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Flywheels; rapidly spinning, magnetized neutron stars in spherical accretion

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SUMMARY

The behaviour of a rapidly spinning, magnetized neutron star embedded in plasma accreting spherically at a supercritical rate is investigated. Friction between the magnetosphere and the surrounding plasma is a source of power. In addition, when the magnetic field is asymmetric with respect to the rotation axis of the star, the rotating magnetosphere acts as a propeller, transferring kinetic energy to the material outside the magnetosphere. We quantify these processes and calculate the values of the Alfvén radius and of the density of the surrounding plasma. We find that, if the neutron star rotates rapidly and has a strong magnetic field, such objects manifest themselves as non-thermal X-ray sources. Otherwise the material outside the magnetosphere shrouds them in such high optical depth that any outgoing radiation is thermalized. Some observational implications are briefly discussed.

1 INTRODUCTION

Much attention has been given to the interaction between a magnetized neutron star and infalling material (for a review, see Lamb 1989). When the magnetic fields have the strengths characteristic of compact objects, plasma generally cannot accrete directly on to the stellar surface. Within a characteristic radius (the Alfvén radius) the plasma dynamics are controlled by the magnetic field. The material finds its way down to the stellar surface either by forcing the field aside in localized regions, or by being channelled towards the magnetic poles (Ghosh & Lamb 1979 and references therein). As well as being important for the accretion flow, the Alfvén radius (defining the magnetopause) is important because it mediates the exchange of angular momentum between the star and the surrounding material. A Keplerian disc can 'spin up' a slowly-rotating star; conversely, if the star is spinning fast, the magnetosphere can act like a flywheel or propeller (Illarionov & Sunyaev 1975), transferring angular momentum to its surroundings and slowing down the star as a consequence.

Theoretical discussion has focused primarily on how a magnetosphere interacts with a *thin disc*, which is relevant to observed sources such as Her X-1. However, the mass transfer in many systems is expected, during the course of their evolution, to build up to a supercritical rate. Radiation pressure would then preclude the existence of any thin disc, and the transferred matter may even smother the neutron star completely. (There have, for instance, been interpretations of Cyg X-3 and SS433 along such lines.)

In this paper, we discuss a model that is in some respects complementary to that which has been discussed in detail by other authors. We consider what happens when mass is supplied to a magnetized neutron star at a supercritical rate. We neglect the angular momentum of the accreted matter, treating the infall as spherical (which may be a valid approximation to a very thick disc predominantly supported by radiation pressure). The material will generally have a large Thomson opacity, and the radiation output will be of order the Eddington luminosity, $L_{\rm Ed}$ – indeed, it could be more if there were radiation-driven jets. The magnetic field controls the flow out to a radius $r_{\rm A}$ (exceeding the stellar radius r_*). The magnetospheric cavity within $r_{\rm A}$ is empty of matter – any material accreting on to the stellar surface would be confined to narrow tubes or diamagnetic blobs with a small filling-factor. If it is difficult for matter to penetrate the magnetosphere, an accretion-powered luminosity $L = L_{\rm Ed}$ requires a high-density envelope to be maintained at $r > r_{\rm A}$, supported quasistatically by magnetic stresses. If the star is spinning, friction or propeller effects provide an additional source of power. We discuss how these processes depend on the magnetic moment and spin rate of the star, and on the pressure and density of the plasma rubbing against the magnetopause. (When the magnetic field is asymmetric with respect to the star's rotation axis, it is important to distinguish between the cases when the rotation velocity is supersonic and when it is subsonic relative to the external plasma.) We quantify these processes, calculating the values of $r_{\rm A}$ and of the external density for which the rotation-generated luminosity would be $L_{\rm Ed}$. Radiation pressure could then 'puff up' the surrounding plasma, and there need be no accretion at all. The self-consistently calculated plasma density outside the magnetopause determines whether these objects would manifest themselves as non-thermal X-ray sources, or whether the material outside the magnetosphere shrouds them in such high optical depth that any outgoing radiation is thermalized.

We briefly discuss some candidate objects that may operate as Eddington-limited flywheels, such as Cyg X-3, Sco X-1, SS433 and Cir X-1. The present study differs from previous work on flywheels (see Treves & Bocci 1987 and references therein) by considering the case when the source is at the Eddington limit. Our presentation is general, the aim being to elucidate the main phenomena and to point out some effects that have not yet received attention in the literature.

2 CONDITIONS OUTSIDE THE MAGNETOPAUSE

When the energy production rate in the vicinity of a neutron star approaches the Eddington Luminosity,

$$L_{\rm Ed} \equiv \frac{4\pi cGM}{\kappa_{\rm es}} \tag{2.1}$$

(where G is the gravity constant, c is the light velocity, M the mass of a neutron star and $\kappa_{\rm es}$ is the electron scattering opacity), radiation pressure gradients are competitive with gravity. When a luminosity very close to $L_{\rm Ed}$ is generated by accretion, the inflow is very slow, being given by the so-called 'settling solution' (Miller 1990 and references therein), and the energy generated at the core (neutron star) is transported outward through a quasi-static envelope (see Fig. 1). When the envelope is very opaque, the situation resembles that of a Thorne–Żytkow object (Thorne & Żytkow 1977). These are stars with massive envelopes and degenerate neutron cores in which most of the power is generated by accretion on to the core. The 'inflowing envelope' in the models of Thorne & Żytkow (1977), which lies just outside the 40-m thick halo above the neutron star core where most of the energy is released, is a high-density version of the envelope that we envisage outside the magnetosphere in the objects discussed here.

The inner and outer boundaries of the envelope depend on the details, but its essential feature is that the pressure scale height is everywhere of order r. This means that the sound speed $c_{\rm s} \approx (p/\rho)^{1/2}$ must scale as $r^{-1/2}$. The densities are generally high enough to ensure that the gas cools to the equivalent blackbody temperature, T. Since this is far below the virial temperature $\sim GMm_{\rm p}/k$ (where $m_{\rm p}$ is the mass of a proton and k the Boltzmann constant), radiation pressure then dominates gas pressure by a large margin. If the envelope outside $r_{\rm A}$ were isentropic, then radiation-pressure-dominated plasma with $(p/\rho) \propto r^{-1}$ must follow the radial dependence $\rho \propto r^{-3}$ and $p \propto r^{-4}$. An isentropic envelope would be ensured by convective motions if the energy were generated by 'propeller-type' effects at $r \approx r_{\rm A}$; the same radial dependence also arises in an accretion-powered source where the luminosity is very close to $L_{\rm Ed}$ (the 'settling solution'). We shall be discussing situations where the luminosity arises from a combination of accretion and rotation, and will assume this form of envelope in all cases.

