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COHERENT RADIATION FROM ENERGETIC ELECTRON STREAMS VIA COLLISIONLESS BREMSSTRAHLUNG IN STRONG PLASMA TURBULENCE

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ABSTRACT

Radiation produced by the scattering of energetic electron streams in regions of plasma turbulence can be more efficient than synchrotron emission. Plasma instability occurs naturally due to counterstreaming plasmas in the environment of galactic jets, and saturates in a turbulent state of intense localized regions of electrostatic field. Energetic electrons accelerate while traversing soliton-like electric field structures. Their radiation is extremely broad-band in frequency, a bremsstrahlung type spectrum in which the minimum impact parameter is the scale size of the soliton, assumed to be a many Debye lengths. Radiation may extend many orders of magnitude higher in frequency than the plasma frequency, and is relativistically beamed. For a stream of electrons, the emission is partially coherent if the beam is bunched. A specific model for beam density fluctuation statistics yields a power-law radiation spectrum. Turbulent radiation in some parameter regimes can equal or exceed synchrotron emission. Such radiation may therefore significantly reduce the overall energy requirements for central engines of galactic radio sources.

Subject headings: galaxies: jets — plasmas — radiation mechanisms — radio sources: variable — turbulence

1. INTRODUCTION

Astrophysical objects such as AGNs contain high-velocity streams of charged particles which radiate when accelerated in ambient magnetic fields. Such plasma environments are highly susceptible to instability, so turbulence should occur often. Except by slowing beam propagation, this will not modify the synchrotron theory for radiation. However, it does open the possibility that there may be other contributions to the emitted spectrum.

Coherent plasma radiation and plasma turbulence have been considered previously in connection with astrophysical objects. Colgate, Lee, & Rosenbluth (1970) computed the emission from the coalescence of electrostatic Langmuir waves as a significant contributor to the radio and infrared emission in QSOs. Recently, turbulence models for galactic jets have been studied in great detail based on advances in plasma turbulence theory. Sol, Pelletier, & Asseo (1989) present a model for beam instability processes and turbulence which accounts for observed structures in galactic jets. A recent paper by Baker et al. (1988) points out the ability of coherent plasma radiation to explain luminosity gaps and other properties of BL Lac objects. Importantly, they speculate significantly on increasing the rate of energy extraction in radiation per particle of jet electron populations. Krishan & Witta (1990) studied coherent continuum emission from active galactic nuclei, showing that it can both produce relativistic electron beams and yield rapidly varying emission.

We describe a new radiation mechanism in which charged particle streams scatter on intense localized electrostatic fields associated with plasma turbulence. These fields arise from a process known as plasma collapse, first described by Zakharov (1972), in which electrostatic energy accumulates in increasingly localized wave packets. Such a plasma is said to be strongly turbulent, as observed and studied in laboratory (Cheung & Wong 1935; Levron, Benford, & Tzach 1987) and space plasmas (Lin et al. 1986). Collapse has been analyzed by computer solution to the Zakharov equations, as well (Degtyarev, Zakharov, & Rudakov 1976; Pereira, Sudan, & Denavit 1977; Nicholson et al. 1978; Robinson, Newman, & Goldman 1988). These solutions characterize the plasma turbulence as an ensemble of soliton-like wave packets. Our proposed radiation mechanism involves scattering of electrons by localized wave packets (see Fig. 1). This resembles "bremsstrahlung" of beam electrons in the turbulence.

This process differs significantly from plasma radiation associated with conversion of electrostatic plasma waves (Ginzburg & Zhelezniakov 1958). The signature of those processes is emission in narrow-frequency bands at the plasma frequency ω_e and its second harmonic. Various plasma wave scattering and collapse scenarios serve as possible conversion mechanisms (Whelan & Stenzel 1985; Smith & Nicholson 1979; Goldman, Reiter, & Nicholson 1980). The possibility of radiation from the beam has not received attention, principally because the beam is perceived as a low-density, incoherent radiator. This changes when the beam is ultrarelativistic, because the radiation is dramatically enhanced by relativistic beaming along the direction of propagation. Another unique feature is that the radiation is highly broad-band in frequency when some coherence is introduced by modulation or bunching in the beam. Thus, the radiation spectrum depends on the nature of the beam modulation. The beam radiation will dominate plasma emission when the beam is dense and highly relativistic (Weatherall 1988), as in energetic astrophysical plasmas.

This paper quantifies emission and spectra of beam radiation in a turbulent plasma in terms of basic parameters, comparing directly with synchrotron radiation. Section 2 puts forward a mathematical model for the soliton electrostatic

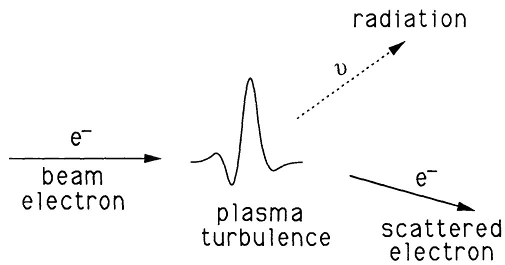

FIG. 1.—Schematic of the turbulent beam-bremsstrahlung radiation mechanism: a relativistic electron scatters off a localized wave packet of electrostatic waves.

fields which act to accelerate the beam electrons. The single electron radiation spectrum, calculated in § 3, is extended to the aggregate radiation by the beam in § 4. An assumed specific model for beam fluctuation statistics then leads to a power-law spectrum. Section 5 compares turbulent beam-bremsstrahlung radiation and synchrotron emission. Summary and conclusions are in § 6.

