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NEUTRON STAR CRUSTAL PLATE TECTONICS. III. CRACKING, GLITCHES, AND GAMMA-RAY BURSTS

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ABSTRACT

When the growing vortex pinning stresses in a cool, spinning-down superfluid neutron star strain the crust beyond its elastic yield strength, large-scale crust cracking events may occur. These would result in pulsar timing glitches with magnitude and recurrence rates near those observed. In old or dead radio pulsars these sudden releases of stored elastic energy should give bursts of X-ray and gamma-rays whose number, energy, and rise time suggest those of the so far unidentified gamma-ray burst sources. The surface magnetic field of a spinning-down crust-cracking neutron star breaks up into large surface patches (platelets) which move apart from each other. Each platelet retains its original surface magnetic field, while the expanding new interplatelet crust regions have little or none. Pulsar spin-down torque observations would reflect the decrease in average surface dipole field, while the field strength inferred from a cyclotron resonance spectral feature above a platelet would remain high and independent of stellar age.

Subject headings: dense matter — gamma rays: bursts — pulsars — stars: neutron

1. INTRODUCTION

Spinning-down (or up) neutron star crusts can be stressed beyond their yield strengths by neutron superfluid vortex line pinning and possibly by core magnetic flux-tube tension. These stresses and their consequences for millisecond pulsars and low-mass X-ray binaries were discussed in the first paper of this series (Ruderman 1991a; hereafter R-I). The second considered applications to the magnetic field evolution of canonical solitary radio pulsars and spin-up "resurrected" pulsars (Ruderman 1991b; hereafter R-II). In none of these applications were details of exactly how a crust yields to a stress which exceeds its strength, whether by lattice crumbling or plastic flow or sudden slipping along faults, important. It was sufficient that such stresses would always move crust toward the stellar equator at an average tangential speed (R-I)

$$v_t = \frac{-\dot{\Omega} R r_\perp}{2\Omega(R^2 - r_\perp^2)^{1/2}}, \tag{1}$$

with $r_\perp$ the radial distance of a crust region from the stellar spin axis, R the stellar radius, and Ω the stellar spin rate. (In the equatorial zone where moving crustal "plates" collide plates were presumed to be mainly subducted into the core, although equatorial compression may be relieved by other kinds of back flow.) The crust motion of equation (1) must cease when spin-down–induced stresses can no longer exceed the crust's yield strength. In R-II equation (18) this was estimated to happen when the spin period (P) exceeds P_2 given by

$$P_2 \sim 10\left(\frac{10^{-4}}{\theta_{\max}}\right)^{1/2} \text{s}, \tag{2}$$

with $\theta_{\max}$ the maximum value of the dimensionless strain θ (fractional change in length) which deep crust matter can bear before yielding. The limiting strain $\theta_{\max}$ is not known quantitatively. The crust is certainly not stronger than the maximum stress before yielding of the microcrystals of which it is formed. Therefore, $\theta_{\max} < 10^{-2}$. Other arguments support $\theta_{\max} < 3 \times 10^{-4}$ (R-II, § 4). The stellar crust is about 10^{17} lattice spacings in length. This is the size of the Earth when measured in lattice spacing of normal terrestrial rock and offers similar difficulties in extrapolating from microcrystal properties to terrestrial crust strength. Not only is the "global" strength unknown for the neutron star crust but the manner of crust yielding is also unclear. We shall examine here some of the special phenomena and structures which result when the crust is assumed to yield to growing global stress with large-scale "cracking"; parts of the crust are assumed to slip suddenly and relieve some significant fraction of an otherwise growing crustal stress. For warm crusts such large cracking would probably not occur because warm crusts' microcrystal yield strengths are so much weaker than those of cold ones. In R-II it was noted that a transition from plastic flow to a more brittle response might be expected near the same ratio of transition temperature (T_t) to crystal melting temperature (T_m) as that measured in the laboratory for (near) Coulomb crystals; then $T_t/T_m \sim 10^{-1}$. The transition regime is over a very small fraction of T_m (cf. Fig. 1). For neutron star crusts relevant T_m are several times 10^9 K and the transition may be expected to occur at around the Crab pulsar inner crust temperature. It almost certainly has already occurred in the older Vela pulsar crust where the temperature at the base has been estimated to be $T = 1.3 \times 10^8$ K ($T/T_m \sim 3 \times 10^{-2}$) (Alpar, Cheng, & Pines 1990a). However, it should be emphasized that the crust might well everywhere limit stress entirely by microscopic crumbling even when each grain responds rather brittlely. The assumption of relatively large-scale crust cracking is a hypothesis which must still be supported mainly by comparisons of its consequences with neutron star observations.