The stresses associated with a magnetic dipole field scale as r^{-6}, so there is a possibility that the quasi-static envelope with $p \propto r^{-4}$ cannot extend closer to the star than some radius $r_{\rm A}$, within which the stresses associated with a distorted magnetic field become dominant. Outside $r_{\rm A}$ therefore, the structure of an isentropic quasi-static envelope would be approximately described by

$$\rho(r) = \rho(r_{\rm A}) \left(\frac{r}{r_{\rm A}}\right)^{-3}, \tag{2.2}$$

$$p(r) = p(r_{\rm A}) \left(\frac{r}{r_{\rm A}}\right)^{-4}, \tag{2.3}$$

$$c_{\rm s}(r)^2 \simeq \frac{p(r)}{\rho(r)} = \frac{GM}{r}. \tag{2.4}$$

In an isentropic flow where radiation pressure is dominant there is, in effect, one free parameter – the entropy of the matter (or, equivalently, the density at a fixed radius). The value of $r_{\rm A}$, for a given stellar magnetic field, then depends on this parameter. For a non-rotating star when accretion is the only energy source available, $\rho(r_{\rm A})$ has to be high enough to ensure that Rayleigh–Taylor instabilities, etc., allow material to leak within the magnetosphere, and down to the star's surface, at a sufficient rate to yield $L \simeq L_{\rm Ed}$. Moreover, in the case of a rotating star, the amount of power generated by friction and propeller effects depends on $\rho(r_{\rm A})$. We can therefore constrain $\rho(r_{\rm A})$ by the self-consistency requirement that such effects do not generate so much power that radiation pressure becomes strong enough to prevent infall of matter from a binary companion.

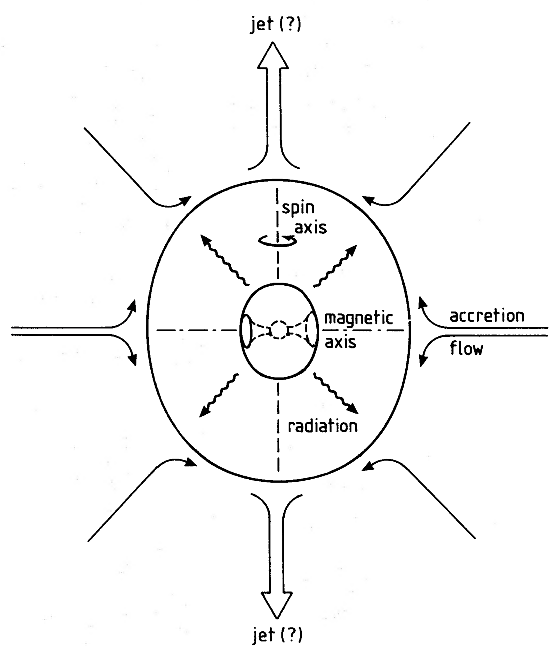

Figure 1. Schematic view of a flywheel. The magnetosphere of a rapidly-spinning, magnetized neutron star extends out to a radius $r_{\rm A}$ and is surrounded by a quasistatic envelope. The density just outside $r_{\rm A}$ adjusts so that the power generated by the rotation (plus the contribution due to accretion) is $\sim L_{\rm Ed}$; the envelope is supported by radiation pressure.

3 RAPIDLY ROTATING NEUTRON STAR AS A FLYWHEEL

3.1 Friction-induced and propeller luminosity

Energy is generated in two different ways by a rapidly spinning neutron star (Davies, Fabian & Pringle 1979; Begelman & Rees 1983, 1984);

(i) *Friction.* If the magnetosphere is axisymmetric (i.e. the magnetic axis is aligned with the spin axis), friction between magnetosphere surface and ambient material will extract rotational energy from the star. The force imposed on the surface of the magnetosphere at the Alfvén radius, r_A, is $fd\theta \sim 2\pi p_A r_A^2 \sin\theta\, d\theta$ (where the suffix A represents the values of physical quantities at r_A and θ is the angle from the spin axis), and the rotational velocity of the magnetosphere at r_A is $v_\varphi \sim r_A\Omega \sin\theta$; the energy generated by this mechanism per unit is therefore

$$L_{\rm fric} \simeq \beta \int_0^\pi f v_\varphi \, d\theta = \beta\pi^2 p_A r_A^3 \Omega, \tag{3.1}$$

where β is a constant much less than 1, representing the efficiency of energy generation by friction. We assume β to be a constant simply by analogy with the 'coefficient of friction' in more mundane contexts. Its actual value depends on much uncertain physics, and it may in practice depend on Ω (for instance, some instabilities may be excited only if Ω is high). The power $L_{\rm fric}$, liberated in a thin layer where the magnetic field changes sharply, may be partially converted into relativistic particles.

(ii) *Propeller.* If the angle between the magnetic axis and the spin axis, i, is not zero, ambient material will be vibrated or stirred by a non-axisymmetric magnetosphere; a 'propeller', with a period, $P_s \equiv 2\pi/\Omega$. The magnetospheric boundary can then be envisaged as a non-axisymmetric surface which rotates rigidly. The shape of the surface depends not only on the stellar magnetic field, but on the dynamics of the interaction with the material outside r_A, which will distort the magnetosphere by an amount depending on Ω. However, the volume of material which is disturbed each time the magnetosphere revolves is of order

$$V = \frac{4}{3}\pi r_A^3 f(i). \tag{3.2}$$

The function $f(i)$ is a measure of how non-axisymmetric the magnetosphere is: we expect it to be ~ 1 for $i \approx \pi/2$, but $\ll 1$ if the misalignment between magnetic and rotation axes is small.