2. ELECTROSTATIC TURBULENCE

The Zakharov equations describing strong turbulence are coupled wave equations for plasma density and the envelope of the high-frequency electrostatic field (for reviews, see ter Haar & Tsytovich 1981; Goldman 1984). The density couples to the electrostatic field through the pondermotive force, which tends to drive plasma electrons from local regions of large field toward regions of lower field. Quasi-neutral coupling to ions expels them as well. Thus, a density cavity forms in which the local plasma frequency of oscillation is depressed, which causes electrostatic waves to refract into the cavity, increasing the pondermotive force. The process is unstable in more than one dimension, increasing localization until at the smallest scales wave energy converts into particle kinetic energy. The localized electrostatic structures in the strongly turbulent state are colloquially called "solitons" or "cavitons."

Collapse decouples the beam from the unstable waves (Hafizi et al. 1982). This happens because ions absorb the momentum of the beam-generated plasma waves and, except for a residual velocity which is not greater than the ion-sound speed, the caviton is at rest in the plasma. Thus the soliton wave packet may be regarded as totally decoupled from the beam, and beam electron trajectories are only slightly perturbed in passage through the soliton.

We propose a specific model for the shape and size of the soliton, in which the characteristic turbulent charge density oscillation is dipolar within a localized region;

$$\rho = \rho_0 \frac{\hat{\boldsymbol{p}} \cdot \boldsymbol{r}}{D} \exp\left(-\frac{r^2}{D^2}\right) \exp(-i\omega_e t) . \tag{1}$$

The density oscillation of amplitude ρ_0 is localized by a Gaussian envelope function of scale length D, oscillates at the plasma frequency ω_e, and has a dipole moment in the direction of the unit vector $\hat{\boldsymbol{p}}$. Because dynamic instability processes drive the cavitons to increasingly smaller size, in practice the scale length D can be associated with the dissipation scale length, approximately 15 Debye lengths. This scale length will tend to dominate because the persistence of density cavitons after the damping or "burnout" of the wave energy allows the regeneration of localized fields at the same scale (Russell, DuBois, & Rose 1986).

This formulation of the field as a localized oscillating dipole is supported by numerical solutions of the Zakharov equations, which show that for a variety of initial conditions modulationally unstable Langmuir waves evolve into dipole-like collapsing solitons (Degtyarev et al. 1976; compare with Weatherall 1988).

The electrostatic fields corresponding to the assumed density configuration can be derived by integrating Poisson's equation. In spherical polar coordinates, where θ is measured relative to the direction of the dipole moment, an exact expression for the soliton's electrostatic field is

$$\begin{aligned}
E_r(r, \theta) &= -\cos\theta \frac{\pi\rho_0 D^4}{r^3} \exp(-i\omega_e t) \\
&\quad \times \left[2\left(1+\frac{r^2}{D^2}\right)\frac{r}{D}\exp\left(\frac{r^2}{D^2}\right) - \sqrt{\pi}\,\mathrm{erf}\left(\frac{r}{D}\right)\right], \\
E_\theta(r, \theta) &= \sin\theta \frac{\pi\rho_0 D^4}{r^3} \exp(-i\omega_e t) \\
&\quad \times \left[-\frac{r}{D}\exp\left(\frac{r^2}{D^2}\right) + \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{r}{D}\right)\right], \\
E_\phi(r, \theta) &= 0 .
\end{aligned} \tag{2}$$

Moreover, near the center of the soliton, the electric field is simply

$$\boldsymbol{E}_0 = -\hat{\boldsymbol{p}}\tfrac{2}{3}\pi D\rho_0 \exp(-i\omega_e t) . \tag{3}$$

The general validity of the Zakharov equations and strong turbulence theory is assured if $E_0^2/8n_0 k_B T \ll 1$. Another implicit assumption will occur in the factoring of the electrostatic oscillation out of the radiation integrals. This requires that the phase of the electric field over the acceleration region is uniform, and is justified when the light travel time across the caviton is small compared with the period of oscillation. This condition is readily satisfied for all frequencies above the plasma frequency as long as $D < 2\pi c/\omega_e$.

3. SINGLE PARTICLE RADIATION

According to classical radiation theory, electrons passing through a soliton will radiate because they are accelerated by the soliton's electrostatic field. If $\dot{\boldsymbol{\beta}}$ is the acceleration, then the formula for the radiation field from the moving charge is (Jackson 1962)

$$\mathbf{E}(x, t) = \frac{e}{c} \frac{\hat{\boldsymbol{n}}}{(1 - \hat{\boldsymbol{n}} \cdot \boldsymbol{\beta})^3 R} [(\hat{\boldsymbol{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]|_{\mathrm{ret}} , \tag{4}$$

where "ret" means evaluated at the retarded time and $\hat{\boldsymbol{n}}$ is the unit vector in the direction of the observation point, a distance R from the source.

