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AVALANCHES AND THE DISTRIBUTION OF SOLAR FLARES

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ABSTRACT

We propose that the solar coronal magnetic field is in a self-organized critical state, thus explaining the observed power-law dependence of solar flare occurrence rate on flare size which extends over more than five orders of magnitude in peak flux. The physical picture that arises is that solar flares are avalanches of many small reconnection events, analogous to avalanches of sand in the models published by Bak and colleagues in 1987 and 1988. Flares of all sizes are manifestations of the same physical processes, where the size of a given flare is determined by the number of elementary reconnection events. The relation between small-scale processes and the statistics of global flare properties which follows from the self-organized magnetic field configuration provides a way to learn about the physics of the unobservable small-scale reconnection processes. We present a simple lattice reconnection model which is consistent with the observed flare statistics. We discuss the implications for coronal heating and suggest some observational tests of this picture.

Subject headings: magnetic fields — plasmas — stars: flare — Sun: activity — Sun: corona — Sun: flares — Sun: magnetic fields — Sun: X-rays

Observations (Dennis 1985; Lin et al. 1984; Datlow, Elcan, & Hudson 1974) show that the distribution of solar flare hard X-ray bursts is a power law in peak photon flux with logarithmic slope 1.8 (see Fig. 1). Although it is difficult to make direct comparisons between these independent observations, they are all roughly consistent with a single power law spanning over five orders of magnitude in peak flux (Dennis 1985). The occurrence rates during different years covering one solar cycle are shown in Figure 2. The power-law index of the distribution is independent of solar cycle, indicating that the underlying mechanism giving rise to the power-law distribution is insensitive to the level of coronal activity. In this *Letter* we suggest a connection between the distribution of solar flares and the physics of individual flares.

We propose that the power-law dependence of flare occurrence rate on flare size is a consequence of the coronal magnetic field being in a self-organized critical state. The concept of self-organized criticality was proposed by Bak, Tang, & Wiesenfeld (1987, 1988) to explain the prevalence of power-law, or scale-invariant, correlations extending over many decades in complex dynamical systems. They show that extended systems with many metastable states can naturally evolve into a critical state with no intrinsic length or time scale. Recent experiments (Babcock & Westervelt 1990), as well as numerical simulations (Carlson & Langer 1989; Kadanoff et al. 1989), have demonstrated the existence of such critical states. A simple example of such a system is a sandpile. As sand is added, the average slope of the sandpile increases until it reaches a state where it remains approximately constant. Once this self-organized critical state is reached, addition of more sand causes avalanches which readjust the local shape. The critical state is insensitive to the initial conditions, requires no fine tuning of parameters, and is an attractor of the dynamics of the system. The system becomes stationary when perturbations cause disturbances which are just able to propagate the length of the system. In the critical state, the system has a distribution of minimally stable regions of all sizes so that small perturbations give rise to avalanches of all sizes from the smallest possible avalanche (a single sand grain), up to the size of the system. There is no characteristic length scale in the system, so a featureless power-law spectrum of avalanche sizes results.

In the numerical sandpile models, sand grains are added randomly to the system, and grains are shifted downward whenever the local slope exceeds some critical value. When the slope is readjusted by shifting grains to neighboring sites, this might force the slope at a neighboring site to exceed the critical value, causing the avalanche to grow. The avalanche continues until the local slope is again everywhere less than the critical value. The spectral index of the resulting power-law avalanche size distribution is robust and insensitive to the value of the critical slope (Bak et al. 1988; Kadanoff et al. 1989). However, different physical systems have different spectral indices, depending upon the number of spatial dimensions and the symmetry of the system (Bak et al. 1988; Kadanoff et al. 1989; O'Brien, Wu, and Nagel 1991). We caution that the underlying physics determining the exact value of the spectral index is not well understood. The essential point is that for systems to exhibit self-organized critical behavior, the exact rules governing how and when the slope is readjusted are not important. The existence of a self-organized critical state requires a local instability which occurs whenever some local parameter exceeds a critical value, resulting in a transport process which changes the value of this quantity at nearby sites with the possibility of causing the value of the parameter at neighboring sites to exceed the critical value.

