ON THE RELATIVISTIC MOTION OF (1566) ICARUS

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ABSTRACT

We collected 727 astrometric observations of Icarus from the interval 1949–1987 to use them for the orbit improvement when studying the motion of the asteroid. To verify the predictions of the general relativity, we determined the relativity factor λ included to the equations of motion; we used the simple form of relativistic equations of motion written in the Painlevé's coordinates, and based on the solution of one-body Schwarzschild problem. We also made an attempt to determine nongravitational effects which appeared to be insignificant in the Icarus' motion. We may conclude that our analysis of the motion of Icarus fully confirmed the predictions of general relativity; we found a value of the relativity factor λ as being close to the theoretical value $\lambda=1$ with the probability of 99.8%, namely: $0.985 \pm 0.033 \leqslant \lambda \leqslant 1.013 \pm 0.018$.

1. INTRODUCTION

Minor planet Icarus, discovered by W. Baade in 1949, is a special object in the family of near-Earth asteroids: it has an eccentric orbit with a small perihelion distance and a short period of revolution around the Sun ($e=0.827$, $q=0.187$ AU, $P=1.12$ y), hence Icarus is the only celestial body, except for Mercury, whose relativistic effects should be reflected in observational data. Using a set of accurate observations of Icarus during 1949–1968, Lieske & Null (1969) carefully examined the motion of the asteroid in order to verify the predictions of general relativity.

To include relativistic terms in the equations of motion, the solution of the one-body Schwarzschild problem was applied based on the isotropic or nonisotropic line element, but those terms had a complicated form and were rather inconvenient for numerical computations. Brumberg (1972), in his handbook of relativistic celestial mechanics, gave a general form of relativistic equations of motion in rectangular coordinates. It appears that using so called Painlevé's coordinates we can modify the solar term only to get an extremely simple form for the relativistic equations of motion (Sitarski 1983). They can be written in the following vectorial form:

$$\ddot{\mathbf{r}}+k^2\left[1+\frac{3}{c^2}(\dot{\mathbf{r}}\cdot\dot{\mathbf{r}}-2\dot{r}^2)\right]\frac{\mathbf{r}}{r^3}=\frac{\partial R}{\partial \mathbf{r}}, \qquad (1)$$

where $\mathbf{r}$ is the radius vector of the asteroid, k is the Gaussian gravitation constant, c is the speed of light in AU/day, $3/c^2=1.00069809\times10^{-4}$, $\dot{r}=\mathbf{r}\cdot\dot{\mathbf{r}}/r$, R is the planetary disturbing function. The form of equations in the Painlevé's coordinates is, of course, equivalent to the equations in the isotropic or nonisotropic coordinates.

It must be emphasized that we have to integrate the equations of motion in the isotropic, nonisotropic, or Painlevé's coordinates if we really want to take into account effects of the general relativity, although an advance of perihelion—the spectacular pure relativistic effect—is small for Icarus and amounts only to 10″ per century. In this work we used all the astrometric positions of Icarus, covering almost the forty-year interval of observation, to investigate the relativistic motion of the asteroid.

2. OBSERVATIONS

Since 1949 Icarus has been observed during several oppositions and we collected 727 astrometric observations of the asteroid (distribution of the observations given in Table 1). Recently, Yeomans (1991) has studied the motion of Icarus using 475 selected astrometric positions and also 9 radar observations for the orbit correction; however, in his earlier paper (Yeomans & Chodas 1989) he wrote: "The radar data provided only a modest absolute improvement for the case when a long history of optical astrometric data exist." Moreover, it appears that radar and optical data are comparable in respect to exactness since in fact $\mathrm{RMS}_{\mathrm{radar}}>\mathrm{RMS}_{\mathrm{optical}}$ (Yeomans 1991, p. 1922, Tab. 2). Therefore, in this work we decided to neglect the scarce radar data of Icarus when improving the dynamical parameters of its motion.

