equatorial plane just outside the surface of the star:

$$\Omega_K \equiv \omega + \frac{\omega'}{2\psi'} + \left[\frac{\nu'}{\psi'}e^{(2\nu - 2\psi)} + \left(\frac{\omega'}{2\psi'}\right)^2\right]^{1/2}\,, \quad (11)$$

where the prime indicates a derivative with respect to r. For a slowly rotating star

$$\Omega_K = \Omega_K^{(o)} - \frac{J}{R_o^3}\,, \quad (12)$$

where $\Omega_K^{(o)} \equiv (M_o/R_o^3)^{1/2}$ is the Keplerian angular velocity for the corresponding nonrotating spherical configuration. (Note that the second term on the right-hand side of eq. [12] acts so as to reduce Ω_K below $\Omega_K^{(o)}$.) In the next section $\Omega_K^{(o)}$ will be used as a scale for Ω.

4. PROPERTIES OF SLOWLY ROTATING STRANGE STARS

Following closely the procedure described in the papers cited at the beginning of the previous section, we have numerically integrated the set of slow-rotation structure equations using the pressure/density relation given by equation (3) with $B = 10^{14}$ g cm^{-3}. A standard fourth-order Runge-Kutta scheme with adjustable step size was used for the integration. A sequence of slowly rotating configurations was generated from the sequence of nonrotating models with central densities ρ_c ranging from $4B$ (for which $M_o = 0$) up to $\rho_c^{\max} = 1.92 \times 10^{15}$ g cm^{-3} (for which $M_o = M_o^{\max} = 2.03\ M_\odot$). Nonrotating models with $\rho_c > \rho_c^{\max}$ would be unstable to radial perturbations.

Figures 1 and 2 show the M against ρ_c and M against R relations obtained in this way both for zero rotation and also for Ω set equal to $\Omega_K^{(o)}$. The latter is an approximation to the condition for material to be shed at the equator and the slow-rotation approximation is not formally valid under these circumstances (see, however, Weber et al. 1991). We here use this value of Ω simply as a scale and to give an indication of the size of changes which could be produced by rotation (following Hartle & Thorne 1968). For smaller Ω, the deformation of the curves is smaller by a factor $(\Omega/\Omega_K^{(o)})^2$. In Figure 2, the solid

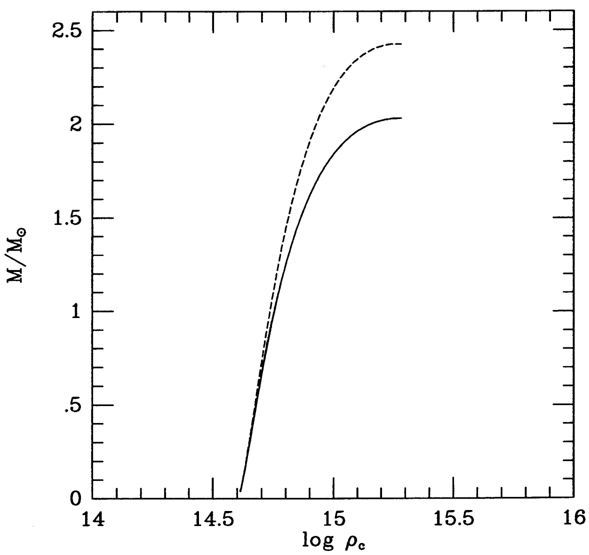

FIG. 1.—Gravitational mass M vs. central density ρ_c (in units of g cm^{-3}) for strange stars. The *solid line* shows the M_o vs ρ_c relation for nonrotating spherical configurations; the *dashed line* shows the M vs. ρ_c relation obtained using the slow rotation method, with Ω set equal to $\Omega_K^{(o)}$.