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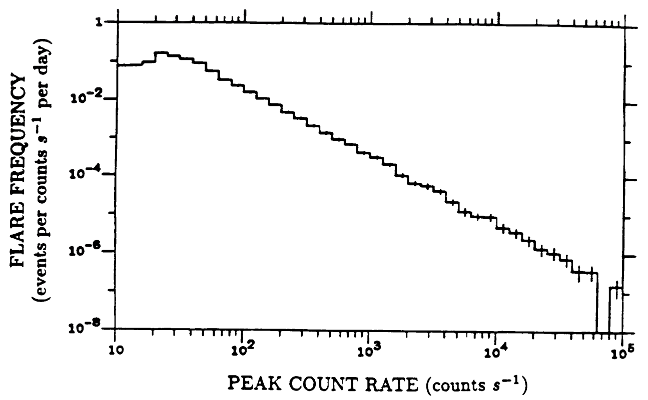

FIG. 1.—Distribution of flare occurrence rate vs. peak count rate for all flares detected with the Hard X-Ray Burst Spectrometer (figure provided by B. R. Dennis). The distribution has logarithmic slope 1.8.

The idea that flares are composed of many smaller events (Parker 1989; Sturrock et al. 1984), or that the corona is heated by many small nonthermal events (Parker 1988; Sturrock et al. 1990), is not new. It is expected that random footpoint motions of magnetic fields anchored in the photosphere will lead to many current sheets in the corona with associated tangential discontinuities in the magnetic field (Low 1990, 1991). It is generally accepted that the magnetic energy stored in the coronal magnetic field is the energy source of flares. Most importantly, if the magnetic energy release process has a local instability, which can trigger the release of magnetic energy at nearby sites, then the coronal magnetic field system can be driven to a self-organized critical state.

We suggest that the behavior of the coronal magnetic field is analogous to that of the sandpile, where the random twisting of the magnetic field by photospheric convective motions plays the role of the addition of sand grains. Parker (1988) has suggested that when the magnetic discontinuity angle θ between the magnetic field vectors on opposite sides of a particular current sheet is less than some critical angle θ_c, magnetic reconnection proceeds slowly due to the high conductivity of the coronal plasma. This allows energy to be stored in the twisted magnetic field. When $\theta > \theta_c$, reconnection can proceed explosively, rapidly reducing θ and dissipating the energy in the transverse magnetic field. The change in the nearby magnetic field strength and topology due to the reconnection thus corresponds to sliding grains of sand to neighboring sites to reduce the local slope. The discontinuity angle at some neighboring current sheets might then be increased above θ_c, causing additional reconnection events. The coronal magnetic field is therefore driven to a state with regions of all sizes which are on the verge of instability. We identify the reconnection avalanches with solar flares.

We stress that other instability criteria besides the one given by Parker (1988) are possible. However, the instability criterion must be a function only of the instantaneous field configuration, and not the history. For example, a model where the instability can trigger additional events at neighboring sites with some probability has a characteristic length scale (the perturbation decay length), and therefore will not exhibit a power law distribution of avalanches. Thus, models where reconnection events are triggered by anomalous resistivity due to accelerated particles and turbulence from nearby previous events will not have a power law flare distribution (see below).

To illustrate the properties of the self-organized field configuration, we have constructed a simple lattice model of reconnection. On a three-dimensional grid of points we specify a three-component vector $\boldsymbol{B}_i$ representing the average magnetic field in a cell. The index i represents the spatial location on the grid. The local magnetic gradient $d\boldsymbol{B}_i$ is defined to be the difference between the local magnetic field and the average of its six