There is a set of 590 observations made during the close approach of Icarus to the Earth (to within 0.04 AU) in 1968. Those observations made by many observers with different instruments were not equivalent and we calculated the weights of the observations according to the O−C residuals obtained for the different observers. The observations also required some classification to eliminate erroneous data.

Bielicki (1972) has elaborated a method for selection of observations, basing it on the objective criterion (from the point of view of calculus of probability) proposed by Chauvenet (1891). Let us denote O−C residuals $\Delta\alpha\cos\delta$ or $\Delta\delta$ by ψ and assume that we have a set of N discrete values

TABLE 1. Distribution of astrometric observations of (1566) Icarus.

Year	Number of observations	Number of residuals	Number of observers
1949	9	18	2
1950	5	10	1
1952	5	10	1
1953	12	24	3
1954	4	8	1
1957	2	4	1
1958	14	28	3
1965	10	20	1
1966	8	16	1
1967	2	4	1
1968	590	1151	26
1976	6	12	2
1977	3	6	2
1982	2	4	1
1985	2	4	1
1986	7	13	2
1987	46	92	6

of the random variable ψ representing the observational errors with the standard normal distribution and with the mean value equal to zero. Let us accept that the root-mean-square (rms) residual may represent the dispersion σ for the set of ψ. We can find the limiting value $\psi_K = \sigma K(N)$ to reject all such values of ψ for which $|\psi| > \psi_K$, if we solve the following equation

$$\sqrt{\frac{2}{\pi}} \int_0^K e^{-\tau^2/2}\, d\tau = 1 - \frac{1}{2N}, \tag{2}$$

where $K(N)$—the upper limit of the integral—is the unknown. The solution of Eq. (2) is a basis for the Chauvenet's criterion. Since the dispersion σ can be calculated for the only set of residuals ψ we have in our disposal, so σ also should be treated as a random variable, therefore, the following Bielicki's criterion is recommended

$$K_B(N) = K(N)/(1 - 0.476\,9363/\sqrt{N}), \quad \psi_{K_B} = \sigma K_B(N).$$

Applying the Bielicki's criterion, we iteratively selected 1151 residuals resulting from the 590 observations made in 1968 to use them for the orbit improvement; the rms of the unit weight amounted to 1ʺ.24, and the residuals for which the weighted absolute values exceeded 4ʺ.43 were rejected. The Bielicki's method for selecting and weighting of observations was also presented in details recently (Bielicki & Sitarski 1991).

3. RELATIVISTIC EFFECTS

Lieske and Null (1969) have investigated the motion of Icarus using 154 selected observations of the asteroid. To detect the relativistic effects, they determined a value of the factor λ included to the Newtonian equations of motion which in the Painlevé's coordinates have the vectorial form

$$\ddot{\mathbf{r}} + k^2 \frac{\mathbf{r}}{r^3} = \frac{\partial R}{\partial \mathbf{r}} - \lambda \frac{3k^2}{c^2} (\dot{\mathbf{r}}\cdot\dot{\mathbf{r}} - 2\dot{r}^2) \frac{\mathbf{r}}{r^3}. \tag{3}$$

To determine a value of λ along with corrections to the orbital elements, we assume that a small correction $\Delta\mathbf{r}$ caused by an inaccuracy of initial parameters of motion and of λ can be presented as

$$\Delta\mathbf{r} = \sum_{i=1}^{7} C_i G_i, \tag{4}$$

where the arbitrary constants C_i for $i=1,\ldots,6$ correspond to the corrections of orbital elements, and $C_7 = \Delta\lambda$. The differential equations for vectors $\mathbf{G}_i$ are:

