

The size distribution of main-belt asteroids from *IRAS* data

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Accepted 1991 July 22. Received 1991 July 22; in original form 1991 June 4

SUMMARY

The *IRAS* data base on asteroid albedos and diameters has been used to derive size distributions for a set of about 4000 numbered main-belt asteroids and for various subsets defined according to the semimajor axis range and family membership. Different procedures have been used to obtain albedo estimates for the (mostly small) members of the sample not observed by *IRAS*, allowing us to assess the sensitivity of the derived size distributions to the assumptions used for this purpose. The results show that, apart from the Flora zone in the inner belt, non-family asteroids always have size distributions characterized by a strong change in slope at diameters ≈ 150 km, with the differential power-law distribution exponent passing from values around 3 or more at large sizes, to ≈ 1 at smaller ones. In the Flora region, where the sample is complete down to about 18 km (contrasting with 44 km for the whole asteroid population), a steep slope is also observed at small sizes. A similar feature is typical of the most populous families.

While the size distribution of large bodies has probably been little affected by collisional evolution, it seems likely that most family members and Flora asteroids are fragments from catastrophic impacts, with a size distribution close to that found after laboratory impact experiments. For the other main-belt asteroids, the flatter slope in the intermediate diameter range between ≈ 30 and 150 km is either due to the influence of self-gravity on the collisional outcomes, or is a remnant of the ‘primordial’ distribution. Collisional evolution models, however, can approximately reproduce the observed overall distribution starting from very different initial populations. Thus, the size distribution data alone do not effectively constrain the initial distribution and the intensity of the collisional process.

1 INTRODUCTION

An outstanding feature of the asteroid belt is that in this zone a vast population of bodies is present, whose number rapidly increases for decreasing brightnesses and sizes. The largest asteroid, Ceres, is 913 km across; about 30 bodies are larger than 200 km, 200 are larger than 100 km, while most of the 4700 asteroids numbered to date are less than 40 km in (mean) diameter.

Up to the 1970s the observational knowledge of the asteroid size distribution was based almost entirely on asteroid (absolute) brightness statistics. These early surveys (Kuiper *et al.* 1958) had correctly inferred the general shape of the distribution, which can be described as a quasi-power law, with a relative excess of bodies at intermediate diameters (around 100 km). In the same way Klačka (1990) found a steeper slope in the most populous families than for non-family asteroids. However, the derivation of asteroid sizes from absolute brightnesses only (by assuming *a priori*

some ‘mean’ value for the albedo) has the drawback of neglecting the possible selection effects and/or systematic trends due to the variable albedos of asteroids. Although the luminosity of an asteroid is proportional to the product of the cross-section (i.e. square of size) and albedo, the latter parameter can span a range of values exceeding a factor of 10, leading to serious uncertainties in the diameters derived from brightnesses values only. Moreover, it is well known that the mean asteroid albedo – along with the distribution of taxonomic types – changes systematically with the orbital semimajor axis in the belt, and overlooking this correlation can introduce strong biases in the derived size distribution (see, e.g. Gradie, Chapman & Tedesco 1989). Today, thanks to the extensive data base on asteroid albedos derived by the *IRAS* satellite (Tedesco 1989), a much better approach is feasible, as we shall discuss.

The asteroid size distribution is important because it can shed some light on both the original size distribution of the planetesimals formed in this part of the Solar System, and the

collisional process which subsequently modified this distribution and eventually produced the currently observed one. Starting in the 1960s, the general shape of the distribution has been interpreted in two different ways, with different implications on our understanding of the history of the asteroid population. Anders (1965) and Hartmann & Hartmann (1968) focused their attention on the ‘bump’ at about 100 km, and interpreted this feature as a direct record of an original bell-shaped planetesimal size distribution, to which a power-law ‘tail’ of fragments was later superimposed by disruptive collisions. On the other hand, Dohnanyi (1969, 1971, 1972) and Hellyer (1970, 1971) were more impressed by the overall quasi-power law trend extending over a wide range of sizes, a property which was confirmed in 1970 by the Palomar–Leyden Survey (PLS; Van Houten *et al.* 1970). These authors developed an analytical theory of the size distribution generated by the impact fragmentation process and showed that, if the collisional parameters (in particular, the impact strength – see Davis *et al.* 1989) are independent of size, any initial size distribution relaxes to a (differential) equilibrium power-law distribution having an exponent of -3.5 . This value does not depend on the size distribution of the fragments generated in an individual impact; since it was found to be close to the slopes derived from the Kuiper *et al.* (1958) and the PLS surveys, the inference was made that the asteroid size distribution has actually relaxed to equilibrium conditions. According to this interpretation, any deviation from the power-law trend (including the ‘bump’ at 100 km, but perhaps with the exception of the very largest sizes) is not a signature of the initial conditions, but is due to the fact that collisional parameters are not really size-independent, in particular due to the onset of self-gravitational effects and to strain-rate dependent scaling of the strength (Davis 1989; Housen & Holsapple 1990). In order to discriminate between the two theories summarized above, a complex numerical simulation of the collisional process appears to be necessary, taking into account observational constraints including not only the present size distribution, but also the properties of families and of peculiar objects such as Vesta (Davis *et al.* 1985) and the distribution of asteroid rotation rates (Binzel *et al.* 1989; Davis *et al.* 1989; Farinella *et al.* 1991a). For this purpose, a re-assessment of the asteroid size distribution based on the most recent data is an obvious prerequisite, and will be the subject of the present paper.

In Section 2 we describe in detail the methods used to derive reliable size distributions by making full use of the *IRAS* data base and also to estimate albedos (hence sizes) for many small asteroids whose albedos have not been directly measured. Section 3 will present the results of this exercise for the whole asteroid set and for several subsets referring to different orbital zones and family memberships, and will discuss possible power-law fits to the observational data. Finally, in Section 4 we will discuss these results from the point of view of their implications on the characteristics of the collisional process.

2 METHODS

The data set used in this work is the same sample of 4100 numbered asteroids that has been recently analysed by Zappalà *et al.* (1990), who applied a multivariate data analysis method to identify asteroid families. In this set of

objects, only about 1700 have an *IRAS*-determined value of the diameter, whereas, for the remaining bodies, only the absolute V magnitude (H parameter) is presently available. In past studies (e.g. Zellner 1979), whenever direct and reliable measurements of the albedo were missing, diameters have been generally inferred from rough criteria exploiting the correlations between albedo, taxonomic type and heliocentric distance, such as assigning C-type albedos to asteroids lying in the outer part of the belt and S-type albedos to objects orbiting in the inner zone. The values derived in this manner, however, are only very tentative guesses, since the relationship between orbital semimajor axis and taxonomic type is not so simple. Moreover, the range of albedos corresponding to a given taxonomic type is often not negligible. For these reasons, and as a consequence of the particular needs of the present analysis, we apply a more refined method to derive better diameter estimates for the objects not directly measured by *IRAS*, while at the same time taking the maximum advantage from the large data base provided by this satellite.

Our method consists of a detailed analysis of the albedo distribution displayed by the asteroids observed by *IRAS*, as a function of the orbital semimajor axis a . In any given range of a , the albedo distribution of the *IRAS*-observed objects is then used in order to assign an albedo value to the asteroids lacking an individual *IRAS* measurement. In this way, from the knowledge of albedo and of the absolute magnitude H , which is independently known for every object of our sample, it is straightforward to obtain the value of the corresponding diameter. However, several other independent estimates of the diameter have been obtained for the asteroids not observed by *IRAS*. We made the alternative choices of either assigning the same average albedo to all objects lying in a narrow range of semimajor axes, or assigning different albedos, chosen at random in such a way to mimic the albedo distribution of the observed objects in the same zone of the belt. The details of these computations are explained in the following. However, we point out that, in this way, we have neglected any possible bias or systematic errors affecting the *IRAS*-derived diameters and albedos. Such effects have been taken into account in a subsequent stage of the computation, as it will be explained later. On the other hand, a recent analysis by Matson *et al.* (1990) has shown that no significant difference can be found between the $25\text{-}\mu\text{m}$ *IRAS*-derived diameters and those obtained by means of occultation observations.