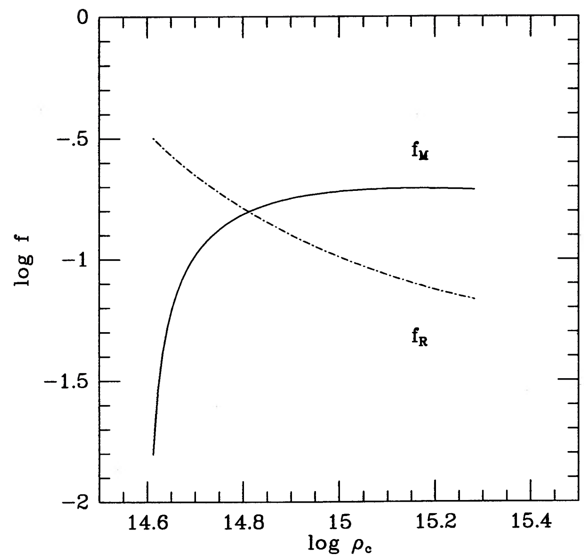

FIG. 3.—Fractional increase of mass (f_M) and of radius (f_R) for strange stars, obtained using the slow rotation method with Ω set equal to $\Omega_K^{(o)}$. The quantities are plotted as functions of central density ρ_c (measured in units of g cm^{-3}).

curve represents the mass versus radius relation for nonrotating models and the dashed curve shows the corresponding relation for Ω set equal to $\Omega_K^{(o)}$ (in this case R refers to the mean radius). The straight lines between the two curves connect models having equal central density. It is of interest to compare these results with those for neutron star models shown in Figure 1 of the paper by Hartle & Thorne (1968). Despite the difference in shape of the curves, in both cases the orientation of the linking straight lines changes smoothly from being horizontal to nearly vertical as one moves upwards in mass through the range of stable models.

Figure 3 shows the fractional change of mean radius R $[f_R \equiv (R - R_o)/R_o]$ and of mass M $[f_M \equiv (M - M_o)/M_o]$

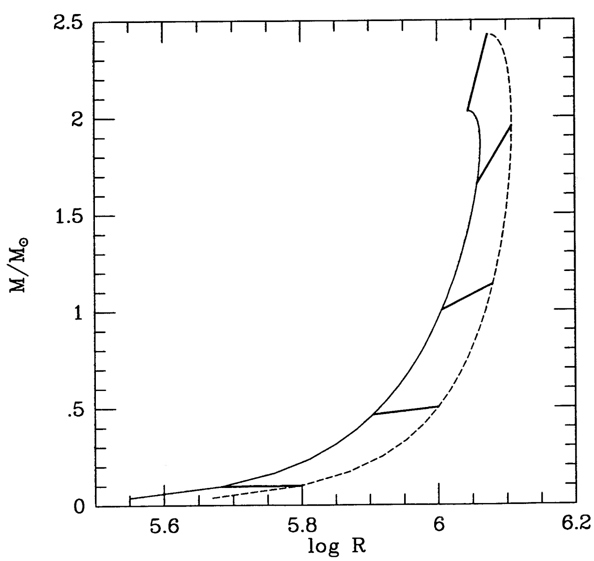

FIG. 2.—Gravitational mass M vs. radius R (in units of cm) for strange stars. The *solid line* denotes the M_o vs. R_o relation for nonrotating spherical configurations. The *dashed line* denotes the M vs. R relation obtained using the slow rotation method, with Ω set equal to $\Omega_K^{(o)}$.

when Ω is raised from zero, while keeping the central density constant; the values of M correspond to $\Omega = \Omega_K^{(o)}$. For very low-mass strange star models ($M \ll M_\odot$), rotation produces only small increases in the equilibrium mass ($f_M \sim 2 \times 10^{-2}$) but there are larger changes in radius ($f_R \sim 0.3$). Along the sequence f_R is a monotonically decreasing function of M_o, reaching its lowest value (0.07) at $M_o^{\max}$ while f_M initially increases but then becomes almost constant (~ 0.2) beyond $M_o \sim 1.4\ M_\odot$. In general, f_M turns out to be smaller for our strange star models than for the standard neutron star models constructed by Ray & Datta (1984) and Datta (1988), also using Hartle's formalism and the curve of f_M against M is significantly different. Lattimer et al. (1990) have reported results for a consistent general relativistic model of a *rapidly* rotating strange star having maximum mass and rotation speed, for which they found that the mass is 30% greater than the maximum for zero rotation (but note that the models being compared do not have the same central density).

The values of various key quantities, for a selection of slowly rotating models, are presented in Table 1.