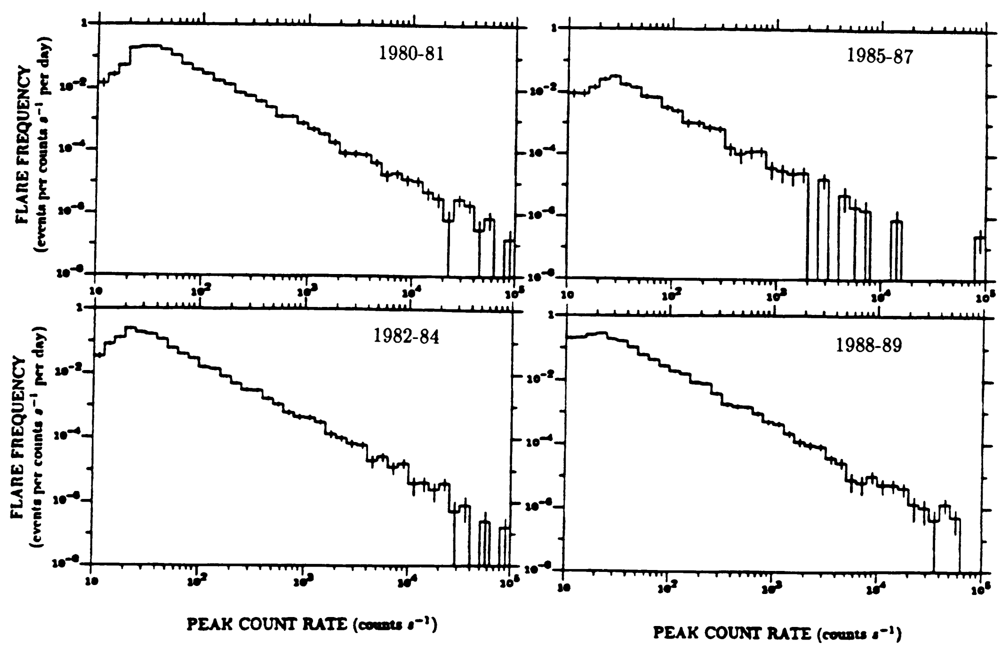

FIG. 2.—Same as Fig. 1, except that only flares detected during the years (*top left*) 1980–1981, (*bottom left*) 1982–1984, (*top right*) 1985–1987, (*bottom right*) 1988–1989 are included, showing the variation in the distribution over a solar cycle (figure provided by B. R. Dennis). The overall occurrence rate is seen to decrease then increase with the solar cycle, but all of the distributions are consistent with the same power law index of 1.8 found in Fig. 1 for all flares.

nearest neighbors $\boldsymbol{B}_{\mathrm{nn}}$,

$$d\boldsymbol{B}_i = \boldsymbol{B}_i - \tfrac{1}{6}\Sigma_{\mathrm{nn}}\boldsymbol{B}_{\mathrm{nn}} \, . \tag{1}$$

The configuration is defined to be unstable to reconnection when the magnitude of the gradient is greater than a critical value, $|d\boldsymbol{B}_i| > B_c$. Note that this criterion is similar to the critical angle condition of Parker if the magnitudes of the vectors $|\boldsymbol{B}_i| = B_i$ are all equal. When a reconnection instability occurs, we vectorially cancel the local magnetic field by transporting one-seventh of the gradient vector to each of its six nearest neighbors,

$$\boldsymbol{B}_i \rightarrow \boldsymbol{B}_i - \tfrac{6}{7}d\boldsymbol{B}_i \, , \quad \boldsymbol{B}_{\mathrm{nn}} \rightarrow \boldsymbol{B}_{\mathrm{nn}} + \tfrac{1}{7}d\boldsymbol{B}_i \, , \tag{2}$$

so that the local field becomes equal to the average of its neighbors, $d\boldsymbol{B}_i \rightarrow 0$. The field at nearby positions might then satisfy the instability criterion, resulting in additional reconnection events. Each reconnection event releases magnetic energy $\Delta\Sigma_j B_j^2 = (\frac{6}{7})|d\boldsymbol{B}_i|^2$.