We consider here consequences of assumed spin-down–induced large-scale crust cracking from the stresses discussed in R-I and R-II for three kinds of neutron star observations. In § 2 we reconsider the proposal (Ruderman 1976) that such crust cracking is the cause of the sudden spin period "glitches"

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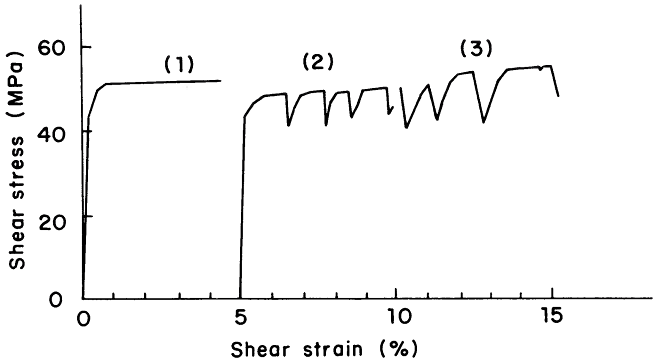

FIG. 1.—Shear stress vs. strain for Li crystals stabilized with 33% Mg. $T = 77$ K for sample (1) and 65 K for (2) and (3) (Siedersleben & Taylor 1989).

observed in some radio pulsars. Section 3 is a discussion of the effect of large-scale crust cracking on the surface magnetic field structure of spinning-down neutron stars. When cracking begins the crustal plate breaks up into smaller platelets. As spin-down continues these platelets move apart from each other. The original magnetic field remains frozen in each platelet, but there is no or, at least, much smaller magnetic field through the new crust which forms between the platelets as they move apart. One consequence is that (average) dipole magnetic field measurements from spin-down torques will become much smaller than the platelet surface field strengths. Section 4 considers the possibility that crust cracking glitches in older neutron stars are also the source of the observed gamma-ray bursts (Pacini & Ruderman 1974; Tsygan 1975; Fabian, Icke, & Pringle 1976; Mitrofonov 1984; Muslimov & Tsygan 1986; Epstein 1988). The magnitude of the sudden energy release from a cracking crust, its time scale, and the expected repetition rate are all suggestive of such an association. (Nonglitch associated bursts may also come from equatorial crust cracking caused by the tension of crustally anchored quantized magnetic flux tubes pushed up to the core surface.)

2. CRUST CRACKING AND PERIOD GLITCHES

When deep crust matter is subject to a small stress σ which compresses or stretches it to a dimensionless strain θ, $\sigma = 3\mu\theta$, with μ the crust lattice's shear modulus (R-I). We assume that the spin-down–induced stress on a pulsar crust discussed in R-II, which forces the crust motion of equation (1), first strains the crust to its elastic yield limit

$$\sigma_{\max} = 3\theta_{\max}\mu \tag{3}$$

(R-I eq. [12]). As σ and θ slightly exceed these limits the crust cracks: the crust strain jumps by $\Delta\theta$ with $\Delta\theta < \theta_{\max} < 10^{-2}$ and the stress is suddenly partly relaxed. (Qualitatively the stress-strain relationship is similar to that of the single crystal behavior of Fig. 1.) A significant fraction of the growing strain from forced crust motion is assumed to be relieved in such sudden cracking and σ does not exceed $\sigma_{\max}$ as the crust moves. The propagation speed of a deep crust crack is just the shear wave velocity in the lower crust,

$$v_s = \left(\frac{\mu}{\rho_L}\right)^{1/2} = \left(0.3\,\frac{(Ze)^2}{b_Z M_Z}\right)^{1/2} \sim 2 \times 10^8 \text{ cm s}^{-1} , \tag{4}$$

with ρ_L the matter density of the crust lattice, b_Z the internucleus spacing ($\sim 50 \times 10^{-13}$ cm), Z the nucleus atomic number (40–50), and M_Z the mass of a lattice nucleus.

Because the spin-down–induced crustal stresses are directed away from the spin axis poles, the sudden $\Delta\theta$ motions of stressed crust generally also move crust outward (except in the equatorial zone). A sudden outward motion of part of the crustal lattice also carries outward those crustal neutron superfluid vortices which are pinned to the moving lattice's nuclei. In this way crustal neutron superfluid which was prevented from spinning down with the crust because its vortices could not move outward suddenly reduces the angular frequency lag ω between its rotation speed Ω_n and the rotation speed of the crustal lattice Ω ($\Omega \equiv \Omega_n - \omega$.) The decrease in Ω_n from an outward displacement of vortices by $R\Delta\theta/2$ is

$$\Delta\Omega_n = -\Delta\theta\Omega_n . \tag{5}$$

The crust must, of course, recoil by spinning-up, a motion which is quickly communicated to and shared with the rest of the star. Then the final shared crust spin-up is