Among the asteroids of our data set, we considered only those having orbital semimajor axes ranging between 2.06 and 3.27 AU, i.e. between the $g=g_6$ secular resonance at the inner edge of the main belt and the 2:1 mean motion resonance with Jupiter (the Hecuba gap). The reason of excluding from our analysis the objects orbiting beyond the 2:1 gap and the inner-planet crossing asteroids, is that observational selection effects are very strong in the former case, while the latter objects are in general so small that they cannot be meaningfully compared with main-belt objects (in other words, their diameters fall in a range where bias-free distributions cannot be derived for any other zone of the belt). Moreover, following the procedure already applied by Zappalà *et al.* (1990), we distributed our asteroid population sample into six zones delimited by the other most important mean-motion resonances with Jupiter; this is useful in order

to separate different parts of the belt, for which the completeness of our sample is different, as will be explained later. The six zones are labelled by numbers from 2 to 7, in agreement with the notation introduced in the previous paper, and their boundaries are specified in Table 1.

Zappalà *et al.* (1990) succeeded in identifying 21 families of asteroids, including 867 objects overall, with most family members belonging to the three major families associated with Eos, Themis and Koronis (the altogether account for 14 per cent of the total sample). Since there are good physical reasons and also observational evidence supporting the hypothesis that the albedos of asteroids belonging to any given family can be non-representative of the 'background' population at the same heliocentric distance (see Gradie, Chapman & Williams 1979; Zellner, Tholen & Tedesco 1985; Chapman *et al.* 1989; Bell 1989), the asteroids belonging to the Zappalà *et al.* (1990) families were treated separately from the 'normal', non-family asteroids.

We scanned each zone, binning together the objects present in intervals of 0.1 AU in semimajor axis (this step appeared small enough to be confident that the diameters computed in this way should be significantly more accurate than those derived from simple considerations on the distribution of taxonomic types, as explained above). In every interval, we computed the average albedo of the non-family, *IRAS*-observed asteroids, and we used this value as the albedo to be assigned to non-family, non-*IRAS*-observed objects. In this way a first diameter estimate could be inferred for these asteroids.

Since the dispersion of the observed albedos is not negligible in some cases even in the narrow semimajor axis strips, we applied an alternative approach, aimed at reproducing the real albedo distribution in each zone. This procedure consisted in assigning at random an albedo value to each object missing direct observations, in such a way that the distribution of the randomly generated albedos strictly fit the actual albedo distribution of *IRAS*-observed objects in each zone (here 'strictly' means that the distribution was constrained to be the same over bins of width 0.01 in albedo). Since in this way an albedo value is assigned to each non-*IRAS*-observed asteroid, which depends on the seed of the random number generator, we repeated for each zone this procedure three times and averaged the results; this was meant to avoid possible anomalies due to particular random assignments of the albedo values.

The asteroids belonging to dynamical families were treated in the following way. In the few cases in which *IRAS*

albedo estimates were available for at least three objects in a family, an average family albedo was computed from these values and then used for deriving the diameters of the remaining members of the family; alternatively, when an *IRAS* albedo was available for less than three objects, we used the average albedo of the non-family, *IRAS*-observed objects having orbital semimajor axes lying in the same range of a spanned by the family. We are aware that this procedure is somewhat questionable, since the two groups of families are treated in a slightly different way, and in particular for the latter group it is assumed that the average albedo of a family is not different from the background value; on the other hand, no better procedure appears to be possible, due to lack of reliable measurements of albedos for most small asteroids. In this respect, collecting reliable physical parameters like albedo and colours for many small asteroids is certainly a high-priority task for asteroid observers in the near future.

In the cases of the three most populous families, it was possible to derive the albedos (and diameters) both by the procedure described above, and also applying the random-generation procedure used for non-family objects. In these cases the random-generated albedos were constrained to strictly fit the distribution of the *IRAS*-observed members of each family; however, as explained above, this procedure was not applicable in the cases of small families having a low number of members.

As remarked by Veeder, Tedesco & Matson (1989), it is well known that the albedo values derived by *IRAS* for faint objects are in general underestimated (see also Veeder 1986). According to Veeder *et al.*, the error can reach a factor of about 2 at the lower brightness cut-off. In order to take into account this effect, we proceeded in the following way: for each sample to be analysed (the different zones of the belt and the major families) we computed the average diameter D' of the five smallest *IRAS*-observed asteroids (we did not take directly the smallest *IRAS* diameter of each sample in order to avoid problems due to the presence of abnormally small individual values). Then we considered the diameter range between D' and $2D'$, multiplied the albedos of all objects in this range (of diameter D) times a factor $(3 - D/D')$, and rederived the albedo histogram of the *IRAS*-observed asteroids of each sample. In this way the objects for which $D = D'$ had their *IRAS* albedo increased by a factor 2, while for $D \geq 2D'$ no correction was made (see Fig. 1, which shows the original and the corrected albedo histograms for zone 2). The corrected albedo distributions were then used for the fit with the distribution of the randomly generated albedos to be assigned to non-*IRAS*-observed asteroids of each sample, as explained above.

Table 1. Ranges [AU] for semimajor axis zones 2–7.

Zone	a_{min}	a_{max}
2	2.065	2.300
3	2.300	2.501
4	2.501	2.825
5–6	2.825	3.030
7	3.030	3.278

3 OBSERVED DISTRIBUTIONS AND POWER-LAW FITS

By applying the procedures described in the previous section, we had at our disposal three different methods for deriving the diameters of asteroids not observed by *IRAS*; accordingly, we could compute in each case the cumulative size distribution of our whole asteroid set, and analyse the trend exhibited by the entire population, as well as by some 'natural' subsamples of it, i.e. those obtained by separately

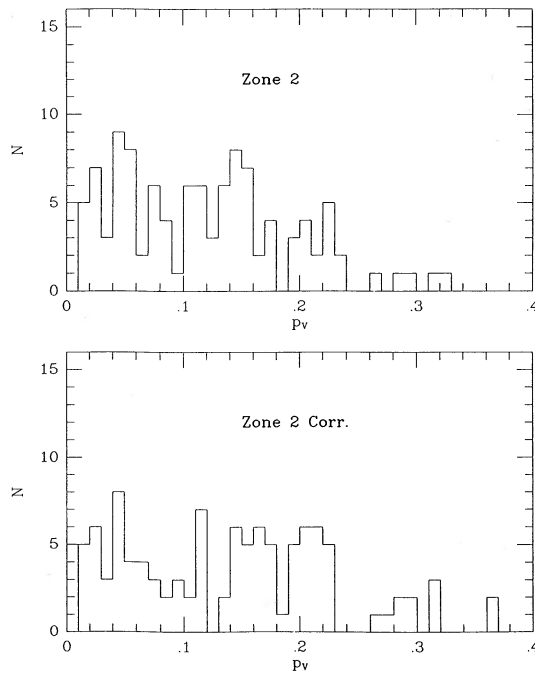


Figure 1. Histograms of the *IRAS*-derived albedos in semimajor axis zone 2; the lower plot includes the correction at small asteroid sizes described in the text.

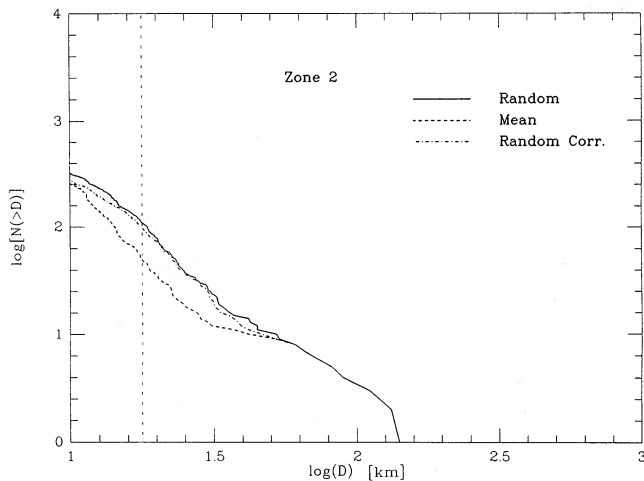


Figure 2. The size distribution of asteroids in zone 2, shown as a log-log plot of the cumulative number of bodies of diameter larger than D versus D . Notice that at small sizes there is some discrepancy between the curves derived according to the different procedures explained in the text to estimate albedos of non-*IRAS*-observed asteroids. The vertical dash-dotted line shows the minimum diameter for which the samples are probably complete (i.e. no, or very few undiscovered asteroids exist).

grouping the asteroids present in the different zones of the belt defined in Table 1, and by separating the family and non-family objects. The results are shown in Figs 2–14, which are all log-log plots of the cumulative number of objects larger than a given diameter versus the diameter itself.