5. THE $T/|W|$ RATIO

The ratio $t \equiv T/|W|$ [where T is the rotational kinetic energy ($\equiv \Omega J/2$) and W is the gravitational potential energy] is a quantity which measures "strength of rotation" and is important for considerations of stability. For uniformly rotating, centrally condensed objects, the range of allowed values for t is severely limited by the condition of no equatorial mass shedding ($\Omega < \Omega_K$). Uniformly rotating Newtonian polytropes with $\Gamma = 4/3$, for example, have $t(\Omega_K) = 0.0074$ which is much smaller than the value for constant density objects, $t(\Omega_K) = 0.5$ (see Shapiro & Teukolsky 1983). The uniformly rotating neutron star models with standard equations of state studied by Friedman et al. (1986) all have $t(\Omega_K) < 0.14$, the value near which the $m = 2$ bar mode is expected to go unstable in the absence of viscosity. (These calculations were general relativistic and allowed consistently for rapid rotation.) For

TABLE 1

KEY QUANTITIES FOR A SELECTION OF STRANGE STAR MODELS

ρ_c (10^{14} g cm^{-3})	$M_o/M_\odot$	R_o/R_s[a]	$\bar{\Gamma}$	f_M	f_R	ϵ[b]	$I/M_o R_o^2$[c]	$\bar{\omega}(R_o)/\Omega$[d]	$\Omega_K^{(o)}$ (rad s^{-1})
4.10	3.8×10^{-2}	31.6	102.2	1.57×10^{-2}	0.317	0.118	0.404	0.99	10626.
5.66	1.0	3.42	8.17	0.134	0.187	0.701	0.442	0.87	11321.
6.84	1.4	2.68	5.68	0.163	0.149	0.757	0.466	0.83	11727.
19.18	2.03	1.85	2.63	0.194	0.068	0.727	0.480	0.74	14073.

[a] $R_s \equiv 2GM_o/c^2$.
[b] Ellipticity in units of $[cJ/(GM_o^2)]^2$.
[c] Moment of inertia in units of $M_o R_o^2$.
[d] $\bar{\omega}(R_o) \equiv \Omega - \omega(R_o)$ in units of Ω.

models with $M \sim 1.4\ M_\odot$, their values of $t(\Omega_K)$ all cluster around 0.1 with the maximum being 0.120.

Strange stars with masses of $\sim 1.4\ M_\odot$ would have radii very similar to those of ordinary neutron stars of the same mass but there is a striking difference between the density profiles. The quantity $\bar{\Gamma}$ can be much larger for strange star models ($=5.68$ for 1.4 $M_\odot$ in our calculations) than for the standard neutron star models ($\sim$2–3) and their density profiles are rather flat right out to the surface at which there is then a discontinuity. In view of this, we expect that $t(\Omega_K)$ for uniformly rotating strange stars with $M \sim 1.4\ M_\odot$ would be intermediate between the values for standard neutron stars and those for constant density configurations, possibly being closer to the latter. Note, however, that relativistic corrections considerably reduce the maximum values for constant density objects below the Newtonian limit of 0.5 (Butterworth & Ipser 1976; for $R \approx 2R_s$, they found 0.31.) The calculations by Lattimer et al. (1990) for a strange star model having maximum mass and rotation speed, give 0.18; for a 1.4 $M_\odot$ model the value would be higher than this. The occurrence of such high values of t for uniformly rotating strange stars opens the possibility for operation of the $m = 2$ bar mode instability.

6. INSTABILITIES OF STRANGE STARS

In this section we examine the stability of rotating strange stars and calculate the minimum rotation periods consistent with stability to nonaxisymmetric perturbations with azimuthal dependence $e^{im\phi}$.

6.1. *Secular Instabilities to Nonaxisymmetric Modes*

Both gravitational radiation reaction (GRR) and viscosity can cause initially axisymmetric stars to become secularly unstable to nonaxisymmetric perturbations (Chandrasekhar 1970; Friedman & Schutz 1978; Comins 1979) and this is important in setting upper limits on the angular velocity. Studies by Lindblom & Detweiler (1977), Comins (1979 a, b), and Lindblom & Hiscock (1983) have demonstrated that the two mechanisms tend to cancel each other out when the dissipation due to viscosity is comparable with that due to gravitational radiation and this causes an increase in the values of the critical angular velocities Ω_m above those which would result if only one of the sources of dissipation were present. For a given model, the size of the increase varies depending on the order of the mode. The interplay between GRR and viscosity determines the value of m for which Ω_m is a *minimum* (corresponding to the period P being a *maximum*) and hence which mode would be the first to go unstable with increasing Ω.