$$\Delta\Omega = \frac{\Delta\theta\Omega_n I_n}{I - I_n} , \tag{6}$$

with I the moment of inertia of the whole neutron star and I_n that of the suddenly spun-down crustal neutron superfluid. If we identify $\Delta\Omega$ with the observed spin-up glitches of Vela-like radio pulsars, put $\Omega \sim \Omega_n$, and use a canonical estimate

$$I_n \sim 10^{-2} I , \tag{7}$$

then

$$\frac{\Delta\Omega}{\Omega} \sim 10^{-2}\Delta\theta . \tag{8}$$

For Vela glitches $\Delta\Omega/\Omega \sim 10^{-6}$. This gives

$$\Delta\theta \sim 10^{-4} \tag{9}$$

for Vela's crust. The interval between glitches (τ_g) should be the length of time needed for further spin-down to rebuild the strain $\Delta\theta$ relaxed by the crust cracking:

$$\tau_g \sim \frac{\Delta\theta}{\dot{\Omega}}\Omega . \tag{10}$$

The combination $(\Delta\Omega/\Omega)(1/\tau_g)$, called the "glitch activity" rate by McKenna & Lyne (1990), is independent of $\Delta\theta$ and $\theta_{\max}$ (Ruderman 1976):

$$\frac{\Delta\Omega}{\Omega} \times \frac{1}{\tau_g} = \frac{\dot{\Omega}}{\Omega}\frac{I_n}{I} \sim \frac{10^{-5}\text{yr}^{-1}}{(\text{Age}/10^3 \text{ yr})} \tag{11}$$

The heuristic argument leading to equation (11) is inconsistent in its treatment of the elastic response of the lattice to growing stress between glitches and in other details. A more careful consideration which leads to equation (11) for spinning-down pulsars with $\Omega \lesssim 10^2$ s^{-1} and to corrections for faster pulsars is given in Appendix A. The model result of equation (11) is compared to observed glitch activity rates of young pulsars in Table 1. Agreement is quite reasonable for the Vela-like family of 10^4 yr old pulsars. We would attribute the very low glitch activity in the younger Crab family solely to the fact that their lower crusts are warmer and respond to growing stress mainly by plastic flow (McKenna & Lyne 1990; Ruderman 1976). A recent estimate (Alpar, Pines, & Cheng 1990b) for the internal Crab pulsar temperature gives $T \sim 4 \times 10^8$ K $\sim 10^{-1}T_m$. (It would appear from Table 1 that the transition from crust plastic flow to a more brittle response takes place after ~ 1700

TABLE 1

GLITCH ACTIVITY IN YOUNG RADIOPULSARS

Pulsar	Age (10^3 yr)	Glitch Activity (10^{-7} yr^{-1}) Observed	Eq. (11)
0531.........	1.2	0.1	80
1509.........	1.5	~0	70
0540.........	1.7	?	60
0833.........	11	8	9
1800.........	16	?	6
1737.........	20	4	5
1823.........	21	5	5

NOTE.—All data entries are taken from McKenna & Lyne (1990).

yr, the estimated age of PSR 0540.) From Figure 1 that transition would be expected during a relatively small interior temperature drop $\Delta T \sim 10^{-1} T \sim$ several times 10^7 K. The radio pulsars much older than Vela should continue to have glitches, but with much longer interglitch intervals τ_g so that few have been observed so far. The observed glitch repetition rates give an effective $\Delta\theta \sim 2 \times 10^{-4}$ for Vela and 4×10^{-5} for PSR 1737. If $\Delta\theta$ may be as large as $\theta_{\max}$ the crust cracking model for pulsar glitches implies $\theta_{\max} \gtrsim 10^{-4}$. This, together with the laboratory measurements of Figure 1 and those shown in Figure 1 of R-II, gives the bounds

$$10^{-4} < \theta_{\max} < 5 \times 10^{-3} . \quad (12)$$

The total number of glitches expected during the spin-down of a pulsar from P_c (the spin period at which crust cracking begins) to P_2 (the spin period of eq. [2] at which crust cracking ceases) is

$$N_g = \frac{1}{\Delta\theta} \ln\left(\frac{P_2}{P_c}\right) \sim 10^5 . \quad (13)$$

The numerical value assumes $P_c = 5 \times 10^{-2}$ s (midway between Crab and Vela periods) $P_2 = 10$ s (eq. [6] with $\theta_{\max} = 10^{-4}$ or R-II, § 5), and $\Delta\theta = 5 \times 10^{-5}$ (for PSR 1737). The glitches will continue after radio pulsar turnoff. Each sudden release of elastic strain energy from the (lower) crust in a glitch is

$$\mathscr{E}_g \sim lR^2\mu\theta_{\max}\Delta\theta \sim 10^{39}\left(\frac{\theta_{\max}}{10^{-3}}\right)\left(\frac{\Delta\theta}{10^{-4}}\right) l_5 \text{ erg} , \quad (14)$$

with R the stellar radius and l the crust thickness ($\sim 10^5$ cm). A roughly equivalent energy release comes from the sudden slowdown of crust neutron superfluid rotation.