The average energy per unit mass which is transferred to the displaced material each revolution is

$$e \simeq \frac{1}{2}(r_A\Omega)^2 f'(i), \tag{3.3}$$

where $f'(i)$ is another function with value $\lesssim 1$ measuring the non-axisymmetry. Taking $f(i) = f'(i) = 1$, we can estimate the power generated by this mechanism

$$L_{\rm prop} \approx e\rho_A(\text{volume of magnetosphere})\frac{\Omega}{2\pi} \approx \frac{1}{3}\rho_A r_A^5 \Omega^3. \tag{3.4}$$

The above is in fact only valid on the *subsonic* case. Since material cannot fall into the region $r < r_A$ at a speed exceeding c_s, the region from which material is displaced each revolution is a shell of thickness $\sim c_s P_s$. In this case we therefore have

$$L_{\rm prop} \approx \frac{4}{3}\pi\rho_A r_A^4 c_s \Omega^2, \tag{3.5}$$

where we have adopted a factor of 2/3 as an approximation for $f'(i)$.

We thus approximately have

$$L_{\rm prop} \simeq \begin{cases} \dfrac{1}{3} p_A r_A^2 \dfrac{(r_A\Omega)^3}{c_s^2} & \text{for } r_A\Omega < 4\pi c_s; \\ \dfrac{4\pi}{3} p_A r_A^2 \dfrac{(r_A\Omega)^2}{c_s} & \text{for } r_A\Omega > 4\pi c_s. \end{cases} \tag{3.6}$$

Here $c_s \equiv \sqrt{p/\rho}$ is the sound velocity taking radiation pressure into account. The 'propeller' mechanism creates shocks which propagate into the settling flow outside r_A. The energy is mainly carried by the matter, but the density is high enough for the shocks to be radiative. Provided that the optical depth is high enough, this energy will be thermalized.

To sum up, both mechanisms are possible when a spin axis and a magnetic axis are not aligned;

$$L_{\rm rot} = L_{\rm fric} + L_{\rm prop}$$

$$\simeq \begin{cases} \beta\pi^2 p_A r_A^2 r_A \Omega, & \text{for } r_A\Omega < \sqrt{3\beta}\pi c_s; \\ \dfrac{1}{3} p_A r_A^2 \dfrac{(r_A\Omega)^3}{c_s^2}, & \text{for } \sqrt{3\beta}\pi c_s < r_A\Omega < 4\pi c_s; \\ \dfrac{4\pi}{3} p_A r_A^2 \dfrac{(r_A\Omega)^2}{c_s}, & \text{for } r_A\Omega > 4\pi c_s. \end{cases} \tag{3.7}$$

3.2 Physical assumptions

We assume that a stationary magnetosphere extends to a radius r_A which adjusts its value so that $L_{\rm rot}$, given by (3.7), is equal to the Eddington Luminosity. If $L_{\rm rot} > L_{\rm Ed}$, the density outside the magnetosphere would be reduced due to the expulsion of material by radiation pressure; conversely, if $L_{\rm rot} < L_{\rm Ed}$, material would fall in and accumulate at the magnetospheric boundary until $L_{\rm rot}$ increased. In either case a state with $L_{\rm rot} = L_{\rm Ed}$ is achieved. At the boundary at $r = r_A$, pressure in the ambient material balances magnetic pressure of the magnetosphere, $p_{\rm mag}$. For a dipole field,

$$p_{\rm mag}(r) \equiv \frac{B(r)^2}{8\pi} = \frac{B_*^2}{8\pi}\left(\frac{r}{r_*}\right)^{-6}, \tag{3.8}$$

where $B(r)$ and B_* are the field strength at r, and at the radius of a neutron star r_*. We can then uniquely determine $\rho(r_A)$ [or p_A] as a function of Ω for given values of r_A or B_*.

3.3 Friction

Suppose $L_{\rm fric}$ is the only energy source. Since $p = p_{\rm mag} \propto r_A^{-6}$ (equation 3.8), $L_{\rm fric} \propto r_A^{-3}$ (equation 3.1). If $L_{\rm fric}$ exceeds the Eddington luminosity, then the magnetosphere expands until

$L_{\rm fric}$ becomes equal to $L_{\rm Ed}$, and vice versa. The size of the magnetosphere, $r_{\rm A}$, is thus determined by a condition, $L_{\rm fric}=L_{\rm Ed}$;

$$r_{\rm A}=\left(\frac{\beta\pi}{16}\frac{\kappa_{\rm es}}{cP_{\rm s}}\frac{B_*^2 r_*^6}{GM}\right)^{1/3}=10^{7.1}\beta_{1.0}^{1/3}m^{-1/3}B_{12}^{2/3}r_{*,6}^{2}P_{\rm s}^{-1/3}\ {\rm cm}, \quad (3.9)$$

for $\beta=0.1\,\beta_{0.1}$, $M=m\,M_\odot$, $B=10^{12}\,B_{12}$ G, and $r_*=10^6\,r_{*,6}^2$ cm. Here $P_{\rm s}(\equiv 2\pi/\Omega)$ is the spin period of a neutron star in seconds. The period, below which the surface velocity of the magnetosphere, $r_{\rm A}\Omega$, exceeds the light velocity, is

$$P_{\rm s,light}\simeq 10^{-1.9}\beta_{0.1}^{1/4}\,m^{-1/4}B_{12}^{1/2}r_{*,6}^{3/2}\ {\rm s}. \quad (3.10)$$

The constant B_* line is expressed as (equations 3.1 and 3.8)

$$\rho_{\rm A}=\left(\frac{2cP_{\rm s}}{\beta\pi^2\kappa_{\rm es}}\right)^{5/3}\left(\frac{8\pi GM}{B_*^2r_*^6}\right)^{2/3} \quad (3.11)$$
$$\simeq 10^{-3.0}\beta_{0.1}^{-5/3}m^{2/3}B_{12}^{-4/3}r_{*,6}^{-4}P_{\rm s}^{5/3}\ {\rm cm}^{-3}.$$

Throughout the following discussion, we make the assumption that $r_{\rm A}$ is the radius at which magnetic stresses are in balance with the *total* external pressure (i.e. radiation pressure as well as gas pressure). This is obviously appropriate if the external density (and hence opacity) is high enough for matter and radiation to be treated as a single composite fluid. Moreover, the estimate of $r_{\rm A}$ is also appropriate, irrespective of the opacity, when $\Omega_{\rm A}\approx c_{\rm s}$ and the propeller effect is important. This is because, even if the thermal pressure of the gas is negligible, magnetic stresses must then balance the 'ram pressure' $\sim\rho_{\rm A}(r_{\rm A}\Omega)^2$ of the external matter [*cf.* (3.3)]; this would preclude having a magnetic pressure as low as the thermal pressure of the gas alone.