A useful coordinate system shown in Figure 2 has the origin at the center of the scattering soliton. The unit vector $\hat{\boldsymbol{e}}_z$ is in the direction of the electron velocity, $\boldsymbol{\beta}$. The coordinate system may be chosen so that the observation point is in the x-z plane. The spherical coordinates (θ, ϕ, r) are convenient for the electron's position vector. The position of the observer at distance R is characterized by the angle χ between $\hat{\boldsymbol{n}}$ and $\boldsymbol{\beta}$.

The radiation from a single beam emerges in a straightforward calculation. Knowing the electron's velocity and position, its acceleration due to the force of the electrostatic field

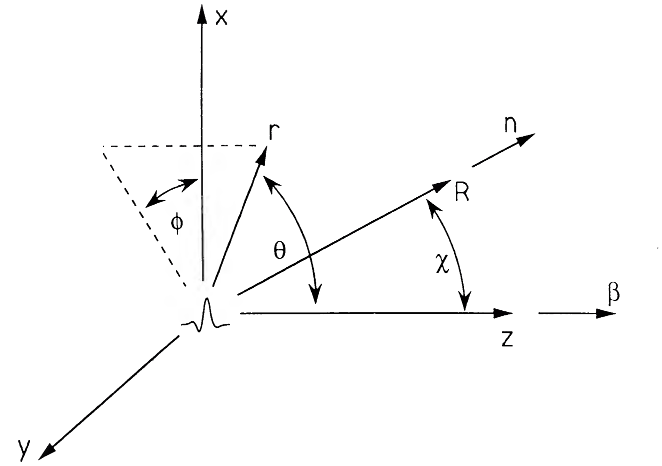

FIG. 2.—Source coordinate system. Soliton wave packet is at the center, beam propagates along z-direction, and observer is positioned in the x-z plane in direction of n.

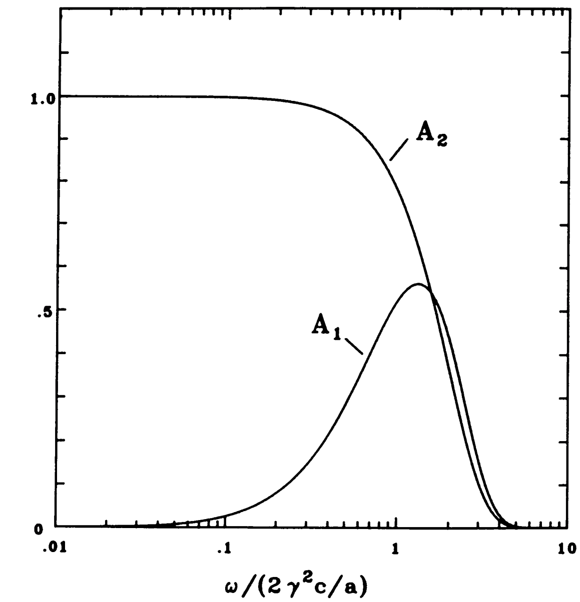

FIG. 3.—Single electron spectral function, $A_1(\omega)$ and $A_2(\omega)$, corresponding to solitons with parallel and perpendicular orientations, respectively, relative to the beam velocity.

given by equation (2) can be computed directly, and inserted into the radiation formula of equation (4).

For certain simple characteristic trajectories, the power spectrum of the field is computed in closed form from the time-dependent Poynting flux. For example, assume that the electron's trajectory is an unperturbed straight-line path through the center of the soliton. We consider two cases for orientations of the soliton's dipole moment parallel to and transverse to the electron's trajectory.

First, let the dipole moment be parallel to the direction of the beam. This special case is interesting because it corresponds to the natural orientation of the fields produced in the plasma by the beam-plasma instability. The electric field of the radiation is polarized along the direction $\hat{\boldsymbol{n}} < (\hat{\boldsymbol{n}} \times \boldsymbol{\beta})$ (the direction of the beam projected on the plane of the sky). The radiated energy per unit frequency per unit solid angle is

$$\frac{dI(\omega)}{d\Omega} = \frac{E_0^2}{8\pi} \sigma_{\mathrm{T}} c \frac{D^2}{v^2} \frac{27}{16\pi} \frac{1}{\gamma^6} \times \left[\frac{\sin \chi}{(1 - \boldsymbol{\beta} \cos \chi)^2}\right]^2 |A_1(\omega - \omega_e)|^2 . \quad (5)$$

Second, assume that the dipole moment is transverse to the direction of the beam. The orientation of the soliton's dipole moment is measured relative to the x-axis by the angle Φ. This case is allowed in a fully developed turbulence in which dipole moments are randomly oriented. The resulting energy spectrum is

$$\frac{dI(\omega)}{d\Omega} = \frac{E_0^2}{8\pi} \sigma_{\mathrm{T}} c \frac{D^2}{v^2} \frac{27}{16\pi} \frac{1}{\gamma^2} \times \left[\frac{(\boldsymbol{\beta} \cos \chi - 1)^2 \sin^2 \Phi + (\boldsymbol{\beta} - \cos \chi)^2 \cos^2 \Phi}{(1 - \boldsymbol{\beta} \cos \chi)^4}\right] \times \left| A_2(\omega - \omega_e) - \frac{1}{2} A_1(\omega - \omega_e) \right|^2 . \quad (6)$$