We start with a uniform magnetic field and drive the system by adding a random vector $\delta\boldsymbol{B}$ to a random position on the grid. If the magnetic gradient then exceeds the critical value B_c, the field is readjusted according to equation (2). The magnetic gradient is then recomputed, and each new unstable point undergoes a reconnection according to equation (2). The field is allowed to relax until the magnetic gradient is again everywhere less than the critical value B_c. Another random vector is then added, and this process is repeated.

As expected, the field is driven to a self-organized critical state with a power-law distribution of event sizes. We show a typical time series of the energy release rate in Figure 3. The occurrence distributions of events versus total energy, peak flux, and duration are shown in Figure 4. The energy release distribution $N(E) \propto E^{-\tau}$ is a power law in total energy released E with logarithmic slope $\tau = 1.4$. The peak flux distribution $N(P) \propto P^{-\alpha}$ is seen to be a power law in peak flux P with logarithmic slope $\alpha = 1.8$, which is in good agreement with observations (see Fig. 1). The duration distribution is expected to be a power law but has a cutoff due to the finite size of the grid (see Kadanoff et al. 1989). In fact, all three distributions show deviations from power-law behavior at large sizes due to the finite-sized 30^3 grid we have used and at small sizes due to the finite resolution of the lattice.

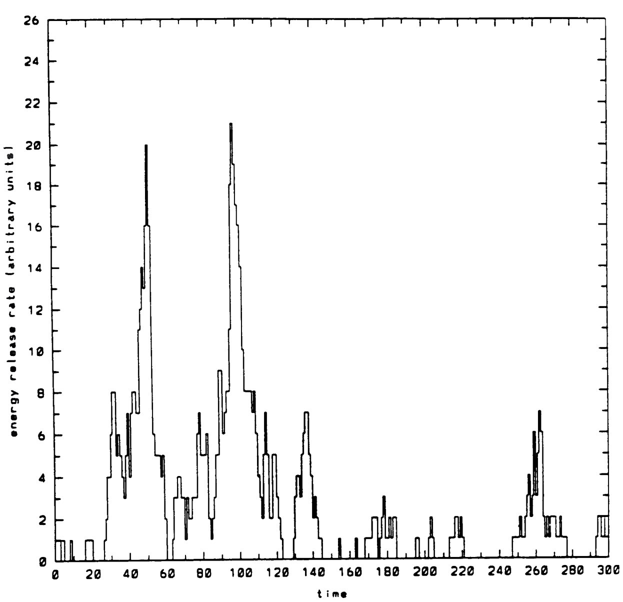

FIG. 3.—Energy release as a function of time for the reconnection model on a 30^3 grid.

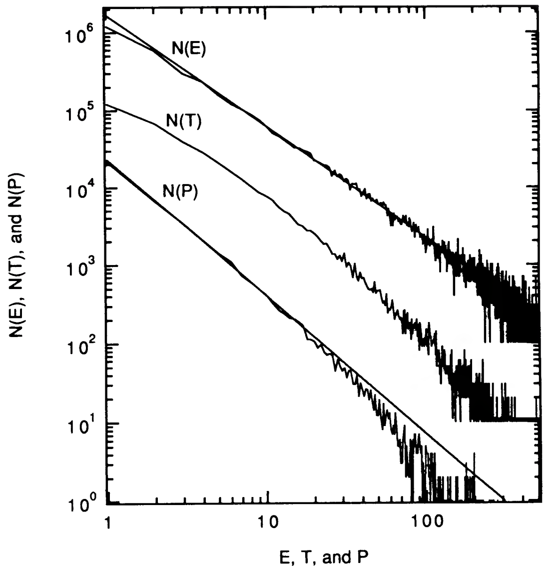

FIG. 4.—Distributions of event energy release $N(E)$, peak flux $N(P)$, and duration $N(T)$ for avalanches in the reconnection model on a 30^3 grid. The energy and peak flux distributions are well approximated by a power law over most of the range. The power-law relations $N(E) \propto E^{-1.4}$ and $N(P) \propto P^{-1.8}$ are also shown for comparison. Note that $N(E)$ and $N(T)$ are offset by factors of 100 and 10, respectively.