We then have the blackbody temperature at $r_{\rm A}$,

$$T_{\rm A}=\left(\frac{1}{a}\rho_{\rm A}\frac{GM}{r_{\rm A}}\right)^{1/4} \quad (3.12)$$
$$\simeq 10^{7.54}\beta_{0.1}^{-1/2}m^{1/2}B_{12}^{-1/2}r_{*,6}^{-3/2}P_{\rm s}^{1/2}\ {\rm K},$$

where a represents the radiation constant.

Once $\rho_{\rm A}$ and $T_{\rm A}$ are known, we can describe the structure of ambient material surrounding a flywheel, using the relations of settling solutions (equations 2.2–2.4):

$$\rho(r)=\rho_{\rm A}\left(\frac{r}{r_{\rm A}}\right)^{-3} \quad (3.13)$$
$$\simeq 10^{-2.7}\beta_{0.1}^{-2/3}\,m^{-1/3}B_{12}^{2/3}r_{*,6}^{2}P_{\rm s}^{2/3}\left(\frac{r}{10^7\,{\rm cm}}\right)^{-3}{\rm cm}^{-3};$$

$$p_{\rm rad}(r)=\rho(r_{\rm A})\,\frac{GM}{r} \quad (3.14)$$
$$\simeq 10^{16.4}\beta_{0.1}^{-2/3}m^{2/3}B_{12}^{2/3}r_{*,6}^{2}P_{\rm s}^{2/3}\left(\frac{r}{10^7\,{\rm cm}}\right)^{-4}{\rm dyn\ cm}^{-2};$$

$$T(r)=\left(\frac{p_{\rm rad}}{a}\right)^{1/4} \quad (3.15)$$
$$\simeq 10^{7.6}\beta_{0.1}^{-1/6}\,m^{1/6}B_{12}^{1/6}r_{*,6}^{1/2}P_{\rm s}^{1/6}\left(\frac{r}{10^7\,{\rm cm}}\right)^{-1}{\rm K}.$$

Thus we estimate

$$\frac{p_{\rm gas}(r)}{p_{\rm rad}(r)}\sim\frac{r}{GM}\,\Re\,T(r)\simeq 10^{-3.5}\beta_{0.1}^{-1/6}\,m^{-5/6}B_{12}^{1/6}r_{*,6}^{1/2}P_{\rm s}^{1/6}\ll 1, \quad (3.16)$$

where $\Re$ is the gas constant. Gas pressure is surely negligible compared with radiation pressure.

The Thomson optical thickness, $\tau_{\rm es}$, and the effective optical thickness, τ_*, are calculated as

$$\tau_{\rm es}(r)\equiv\int_r^\infty\kappa_{\rm es}\rho(r)\,dr=\frac{\kappa_{\rm es}\rho_{\rm A}r_{\rm A}}{2}\left(\frac{r}{r_{\rm A}}\right)^{-2}, \quad (3.17)$$

and

$$\tau_*(r)\equiv\int_r^\infty\sqrt{\kappa_{\rm ff}\kappa_{\rm es}}\rho(r)\,dr=\frac{4}{7}\sqrt{\kappa_{\rm es}\kappa_0}\rho_{\rm A}^{3/2}T_{\rm A}^{-7/4}r_{\rm A}\left(\frac{r}{r_{\rm A}}\right)^{-7/4}, \quad (3.18)$$

where $\kappa_{\rm ff}=\kappa_0\rho T^{-7/2}$, $\kappa_0=7.5\times10^{22}$ (c.g.s.), and we used equation (3.13). From equations (3.11), (3.12), (3.17) and (3.18), we have

$$\tau_{\rm es}(r_{\rm A})\simeq 10^{3.4}\beta_{0.1}^{-4/3}m^{1/3}B_{12}^{-2/3}r_{*,6}^{-2}P_{\rm s}^{4/3}, \quad (3.19)$$

and

$$\tau_*(r_{\rm A})\simeq 10^{0.4}\beta_{0.1}^{-31/24}m^{-5/24}B_{12}^{-11/24}r_{*,6}^{-11/8}P_{\rm s}^{31/24}. \quad (3.20)$$

Using the relation, $\tau_*(r)=\tau_*(r_{\rm A})(r/r_{\rm A})^{-7/4}$, the effective temperature (the temperature at the radius where $\tau_*=1$) is estimated as

$$T_{\rm eff}\simeq 10^{7.31}\beta_{0.1}^{5/21}\,m^{13/21}B_{12}^{-5/21}r_{*,6}^{-5/7}P_{\rm s}^{-5/21}\ {\rm K}. \quad (3.21)$$

These relations are summarized in Fig. 2 for the case of a strong magnetic field; $B_*=10^{12}$ G, and in Fig. 3 for a case of a weak field; $B_*=10^{10}$ G. The envelope is optically (Thomson) thick unless $P_{\rm s}$ is very short. As Ω increases, the envelope shrinks, and $\rho_{\rm A}$ decreases in order to keep the propeller luminosity at the Eddington luminosity.