The electric field producing the $\cos \Phi$ term is polarized in direction of $\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \boldsymbol{\beta})$; the $\sin \Phi$ term is due to polarization along $\hat{\boldsymbol{n}} \times \boldsymbol{\beta}$. The spectral functions A_1 and A_2 in the above formulas are Fourier integrals:

$$A_1(\omega) = \left[\frac{D\omega}{2\gamma^2 c}\right]^2 \int_0^\infty \frac{\operatorname{erf}(t)}{t} \cos \left[\frac{D\omega}{2\gamma^2 c} t\right] dt$$

$$A_2(\omega) = \exp\left[-\frac{1}{4}\left(\frac{D\omega}{2\gamma^2 c}\right)^2\right] \quad (7)$$

The spectral functions are plotted in Figure 3.

A few facts are worthy of summary:

1. A single electron encounter produces a range of frequencies, from the plasma frequency, up to the frequency $2\gamma^2 c/D$. This spectrum would be characteristic of collisional bremsstrahlung, except here the role of the minimum impact parameter is played by the soliton scale size D. Comparing the turbulent cutoff frequency to the synchrotron peak frequency:

$$\frac{2\gamma^2 c/D}{\gamma^2 \omega_{ce}} = \frac{\omega_e}{\omega_{ce}} \frac{c}{v_{\mathrm{T}e}} \frac{\lambda_{\mathrm{D}}}{D} , \quad (8)$$

where ω_{ce} is the electron cyclotron frequency, $v_{\mathrm{T}e}$ is the electron thermal velocity, and λ_{D} is the Debye length.

2. Radiation is relativistically beamed. For polarization $\hat{\boldsymbol{n}} \times \boldsymbol{\beta}$, as in the second case, the maximum intensity occurs looking directly into the beam at $\chi = 0$, with narrow half-widths of order $1/\gamma$. For the other polarization, $\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \boldsymbol{\beta})$, the radiation is in a cone having emission maximum near $1/\gamma$.

3. Radiation is substantially enhanced by the relativistic beaming for the case of transverse orientation of the dipole moment.

4. Radiation will be polarized. With no special orientation of soliton dipole moments, the ratio of polarization is given by

$$\frac{\mathbf{E} \cdot (\hat{\boldsymbol{n}} \times \boldsymbol{\beta})}{\mathbf{E} \cdot [\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \boldsymbol{\beta})]} = \frac{(\beta \cos \chi - 1)}{(\beta - \cos \chi)} . \quad (9)$$

An average over observation angles χ indicates a 75% net polarization in the direction $\hat{\boldsymbol{n}} \times \boldsymbol{\beta}$.

4. BEAM RADIATION

The single particle radiation calculation shows that the impulsive acceleration of an electron in the soliton field launches an electromagnetic wave packet containing a broad spectrum of frequencies. If the acceleration of more than one charge is involved, as with the passage of a beam of electrons through the soliton field, a superposition of radiation from each charge occurs. Radiation from one electron at the frequency ω will be out of phase with the radiation of a second electron accelerated at a time π/ω later, for any part of the spectrum. Therefore, a uniform electron stream will radiate without phase coherence, and the resulting incoherent scattering will be intrinsically weak.

On the other hand, a beam which is nonuniform in density can maintain a degree of phase coherence over a range of wavelengths comparable with the scale length of the density variation. Modulation of electron beam density in laboratory radiation sources illustrates this principle. In devices such as klystrons, free-electron lasers, Cerenkov masers, and others, electrons are bunched by resonant feedback with the electromagnetic wave, and coherence is maintained by external constraints established by electromagnetic cavities, wiggler fields, dielectric-lined wave guides, virtual cathodes, etc. Electron bunching by trapping in the potential fields of plasma waves has been hypothesized to enhance synchrotron and Compton emission from beams in unbounded plasmas as well (Kato, Benford, & Tzach 1983). They observe just the sort of collective Compton-boosted emission up to $\omega \sim \gamma^2 \omega_e$ we explore here; however, their explanation hinges upon a high-beam bunching factor and long-range coherence, perhaps resulting from a transient response of the system. In the steady state strong turbulence postulated in the present case, modulation of the beam by wave trapping is mitigated by the self-focusing and collapse of the wave packets. Instead, we describe dynamical plasma processes affecting beam density fluctuations with a statistical formalism.