Because of the sudden $\Delta\theta$ motion of crust in a large glitch, pulsar magnetic fields, spin-down torques, and spin-down indices should have permanent glitch-associated changes of order $\Delta\theta \sim 10^{-4}$. In Crab-like pulsars, where warmer crust relaxation is less discontinuous than in the cooler Vela family, these changes should be accomplished more smoothly. A permanent 10^{-4} change in magnetic spin-down torque after the sudden movement of crustal B in a Vela glitch is very much smaller than other contributions to the postglitch $\Delta\dot{\Omega}/\dot{\Omega}$ healing from internal angular momentum exchange (see Appendix B). It would, therefore, be difficult to confirm it. However, a second kind of crust-cracking response to overstressing should come from the spin-down–driven accumulation of magnetic flux tubes anchored to the base of the equatorial zone crust (R-II § 5). These will give a large azimuthal stress there from the tension along the flux tubes. If this stress is relaxed by crust-cracking, large Vela glitchlike $\Delta\Omega/\Omega$ would not be expected since there is no sudden large ($R\Delta\theta \sim 10^2$ cm) outward motion of pinned vorticity. However, the change in $\boldsymbol{B}$ from such a crack would give $\Delta\dot{\Omega}/\dot{\Omega} \sim \Delta\theta \sim 10^{-4}$. Numerous microglitch events of this sort appear to exist in Vela (Cordes, Downs, & Krause-Polstorff 1988). Postglitch healing after crust cracking and sudden movement of vortices which do not unpin is discussed in Appendix B.

3. EVOLVING MAGNETIC FIELD STRUCTURE IN SPINNING-DOWN NEUTRON STARS

If a post-Crab pulsar crust relaxes with large-scale cracking the crust breaks up into "platelets" (R-I). "New" matter flows into the cracks between platelets (e.g. R-I, Fig. 5). This new matter differs in important ways from the old crust matter of a platelet. First, it does not have much of the crustal magnetic field, which remains frozen in the platelets. Second, the atomic number of new nuclei can be smaller than that of the original deep crust matter in the platelets. [The original nuclei were formed at high temperatures ($kT \sim 1$ MeV) which facilitated the formation of those nuclei which minimized the free energy of crustal matter. In the deep crust these nuclei have large "magic numbers" of protons $Z = 40$ or 50. It is difficult to find accessible paths to build such nuclei from smaller units if new deep crust formation takes place at the much lower crust temperature ($kT \sim 10$ keV) of a Vela-like pulsar. As in the much lower temperature terrestrial world low Z matter can generally be quite stable against fusion.] In the Coulomb lattice of the deep crust of a neutron star $\theta_{\max}$ is not sensitive to Z, but $\sigma_{\max}$ is proportional to $Z^{2/3}$. Thus interplate crust may be somewhat weaker than platelet crust if its nuclei are effectively frozen at smaller Z (e.g., the magic $Z = 28$). One consequence may be that continued spin-down–induced cracking would be expected to take place at platelet boundaries and beyond in the interplatelet deep crustal matter. The platelets themselves should then retain their integrity and their original magnetic fields during spin-down. The resulting evolution of the surface magnetic field of a such spinning-down pulsar crust is indicated in Figure 2. When the crust is so warm that it stretches plastically without large cracking (i.e., Crab-like or younger), the original crust is stretched (and thus thinned) by the motion toward the equator of eq. (1). As a young pulsar spins-down from its initial spin period and cools, the initial polar cap fields initially spread with the highly conducting crust in which they are embedded to fill much of each hemisphere. At $P = P_c$ cracking begins; thereafter, platelets move apart until the spin-down period reaches the P_2 of eq. (2). Each platelet has frozen into it the field $B(P_c)$ which it had when cracking first began. This field should be quite uniform over a platelet surface since it came from the initial stretching of the field of a relatively small polar cap. Then in the spin-down regime $P_c < P < P_2$ while the *average* surface dipole field $\langle B(P) \rangle$ decreases inversely with P (R-II; Srinivasan et al. 1990)

$$\langle B(P) \rangle = \langle B(P_c) \rangle P_c/P , \quad (15)$$

the rather uniform platelet fields (B_p) remain frozen at

$$B_p = \langle B(P_c) \rangle \sim B(\text{Vela}) \sim 3 \times 10^{12} \text{ G} . \quad (16)$$