3.4 Subsonic propeller

When $L_{\rm prop}$ becomes dominant, the rotationally-induced luminosity is independent of $r_{\rm A}$ in the subsonic regime (equations 3.6 and 3.8);

$$L_{\rm rot}=\frac{\pi^2}{3GM}\frac{B_*^2r_*^6}{P_{\rm s}^3}. \quad (3.22)$$

This is because, for $p\propto r^{-4}$, an increase in the size of the propeller ($r_{\rm A}$) is cancelled by a decrease in the pressure, and

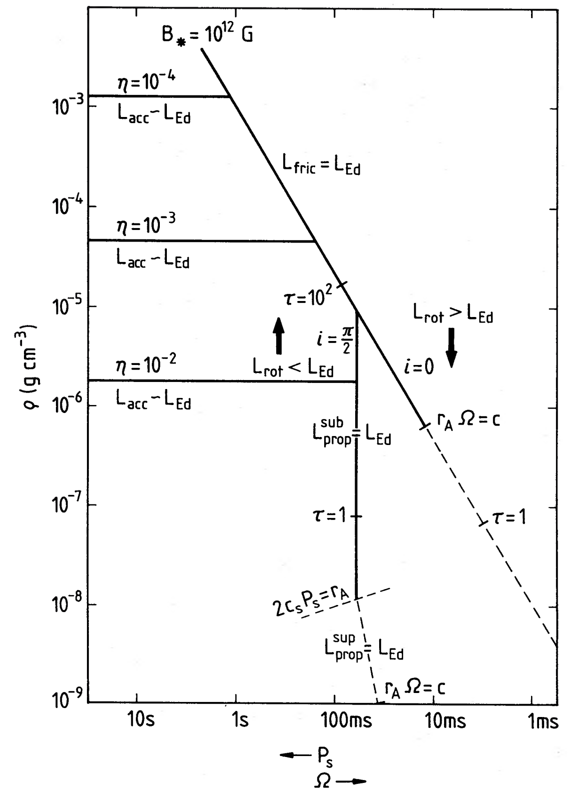

Figure 2. Structure of a flywheel with strong magnetic fields, $B_* = 10^{12}$ G. The density just outside r_A is depicted by the solid lines as a function of the spin period (P_s). We distinguish following four states: the accretion-dominated state (L_{acc}), the friction-dominated state (L_{fric}), the subsonic propeller-dominated state (L_{prop}^{sub}), and the supersonic propeller-dominated state (L_{prop}^{super}). The places where $\tau_{es} = 100$ and 1, and where the propeller velocity reaches the light velocity are indicated. Adopted parameters are $M = 1\ M_\odot$ and $r_* = 10^6$ cm.

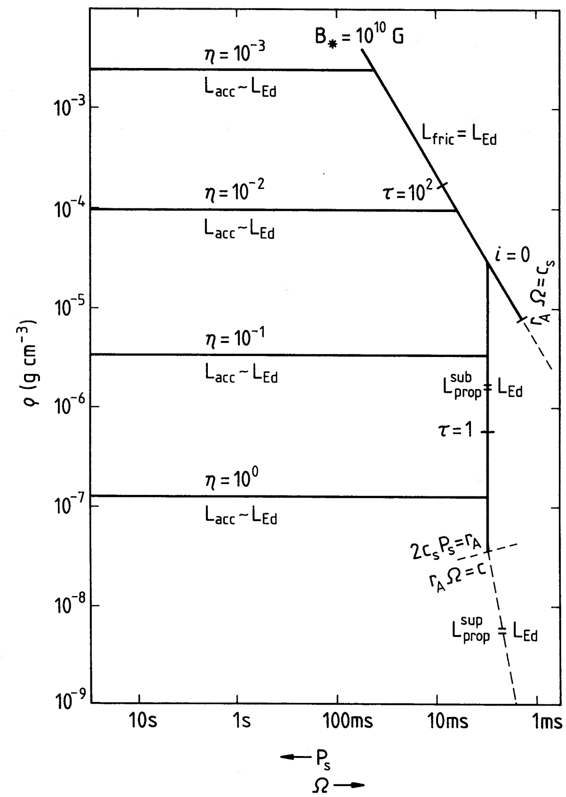

Figure 3. As Fig. 2, but with weak magnetic field, $B_* = 10^{10}$ G.

vice versa. The value of P_s (or Ω) is independent of r_A, and is found from the energy balance, $L_{rot} = L_{Ed}$,

$$P_{s,crit} = \left(\frac{\pi \kappa_{es}}{12c}\right)^{1/3} \left(\frac{B_* r_*^3}{GM}\right)^{2/3} \tag{3.23}$$

$$\simeq 10^{-1.2} m^{-2/3} B_{12}^{2/3} r_{*,6}^2 \text{ s.}$$

The Alfvén radius and the Thomson optical depth are given as functions of ρ_A as

$$r_A = \left(\frac{1}{8\pi\rho_A} \frac{B_*^2 r_*^6}{GM}\right)^{1/5} \tag{3.24}$$

$$\simeq 10^{7.7} \rho_{-6}^{-1/5} m^{-1/5} B_{12}^{2/5} r_{*,6}^{6/5} \text{ cm,}$$

and

$$\tau_{es} = \frac{\kappa_{es}}{2} \left(\frac{\rho_A^4}{8\pi} \frac{B_*^2 r_*^6}{GM}\right)^{1/5} \tag{3.25}$$

$$\simeq 10^{1.0} \rho_{-6}^{4/5} m^{-1/5} B_{12}^{2/5} r_{*,6}^{6/5},$$

where $\rho_{-6} = \rho_A/(10^{-6}\ \text{g cm}^{-3})$.

The system is unstable for changing Ω (P_s) in this stage (see Fig. 2). In the case of slow rotation ($P_s > P_{s,crit}$), the subsonic propeller luminosity is always sub-Eddington, which leads to an increase in the density until L_{fric} dominates over L_{prop}. On the other hand, in the case of rapid rotation ($P_s < P_{s,crit}$), the luminosity produced exceeds the Eddington luminosity. The density is then decreased until the propeller becomes supersonic. The subsonic-propeller phase is therefore never observed.

3.5 Supersonic propeller

When $r_A\Omega$ become supersonic, the efficiency of producing energy drops down by a factor $r_A\Omega/c_s$ (>1). The condition for the subsonic flow is

$$\frac{r_A\Omega}{c_s} \leq 4\pi, \tag{3.26a}$$

or

$$P_s \geq r_A/(2c_s). \tag{3.26b}$$

This condition is rewritten as

$$\rho_A \geq \frac{3c}{8\pi^2 \kappa_{es}} (4GM)^{-2/3} P_s^{-1/3} \simeq 10^{-8.3} m^{-2/3} P_s^{-1/3} \text{ cm}^{-3}, \tag{3.27}$$

where we used equations (2.4), (3.23) and (3.26b).