$$\ddot{\mathbf{G}}_i = \mathbf{g}\cdot\mathbf{G}_i \quad \text{for} \quad i=1,\ldots,6,$$
$$\ddot{\mathbf{G}}_7 = \mathbf{g}\cdot\mathbf{G}_7 - \frac{3k^2}{c^2} (\dot{\mathbf{r}}\cdot\dot{\mathbf{r}} - 2\dot{r}^2) \frac{\mathbf{r}}{r^3}; \tag{5}$$

the analytical forms of elements of the symmetric matrix $\mathbf{g}$ were published (Sitarski 1971, 1981). A numerical integration of the above differential equations (together with the equations of motion) yields the values of $\mathbf{G}_i$ for the observation moments and allows us to compute the values of differential coefficients in observational equations (Sitarski 1971).

Lieske and Null determined λ using the either isotropic or nonisotropic coordinates and found that $0.87 \pm 0.08 \leqslant \lambda \leqslant 1.26 \pm 0.11$; based on their set of 154 Icarus observations and using the Painlevé's coordinates, we got $\lambda = 0.93 \pm 0.06$ (Sitarski 1983).

In the present work we used the 727 observations of Icarus and determined λ along with orbital elements solving 1424 observational equations by the least-squares method. We obtained

$$\lambda = 0.985 \pm 0.033.$$

We also made an attempt to take into account a hypothetical "nongravitational" effect in the motion of Icarus. We assumed that such an effect could manifest itself by a secular change $da/dt = \dot{a}$ of the semi-major axis a of the orbit. Hence, together with orbital elements we determined λ and $\dot{a}$, and we got

$$\lambda = 1.013 \pm 0.018,$$

$$\dot{a} = (-0.206 \pm 0.109) \times 10^{-11}\ \text{AU/day}.$$

We can see that in both cases (either without or with $\dot{a}$) the determined value of λ is very close to the theoretical $\lambda = 1$, and a difference $|1 - \lambda|$ is contained within the range of the mean error of λ.

TABLE 2. Probabilities $P(n,l)$ in [%] (after Drożyner 1981).

n	1μ	2μ	3μ	4μ	5μ
1	68.3	95.5	99.7	100.0	100.0
2	39.3	86.5	98.9	100.0	100.0
3	20.0	73.9	97.1	99.9	100.0
4	9.0	59.4	93.9	99.7	100.0
5	3.7	45.0	89.1	99.3	100.0
6	1.4	32.3	82.6	98.6	100.0
7	0.5	22.0	74.7	97.5	100.0
8	0.2	14.3	65.8	95.8	99.8

Drożyner (1981) has considered n-dimensional hyperellipsoid of probabilities for values of unknowns determined from observational equations by the least-squares correction. Assuming a normal distribution of errors, he could estimate the probability $P(n,l)$ that a value of any parameter among the n ones determined by the least-squares method is contained within the ranges of $l\mu$ given by the mean error μ of the parameter. Drożyner found two different analytical forms of $P(n,l)$ for an even and odd number of parameters, i.e., for $n=2k$ or $n=2k+1$:

$$P_{2k}=\frac{1}{2^{k-1}(k-1)!}\int_0^l \rho^{2k-1}e^{-\rho^2/2}\,d\rho,$$
$$P_{2k+1}=\frac{\sqrt{2/\pi}}{(2k-1)!!}\int_0^l \rho^{2k}e^{-\rho^2/2}\,d\rho. \tag{6}$$

By means of the above relations the $l\mu$ probabilities for $l=1,...,5$ and $n=1,...,8$ are calculated and given in Table 2.

We can use the data of Table 2 to estimate the probability that $\lambda\neq 1$: in both cases (i.e., for $\lambda=0.985$ and $\lambda=1.013$) this probability amounts to about 0.2%. Hence, we may draw the following conclusion: it is possible to confirm the predictions of general relativity when analyzing the motion of Icarus, since from the positional observations of the asteroid we are able to determine a value of the relativity factor λ as equal to 1 with the probability of 99.8%.