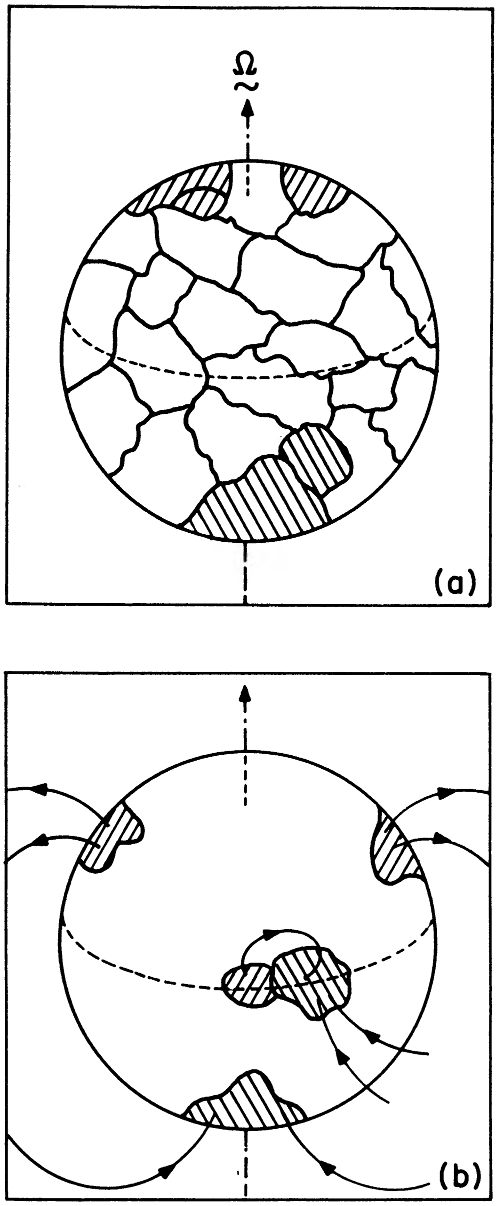

FIG. 2.—Evolution of surface platelets and magnetic field after crust cracking begins. (*a*) An initial configuration of cracks; the magnetic field is continuously distributed through the surface. (*b*) Separation of original platelets (hatched) after further spin-down and equatorial zone subduction. At the original platelets $\boldsymbol{B}$ maintains the value it had in (*a*). $\boldsymbol{B} = 0$ at new interplatelet crust.

Therefore measurements of B in older spun-down neutron stars should give conflicting results. Those based upon the magnitude of the magnetic dipole moment (spin-down in radio pulsars, accretion torques in X-ray pulsars) should give smaller values of B than those inferred from surface cyclotron resonance features in the X-ray spectra of X-ray pulsars and gamma-ray burst sources). The predicted extreme uniformity of a platelet field could also make a cyclotron resonance feature much narrower than would be expected otherwise. It should be emphasized, however, that this view of magnetic field evolution assumes that the field is not perturbed by backflow of highly conducting crustal matter *above* the moving platelets.

4. GAMMA-RAY BURSTS

If radio pulsar glitches are a manifestation of crust cracking, they are associated with sudden releases of stored elastic strain energy in the deep crust. The total rate of such events from the neutron star population of the Galaxy, the amount of energy released per glitch, and the initial time scale for that release are all near what seem to be required for the sources of observed gamma-ray bursts (GRBs).

Because efficient generation of gamma-rays from this elastic energy release may be high only in the older, longer period, neuton stars which have relatively charge-starved magnetospheres (Blaes et al. 1989), observed burst sources may be limited to dead radio pulsars which no longer have steady copious sources of $e^{\pm}$ pairs in their magnetospheres. This is an old neutron star population which should, therefore, be distributed broadly, a kiloparsec or more above and below the Galactic disk since the younger active radio pulsars typically have very high velocities (~ 150 km s^{-1}) Because the so far observed GRB sources seem to be isotropically distributed about us only those neutron stars nearer than about a kiloparsec should be in the candidate source population. These could number 10^7, about 10^{-2} of all Galactic neutron stars. Since GRBs are observed at a rate of about 10^2 yr^{-1} each candidate source must give 10^2 yr^{-1} $\times$ 10^{10} yr $\times$ $10^{-7} = 10^5$ GRBs. The total number of glitches remaining in a spinning-down neutron star period P follows from eq. (13):

$$N_g(>P) = 2 \times 10^4 \ln (P_2/P) \qquad (17)$$

with $P_2(>P)$ given by eq. (2). If the needed large GRB source population consists mainly of dead radio pulsars, $P \gtrsim 2$ s. With $\theta_m \sim 10^{-4}$, the smallest value compatible with the limits of eq. (12), equation (17) gives $N_g = 5 \times 10^4$, quite near (but perhaps disturbingly less than) the above estimate of 10^5 GRBs per neutron star.