We have always excluded the four largest asteroids from our analysis, on the basis of a physical argument. According

to the current collisional evolution models (Davis *et al.* 1985, 1989), the mass of these objects is unlikely to have been much affected by impacts since the end of the planetesimal growth process, contrasting with the history of most smaller asteroids; in a sense, the few biggest asteroids are small *planets*, i.e. they belong to a physically distinct population which should not be mixed with the general asteroid population. Of course, the limiting diameter we have chosen ($D_{\max} = 350$ km) is subject to some arbitrariness; however, since these large objects are anyway just a few, our results are not very sensitive to the specific choice we have made.

Another, more important problem arises from the obvious observational selection effect against the discovery of faint asteroids, which has the consequence that the observed size distributions can be considered reliable only for diameters greater than some minimum value, below which our samples are likely to be incomplete. The threshold diameter $D_{\min}$ (indicated by a dash-dotted, vertical line in the figures) obviously depends upon the heliocentric distance, hence it is different for each of the different zones. In order to quantify what exactly is the limiting diameter, we adopted the following simple criterion. Assuming that $V = 16.0$ mag is the maximum apparent magnitude for which our samples are complete and that the minimum albedo of asteroids is 0.02, and using an heliocentric distance equal to the outer boundary of each zone, we could compute the corresponding diameter threshold $D_{\min}$, which should give a fairly good estimate of the minimum size for ‘completeness’ in each zone. While the adopted maximum value of V can be considered somewhat conservative, being based on the value of the mean opposition magnitude cut-off adopted by Zellner (1979), we preferred to be on the safe side when avoiding observational selection effects on the results of our analysis. In most cases, however, the trends of the cumulative size distributions shown in the figures indicate by themselves that our estimates of $D_{\min}$ are reasonable, as the lower slope for $D < D_{\min}$ suggests that part of the existing objects are actually missing. In the case of the three major families (Eos, Themis and Koronis), $D_{\min}$ was computed in a slightly different way, namely on the basis of the actual range of semimajor axes and of the observed distribution of albedos for each family (the limiting albedo being such that more than about 90 per cent of the members of each family have albedos larger than it). The $D_{\min}$ value for the whole sample of family objects is lower than that for the sample of ‘background’ objects, since the largest semimajor axis value of the most distant family is smaller than the outer border of the belt used for the non-family objects.

Fig. 2 shows the difference between the cumulative distributions obtained by our three different methods for estimating the diameters of non-*IRAS*-observed bodies for one particular case. In general, the differences are small, though not completely negligible.

In order to summarize the results in a simple quantitative way, a well-known method used is to fit the size distributions with power laws, or *truncated Pareto distributions*, spanning a diameter range between a lower limit $D_{\min}$ and an upper limit $D_{\max}$ (e.g. see Kresák 1977; Hughes 1982; Donnison & Sudgen 1984). If the probability density function $\zeta(D)$ – i.e. the fraction of objects lying in the diameter interval δD centred at a given value D – is assumed to be proportional to $D^{-(\alpha+1)}$ (with α a constant exponent), integration over the

range $D_{\min}$ to $D_{\max}$ yields the normalization coefficient, so that we have

$$\zeta(D) = \frac{\alpha}{D_{\min}} \left(\frac{D}{D_{\min}} \right)^{-(\alpha+1)} \left[1 - \left(\frac{D_{\min}}{D_{\max}} \right)^{\alpha} \right]^{-1}.$$

In order to obtain the usual cumulative size distribution $\theta(D)$, namely the fraction of bodies larger than some given diameter D , $\zeta(D)$ must be integrated between D and $D_{\max}$. As a consequence, we find that $\theta(D)$ is proportional to $(D^{-\alpha} - D_{\max}^{-\alpha})$, so that, in a log-log plot, the relationship between $\theta(D)$ and D is *not* represented by a straight line, as is usually assumed, unless $D_{\max} \rightarrow \infty$. In order to match a Pareto distribution to a given data set, i.e. to find the best-fitting value of the exponent α and the corresponding standard error, we cannot use a simple least-square procedure to either the cumulative or the differential distribution, because the different data points would not be independent (Hughes 1982; Donnison & Sudgen 1984). Instead, as suggested by Donnison & Sudgen, an appropriate procedure is provided by the *maximum likelihood estimation* method, based on the idea of finding the maximum of the function $L(\alpha) = \prod \zeta(D_i)$, where D_i is the given set of diameters (the index i labelling all the objects included in the sample). The maximum of $l = \ln L$ is actually more easily found, by solving the equation $\partial l / \partial \alpha = 0$. This yields the equation

$$\frac{1}{\alpha} = \frac{\sum \ln(D_i/D_{\min})}{n} + \frac{r \exp(-ar)}{1 - \exp(-ar)},$$

which has to be solved by numerical iterations for the best fitting value of α [here n is the total number of objects in the sample, the sum over i is extended to all of them, and $r = \ln(D_{\max}/D_{\min})$]. The standard error of the estimate of α is

$$\delta\alpha = n^{-1/2} \left[\alpha^2 - \frac{r^2 \exp(-ar)}{1 - \exp(-ar)^2} \right]^{1/2}.$$

The goodness of the fit can now be assessed by a χ^2 test, after having grouped the data in such a way that no group includes less than 10 objects (five when the complete sample consists of less than 100 objects).

Figs 3(a) and 4 and Tables 2 and 3 show the results of the procedure described above for the entire data set, and for the same set but excluding the family asteroids and those belonging to semimajor axis zone 2 (the so-called Flora region), respectively. The latter sample was selected because, as we shall see later, there is evidence that both zone 2 and the families are characterized by peculiar size distributions, steeper than usual especially at small sizes (as pointed out by Zappalà *et al.* (1990), it is likely that a large fraction of the population of zone 2 would be recognized as one or a few families, were it possible to use more accurate and stable proper elements in the family identification analysis). The upper part of the tables gives the total number of objects belonging to each sample, the value of $D_{\min}$ (44 km in the case of the whole sample), the number n of objects larger than $D_{\min}$ according to any given diameter derivation procedure (from the *mean* albedo in each semi-major axis strip, or, alternatively, from the *random-generated* albedo, or from the *random-generated* albedo *corrected* for the IRAS bias at small sizes, as discussed in Section 2). The tables give