4.1. *Correlations in Beam Density*

Although correlations in beam density is the physical quantity of interest, fluctuation statistics are usually described via a power spectrum (Tatarski 1967). These two quantities are related as follows. The density correlation function is defined:

$$B(x_1, x_2) = [n(x_1) - n_0][n(x_2) - n_0] . \tag{10}$$

If the fluctuations are homogeneous and isotropic, the correlation function only depends on the separation of the two points, $|x_1 - x_2|$. The one-dimensional spectral density is the Fourier transform of this function along one direction:

$$V(k) = \tfrac{1}{2} \int_{-\infty}^{\infty} B(x_1 - x_2) \exp [ik(x_1 - x_2)] d(x_1 - x_2) . \tag{11}$$

4.1.1. *Thermal Fluctuations*

In the case of a uniform beam, the spectrum of beam density correlations is given by

$$V(k) = n_b^2 \frac{1}{n_b \lambda_{Db}^3} \frac{\lambda_{Db}}{1 + k^2 \lambda_{Db}^2} , \tag{12}$$

where λ_{Db} is the Debye length of the beam plasma species.

4.1.2. *Beam Modulation*

Nonthermal fluctuations of beam density arise from plasma turbulence. Soliton collapse models predict isolated islands of turbulence of high values of E_0^2 which act as scattering centers for the radiation. Between collapsing cavitons is a low-wave density, E_T^2, which remains coupled to the beam, and which is maintained through the beam plasma instability. If r_0 is the plasma instability wavelength, density correlations in the beam which are resonant with the plasma wave will have a monochromatic spectrum:

$$V(k) = n_b^2 \left(\frac{\Delta n_b}{n_b}\right)^2 \delta(k - 2\pi/r_0) . \tag{13}$$

We can estimate the magnitude of the density modulation by computing the bunching during the convective flow of the beam through the wave of amplitude E_T. In the fluid beam instability regime $[1/\gamma^2 \Delta E/E < (n_b/\gamma n_0)^{1/3}$, where $\Delta E/E$ is the energy spread of the beam], there is a net flow velocity between the beam and the fastest growing wave, given by $(\frac{2}{5})^{1/3}(n_b/\gamma n_0)^{1/3} v_b$. In the frame of reference stationary with the wave, the velocity difference is $\Delta\tilde{v} = 0.26c$, assuming $\gamma = 100$ and $n_b/n_0 = 10^{-5}$. We use a linearized fluid continuity equation to compute the modulation of the beam as it speeds up and slows down in the beam-generated wave potential:

$$\left(\frac{\Delta n_b}{n_b}\right)^2 = 2\left(\frac{v_b}{\Delta\tilde{v}}\right)^4 \frac{k_B T}{mc^2} \gamma^2 W_T . \tag{14}$$

In order for a strong turbulent state to develop, the background energy density, $W_T = E_T^2/8\pi n_0 k_B T$, must be large enough to induce collapse. In the present case, the collapse threshold requires $W_T > (k\lambda_D)^2$, or W_T on the order of 4×10^{-5} (Nicholson & Goldman 1978). Comparable energy densities of $W_T = 10^{-6}$ are measured in situ in the solar wind in the presence of solar flare electron streams (Lin et al. 1986). Thus, within these bounds for W_T, we estimate the magnitude of the beam density fluctuations from equation (14) as $(\Delta n_b/n_b)^2 > 2 \times 10^{-4}$. We have assumed an ambient plasma temperature of 20 eV.

4.1.3. *Turbulent Spectrum*

A broad spectrum of density fluctuations will develop from the beam modulation through nonlinear effects. In classical turbulence theory, a fluctuation spectrum is established by the nonlinear flow of energy from one scale, where wave energy is being generated, to another scale, where energy is being dissipated. According to weak turbulence theory (Zakharov 1967; Davidson 1968), a four-wave scattering interaction in a single fluid will produce a plasmon condensate spectrum with a power-law index, in one dimension, of $\alpha = -5/3$. We therefore parameterize this spectrum in terms of the power law index, α; the mean squared density variation, $\Delta n^2 = 1/T \int^T [n(x) - n_0]^2 \, dt$; and a perturbation scale length r_0 (Tatarski 1967):

$$V(k) = n_b^2 \left(\frac{\Delta n_b}{n_b}\right)^2 \frac{\Gamma(\alpha)}{\sqrt{\pi}\Gamma(1/2)} \frac{r_0}{(1 + k^2 r_0^2)\alpha} , \tag{15}$$

where Γ is the gamma function.

The turbulent spectrum of density fluctuations imparts partial coherence to a wide-frequency spectrum of emissions during beam scattering on the spiky turbulence. The radiation will have a power-law spectrum, even though the single particle radiation spectrum is not a power law. The role played here by density fluctuations is analogous to that of electron velocity dispersion in the synchrotron model.