The Alvén radius is obtained from equations (3.6) and (3.8):

$$r_A = \left(\frac{\pi}{6} \frac{\kappa_{es}}{cP_s^2}\right)^{2/3} \frac{(B_* r_*^3)^{4/3}}{GM} = 10^{6.4} m^{-1} B_{12}^{4/3} r_{*,6}^4 P_s^{-4/3} \text{ cm,} \tag{3.28}$$

and the spin period over which the propeller velocity becomes larger than the light velocity is

$$P_{\rm s,light}=10^{-1.4}m^{-3/7}B_{12}^{4/7}r_{*,6}^{12/7}\ {\rm s}. \tag{3.29}$$

The structure of the envelope is (equations 2.1, 3.6, 3.8 and 3.28), for a given value of B_*, expressed as

$$\rho_{\rm A}=\frac{1}{8\pi}\left(\frac{6c}{\pi}\frac{P_{\rm s}^2}{\kappa_{\rm es}}\right)^{10/3}(GM)^4(B_*r_*^3)^{-14/3} \tag{3.30}$$

$$\simeq 10^{0.3}m^4B_{12}^{-14/3}r_{*,6}^{-14}P_{\rm s}^{20/3}\ {\rm cm}^{-3},$$

$$T_{\rm A}\simeq 10^{8.5}m^{3/2}B_{12}^{-3/2}r_{*,6}^{-9/2}P_{\rm s}^2\ {\rm K}, \tag{3.31}$$

$$\rho(r)=\rho_{\rm A}\left(\frac{r}{r_{\rm A}}\right)^{-3} \tag{3.32}$$

$$\simeq 10^{-1.4}mB_{12}^{-2/3}r_{*,6}^{-2}P_{\rm s}^{8/3}\left(\frac{r}{10^7\ {\rm cm}}\right)^{-3}\ {\rm cm}^{-3},$$

$$p_{\rm rad}({\rm r})=\rho(r)\frac{GM}{r} \tag{3.33}$$

$$\simeq 10^{17.7}m^2B_{12}^{-2/3}r_{*,6}^{-2}P_{\rm s}^{8/3}\left(\frac{r}{10^7\ {\rm cm}}\right)^{-4}\ {\rm dyn\ cm}^{-2},$$

$$T(r)=\left(\frac{p_{\rm rad}}{a}\right)^{1/4} \tag{3.34}$$

$$\simeq 10^{8.0}m^{1/2}B_{12}^{-1/6}r_{*,6}^{-1/2}P_{\rm s}^{-2/3}\left(\frac{r}{10^7\ {\rm cm}}\right)^{-1}\ {\rm K},$$

$$\frac{p_{\rm gas}(r)}{p_{\rm rad}(r)}\simeq 10^{-3.2}m^{-1/2}B_{12}^{1/6}r_{*,6}^{1/2}P_{\rm s}^{-2/3}\ll 1, \tag{3.35}$$

$$\tau_{\rm es}\simeq 10^{6.0}m^3B_{12}^{-10/3}r_{*,6}^{-10}P_{\rm s}^{16/3}, \tag{3.36}$$

and

$$\tau_*\simeq 10^{2.1}m^{19/8}B_{12}^{-73/24}r_{*,6}^{-73/8}P_{\rm s}^{31/6}. \tag{3.37}$$

The envelope is thus optically thin (to absorption, rather than scattering) at short periods, $P_{\rm s}<100$ ms.

3.6 Angular momentum effects

The frictional and propeller effects transfer angular momentum as well as energy from the spinning star to its surroundings. The material outside $r_{\rm A}$ would therefore be rotating. (This is an extra reason, in addition to the fact that the magnetospheric cavity is in any case non-spherical, why the actual situation cannot be spherically symmetrical.) This angular momentum could be transferred outwards by viscosity as in an accretion disc, the material then taking the form of an oblate differentially-rotating cloud. The cloud's rotation would be sub-Keplerian, because part of its support would still come from radiation pressure. The angular velocity of the cloud (and its degree of flattening) would depend on the viscosity. Allowance for the rotation and flattening would change the conditions at $r_{\rm A}$, and therefore our estimates of L, by a factor of order unity.

In the supersonic propeller case, some of the angular momentum could be transferred to material which acquires more than the escape velocity and is expelled from the system.

4 THE RELATIVE IMPORTANCE OF ROTATION AND ACCRETION

4.1 Accretion-energy dominated case

If the rotation of a neutron star is sufficiently slow, energy is mainly supplied by accretion instead of by rotation of the magnetosphere; in other words, $L_{\rm acc}\gg L_{\rm rot}$. We therefore aim to consider what factors determine the importance of accretion relative to the effects of rotation explained in the preceding section.

Let us now, therefore, consider the contribution from accretion. When there is also a rotational contribution to L, the radial inflow is no longer described by a modified free fall velocity,

$$v_{\rm mff}=\sqrt{\frac{2GM\varepsilon}{r}\frac{r_*}{r}}, \tag{4.1}$$

where ε denotes the fractional difference between the energy generation around a neutron star and the critical value,

$$\varepsilon\equiv 1-\mu-\frac{L_{\rm ex}}{L_{\rm Ed}} \tag{4.2}$$

and r_* is the radius of the neutron star. Here $L_{\rm ex}$ is the additional energy gain, such as $L_{\rm rot}$, where

$$\mu\equiv\frac{\dot{M}}{\dot{M}_{\rm crit}}. \tag{4.3}$$

For sufficiently small radii,

$$r\ll(\mu/\varepsilon)r_*, \tag{4.4}$$

The accretion velocity, $v_{\rm acc}$, then becomes proportional to r (Miller 1990):

$$\frac{v_{\rm acc}(r)}{v_{\rm esc}(r_*)}\simeq\sqrt{\frac{50\varepsilon}{1-\varepsilon}}\left(\frac{\varepsilon}{\mu}\right)^{3/2}\left(\frac{r}{r_*}\right), \tag{4.5}$$

where

$$v_{\rm esc}(r)\equiv\sqrt{\frac{2GM}{r}}. \tag{4.6}$$

In general, matter cannot accrete freely on to a star, owing to the radiation drag force, and the presence of a magnetosphere. Here we define a parameter, η, representing how easily matter can accrete on to a star:

$$v_{\rm acc}(r_{\rm in})\equiv\eta v_{\rm esc}(r_{\rm in}) \tag{4.7}$$

and $v_{\rm acc}(r)=v_{\rm acc}(r_{\rm in})(r/r_{\rm in})$, where $r_{\rm in}$ is either of r_* (in the case of non-magnetized neutron stars) or $r_{\rm A}$ (in the case of magnetized neutron stars).