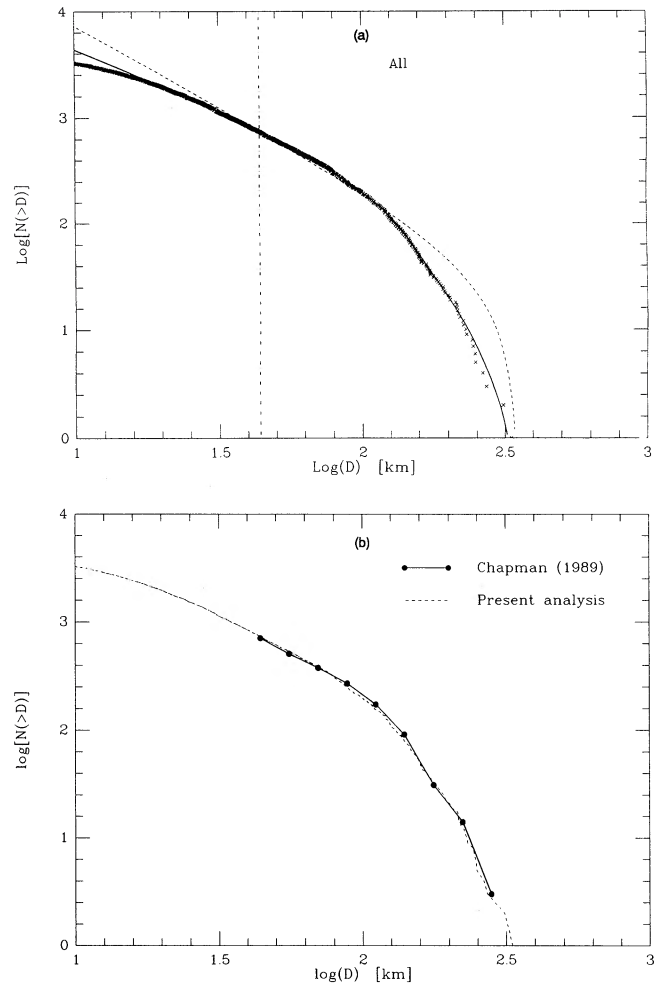


Figure 3. (a) Power-law fits to the observed size distribution (derived from the corrected IRAS albedos) for the entire asteroid sample. The dashed curve is the best-fitting single truncated Pareto distribution, while the full line corresponds to the best-fitting double truncated Pareto distribution (see text). (b) The cumulative size distribution for the whole asteroid population as derived in this paper (dashed line), compared with that obtained by Chapman (1989; solid line and dots). In this representation, we have used a small number of diameter values with equal logarithmic spacings.

the best-fitting value of α with its standard error, the corresponding exponent δ of the differential mass distribution (assuming size-independent densities and shapes; we have $\delta = 1 + \alpha/3$), the extrapolated number of objects of diameter larger than 10 km, and the results of a χ^2 test (value of χ^2 statistics, number of degrees of freedom, probability P that the observed residuals are *not* due to chance, and result of the test – ‘reject’ the fit when $P > 95$ per cent, ‘marginally reject’ for $85 \text{ per cent} < P < 95$ per cent, and ‘no reject’ for $P < 85$ per cent). In the figures, which are log-log plots of the cumulative size distributions, the best-fitting truncated Pareto distributions are shown by the dashed curves, which approximate straight lines only for $D \ll D_{\max}$.

As it can be easily seen from both the figures and the tables, for both of these two very populous samples, the Pareto fit must be rejected. The main reason is that the Pareto distribution curve, while providing a good match for

Table 2. Results of the power-law fits for the whole sample considered in the present work; for the meaning of the listed quantities, see text.

Whole Population (3952 objects)										$D_{\min} = 44 km$		
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)		χ^2	ν	P (%)	Interp.			
“mean”	741	1.51 ± 0.06	1.50 ± 0.02	$7.2 \cdot 10^3$		48.62	25	99	Reject			
Double Truncated Pareto Distribution												
Small objects					Large objects							
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	145	668	1.07 ± 0.04	1.36 ± 0.01	$4.3 \cdot 10^3$	73	3.7 ± 0.4	2.24 ± 0.15	11.58	23	2	No R.

Table 3. Same as in Table 2, but for the sample obtained excluding both family and zone 2 asteroids.

Whole (no Zone 2, no families) (2446 obj.)										$D_{\min} = 44 km$		
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)		χ^2	ν	P (%)	Interp.			
“mean”	678	1.48 ± 0.06	1.49 ± 0.02	$6.4 \cdot 10^3$		51.82	24	99	Reject			
“random”	707	1.52 ± 0.06	1.51 ± 0.02	$7.0 \cdot 10^3$		50.35	24	99	Reject			
“random corr.”	703	1.51 ± 0.06	1.50 ± 0.02	$6.9 \cdot 10^3$		52.36	24	99	Reject			
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	160	633	1.02 ± 0.04	1.34 ± 0.01	$3.8 \cdot 10^3$	45	3.6 ± 0.5	2.22 ± 0.18	13.86	22	9	No R.
“random”	160	662	1.08 ± 0.04	1.36 ± 0.01	$4.2 \cdot 10^3$	45	3.6 ± 0.5	2.22 ± 0.18	13.66	22	9	No R.
“random corr.”	160	658	1.06 ± 0.04	1.35 ± 0.01	$4.1 \cdot 10^3$	45	3.6 ± 0.5	2.22 ± 0.18	14.15	22	10	No R.

the slope at small sizes (where most objects are concentrated), fails badly at diameters larger than $\approx 100 \text{ km}$, where the real distribution is much steeper. This is not surprising, because a change of slope when passing from 'small' to 'big' objects has long been suspected for the general asteroid population (see the debate on the 'bump' at $D \approx 100 \text{ km}$ summarized in Section 1), and has been both predicted by theoretical models of the asteroid collisional evolution (Davis 1989; Davis *et al.* 1989), and actually observed in the size distribution of fragments from hypervelocity impact experiments (Fujiwara, Kamimoto & Tsukamoto 1977; Fujiwara *et al.* 1977).

Therefore, we have attempted an alternative fit to the observed distributions, using a pair of truncated Pareto distributions with different exponents, connected at some transition diameter D_{tr} . The two values of the exponent α and the value of D_{tr} are treated in this case as three free parameters to be found. We have again used the maximum

likelihood method to separately estimate α in the two ranges ($D_{\min}, D_{tr}$) and ($D_{tr}, D_{\max}$); then, we have varied D_{tr} with steps of 5 km, looking for the maximum of the logarithm of the likelihood function, as defined above (with the constraint that at least five objects should always be included in either size range). The lower parts of Tables 2–13 show the results of this exercise: values of n , α and δ are given for both the 'small' and the 'large' asteroids, along with the transition diameter D_{tr} , the extrapolated number of objects larger than 10 km and the new results of the χ^2 test (in the same format as described above). For both the entire asteroid population, and the population excluding zone 2 and family objects (in the latter case we have used all the three different diameter derivation methods) the fit is now satisfactory. The corresponding 'double' Pareto distributions are plotted in the figures as full lines.

We shall now comment in some detail on the results shown in the figures and tables.

Whole population (Fig. 3a, Table 2). As we have already noticed, there is a clear change of slope at about $D = 145$ km. The large objects have a much steeper slope, $\alpha = 3.7$ contrasting with $\alpha = 1.1$ at smaller sizes. Between 44 km (minimum size for completeness) and 145 km, the mass distribution index δ has a value of about 1.36, much lower than both Dohnanyi's equilibrium value (11/6) and the values observed for the fragments from laboratory impact experiments (typically, 1.7 to 2; see Fujiwara *et al.* 1977; Capaccioni *et al.* 1986). This appears to rule out in this size range either an intense, strength-dominated collisional process, or a high abundance of competent fragments from a few discrete breakup events.

We have to notice that we have also compared the distribution for the whole population derived by means of the procedure described in Section 2 with that derived by Chapman (1989) (see Fig. 5, p. 330, in Gradie *et al.* 1989) and used by Farinella *et al.* (1991a) for their comparison with the outputs of numerical simulations on the collisional history of asteroids. For this purpose, we used a representation consistent with that adopted in the papers described above, i.e. a small number of discrete diameter values separated by equal logarithmic intervals (of about 0.1). As apparent from Fig. 3(b), the discrepancies between the two distributions are very small.

Whole population, without families and zone 2 (Fig. 4, Table 3). The results are similar to the previous ones. It is interesting to note that they are not sensitive to the method used to infer albedos for non-IRAS-observed bodies.