The elastic energy released in each glitch–GRB is given by equation (14) as $\mathscr{E}_g \sim 10^{38}$ erg. If most of this energy is ultimately converted into energetic radiation from the neutron star's disturbed magnetosphere, the observed GRB fluence would be $\mathscr{E}_g/4\pi/(1 \text{ kpc})^2 \sim 10^{-6}$ ergs cm^{-2}, typical of that in a GRB. The time scale for initiating this energy release (Δt) is that for a crack (which moves with the shear wave velocity of eq. [3]) to propagate through a lower crust thickness (l). The rise time

$$\Delta t \sim \frac{l}{v_s} \sim 5 \times 10^{-4} \text{ s} \qquad (18)$$

is consistent with GRB observations. The time structure of the whole GRB depends upon unknown details of how a crack network develops within the crust. A shear wave carrying energy away from a deep crust crack will also travel around the star and be continually refocused to its emission point with a period $P_s \sim 2\pi R/v_s \sim 30$ ms. It is tempting to suggest this as the origin for the reported 23 ms quasi-period observation of the 1979 March 5 GRB (Barat et al. 1979). After the crack the released elastic energy is initially in the form of crust shear vibrations with frequencies ($\sim v_s/l$) in the KHz regime. Blaes et al. (1988) have shown how such high-frequency elastic vibrations reach the stellar surface as Alfvén waves with amplitudes greatly amplified from those of the initial deep crust oscillations. In the GRB model of Blaes et al. elastic energy release by a midcrust mechanical instability is replenished by stress from continued cold mass accretion onto the star surface rather than spin-down as suggested here. The spin-down–powered crust cracking is deeper and thus can involve greater elastic energy storage and release. Many details of the Blaes et al. model can, however, be carried over.

No GRBs were observed during the Vela radio pulsar's glitches, implying not much more than 10^{38} ergs s^{-1} into any glitch associated GRB from this pulsar. The model's $\mathscr{E}_g \sim 10^{38}$ ergs might have escaped detection. Moreover, even if the plasma density in Vela's magnetosphere were only the minimum Goldreich-Julian (1969) charge density needed to keep the magnetosphere electric field near zero in the star's corotating reference frame, potential GRB emission from this pulsar might be suppressed (Appendix C).

A second large source for replenishing the stored elastic energy released in large-scale crust cracking can be the interior core magnetic field. During spin-down this field may be carried into the equatorial zone around the bottom of the crust as indicated in R-II, Figure 2. The crust there may be compressed (azimuthally) by the tension (T_Φ) of each of the large number (N_Φ) of crust anchored quantized core magnetic flux tubes whose crust entry points have moved down along fixed meridians toward the equator. From R-II, § 5 equation (16) this magnetic stress ($\sim \langle B \rangle B_c/8\pi$ with $B_c \sim 10^{15}$ G and $\langle B \rangle$ the average field) can crack the crust if the average crust field $\langle B \rangle$ measured by the spin-down torque satisfies

$$\langle B \rangle \geq 3 \times 10^{10} \left(\frac{\theta_{\max}}{10^{-3}} \right) . \tag{19}$$

This inequality holds for most extinct radio pulsars. The total number of such core magnetic field–induced equatorial crust cracking events is

$$E'_B \sim \frac{\langle B \rangle B_c}{8\pi} \frac{4\pi}{3} R^3 \sim N_\Phi T_\Phi R \times 10^{43} (\langle B \rangle / 10^{11}\ \mathrm{G})\ \mathrm{ergs} . \tag{20}$$

If the amount of elastic crust energy liberated in each large-scale crust cracking ($\mathscr{E}'_g$) equals the $\mathscr{E}_g$ of equation (14), the number of such core magnetic field–induced equatorial crust cracking events is

$$N'_g = \frac{E'_B}{\mathscr{E}'_g} = \frac{E'_B}{\mathscr{E}_g} \sim 10^5 \frac{\langle B \rangle}{10^{11}\ \mathrm{G}} \left(\frac{10^{-4}}{\theta_{\max}} \right) \left(\frac{10^{-4}}{\Delta\theta} \right) . \tag{21}$$

In slowly rotating neutron stars the number of large-scale cracking events caused by magnetic flux tension may be comparable to or even greater than the number of remaining glitches directly induced by spin-down. They may therefore, prove a significant additional source for GRBs. However, while direct spin-down–induced vortex pinning stress on a crust is generally always reduced by the small outward crust motion in a canonical glitch, this need not be the case for azimuthal magnetic flux-tube tension. While the interval between glitches is, therefore, the long τ_g of equation (9), the interval between equatorial cracking events from the azimuthal pull of flux tubes is not easily predicted. They may well come in bunches with much smaller intervals between bursts.

We note, finally, that although glitch observations imply $\Delta\theta \sim 10^{-4}$ they do not much constrain $\theta_{\max}$ as long as it is greater than $\Delta\theta$ (and small enough so that if crust pinning causes the crack-inducing stress it is not preempted by vortex line unpinning). However, the total number of GRB events from slowly spinning neutron stars does depend on $\theta_{\max}$ and seems to need a small $\theta_{\max} \sim 10^{-4}$. Then in each glitch a very large fraction of the crust elastic strain energy would be liberated. Such a small $\theta_{\max}$ would apparently also ensure that much of the crust strain is relieved by crust breaking rather than vortex unpinning for all radio pulsars.