In the case of a non-magnetized neutron star ($L_{ex}=0$), we have from equation (4.7),

$$\eta \equiv 5\sqrt{2}\left(\frac{1-\mu}{\mu}\right)^2. \tag{4.8}$$

For $\mu=0.9$ and 0.99, we find $\eta=0.09$ and 7×10^{-4}, respectively. Once we know the values of density and pressure at $r=r_*$, the structure of the ambient gas is given by the relations of the settling solution (see Section 2).

In the presence of a magnetosphere, settling solutions cannot be applied to the region inside r_A because magnetic pressure then dominates radiation pressure and because matter will probably be in the form of blobs. The value of η then depends on the complicated plasma processes whereby blobs of plasma can penetrate the magnetospheric cavity. We do not know how to evaluate η accurately, and so leave it as a parameter much smaller than 1.

Suppose that $\mu\sim1$ and $\varepsilon\sim1-\mu\ll1$. Then

$$\dot{M}\equiv 4\pi r^2\rho(r)v_{acc}(r)\sim \dot{M}_{Ed}, \tag{4.9}$$

where $\dot{M}_{Ed}$ represents a critical accretion rate defined as

$$\dot{M}_{Ed}\equiv L_{Ed}\frac{r_*}{GM}=10^{18.0}r_{*,6}\ \mathrm{g\,s^{-1}}. \tag{4.10}$$

Assuming $p_A/\rho_A\sim c_s^2\sim GM/r_A$ (equation 2.4), we finally obtain

$$r_A=\left(\frac{\sqrt{2}\eta\kappa_{es}}{cr_*}\right)^{2/7}(GM)^{-1/7}\left(\frac{B_*^2r_*^6}{8\pi}\right)^{2/7}$$
$$\simeq 10^{7.1}\left(\frac{\eta}{10^{-4}}\right)^{2/7}m^{-1/7}B_{12}^{4/7}r_{*,6}^{10/7}\ \mathrm{cm}. \tag{4.11}$$

This equation tells us how, for an accretion-powered source, r_A depends on the parameter η, which determines how easily material can force its way into the magnetosphere. If η is very small, a high external pressure is needed in order for accretion to occur at a sufficient rate to yield $L\simeq L_{Ed}$, and thus squeeze r_A down to a small value.

Similarly we find

$$\rho_A=\left(\frac{cr_*}{\sqrt{2}\eta\kappa_{es}}\right)^{10/7}(GM)^{-2/7}\left(\frac{B_*^2r_*^6}{8\pi r_*^8}\right)^{-3/7} \tag{4.12}$$
$$\simeq 10^{-2.9}\left(\frac{\eta}{10^{-4}}\right)^{-10/7}m^{-2/7}B_{12}^{-6/7}r_{*,6}^{-8/7}\ \mathrm{cm^{-3}},$$

$$T_A\simeq 10^{7.56}\left(\frac{\eta}{10^{-4}}\right)^{-3/7}m^{3/14}B_{12}^{-5/14}r_{*,6}^{-9/14}\ \mathrm{K}, \tag{4.13}$$

$$\tau_{es}\simeq 10^{3.5}\left(\frac{\eta}{10^{-4}}\right)^{-8/7}m^{-3/7}B_{12}^{-2/7}r_{*,6}^{2/7}, \tag{4.14}$$

$$\tau_*\simeq 10^{0.5}\left(\frac{\eta}{10^{-4}}\right)^{-31/28}m^{-53/56}B_{12}^{-5/56}r_{*,6}^{47/56}, \tag{4.15}$$

and

$$T_{eff}\simeq 10^{7.27}\left(\frac{\eta}{10^{-4}}\right)^{10/49}m^{74/98}B_{12}^{-15/49}r_{*,6}^{-55/49}\ \mathrm{K}. \tag{4.16}$$

Note that the density is independent of Ω as long as $L_{acc}>L_{rot}$. The line which represents the density in the accretion era is also displayed in Figs 2 and 3. For periods longer than the critical value, which is a strong function of a parameter η, the accretion luminosity exceeds the propeller luminosity. The more effectively the magnetic field inhibits the inflow, the smaller Ω has to be before the propeller luminosity takes over.

Let us consider objects with progressively increasing rotation rates Ω. The contribution of L_{rot} is small compared with L_{acc} as long as a neutron star rotates slowly; thus the structure of the ambient material is then independent of Ω. For the case of a strong field, $B_*=10^{12}$ G (Fig. 2), however, eventually L_{fric} will exceed L_{acc}, as Ω goes up. In this stage (friction-energy dominated era), the system is optically thick and thus looks like a giant star. The density of ambient material then decreases with increase of Ω. If the shape of the magnetosphere is non-axisymmetric, eventually the motion of the rotating 'paddle-wheels' becomes the dominant energy source, when an instability takes place (subsonic propeller-dominated era). The density of the ambient gas rapidly decreases until the propeller velocity, $r_A\Omega$, becomes supersonic. In this stage the ambient material becomes optically thin. The radiation then emerges predominantly as hard X-rays, in contrast to the thermal (soft X-ray) spectrum expected in the friction- or accretion-dominated cases.