Zone 2 (Fig. 5, Table 4). Here $D_{\min} = 18$ km, i.e. our sample is complete down to fairly small sizes; moreover, our estimate of $D_{\min}$ is probably conservative, as in this region low-albedo asteroids are rather rare (actually, we can see that the distribution possibly starts to 'flatten' only at about 10 km). While the fit with a single Pareto distribution is not good (although it can only marginally be rejected when albedos are derived by correcting the IRAS data at small sizes), the situation improves considerably with the double Pareto

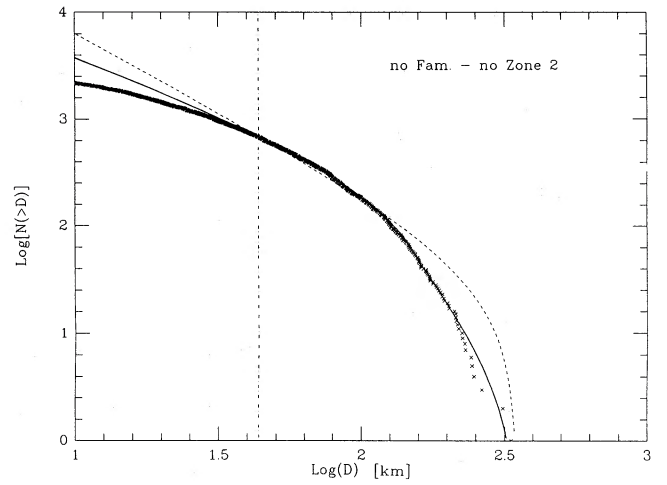


Figure 4. Same as Fig. 3(a), but for the sample of asteroids not belonging to dynamical families or to the Flora region.

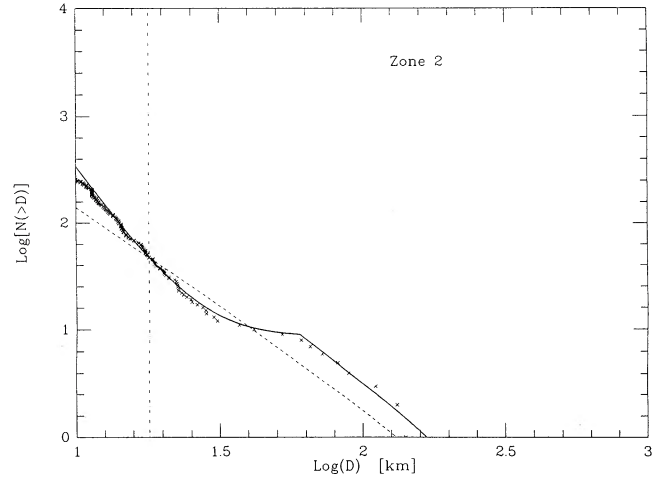


Figure 5. Same as Fig. 3(a), but for the sample of asteroids belonging to zone 2.

Table 4. Same as in Table 2, but for the sample of asteroids belonging to zone 2.

Zone 2 (639 obj.)					$D_{min} = 18 km$							
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)	χ^2	ν	P (%)	Interp.				
“mean”	47	1.9 ± 0.3	1.62 ± 0.09	$1.4 \cdot 10^2$	10.87	5	95	Reject				
“random”	106	2.5 ± 0.2	1.83 ± 0.08	$4.6 \cdot 10^2$	12.39	6	95	Reject				
“random corr.”	94	2.4 ± 0.2	1.79 ± 0.08	$3.8 \cdot 10^2$	9.55	6	86	M.Rej.				
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	60	38	3.7 ± 0.6	2.22 ± 0.20	$3.4 \cdot 10^2$	9	1.9 ± 0.6	1.64 ± 0.21	1.04	3	21	No R.
“random”	80	100	3.0 ± 0.3	2.01 ± 0.10	$6.1 \cdot 10^2$	6	2.6 ± 1.0	1.85 ± 0.35	7.05	4	87	M.Rej.
“random corr.”	80	88	2.9 ± 0.3	1.97 ± 0.10	$5.0 \cdot 10^2$	6	2.6 ± 1.0	1.85 ± 0.35	5.84	4	79	No R.

distribution; the best-fit is found for $D_{tr} = 60$ km. Anyway, the estimates of δ are never far from the 11/6 value. Interestingly, in this zone the size distribution resembles those of the major families (see later) more than those seen in other zones of the asteroid belt, where usually the distribution is steeper at larger sizes; it is thus possible that most bodies in this zone are members of one – or perhaps a few – populous Flora families, which are difficult to recognize because of problems in both the proper elements theory and the family identification procedure (Zappalà *et al.* 1990).

Zone 3 (Fig. 6, Table 5). The fit with a single Pareto distribution cannot be rejected by the χ^2 test; however, the figure shows clearly that at large sizes there are systematic discrepancies, and the fit is considerably improved when a double distribution is used, with a steeper slope at large sizes. It is also interesting to notice that in spite of the observational bias, for $D < D_{min}$ the observed distribution lies

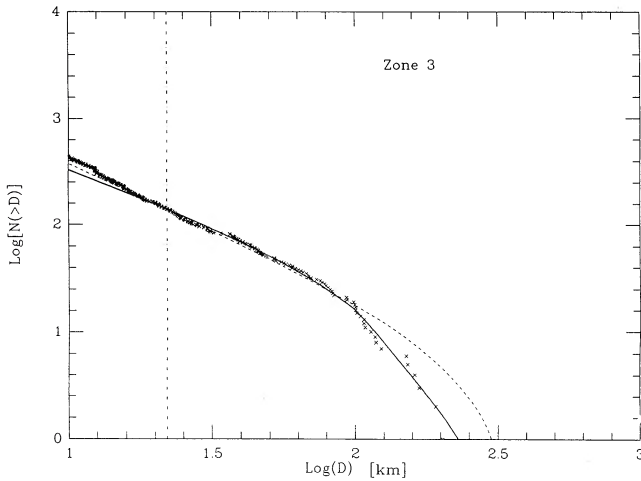


Figure 6. Same as Fig. 3(a), but for the sample of asteroids belonging to zone 3.

above the best-fitting curve, i.e. it becomes somewhat steeper than at larger sizes, and not flatter as it would be expected (and as it happens for all the other samples); this suggests that below ≈ 20 km the slope probably gets significantly higher than the value seen at intermediate sizes.

Zone 4 (Fig. 7, Table 6). Here there is a strong evidence of a transition between two different behaviours: an almost flat distribution (α less than 1, $\delta \approx 1.3$) for $30 \text{ km} < D < 155 \text{ km}$ and a steep distribution ($\alpha \approx 5$) from the 21 objects larger than 155 km. Neither of these slopes is consistent with the Dohnanyi equilibrium value or with a fragmental size distribution.

Zone 5 + 6 (Fig. 8, Table 7). Due to the scarcity of non-family asteroids in zones 5 and 6 defined in the Zappalà *et al.* (1990) paper, we have merged these two zones for the present analysis. The situation is rather similar to that seen in

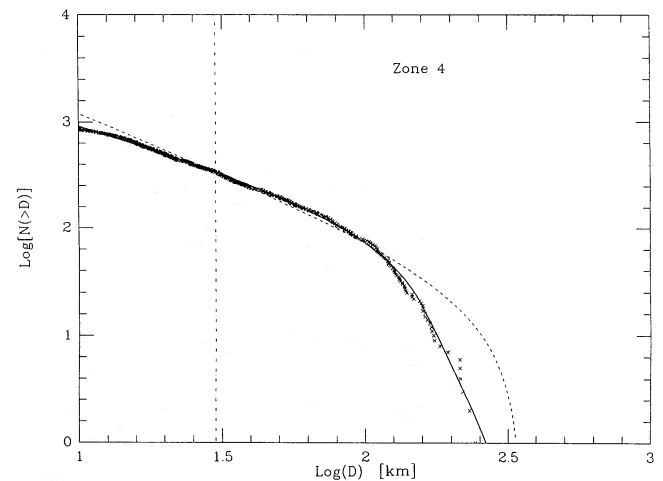


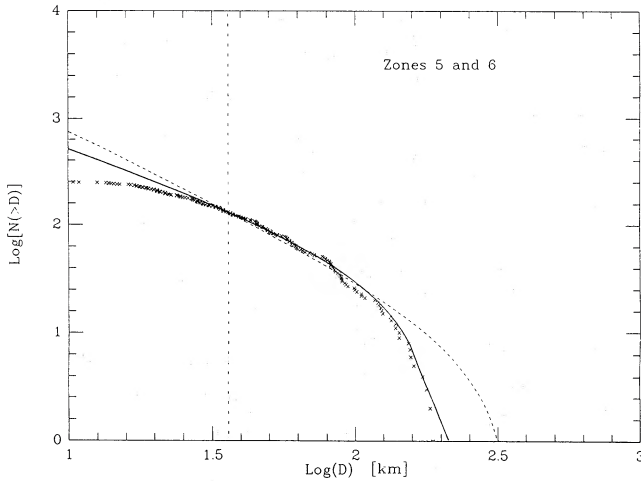
Figure 7. Same as Fig. 3(a), but for the sample of asteroids belonging to zone 4.