4.2. *Beam Radiation Spectrum*

We computed the energy radiated per unit solid angle per unit frequency interval in § 3 for a single accelerated particle. For the accelerated motion of more than one charge, a coherent sum of radiation fields must be incorporated into the energy spectrum. In this case, the contribution of each electron to the sum has a phase which depends only on the relative timing, t_k, of its transit of the soliton:

$$\frac{dE(\omega)}{d\Omega} = \frac{dI(\omega)}{d\Omega} \left| \sum_k \exp(i\omega t_k) \exp(-i\omega_e t_k) \right|^2 . \quad (16)$$

For simplicity of calculation, the single particle acceleration, which was computed for a crossing through the center, will be used for all electrons passing within a transverse distance D of the center. $dI(\omega)/d\Omega$ is the single particle radiation spectrum. If x is the direction of beam's propagation, the crossing time t_k of the kth electron is just a function of the electron's position x_k measured relative to the soliton's center at some fiducial time: $t_k = x_k/v_0$. The sum can then be written as an integral over the beam density:

$$\frac{dE(\omega)}{d\Omega} = \frac{dI(\omega)}{d\Omega} (\pi D^2)^2 \left| \int n(x) \exp\left[\frac{i(\omega - \omega_e)x}{v_0}\right] dx \right|^2 . \quad (17)$$

The cross-sectional area πD^2 replaces the integration over y and z. The right-hand side can be written out as a double integral, which explicitly incorporates the density correlation function:

$$\left| \int n(x) \exp[i(\omega - \omega_e)x/v_0] dx \right|^2$$
$$= \iint n(x_1)n(x_2) \exp[i(\omega - \omega_e)(x_1 - x_2)/v_0] dx_1 \, dx_2 . \quad (18)$$

Since the integration is the Fourier transform of the density correlation function, the radiation formula contains the density power spectrum, $V(k)$, as defined by equation (15) for turbulent beam fluctuations. The total energy radiated by the soliton is given by

$$\frac{dE(\omega)}{d\Omega} = \frac{dI(\omega)}{d\Omega} (\pi D^2)^2 2\pi L \times \left\{ v_0 n_b^2 \, \delta(\omega - \omega_e) + V\left[\frac{(\omega - \omega_e)}{v_0}\right] \right\} . \quad (19)$$

The beam length, L, enters the formula for energy radiated from a single soliton for the duration of the beam; L factors out in a calculation of power. The first term is the coherent part of the spectrum, relating to the radiation at the plasma frequency. The second term is the partially coherent, broadband component of the spectrum, deriving from the density clumping in the beam.

The spectrum is indicated schematically in Figure 4 with lower bound at the plasma frequency. The upper bound, $2\gamma^2 c/D$ is the highest frequency emitted by electrons scattering on solitons of scale length D. The slope of the spectrum, here shown as $-5/3$, depends in this model on the statistics of fluctuations in the beam. The assumed turbulent spectrum of beam density causes a substantial enhancement in emission over the thermal fluctuations alone, which can be computed

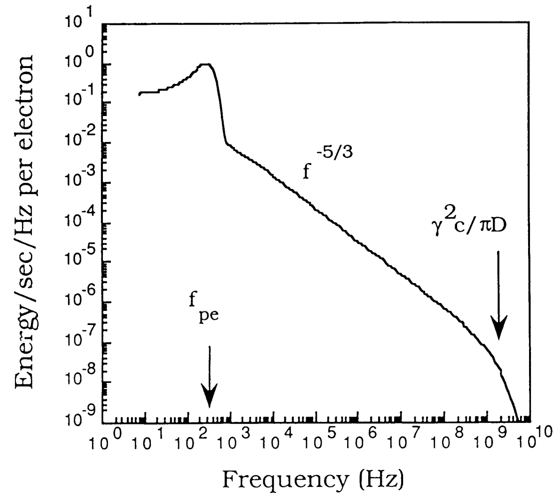

FIG. 4.—Beam radiation power spectrum, normalized to the plasma line power. Assumed plasma parameters: plasma line width $\Delta f/f_{pe} = \frac{1}{2}$; $n_0 = 10^{-3}$ cm^{-3}; $\gamma = 100$; $T_e = 20$ eV.

using equation (12). Emission from a uniform beam is smaller by

$$\frac{(n_b \lambda_{Db}^3)^{-1}}{(\Delta n_b/n_b)^2} \frac{\lambda_{Db}}{r_0} \sim 3 \times 10^{-19} . \quad (20)$$

The numerical value is computed using an assumed beam temperature of $\Delta\gamma/\gamma = 10\%$, and beam density of 10^{-8} cm^{-3}.

In reality, the angular dependence of the single particle radiation does not apply to the net radiation from the beam because a large number of soliton geometries will be encountered, and a finite spread in the transverse component of the beam velocity is also to be expected. A useful measure of the radiation is the total energy radiated over all angles, recognizing that the radiation will be forward beamed according to the beam pitch angle and Lorentz factor. We integrate the single electron intensity function over solid angle:

$$I(\omega) = \frac{E_0^2}{8\pi} \sigma_T c \frac{D^2}{c^2} \frac{9}{4} \times \left[(3\hat{p}_y + \hat{p}_x) \left| A_2 - \frac{A_1}{2} \right|^2 + \frac{2\hat{p}_z}{\gamma^2} |A_1|^2 \right] . \quad (21)$$

The orientation of the soliton relative to the beam direction is given by the unit vector $\hat{p}$. Because of the γ^{-2} factor, we can ignore the second term in the brackets. We can also approximate the spectral function $|A_2 - A_1/2|$ as unity between ω_e and $2\gamma^2 c/D$ (see, for instance, Fig. 3). Assuming no preferred direction for the vector $\hat{p}$, we average over an ensemble of soliton orientations, finding

$$\langle I(\omega) \rangle = \frac{3}{4\pi} E_0^2 \sigma_T c \frac{D^2}{c^2} \quad \omega_e < \omega < \frac{2\gamma^2 c}{D}$$
$$= 0 \quad \omega \text{ other} . \quad (22)$$

5. BEAM PLASMA RADIATION VERSUS SYNCHROTRON RADIATION

We compute the power radiated by the beam from the energy radiated per electron, taking into account the frequency of electron/soliton collisions.