The temperature dependence of the viscosity coefficients leads also to a temperature dependence of the values of the Ω_m and of the order of the mode for which Ω_m is a minimum. At some temperatures, viscosity can be sufficiently large that none of the modes become unstable for rotation speeds below the Keplerian limit.

We have calculated the critical values of Ω at the points of onset of secular instability for models of strange stars, using the same equation of state as in the previous sections and following the approach described by Lindblom (1986) with modifications contained in the papers by Cutler & Lindblom (1987) and Ipser & Lindblom (1991).

For any m, the value of Ω at the instability point can be determined by a linear perturbation calculation, starting from an analysis of the normal modes $e^{i(\omega_m t + m\phi)}$. The eigenfrequency ω_m can be written as $\omega_m = \sigma_m + i/\tau_m$ where σ_m and τ_m are the oscillation frequency and characteristic damping time, respectively, of the mth mode. Instability corresponds to τ_m being negative and sets in when $1/\tau_m = 0$. This condition gives rise to an expression for the critical values of Ω:

$$\Omega_m = \frac{\sigma_m(0)}{m}\left\{\alpha_m(\Omega_m) + \gamma_m(\Omega_m)\left[\frac{\tau_m^{\rm GRR}}{\tau_m^\eta}\right]^{1/(2m+1)} \times \left[1 + \left(\frac{\tau_m^\eta}{\tau_m^\zeta}\right)\frac{\epsilon_m(\Omega_m)}{1-(\Omega_m/\Omega_K)^4}\right]^{1/(2m+1)}\right\}, \quad (13)$$

(Ipser & Lindblom 1991) where $\sigma_m(0)$, $\tau_m^{\rm GRR}$, τ_m^η, and τ_m^ζ are the frequency and the dissipation time scales (due to GRR, shear viscosity and bulk viscosity, respectively) for the mth mode of a corresponding nonrotating model and the corrections for rotation are contained in the functions $\alpha_m(\Omega)$, $\gamma_m(\Omega)$, and $\epsilon_m(\Omega)$. Cutler & Lindblom (1987) used for $\sigma_m(0)$, $\tau_m^{\rm GRR}$ and τ_m^η values obtained from fully general relativistic numerical calculations with the appropriate equation of state (see also Lindblom & Detweiler 1983; Detweiler & Lindblom 1985), but used Newtonian Maclaurian spheroid expressions for α_m and γ_m which can be derived from formulae given by Comins (1979a, b). They did not include bulk viscosity. Recently, Ipser & Lindblom (1990, 1991) have computed α_m, γ_m, and ϵ_m for Newtonian polytropic models and Cutler & Lindblom (1991) have made an investigation in the post-Newtonian regime.

For calculating $\sigma_m(0)$, $\tau_m^{\rm GRR}$, τ_m^η, and τ_m^ζ we have constructed a general relativistic computer code using broadly the same strategy as Detweiler & Lindblom (1985), but with slightly different numerical techniques and using the formulae for the viscous dissipation times given by Cutler et al. (1990). Our code has been tested against previous results for standard neutron star equations of state. We here present results for strange star models with the canonical mass $M_o = 1.4\ M_\odot$ and the maximum mass $M_o^{\rm max} = 2.03\ M_\odot$. In each case we have used the Newtonian Maclaurin spheroid expressions for α_m and γ_m.

The 1.4 $M_\odot$ model has a very flat density profile and so the use of these expressions is quite appropriate; the 2.03 $M_\odot$ model has a less flat density profile and so the use of these expressions is less good there but nevertheless our method is equivalent to that followed previously for the standard neutron star equations of state.

Dissipation due to bulk viscosity vanishes in the limit of incompressibility and so there is no Maclaurin spheroid expression for ϵ_m. In view of this, we have used (in tabulated form) the values of ϵ_m given by Ipser & Lindblom (1991) for the stiffest equation of state which they considered ($\Gamma = 7/3$), taking Ω/Ω_K as the independent variable. For consistency with the treatment for α_m and γ_m, we then set Ω_K equal to the value for the comparison Maclaurin sequence in our calculations. This procedure will give an upper limit on the effect of bulk viscosity in determining the critical angular velocities.