Table 5. Same as in Table 2, but for the sample of asteroids belonging to zone 3.

Zone 3 (645 obj.) $D_{min} = 22km$												
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10km$)	χ^2	ν	P (%)	Interp.				
“mean”	139	1.23 ± 0.10	1.41 ± 0.03	$3.7 \cdot 10^2$	10.64	10	61	No R.				
“random”	183	1.50 ± 0.11	1.50 ± 0.04	$6.0 \cdot 10^2$	16.64	14	72	No R.				
“random corr.”	182	1.53 ± 0.11	1.51 ± 0.04	$6.1 \cdot 10^2$	13.76	14	53	No R.				
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	100	123	0.99 ± 0.09	1.33 ± 0.03	$3.3 \cdot 10^2$	16	2.9 ± 0.7	1.98 ± 0.24	7.81	8	55	No R.
“random”	100	167	1.40 ± 0.11	1.47 ± 0.04	$5.7 \cdot 10^2$	16	2.9 ± 0.7	1.98 ± 0.24	16.87	12	85	M.Rej.
“random corr.”	100	166	1.48 ± 0.12	1.49 ± 0.04	$5.9 \cdot 10^2$	16	2.9 ± 0.7	1.98 ± 0.24	14.12	12	71	No R.

Table 6. Same as in Table 2, but for the sample of asteroids belonging to zone 4.

Zone 4 (908 obj.)					$D_{\min} = 30 km$							
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)	χ^2	ν	P (%)	Interp.				
“mean”	338	1.11 ± 0.06	1.37 ± 0.02	$1.2 \cdot 10^3$	50.04	27	99	Reject				
“random”	371	1.23 ± 0.06	1.41 ± 0.02	$1.5 \cdot 10^3$	45.34	27	99	Reject				
“random corr.”	369	1.22 ± 0.06	1.41 ± 0.02	$1.5 \cdot 10^3$	44.31	27	98	Reject				
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	155	317	0.75 ± 0.05	1.25 ± 0.02	$9.1 \cdot 10^2$	21	5.3 ± 1.2	2.76 ± 0.38	24.00	25	48	No R.
“random”	155	350	0.94 ± 0.05	1.31 ± 0.02	$1.2 \cdot 10^3$	21	5.2 ± 1.2	2.76 ± 0.38	26.46	25	62	No R.
“random corr.”	155	348	0.93 ± 0.05	1.31 ± 0.02	$1.2 \cdot 10^3$	21	5.3 ± 1.2	2.76 ± 0.38	25.16	25	55	No R.

**Figure 8.** Same as Fig. 3(a), but for the sample of asteroids belonging to zones 5 and 6.

zone 4. Here, however, the distribution for the eight objects larger than 155 km is even steeper, while at about 100 km it is somewhat irregular. As a consequence, even with the double distribution the fit is not satisfactory.

Zone 7 (Fig. 9, Table 8). In this zone a single Pareto distribution cannot be rejected, though the systematic discrepancies at large sizes are removed when a double distribution is fitted. The reason for this behaviour is that the distribution is again steeper at large sizes, but much less so than in the two previous cases. In this respect, the situation is similar to that of zone 3. Here, however, the selection effect below $D_{\min}$ is quite strong.

All families (Fig. 10, Table 9). As we have already mentioned, the situation is similar to that of zone 2, with the slope decreasing for larger sizes. The single Pareto distribu-

tion cannot be rejected, but again the fit looks much better with the double distribution. Notice that, especially in the latter case, the high value of α at small sizes results in a prediction of a very substantial number of small bodies when extrapolated to $D \approx 10$ km. The observed distribution is consistent with those found in the laboratory for fragments generated by high-velocity impact breakups, and the value of δ at small sizes is close to the values derived by Klačka (1990).

Families 51 + 61 + 71 (Fig. 11, Table 10). These are the three populous Koronis, Eos and Themis families. As they account for almost 2/3 of all the family asteroids, it is not surprising that their behaviour is similar to that of the previous sample. Notice, however, that the best-fitting exponents are fairly sensitive to the method adopted to infer albedos.

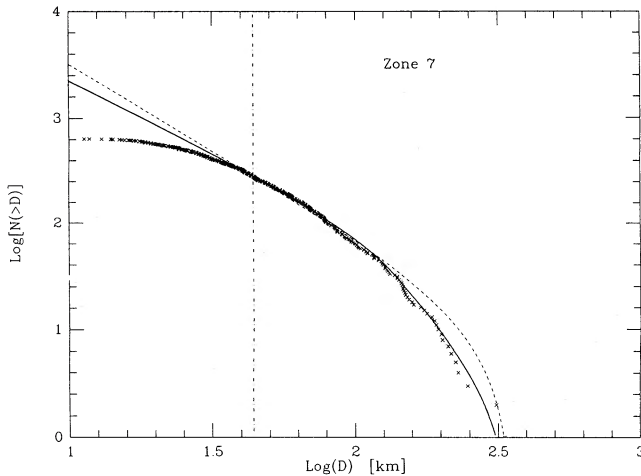
Individual families 51, 61, 71 (Figs 12–14, Tables 11–13). For the three individual families the size distributions can be fitted by single power laws; both the goodness of the fit and the best-fitting exponents, however, are method-sensitive. It is thus impossible to make a reliable estimate of the number of undiscovered family members at sizes ≈ 10 km and smaller.

4 DISCUSSION

We can summarize the results discussed above as follows. The size distribution of the whole asteroid population *cannot* be fitted by a single-exponent power law, due to a large change of slope at about 150 km diameter: larger asteroids have a steep distribution, with δ close to or larger than 2, while smaller asteroids (down to the completeness limit diameter of 44 km) show a remarkably flat distribution, with $\delta \approx 1.35$. Surprisingly enough, neither of these values is close to the equilibrium slope of about 1.8 predicted by the analytical theory assuming size-independent collisional

Table 7. Same as in Table 2, but for the sample of asteroids belonging to zones 5 and 6.

Zone 5 + 6 (250 obj.) $D_{\min} = 36km$												
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10km$)		χ^2	ν	P (%)	Interp.			
“mean”	131	1.34 ± 0.12	1.44 ± 0.04	$7.5 \cdot 10^2$		22.71	11	98	Reject			
“random”	138	1.36 ± 0.12	1.45 ± 0.04	$8.2 \cdot 10^2$		27.05	11	99	Reject			
“random corr.”	129	1.27 ± 0.11	1.42 ± 0.04	$6.9 \cdot 10^2$		31.17	10	99	Reject			
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	155	123	0.92 ± 0.09	1.31 ± 0.03	$5.1 \cdot 10^2$	8	6.5 ± 2.3	3.18 ± 0.77	13.11	9	84	No R.
“random”	155	130	0.96 ± 0.09	1.32 ± 0.03	$5.5 \cdot 10^2$	8	6.5 ± 2.3	3.18 ± 0.77	17.96	9	96	Reject
“random corr.”	155	121	0.81 ± 0.08	1.27 ± 0.03	$4.5 \cdot 10^2$	8	6.5 ± 2.3	3.18 ± 0.77	21.72	8	99	Reject

**Figure 9.** Same as Fig. 3(a), but for the sample of asteroids belonging to zone 7.

outcomes, and which is also typical for fragments from individual catastrophic impacts. (We have found some clues, however, that the slope may become again steep at sizes smaller than about 20 km, which are poorly sampled with the currently available data.) Moreover, the size distribution undergoes substantial changes in the different semimajor axis zones and when families are taken into account. In particular, in the Flora region, where the sample is complete down to about 18 km, a steep slope ($\delta \approx 2$) is observed even at small sizes, and a similar behaviour is typical of the most populous families.