The analysis of radiation from the full system requires knowledge of the fraction of the volume filled by the solitons, which is denoted by the packing fraction f. A simple dimensional analysis yields a value for f on the order of $\omega_e^3 D^3/c^3 \sim 10^{-2}$, based on an assumed spacing between cavitons of a half-wavelength of the instability length $r_0 = c/\omega_e$. A further indication of the packing fraction is provided through laboratory beam plasma experiments. The Levron et al. (1988) experiments concern turbulence with $0.1 < E_0^2/8\pi n_0 k_B T < 1$. Using two methods—the ratio of satellite spectral lines to allowed lines, and the experimental optical efficiency of their diagnostic system—they find $0.5 < f < 0.3$.

The time-averaged radiated power per frequency interval per electron in the beam is

$$P_{e,\omega} = \langle I(\omega)\rangle f \frac{3}{2}\pi^2 D n_b v_0^2\left[\delta(\omega - \omega_e) + \frac{V[(\omega - \omega_e)/v_0]}{n_b^2 v_0}\right]. \tag{23}$$

From equation (15) for $V(k)$, it is simple to integrate over frequency to get the time-averaged radiated power per electron in the beam:

$$P_e = \left[\frac{E_0^2}{8\pi}\sigma_T c\right]\left[\frac{4}{3} n_b \pi D^3\right]\frac{27\pi}{4} f \times \left\{1 + \frac{0.24}{\alpha - 1}\left(\frac{\Delta n_b}{n_b}\right)^2\left[\left(\frac{c}{r_0 \omega_e}\right)^{\alpha-1} - \left(\frac{D/r_0}{2\gamma^2}\right)^{\alpha-1}\right]\right\}. \tag{24}$$

The first term in the power formula, $(E_0^2/8\pi)\sigma_T c$, is the classical Thomson scattering rate (Ginzburg & Zhelezniakov 1958). The second term is the number of beam electrons in the soliton volume—the coherence effect which can be very large. The fraction of volume filled by solitons is f. In the last bracketed term, the unitary term represents the power radiated at the plasma line, and the second term is the power radiated in the power-law tail. The latter term involves the mean square beam density fluctuation, $\Delta n_b^2/n_b^2$; the ratio of the plasma skin depth, c/ω_e, to the fluctuation scale length, r_0; and the ratio D/r_0 of the minimum to the maximum turbulence scale lengths.

In the following discussion, we will disregard radiation produced at the plasma frequency because wave energy at this frequency is likely to be trapped and absorbed in the jet plasma, particularly if the radiation is produced in an underdense region of the jet: thus, we drop the unitary term in the brackets in equation (24).

We also observe that the total radiated power is relatively insensitive to the value of α when α is near unity. Hence, we use the limit

$$\lim_{\alpha \to 1} \frac{0.24}{\alpha - 1}\left\{\left(\frac{c}{r_0 \omega_e}\right)^{\alpha-1} - \left(\frac{D/r_0}{2\gamma^2}\right)^{\alpha-1}\right\} = 0.24 \ln\left(\frac{2\gamma^2 c}{D\omega_e}\right). \tag{25}$$

The argument of the logarithm is the frequency bandwidth, which, by inspection of Figure 4, is of order 10^7.

Now compare the power in the turbulent beam-bremsstrahlung radiation directly with the synchrotron power. The power radiated per beam electron in incoherent synchrotron emission is (Rybicki & Lightman 1979)

$$P = \frac{4}{3}\left(\frac{B_0^2}{8\pi}\sigma_T c\right)\beta^2\gamma^2 . \tag{26}$$

The turbulent radiation can greatly exceed synchrotron emission, principally because astrophysical plasmas have a tremendous number of electrons in a Debye sphere. Scaling to typical parameters, the ratio of beam radiation power by scattering on electrostatic turbulence to beam radiation power by synchrotron emission is

$$\frac{P\ (\text{turbulent})}{P\ (\text{synchrotron})} = 2 \times 10^5 f \frac{E_0^2}{8\pi n_0 k_B T}\left[\frac{\Delta n_b^2/n_b^2}{2 \times 10^{-4}}\right]\left[\frac{D}{15\lambda_D}\right]^3 \times \left[\frac{T}{20\ \text{eV}}\right]^{5/2}\left[\frac{n_0}{10^{-3}\ \text{cm}^{-3}}\right]^{1/2}\left[\frac{10^{-6}\ \text{G}}{B_0}\right]^2\left[\frac{n_b/n_0}{10^{-5}}\right]\left[\frac{100}{\gamma}\right]^2 \times \left[\frac{\ln(2\gamma^2 c/D\omega_e)}{16}\right]. \tag{27}$$

The numerical parameters correspond to hypothetical conditions in galactic jets. The scaling values used for density, temperature, and magnetic field give a ratio for thermal to magnetic energy of approximately 1. The turbulent emission has considerable advantage in efficiency over synchrotron emission. The dimensionless electrostatic energy within soliton structures is $W_0 = [E_0^2/8\pi n_0 k_B T]$. Note that the product $f W_0$ is the normalized field energy density in cavitons in three dimensions. In Langmuir collapse, this is conserved even as cavitons shrink. Thus, the average over a time ensemble of cavitons should reflect a value for this product and the mean caviton volume D^3. Unfortunately, no experiment measured all of these quantities.