The simulation results shown in Figures 3 and 4 were obtained using a critical slope $B_c = 7$. However, these results are completely insensitive to the value of B_c. In fact, we have run models where the critical slope is a function of position with B_c randomly varying between 6.5 and 7.5 and find no differences in the distributions shown in Figure 4. Thus, the solar flare distributions are expected to be unaffected by changes in coronal conditions which affect B_c. The three components of the randomly added driving vector $\delta\boldsymbol{B}$ are each random numbers between -0.2 and 0.8. Note that this implies that there is a directionality to the photospheric motions which on average tends to increase the magnetic field in a certain direction. If the random values of the components of δB are instead symmetric about zero, then the magnetic field is decreased as often as it is increased, and the field configuration is not driven to a self-organized critical state. This would be analogous to a situation where sand grains are subtracted from a sandpile as often as they are added, so that the sandpile never reaches the critical state. Decreasing the driving rate by making the components of $\delta\boldsymbol{B}$ smaller merely increases the average time from one event to another (see Fig. 3) but does not affect any of the distributions. We also find that our results are insensitive to how the magnetic field is redistributed after a reconnection event (eq. [2]) provided that the sum of the magnetic field vectors is conserved in each reconnection, and on average the field is redistributed with no preferred direction. In other words, the field may be transported anisotropically, but the amount of redistributed field is on average symmetric in the $\pm x$ directions (similarly for the $\pm y$ and $\pm z$ directions). If we modify the instability conditions so that avalanches spread by triggering neighboring cells with some probability, or by reducing B_c at neighboring sites, we find that, as expected, the distributions shown in Figure 4 are no longer power laws. These results and other properties of the self-organized field configuration will be discussed more fully in a later publication.

This picture of flares is of course oversimplified, but we believe it contains the essential physics underlying the distribution of flares. As in the simple lattice model, we expect the solar flare avalanche size distribution to be insensitive to much of the microphysics (e.g., the exact value of the critical angle θ_c or B_c). The fact that the flare size spectrum is a featureless power law, and that the spectral index of the flare distribution is constant over the solar cycle, even though the total flare occurrence rate changes, are natural consequences of the self-organized critical state of the magnetic field. We therefore expect that the flare distribution function will have the same slope in active and quiescent regions. The variation in flare occurrence rate with solar cycle (see Fig. 2) can be explained if either the average energy released in an elementary reconnection event, or the rate of energy input to the magnetic field varies with the solar cycle. We also note that the energy distributions of stellar flares are also power laws with spectral indices near 2 (Shakhovskaya 1989), suggesting a similar scenario on other stars.

Thus the classification of flares into nanoflares, microflares, giant flares and so on, is arbitrary since there is no preferred scale and the fundamental energy release mechanism is the same for flares of all sizes. However, it is known that flares of different size have different observed characteristics. For instance, two well-known properties of impulsive X-ray bursts are that larger flares tend to have harder X-ray spectra, and the X-ray spectral index is hardest during the peak of the flare (Dennis 1988). This may be qualitatively explained in our picture because electrons initially accelerated in a particular reconnection region can be further accelerated as they propagate through other reconnecting regions. Larger flares will have more reconnection events, and the number of active reconnection sites will be greatest at flare maximum. We are currently investigating the expected accelerated particle distribution in this scenario. Furthermore, the duration of the elementary reconnection events must be much less than the rise time of a solar flare which consists of many elementary events because the time it takes for the reconnection avalanche to spread to neighboring sites is roughly the duration of a single event. We thus expect that the energy release time of an elementary reconnection even is much less than 1 s.

The distribution of solar flare avalanche (or energy release) times T is also predicted to be a power law, $N(T) \propto T^{-\eta}$. As mentioned earlier, the extent of the power-law behavior of the lifetime distribution in Figure 4 is limited but will be greater for larger systems. Unfortunately, determination of the energy release time distribution from X-ray observations may be difficult. This is because the duration of flare emissions can be substantially longer than T due a number of effects including thermal emission from plasma heated during the impulsive phase of the burst (Lu & Petrosian 1990). In addition, measuring the lifetime of small flares is difficult due to the obscuring effect of background counts from nonsolar sources. We suggest, however, that the rise time in hard X-rays may be a useful measure of the energy release time.