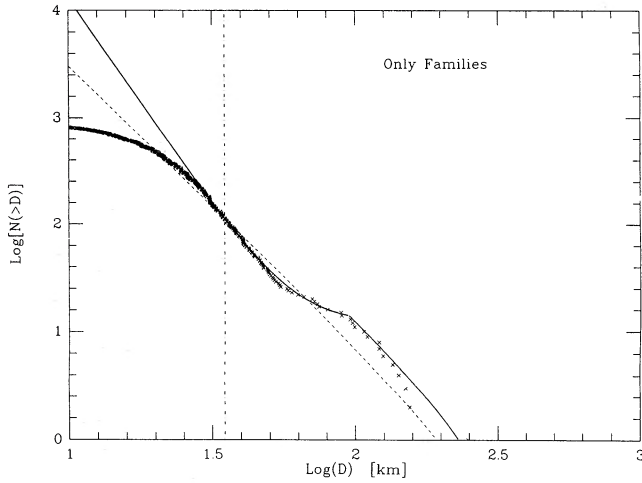
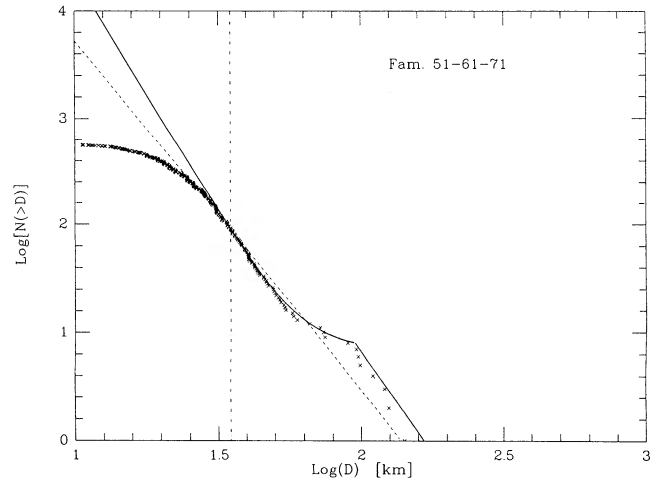
Can these results be interpreted in terms of collisional evolution? While the answer is probably positive, it is also going to be more complex than previously thought, and affected by substantial uncertainties on the intensity of the past collisional process.

Families are relatively simple: their size distributions are clearly distinct from those of the 'background' objects in the same zones of the belt, and this is likely to mean that these distributions 'remember' well the initial distribution generated by the catastrophic disruption of a parent body. Changes of slope, if present, are probably similar to those observed for fragments from laboratory experiments, and/or may be due to gravitational reaccumulation of slowly-escaping fragments (Zappalà *et al.* 1984). The well-preserved size distributions of families probably put an upper bound to their age, which might be estimated by suitable collisional evolution models (Davis 1990). Finally, it is interesting to notice that a simple extrapolation of the distributions observed at ≈ 50 km would lead us to predict that the asteroid population is dominated by family members for sizes smaller than about 10 km (Klačka 1990). However, we believe that this conclusion is premature, due to the uncertainties of the best-fitting slopes and the likelihood of further changes of slope below the completeness limits. More generally, it should be pointed out that at present we cannot reliably extrapolate from the available data to give the abundance of small asteroids, since even at sizes of a few km the uncertainty is already at least a factor 10. This means that we are currently unable to estimate the collisional lifetimes and the likely ages of all the asteroids smaller than several tens of km, including those which in the next decade will be encountered by interplanetary probes (Farinella *et al.* 1991b).

In zone 2, the Flora region, we have a dilemma: either we are seeing a collisionally relaxed population with a slope approximating the prediction of Dohnanyi's model, or we are just looking at a population dominated by one or more families, behaving as described above and not yet unequivocally recognized as such owing to problems in the dynamical theories. We think that the latter hypothesis is more likely, because there would be no *a priori* reason for expecting different collisional response parameters in this zone with

Table 8. Same as in Table 2, but for the sample of asteroids belonging to zone 7.

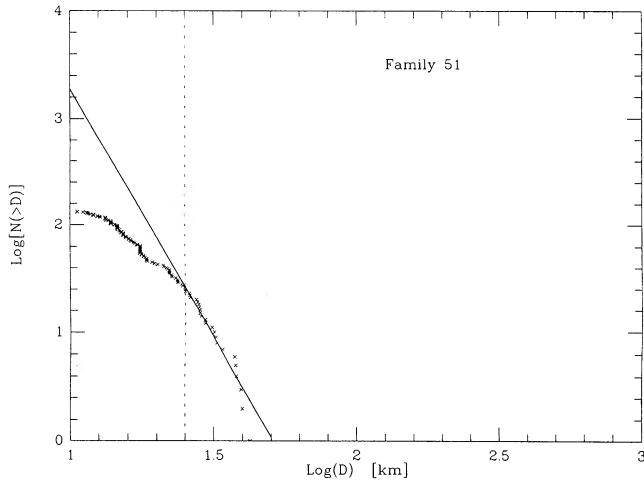
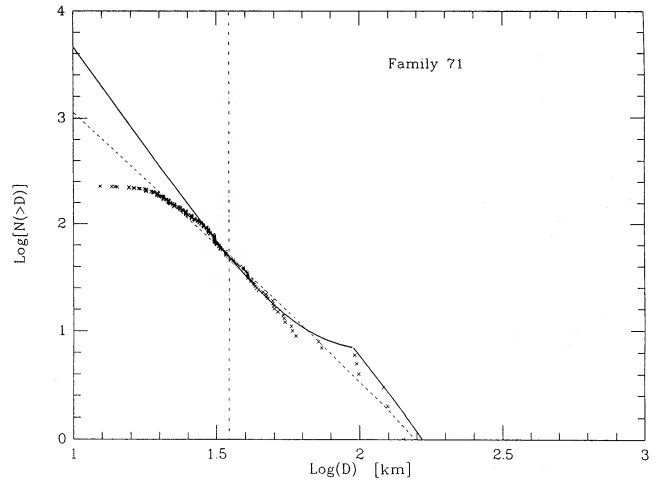
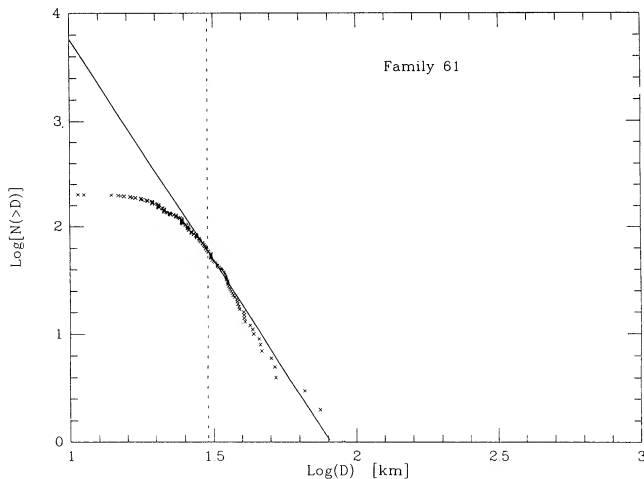
Zone 7 (643 obj.) $D_{\min} = 44 km$													
Single Truncated Pareto Distribution													
	n	α	δ	N ($> 10 km$)	χ^2	ν	P (%)	Interp.					
“mean”	282	1.62 ± 0.10	1.54 ± 0.03	$3.2 \cdot 10^3$	28.16	22	83	No R.					
“random”	293	1.63 ± 0.09	1.54 ± 0.03	$3.4 \cdot 10^3$	22.48	24	45	No R.					
“random corr.”	298	1.64 ± 0.09	1.55 ± 0.03	$3.5 \cdot 10^3$	30.51	24	83	No R.					
Double Truncated Pareto Distribution													
	Small objects					Large objects							
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.	
“mean”	140	252	1.32 ± 0.08	1.44 ± 0.03	$2.2 \cdot 10^3$	30	2.8 ± 0.5	1.94 ± 0.17	21.92	20	66	No R.	
“random”	140	263	1.30 ± 0.08	1.43 ± 0.03	$2.3 \cdot 10^3$	30	2.8 ± 0.5	1.94 ± 0.17	15.51	22	16	No R.	
“random corr.”	140	268	1.31 ± 0.08	1.44 ± 0.03	$2.3 \cdot 10^3$	30	2.8 ± 0.5	1.94 ± 0.17	23.48	22	63	No R.	