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APPENDIX A

THE GLITCH ACTIVITY RATE

The spinning-down crust cracks when $\omega \equiv \Omega_n - \Omega$ grows to reach a critical ω_B. Then Ω_n suddenly drops by $-\Delta\Omega_n (> 0)$ as part of the pinning lattice moves outward. The rest of the star immediately spins-up by

$$\Delta\Omega = -\frac{\Delta\Omega_n I_n}{I - I_n} . \tag{A1}$$

Then in the spin-up glitch

$$\Delta\omega = \Delta\Omega_n - \Delta\Omega = -\frac{I}{I_n} \Delta\Omega . \tag{A2}$$

The interval between glitches (τ_g) is the time needed for further stellar spin-down to increase ω again to the breaking value ω_B:

$$\tau_g = -\frac{\Delta\omega}{\dot{\omega}} . \tag{A3}$$

Beteeen glitches the rate of increase of ω is

$$\dot{\omega} = \dot{\Omega}_n - \dot{\Omega} = -\frac{2\dot{r}_\perp}{r_\perp}\Omega_n - \dot{\Omega}$$

$$= -\frac{2\dot{s}}{r_\perp R}(R^2 - r_\perp^2)^{1/2} - \dot{\Omega}\,, \tag{A4}$$

with $\dot{s}$ the tangential displacement velocity of the crust between glitches. From R-I, equation (15) (with $f = 1$),

$$\dot{s} = \frac{R^3 \rho_n \Omega_n \dot{\omega}}{12\mu} \frac{r_\perp (R^2 - r_\perp^2)^{1/2}}{R^2}\,. \tag{A5}$$

Then

$$\dot{\omega} = -\frac{(R^2 - r_\perp^2)\rho_n \Omega_n^2}{6\mu}\dot{\omega} - \dot{\Omega} \tag{A6}$$

and, for $r_\perp^2 \sim R^2/2$,

$$\frac{\Delta\Omega}{\Omega}\cdot\frac{1}{\tau_g} \sim \frac{I_n}{I}\left(1 + \frac{R^2 \rho_n \Omega_n^2}{12\mu}\right)^{-1}\,. \tag{A7}$$

For $\rho_n = 5 \times 10^{13}$ g cm^{-3}, $\Omega^2 = 10^4$ s^{-2} (Vela), $R^2 = 10^{12}$ cm^2 and 12 $\mu = 4 \times 10^{30}$ ergs cm^{-3} the second term within the right-hand side parentheses is 10^{-1}. It is neglected in equation (11). If there are N separate regions of the strained crust which alternatively crack, each glitch $\Delta\Omega$ is smaller by N^{-1} than the value it would have if all of the pinned crustal superfluid suddenly moves, but the total glitch rate would be N times larger than that for any one of these separate regions. Then equation (A7) and equation (11) would still hold with I_n the total pinned crust superfluid moment of inertia.

APPENDIX B

POSTGLITCH RELAXATION

If crust cracking and the pinned vortex motion it causes are the origin of pulsar timing glitches, simple models for postglitch relaxation could involve four different stellar components:

1. The core, crust lattice, and crust electrons, all strongly coupled together and containing almost all of the stellar moment of inertia.

2. That part of the crust's internuclear neutron superfluid whose vortex lines are so strongly pinned to lattice nuclei that they do not unpin during or after a crust cracking glitch. From equation (11) and Table 1 this part of the crustal neutron superfluid has $I_n = 9 \times 10^{-3} I$. Its pinnings cause glitches.

3. Crustal neutron superfluid, above or below the strongly pinned superfluid of (2), that can be approximated as having perpetually unpinned vortices which, however, interact strongly with the crust (Jones 1990). These do not initially move outward with the crust motion of the glitch. Rather, this component, with moment of inertia I_p accommodates to the initial crust spin-up $\Delta\Omega$ by continuously increasing its own spin until the difference between its spin and that of the crust lattice again approaches the much smaller preglitch steady-state value. This superfluid component spin-up must be accompanied by a postglitch change in the observed stellar spin-down rate ($\Delta\dot{\Omega}_1$) which satisfies

$$\int_0^\infty \Delta\dot{\Omega}_1(t)dt = -\frac{I_p}{I}\Delta\Omega\,. \tag{B1}$$

In their analysis of the Vela "Christmas glitch's" most rapid postglitch response, Alpar et al. (1990b) fit timing observations to a single exponential for $\Delta\dot{\Omega}_1(t)$. They find $I_p = 5 \times 10^{-3} I$. The total moment of inertia in the pinned superfluid plus that of the rapidly spun-up superfluid after a glitch is $I_1 = I_n + I_p \sim 1.4 \times 10^{-2} I$. The remaining much longer and larger postglitch healing is the response of the fourth component.