6.2. *Minimum Values of* Ω_m

In the absence of viscosity and as long as the Keplerian limit does not intervene, Ω_m is a monotonically decreasing function of m and so the highest m modes are the first to become unstable with increasing Ω. However, the growth time is a monotonically *increasing* function of m and the upper limit on Ω is set by comparing the growth time of the mode with the lifetime of the star. The presence of viscosity acts to increase Ω_m with the increase factor being largest for large m. Ω_m is then a decreasing function of m for small m but becomes an increasing function for higher order modes with the minimum critical value coming at some intermediate order. It is this mode which sets the limit for the rotation velocity as long as its growth time is less than the lifetime of the star.

Table 2 contains results from our calculations for two representative central temperatures ($T = 10^8$ and 10^{10} K). At the lower temperature, the effects of shear viscosity dominate over those of bulk viscosity while the reverse is true at the higher temperature. The asterisks (*) in Table 2 indicate that the mode remains stable up to the angular velocity at which the relevant comparison sequence (involved in calculating α_m, γ_m, and ϵ_m) terminates due to mass shedding. The quantity of greatest physical interest is the minimum value of Ω_m (corresponding to the maximum P). For each of the two masses, when $T = 10^8$ K, shear viscosity causes the minimum to occur for $m = 3$ and so it is this mode which becomes unstable first with increasing Ω. The corresponding periods are $P = 0.994$ and 0.843 ms for 1.4 and 2.03 $M_\odot$, respectively. At $T = 10^{10}$ K, the minimum again comes at $m = 3$ with the corresponding periods being $P = 0.989$ and 0.800 ms. From the results of Lattimer et al. (1990) we know that the shedding limit for 2.03 $M_\odot$ comes at $\Omega_K = 0.65\Omega_K^{(o)}$ ($=9217$ rad s^{-1}) and so, using this, we see that the minimum Ω_m is equal to 0.81 Ω_K when $T = 10^8$ K and 0.85 Ω_K when $T = 10^{10}$ K. Ray & Datta (1984) use as their rotation limit an estimate of the Ω at which the $m = 2$ mode would become unstable based on the comparison with Maclaurin spheroids [$\Omega_s = (0.27)^{1/2}\Omega_K^{(o)}$]. For our models this corresponds to $\Omega_s = 6094$ rad s^{-1} for 1.4 $M_\odot$ and to $\Omega_s = 7313$ rad s^{-1} for 2.03 $M_\odot$ which can be seen to agree quite well with our results for the minimum Ω_m.

Figure 4 shows the critical periods P, as a function of temperature T, for strange star models of 1.4 $M_\odot$ (*upper curves*) and 2.03 $M_\odot$ (*lower curves*). For each mass, any point above the corresponding continuous curve lies in the allowed range of rotation speeds compatible with stability while the region beneath the curve is forbidden. The solid lines represent results of the full calculation, while the dashed ones are for calculations in the absence of bulk viscosity. At $T = 10^7$ K, shear viscosity is completely dominant over bulk viscosity and the critical periods are those for the $m = 2$ mode. At slightly higher temperatures the critical mode shifts to $m = 3$ (the discontinuities in the gradients of the curves correspond to changes in the order of the critical mode). Between 10^8 and 10^9 K, bulk viscosity first starts to become important and then rapidly dominates. This change produces a *maximum* in the P versus T relation which would not appear in the presence only of shear viscosity. At very high temperatures, P again becomes an increasing function of T on account of the rising contribution of the second term in the denominator of equation (7) with increasing temperature.

It is interesting to compare Figure 4 of this paper with Figures 17–19 of the paper by Ipser & Lindblom (1991) which are for Newtonian polytropic models having expressions for the shear and bulk viscosities appropriate for standard neutron star matter. The contribution of bulk viscosity becomes important at a higher temperature for these "standard" models