**Figure 10.** Same as Fig. 3(a), but for the sample of asteroids belonging to dynamical families.**Figure 11.** Same as Fig. 3(a), but for the sample of asteroids belonging to families 51, 61 and 71.**Table 9.** Same as in Table 2, but for the sample of family asteroids.

All Families (867 obj.) $D_{min} = 35 km$												
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)		χ^2	ν	P (%)	Interp.			
“mean”	112	2.6 ± 0.2	1.88 ± 0.08	$3.1 \cdot 10^3$		16.65	14	73	No R.			
Double Truncated Pareto Distribution												
Small objects					Large objects							
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	95	98	3.9 ± 0.4	2.29 ± 0.13	$1.3 \cdot 10^4$	14	2.5 ± 0.7	1.85 ± 0.23	7.38	12	17	No R.

Table 10. Same as in Table 2, but for the sample of asteroids belonging to families 51, 61 and 71.

Families 51+61+71 (567 obj.)					$D_{min} = 35km$							
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10km$)		χ^2	ν	P (%)	Interp.			
“mean”	90	3.2 ± 0.3	2.08 ± 0.11	$5.2 \cdot 10^3$		13.49	12	67	No R.			
“random”	109	3.2 ± 0.3	2.06 ± 0.10	$5.8 \cdot 10^3$		8.71	8	63	No R.			
“random corr.”	82	2.6 ± 0.3	1.87 ± 0.10	$2.2 \cdot 10^3$		7.52	11	25	No R.			
Double Truncated Pareto Distribution												
	Small objects					Large objects						
	D_{tr} (km)	n	α	δ	N ($> 10km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	95	82	4.4 ± 0.5	2.48 ± 0.16	$2.1 \cdot 10^4$	8	3.6 ± 1.3	2.21 ± 0.43	5.10	10	12	No R.
“random”	95	101	3.8 ± 0.4	2.28 ± 0.13	$1.3 \cdot 10^4$	8	3.6 ± 1.3	2.21 ± 0.43	6.31	6	61	No R.
“random corr.”	90	73	3.1 ± 0.4	2.02 ± 0.12	$3.5 \cdot 10^3$	9	3.4 ± 1.1	2.12 ± 0.37	6.59	9	32	No R.

**Figure 12.** Same as Fig. 3(a), but for the sample of asteroids belonging to family 51.**Figure 14.** Same as Fig. 3(a), but for the sample of asteroids belonging to family 71.**Figure 13.** Same as Fig. 3(a), but for the sample of asteroids belonging to family 61.

respect to the rest of the belt (although it should be kept in mind that this sample is complete to a smaller limiting size than all the other ones).

Finally, for all the other zones and for the overall samples, where there is always a clear change of slope and a comparatively flat distribution (not a real 'bump'!) at $D \approx 100\text{ km}$, we have another dilemma. Even if we take it for granted that the steep distributions at large sizes are 'primordial' (i.e. that only a minor fraction of asteroids larger than 150 km have been eroded or disrupted by collisions, in agreement with the conclusions of recent numerical collisional evolution models; see Davis *et al.* 1989), at intermediate sizes it is difficult to discriminate between the two alternative possibilities which we already mentioned in Section 1. These can be formulated as follows: (1) in this size range also, the shape of the distribution 'remembers' the original distribution of asteroidal planetesimals, which was possibly peaked at about 100 km and was later supplemented by a 'tail' of small collisional

fragments; (2) the size distribution has relaxed to an equilibrium one, but the slope is different from that predicted by Dohnanyi's model because collisional outcomes are not invariant with size, and in particular at about 100 km self-gravitation plays an important role in compressing asteroid interiors (hence making them more resistant to impacts) and/or causing partial reaccumulation of the ejected fragments. In the latter case, it is possible that the primordial asteroid belt

contained much more material than the present one, in particular at intermediate and small sizes. Both these alternative scenarios can be reproduced by numerical simulations of the collisional process, provided suitable sets of collisional parameters *and* appropriate – completely different – starting size distributions are assumed (Davis *et al.* 1979; Farinella *et al.* 1991a; Davis *et al.*, in preparation). This means that the present size distribution alone is unable to tell us unequivocally the collisional history of the asteroid belt; other observational constraints are necessary, and are going to be used in future studies.

Table 11. Same as in Table 2, but for the sample of asteroids belonging to family 51.

Family 51 (137 obj.)				$D_{\min} = 25km$				
Single Truncated Pareto Distribution								
	n	α	δ	N ($> 10km$)	χ^2	ν	P (%)	Interp.
“mean”	27	4.6 ± 0.9	2.54 ± 0.30	$1.9 \cdot 10^3$	1.11	2	43	No R.
“random”	37	3.4 ± 0.6	2.14 ± 0.19	$8.5 \cdot 10^2$	2.33	4	32	No R.
“random corr.”	22	2.8 ± 0.6	1.92 ± 0.20	$2.8 \cdot 10^2$	1.31	2	48	No R.

Table 12. Same as in Table 2, but for the sample of asteroids belonging to family 61.

Family 61 (202 obj.)				$D_{\min} = 30 km$				
Single Truncated Pareto Distribution								
	n	α	δ	N ($> 10 km$)	χ^2	ν	P (%)	Interp.
“mean”	61	4.1 ± 0.5	2.38 ± 0.18	$5.8 \cdot 10^3$	9.35	7	78	No R.
“random”	62	3.8 ± 0.5	2.28 ± 0.16	$4.2 \cdot 10^3$	7.72	6	74	No R.
“random corr.”	39	3.2 ± 0.5	2.07 ± 0.17	$1.3 \cdot 10^3$	2.10	4	28	No R.

Table 13. Same as in Table 2, but for the sample of asteroids belonging to family 71.

Family 71 (228 obj.)					$D_{\min} = 35 km$							
Single Truncated Pareto Distribution												
	n	α	δ	N ($> 10 km$)	χ^2	ν	P (%)	Interp.				
“mean”	49	2.5 ± 0.4	1.84 ± 0.12	$1.1 \cdot 10^3$	8.56	7	71	No R.				
“random”	58	2.5 ± 0.3	1.84 ± 0.11	$1.3 \cdot 10^3$	9.03	8	66	No R.				
“random corr.”	46	2.1 ± 0.3	1.70 ± 0.10	$6.5 \cdot 10^2$	18.48	6	99	Reject				
Double Truncated Pareto Distribution												
	Small objects				Large objects				χ^2	ν	P (%)	Interp.
	D_{tr} (km)	n	α	δ	N ($> 10 km$)	n	α	δ	χ^2	ν	P (%)	Interp.
“mean”	95	42	3.7 ± 0.6	2.24 ± 0.19	$4.6 \cdot 10^3$	7	3.4 ± 1.3	2.12 ± 0.42	6.66	5	75	No R.
“random”	95	51	3.2 ± 0.5	2.07 ± 0.15	$3.0 \cdot 10^3$	7	3.4 ± 1.3	2.12 ± 0.42	10.45	6	89	M.Rej.
“random corr.”	90	38	2.7 ± 0.4	1.89 ± 0.14	$1.2 \cdot 10^3$	8	3.2 ± 1.1	2.07 ± 0.38	18.36	4	99	Reject

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