So far we have assumed $L_{rot}\simeq L_{Ed}$, but a similar transition could occur even for the same P_s if we reduce L_{rot}. This may happen when the mass transfer rate is greatly enhanced and part of the power is supplied by accretion. In Fig. 4 we plot ρ_A as a function of L_{rot}/L_{Ed} for several values of $P_s=1$, 0.32

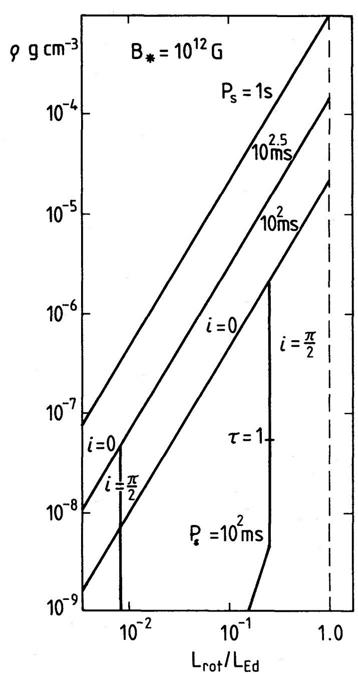

Figure 4. Structure of a flywheel as a function of L_{rot}/L_{Ed}. We assume $B_*=10^{12}$ G and $P_s=1$, 0.32 and 0.1 s.

and 0.1 s. We see in this figure that, for the case of $P_s = 0.1$ s, the propeller is subsonic at $L_{rot} \simeq L_{Ed}$, but becomes supersonic when $L_{rot}/L_{Ed} < 1/4$. This transition is rather abrupt; a small change in L_{rot} will yield large changes in observational appearance.

In contrast, accretion energy always dominates over L_{rot} in the case of weak fields (Fig. 3). The density will not change so much even when the spin period of the neutron star increases. Note that, similarly to the case of a strong field strength, if we decrease L_{rot} we also have a subsonic–supersonic transition at much smaller values of P_s; e.g. for $P_s = 1$ ms, the transition occurs when $L_{rot}/L_{Ed} < 1/4$.

Although the surrounding material is better approximated as a sphere than as a disc, there is, in any realistic case, certain to be some anisotropy: this could be associated with rotational flattening, or could arise because the magnetospheric cavity created by a dipole field would in any case be ellipsoidal rather than spherical. Large-scale convective or circulatory motions would then occur within the plasma outside r_A in order to maintain pressure balance. There is even then the possibility of jets (*cf.* Begelman & Rees 1984). If radiation or kinetic energy can escape easily in some preferred directions, it is in principle possible for L_{rot} (or even L_{acc}) to exceed L_{Ed}.

4.2 Observational implications

The operation of a flywheel has previously been considered by Priedhorsky (1986) for Sco X-1 and by Treves & Bocci (1987; see also references therein) for Cyg X-3. These studies show that the extraction of spin energy from rapidly-rotating neutron stars in thin discs may be of importance. They do not consider what happens when the power reaches the Eddington limit, as we assume here. Both objects do have luminosities comparable with L_{Ed} and, if they contain neutron stars, may be compared with our spherical accretion model. It may also apply to other Eddington or super-Eddington luminosity sources such as SS433 and Cir X-1.

We are not, however, able to produce a detailed picture of the appearance of an Eddington-limited flywheel since it depends on the outermost structure of the envelope. Certainly in the friction case or the accretion-powered case in which η is small, there will be a large envelope, very optically-thick, and the object may resemble a star. Such objects may occur in the normal evolution of high-mass X-ray binaries undergoing Roche-lobe mass transfer. A substantial wind probably occurs from the envelope around the neutron star, taking away most of the mass transferred from the more massive primary. If the magnetosphere maintains a non-spherical, transparent region immediately around the neutron star then jets may be set up by motions in the surrounding gas necessary to maintain pressure equilibrium. SS433 may be such an object.

Continued strong accretion on to a neutron star in a low-mass binary system may spin it up to millisecond periods. In that case, the object may evolve on to the subsonic-propeller line in Fig. 2. It may remain close to that line since spin energy is tapped at faster periods. Alternatively, the object may oscillate between a high optical-thickness accretion (or friction-powered) state and a much thinner propeller state. Jets may again form, possibly with radio outbursts from relativistic electrons accelerated by the propeller, in either state. Cyg X-3 and Sco X-1, both of which have (or have had) jets or outbursts (Strom, van Paradijs & van der Klis 1989; Geldzahler & Fomalont 1986), are candidates for such an object.

Perhaps the best candidate for the processes described above is Cir X-1. This puzzling source shows strong X-ray and radio outbursts every 16.6 d, chaotic millisecond variability, an ultrasoft high-intensity state and X-ray bursts (see Tennant 1988; Inoue 1990 and references therein). During some outbursts, the flux from the source is apparently super-Eddington, where the Eddington limit is defined from the peak flux in the X-ray bursts (when radius expansion occurs). Outbursts are sometimes preceded by large absorption events, and when the source is very bright it may show extreme intensity and spectral changes on the time-scale of minutes.

We know that Cir X-1 contains a neutron star with a weak magnetic field (i.e. $B \lesssim 10^{10}$ G) because of the observation of X-ray bursts (Tennant, Shafer & Fabian 1986). The 16.6-d cycle is probably due to periodic mass transfer from a binary companion, possibly in an eccentric orbit. The neutron star is therefore subject to a varying mass influx which drives the object from a weak accretion state, through an Eddington-limited state and into the supersonic-propeller regime where it can be super-Eddington until it clears away the surrounding material. Such behaviour requires the spin rate of the neutron star to be in the millisecond range. We have not (and cannot at this stage) pursued the stability of accretion on to a flywheel, but would not be surprised if it led to behaviour such as observed from Cir X-1.

Finally, we note that there may be many objects in the Eddington-limited subsonic-propeller regime in the Galaxy. They may occur in massive ex-X-ray binaries (Cen X-3 may become one) and in low-mass X-ray binaries, particularly in globular clusters, where the circumstances allow super-Eddington rates of mass transfer. Such objects should appear as stars with luminosities at the Eddington limit for the central neutron star (i.e. for about 1.4 $M_\odot$). They may resemble Thorne–Żytkow objects and appear as giant stars, but the likely high angular momentum of the core, being a close binary, and strong mass loss, may cause them to be smaller, bluer and possibly to have jets.

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