TABLE 2

TIME SCALES AND CRITICAL ANGULAR VELOCITIES FOR STRANGE STAR MODELS

M_o ($M_\odot$)	T (K)	m	$\Omega_m^{\rm GRR}$ (rad s^{-1})	Ω_m (rad s^{-1})	$P_m \equiv 2\pi/\Omega_m$ (10^{-3} s)	$\tau_m^{\rm GRR}$ (s)	τ_m^η (s)	τ_m^ζ (s)
1.40	10^8	2	6334	6477	0.970	2.39×10^{-1}	2.09×10^5	3.16×10^8
...	...	3	5476	6324	0.994	1.96×10^1	7.47×10^4	3.76×10^8
...	...	4	4865	6578	0.955	1.60×10^3	3.89×10^4	5.30×10^8
...	...	5	4417	*	*	1.45×10^5	2.40×10^4	7.54×10^8
1.40	10^{10}	2	6334	6496	0.967	2.39×10^{-1}	2.09×10^9	2.11×10^5
...	...	3	5476	6352	0.989	1.96×10^1	7.47×10^8	1.42×10^5
...	...	4	4865	6638	0.946	1.60×10^3	3.89×10^8	1.47×10^5
...	...	5	4417	*	*	1.45×10^5	2.40×10^8	1.71×10^5
2.03	10^8	2	7828	7779	0.808	1.40×10^{-1}	1.33×10^5	2.14×10^7
...	...	3	6589	7455	0.843	1.22×10^1	4.83×10^4	3.37×10^7
...	...	4	5751	7722	0.814	1.02×10^3	2.55×10^4	5.71×10^7
...	...	5	5165	*	*	9.29×10^4	1.59×10^4	9.22×10^7
2.03	10^{10}	2	7828	7927	0.793	1.40×10^{-1}	1.33×10^9	6.43×10^3
...	...	3	6589	7855	0.800	1.22×10^1	4.83×10^8	6.55×10^3
...	...	4	5751	*	*	1.02×10^3	2.55×10^8	8.93×10^3
...	...	5	5165	*	*	9.29×10^4	1.59×10^8	1.27×10^4

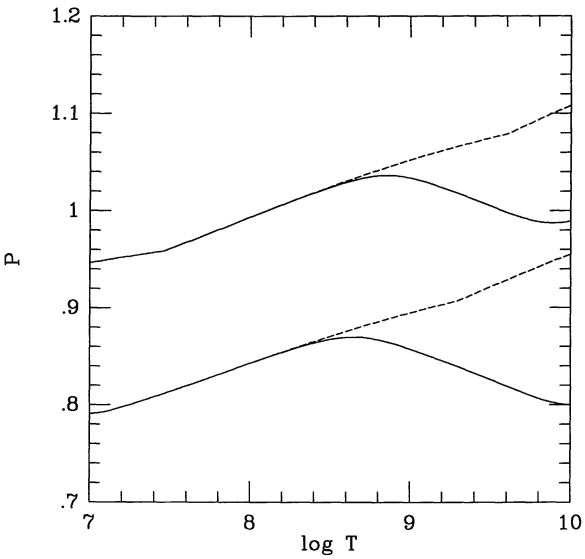

FIG. 4.—Critical periods P (in units of ms) for the onset of nonaxisymmetric instability as a function of temperature T (in units of K). The upper curves correspond to $M_o = 1.4\ M_\odot$ and the lower curves to $M_o^{max} = 2.03\ M_\odot$. For each mass, the *solid line* represents the result of the full calculation, the *dashed line* is for calculations neglecting bulk viscosity.

($\sim 4 \times 10^9$ K) but is then more dramatic as a result of the continuing sharp temperature dependence of ζ ($\propto T^6$) even up to the highest temperatures. From our calculation, the smallest critical periods for the two masses occur at $T = 10^7$ K and are $P = 0.95$ ms (for 1.4 $M_\odot$) and $P = 0.79$ ms (for 2.03 $M_\odot$) both of which correspond to rotation speeds well below the respective shedding limits.

The critical periods for the canonical mass strange star model (between 0.946 and 1.036 ms for the temperature range of Fig. 4) are considerably shorter than the periods of the fastest known millisecond pulsars PS 1937+214 ($P = 1.558$ ms) PS 1957+20 ($P = 1.607$ ms). The existence of these two pulsars has led to the necessity for discarding some of the stiffest equations of state proposed for neutron star matter on the grounds that they would give minimum allowed periods longer than the observed ones (Lindblom 1986). Here, the 1.4 $M_\odot$ model is composed of very stiff matter indeed but it cannot be ruled out in this way.

In Table 3, we show the ratios of $\sigma_m(0)$, τ_m^{GRR} and τ_m^η, as calculated from the general relativistic computer code, to the corresponding quantities calculated for Newtonian Maclaurin spheres having the same mass, radius and viscosity law as the relativistic models. Expressions for the latter are given in the papers by Lindblom (1986) and Cutler & Lindblom (1987). We follow the notation of these previous papers with ω_m being the σ_m ratio, χ_m the ratio of τ_m^{GRR} and Υ_m the ratio of τ_m^η. The numerical values of these correction factors (which include both general relativistic an density profile related contributions) are of interest in assessing the level of accuracy to be expected from approximate estimates based on the analytic Newtonian formulae for constant density models. As mentioned above, there is no corresponding ratio for ϵ_m.