The self-organized critical system has randomly occurring events which release energy E in time T giving rise to an energy release rate $L(\boldsymbol{x}, t)$ which is a function of both position and time. The energy release rate $L(t)$ integrated over position is a time-varying signal whose spectral power $S(\omega)$ is a power law (Bak et al. 1988) in frequency ω,

$$S(\omega) = \left| \int_{-\infty}^{\infty} L(t) \exp(i\omega t) dt \right|^2 \propto \omega^{-\beta}, \tag{3}$$

where $\beta = (\tau - 3)(1 - \eta)/(\tau - 1)$. This gives a direct relation between the power-law indices which can be tested by observations. Our simple lattice model leads to a power law $S(\omega)$ with $\beta \approx 2$ in spite of the fact that the duration distribution is only approximately a power law. In a future publication, we will describe results from simulations on larger grids and discuss this relation in greater detail.

The total energy released by all flares in the corona is of interest because of the possibility that it contributes to coronal heating. If all flares arise from regions where the magnetic field is in a self-organized critical state, then the differential occurrence rate of flares will be a power law when plotted as a function of total flare energy. More sensitive detectors on the *Solar-A* satellite and forthcoming balloon flights will then find that the flare distribution continues with the same spectral index to lower flare energy. Recent studies (Petrosian 1991; Hudson 1991) find that in order to account for coronal heating, the number of small (presently unobserved) flares must deviate above a single power-law distribution if the logarithmic slope of the flare energy distribution is less than 2. If the logarithmic slope of this distribution is less than 2 (as it is in our simple model) then the total energy released by all flares is dominated by flares near the maximum flare energy, and the smallest flares contribute an insignificant amount of heating.

We have argued that flares are avalanches of many small reconnection events, and that the coronal magnetic field is in a self-organized critical state. The observed power-law distribution of solar flares is a direct consequence of the physics of such a state. Furthermore, this model makes strong predictions about the statistics of flares which are testable by observation.

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REFERENCES

Babcock, K. L., & Westervelt, R. M. 1990, Phys. Rev. Letters, 64, 2168
Bak, P., Tang, C., & Wiesenfeld, K. 1987, Phys. Rev. Letters, 59, 381
———. 1988, Phys. Rev. A, 38, 364
Carlson, J. M., & Langer, J. S. 1989, Phys. Rev. Letters, 62, 2632
Datlow, D. W., Elcan, M. J., & Hudson, H. S. 1974, Solar Phys., 39, 155
Dennis, B. R. 1985, Solar Phys., 100, 465
———. 1988, Solar Phys., 118, 49
Hudson, H. S. 1991, Solar Phys., 133, 357
Kadanoff, L., Nagel, S. R., Wu, L., & Zhou, S. 1989, Phys. Rev. A, 39, 6524
Lin, R. P., Schwartz, R. A., Kane, S. R., Pelling, R. M., & Hurley, K. C. 1984, ApJ, 283, 421
Low, B. C. 1990, ARA&A, 28, 491
Low, B. C. 1991, preprint
Lu, E. T., & Petrosian, V. 1990, ApJ, 354, 735
O'Brien, K., Wu, L., & Nagel, S. R. 1991, Phys. Rev. A, 43, 2052
Parker, E. N. 1988, ApJ, 330, 474
———. 1989, Solar Phys., 121, 271
Petrosian, V. 1991, in Dynamics of Solar Flares: Proc. Flares 22 Workshop, ed. B. Schmeider & E. Priest (Observatoire de Paris: DASOP), 109
Shakhovskaya, N. I. 1989, Solar Phys., 121, 375
Sturrock, P. A., Dixon, W. W., Klimchuk, J. A., & Antiochos, S. K. 1990, ApJ, 356, L31
Sturrock, P. A., Kaufmann, P., Moore, R. L., & Smith, D. F. 1984, Solar Phys., 94, 341