4. That part of the crust superfluid whose vortices more gradually move out in response to the postglitch growth of $\omega = \Omega_n - \Omega_c$. (Elastic crust deformation motion of pinned vortices is not relevant here.) This long time-scale motion is attributed to *vortex* creep in the extensive analyses of Alpar, Cheng, & Pines (1984), and Alpar et al. (1988) which allow vortex and lattice parameters to be inferred directly from observations. (See also the postglitch healing model of Jones [1990] in which all postglitch healing is associated with unpinned vortices.) We note here that similar vortex array motion might, alternatively (or additionally), be attributed to local *crust* lattice creep (Ruderman 1976) or to local lattice crumbling. In these cases the relevant vortices do not unpin. An analysis as complete as that of Alpar et al. (1984, 1988, 1990) for vortex creep would then need a better description of crustal lattice stress response than is available.

If superfluid of this sort has reduced its ω through any of the above ways of moving its vortex array outward in response to spin-down, the effect on observed post-glitch $\dot{\Omega}$ may be qualitatively similar. For a relevant superfluid moment of inertia I_2, the change in $\dot{\Omega}(\Delta\dot{\Omega}_2)$ and the total amount of ω which is so healed between one glitch and the next ($\Delta\omega$) are related by

$$\int_0^{t_g} \Delta\dot{\Omega}_2 \, dt = \frac{I_2}{I} \Delta\omega = \frac{I_2}{I} \times \frac{I}{I_1} \Delta\Omega = \frac{I_2}{I_1} \Delta\Omega \, . \tag{B2}$$

The coefficient of $\Delta\Omega$ is of order unity but its exact value depends upon the relative amounts of component (4) and the sum of components (2) and (3). This result is implicit in the Alpar et al. analyses.

APPENDIX C

MAGNETOSPHERIC INDUCTANCE SUPPRESSION OF GRBs

The potential drop between B field lines above a magnetized platelet oscillating with frequency $\hat{\omega}$ is simply related to the Alfvén wave power ($\mathscr{P}_A$) through it:

$$\Delta V = \left(\frac{8\mathscr{P}_A}{c}\right)^{1/2} \cos \hat{\omega} t \, . \tag{C1}$$

For $\mathscr{P}_A = 10^{38}$ ergs s^{-1}, $\Delta V \sim 8 \times 10^{16} \cos \hat{\omega} t V$. A necessary condition for efficient conversion of $\mathscr{P}_A$ into GRB power is that this ΔV and $\mathscr{P}_A$ be imposed on a magnetosphere with very few ambient particles to absorb the input energy. Ultrarelativistic particle acceleration as a response of the magnetosphere to the oscillating platelet electric field of equation (A1) could be suppressed by the current flow response along $\boldsymbol{B}$ of the minimum ambient Goldreich-Julian $\boldsymbol{\Omega} \cdot \boldsymbol{B}/2\pi c$ charge density already present in the magnetosphere near the stellar surface. Because of that current flow and its associated magnetic field, the response of the near magnetosphere to ΔV could be largely inductive. A necessary condition for most of $\mathscr{P}_A$ and ΔV being available for ultrahigh energy particle acceleration is that the $\Delta V > \mathscr{L}\dot{J}$ with $\mathscr{L}$ the relevant magnetosphere inductance and $\dot{J}$ the time derivative of the current from the ΔV induced motion of the Goldreich-Julian charge density. This condition is

$$\Delta V > \hat{\omega}\left(\frac{\Omega B}{2\pi}\right)(\pi \hat{R}^2)\left(\frac{2L}{c^2}\right) , \tag{C2}$$

with $\hat{R}$ the platelet radius, and L the length of the "closed" B-field line bundle from the platelet to its reentry location back into the crust. With platelet $B = 3 \times 10^{12}$ G, $L = 10^6$ cm, $\hat{\omega} = 10^4$ s^{-1}, $\mathscr{P}_A = 10^{38}$ ergs s^{-1}, and $\Omega = 10^2$ s^{-1}, the inequality (C2) is satisfied for $\hat{R} < 10^5$ cm. If crust-cracking broke Vela's surface into platelets larger than about 10^5 cm in radius strong GRBs might not be associated with Vela glitches just because of the large Goldreich-Julian charge density in the magnetosphere. Alternatively, large GRBs may accompany glitches only in magnetospheres starved of charge because outer magnetosphere pair production and γ-ray emission has turned off or, probably more significantly, because there is no longer the steady $e^{\pm}$ production of canonical radio pulsars which have not yet reached the canonical "death line." This would restrict strong GRB sources to slowly rotating extinct radio pulsars, a result already implied by Blaes et al. (1989).

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