We also can calculate the probability that the "nongravitational" effect expressed by $\dot{a}$ is significant, i.e., that $\dot{a}\neq 0$. According to the mean error of $\dot{a}$ we can estimate that this probability amounts to about 15%.

4. THE ORBIT

Orbital elements given by Lieske & Null (1969) for June 13.0 ET 1949 as determined by them from 154 observations made during 1949–1968, appear to be very good initial data for extrapolation of the motion of Icarus till 1992: the residuals of 66 observations from 1976–1987 show no systematic trend (the rms is 1$''$.56), and the extrapolated perihelion time T in 1992 is Dec. 14.90900 ET whereas T found by us using 727 observations from 1949–1987 is Dec. 14.90902 ET.

TABLE 3. Orbital elements of (1566) Icarus. The mean anomaly M and the mean daily motion n are given in degrees; ω, Ω, i are given in degrees and referred to the equinox 2000.0. Rectangular equatorial coordinates x, y, z in AU, and velocity components $\dot{x}$, $\dot{y}$, $\dot{z}$ in AU/day also are referred to the equinox 2000.0.

Epoch	1992 June 27.0 ET	1996 June 6.0 ET
M	209.505078	37.520708
n	0.88055575	0.88065589
a	1.07803157	1.07794984
e	0.82676057	0.82687205
ω	31.21723	31.22054
Ω	88.16495	88.15395
i	22.88336	22.88379
x	+1.003000537015	−0.129381255856
y	−1.284053443630	−0.921218124338
z	−1.041341597719	−0.354522379058
$\dot{x}$	+0.00284847162635	+0.00728361210820
$\dot{y}$	+0.00477128999656	−0.01337361522553
$\dot{z}$	+0.00082822221075	−0.00938438716564

The next close approach of Icarus to the Earth should occur in June 1996 with the minimum distance of 0.101 AU reached on 11 June. The improved orbital elements of Icarus for two epochs are given in Table 3. The relativistic equations of motion of Icarus were integrated by the recurrent power series (Sitarski 1979) including the perturbations due to all the nine planets.

5. QUESTION OF A COMETARY ORIGIN

Yeomans (1991) has considered the problem of comets among the near-Earth asteroids looking for dynamical arguments when studying the motion of the asteroids. The eccentric orbit of Icarus might point out to its cometary origin, so detection of a slight nongravitational effect in the Icarus' motion could confirm the hypothesis.

TABLE 4. Nongravitational parameters determined for (1566) Icarus: $\dot{a}=da/dt$ is a daily change of the semi-major axis a of the orbit; A_1, A_2 are the Marsden's cometary nongravitational parameters.

	Newtonian solutions			Relativistic solutions		
Case	(a)	(b)	(c)	(A)	(B)	(C)
$10^{11}\,\dot{a}$	—	-1.16 ± 0.12	—	—	-0.18 ± 0.10	—
$10^{10}\,A_1$	—	—	-1.58 ± 0.06	—	—	-0.03 ± 0.06
$10^{15}\,A_2$	—	—	-4.99 ± 2.50	—	—	-4.72 ± 2.49
rms	1$''$.59	1$''$.54	1$''$.24	1$''$.24	1$''$.24	1$''$.24

Icarus is a special case among the near-Earth asteroids because of the effects of general relativity distinctly influencing its motion. To show how important rôle play the relativistic effects in the motion of Icarus, we made an attempt to determine nongravitational parameters for the asteroid in two cases: either considering the pure Newtonian motion of Icarus or taking into account the relativistic effects. Results are given in Table 4. We can see that the Newtonian solutions (cases a, b, c) might suggest existing of some nongravitational effects especially as expressed in the form of the Marsden's parameters A_1, A_2, whereas the relativistic solutions (cases A, B, C) indicate no need of including any additional terms to the equations of motion to describe the motion of Icarus sufficiently well over almost its forty revolutions around the Sun (a value of rms is the same in all three cases A, B, C).

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