TABLE 3

FREQUENCY AND DAMPING TIME RATIOS FOR STRANGE STAR MODELS

M_o ($M_\odot$)	m	ω_m	χ_m	Υ_m
1.40	2	1.035	2.172	0.489
...	3	0.971	4.473	0.489
...	4	0.946	8.484	0.491
...	5	0.933	15.69	0.494
2.03	2	1.059	3.757	0.365
...	3	0.954	11.86	0.371
...	4	0.913	32.91	0.378
...	5	0.892	88.28	0.384

As for standard neutron star models, the values of ω_m are all rather close to unity. On the other hand, the GRR time-scale correction factors deviate considerably from unity, increasing with increasing m (as also is the case for neutron stars). The differences between neutron star and strange star models are most apparent in the viscous time-scale correction factor. For our 1.4 $M_\odot$ model, with its very flat density profile, the values of Υ_m are much higher than those for neutron stars at low m but they then vary less with increasing m. The 2.03 $M_\odot$ model is intermediate between the 1.4 $M_\odot$ strange star and the neutron stars.

7. DISCUSSION AND CONCLUSION

In this paper, we have investigated first the properties of uniformly rotating strange star models using Hartle's slow rotation technique. As far as rotational deformations are concerned, there are a number of detailed differences between the strange star models and standard neutron stars, but on the whole the similarities are more striking than the differences as also was the case for the properties of the nonrotating models.

We have noted that uniformly rotating strange stars would be able to support large amounts of rotational kinetic energy before reaching the mass shedding limit. For the 1.4 $M_\odot$ model, the maximum value of $T/|W|$ before mass shedding will certainly be well in excess of the value 0.14 near which the $m = 2$ bar mode is likely to become unstable in the absence of viscosity, thus permitting the growth of this mode which appears to be forbidden for standard neutron stars (Friedman et al. 1986).

As far as instability to nonaxisymmetric modes is concerned, we have shown that, for the choice of parameters used here, these instabilities would limit the rotational periods of strange stars in the range of temperature considered (as also is the case for neutron stars). There are two different contexts within which these instabilities can operate. (1) The case of *young* (*hot*) strange stars born rapidly rotating for which the relevant temperatures are initially $\sim 10^{10}$ K with subsequent rapid cooling to $\sim 10^8$ K: here, for the higher temperatures, bulk viscosity moderates (but does not suppress) the GRR instability and the $m = 3$ mode is preferred throughout the range. The effect of bulk viscosity for strange stars does not appear to be as dramatic as previously predicted (Sawyer 1989), and this can be traced to the *very low compressibility* of strange matter particularly near the surface where the dissipation integral peaks. In order for it to suppress the instabilities at any part of the temperature range, ζ would need to be larger by about five orders of magnitude for the 1.4 $M_\odot$ model and four orders of magnitude for the 2.03 $M_\odot$ model. (2) The case of *old* (*cooler*) strange stars spun up by accretion, for which the relevant temperatures are probably $\sim 10^7$ K or slightly more (Miralda-Escudé, Haensel, & Paczyński 1990): here, shear viscosity moderates the GRR instability and the preferred mode will be either $m = 2$ or $m = 3$. This point is worth emphasizing. For old pulsars spun up by accretion, it is possible that the limiting rotation speed may be set by the $m = 2$ bar mode if the pulsar is a strange star similar to the models considered here, whereas it seems probable that no standard neutron star equation of state would allow instability to set in by this mode.

There seem then to be three important ways in which strange stars might be positively distinguished from standard neutron stars. First, strange stars should cool more quickly than neutron stars (see Pizzochero 1991). Second, there is no minimum mass for strange stars. Third, strange stars can become unstable to the $m = 2$ bar mode. If future observations of gravitational waves were to reveal the signature of a bar mode from an old pulsar spun up by accretion, then this would be quite strong evidence in favor of it being a strange star, and hence would also be in support of Witten's hypothesis that strange quark matter is the true ground state of hadronic matter.

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