We can apply to equation (27) nonlinear thermodynamic bounds which exist for unstable beam-plasma systems. Davidson & Yoon (1989) found such limits for the relativistic case, including kinetic energy perpendicular to the beam direction. For cosmic jets this is probably a minor element, since most of the energy available for emission lies in the beam kinetic energy. For astrophysical cases, deceleration of the beam can supply more free energy than reordering of transverse momenta. Assuming this to be so, the absolute thermodynamic bound on available electrostatic energy density (from which beam electrons can scatter) is set by requiring that it not exceed the kinetic energy density of the beam itself. We write this constraint as

$$f W_0 = q\frac{(\gamma - 1)n_b mc^2}{8\pi n_0 k_B T}, \tag{28}$$

where the steady state factor $q < 1$ represents the efficiency of transfer of beam kinetic energy to electrostatic waves.

Many physical circumstances influence q. In galactic jets, with boundaries far away, electrostatic wave convection cannot readily deplete the energy reservoir. Similarly, in many laboratory experiments, wave convection cannot occur before the experiment is over. This allows a high electrostatic wave energy density. Here we estimate the importance of the equation (28) constraint and compare with recent laboratory work. We may recast equation (28) as

$$f W_0 = q\left(\frac{n_b/n_0}{10^{-5}}\right)\left(\frac{\gamma}{100}\right)\left(\frac{T}{20\ \text{eV}}\right)^{-1}. \tag{29}$$

if all the physical parameters are of order unity, equation (27) yields $P(t)/P(s) \sim 160{,}000q$.

Recent laboratory work has studied generation of strong Langmuir turbulence from relativistic beams, including microwave emission. Stark effect measures of E^2, together with model-dependent analysis of the microwave power, can estimate the turbulent volume fraction f. These fields contribute the high-density regions which dominate the statistical ensemble leading to an average of W (Levron et al. 1988; Zhai & Benford 1990). These methods converge on a range of values for q:

$$10^{-3} < q\left(\frac{n_b/n_0}{10^{-5}}\right)\left(\frac{\gamma}{100}\right)\left(\frac{T}{20 \text{ eV}}\right)^{-1} < \frac{1}{2}. \quad (30)$$

Taking over this range to the cosmic jet case yields a lower bound of $P(t)/P(s) \sim 160$ and an upper bound of about 80,000.

Of course, there may be vast differences between the turbulence in galactic jets and those of laboratory relativistic beam-plasma systems. Since we cannot hope to observe the underlying turbulence in astrophysical cases, laboratory work remains a guide which can place constraints on imagination. Interestingly, the turbulent beam-bremsstrahlung radiation is significant compared with the synchrotron power for quite ordinary parameters for the jet plasma, and equally unremarkable assumptions about the plasma turbulence in the beam-plasma. Our results are based in the main part on the assumptions of strong plasma turbulence in the jet, in which the caviton size is set by the burnout/regeneration scale length of $D = 15\lambda_D$ and a packing fraction of 5% or greater; and beam density fluctuations on the order of 10%, which arise from interaction of the beam with the background plasma.

6. CONCLUSION

Because plasmas containing beams should be turbulent, we have considered radiation produced by beam scattering on turbulent electrostatic fields, compared with beam synchrotron emission. We used a specific model representing turbulent fields as solitary wave structures, although qualitatively the calculation applies to other cases in which plasma electric fields scatter beam electrons.

We extended a radiation calculation for a single electron to a stream of electrons. For uniform beams, the radiation is incoherent. However, by allowing density inhomogeneity in the beam, the radiation becomes partially coherent over a broad band of frequencies. A specific power-law model for the beam fluctuations leads to power-law radiation spectra.

This "collisionless bremsstrahlung" beam radiation mechanism differs from other plasma radiation models. The radiation is relativistically beamed, and has frequency components many orders of magnitude above the local plasma frequency $[\omega_{\max} = \gamma^2(T_e/mc^2)^{-1/2}\omega_e]$. In diffuse astrophysical plasmas, this can extend into the microwave spectral region.

Depending on the values of the turbulence parameters, and in particular the fraction of the plasma volume filled with nonlinear electric field wave packets and the degree of beam inhomogeneity, it seems possible that the radiation efficiency per fast electron for the turbulent mechanism could be greater than the synchrotron efficiency. This has significant consequences for long-standing problems relating to the total energy required to drive astrophysical radio sources.

For example, increasing radiative efficiency in galactic jets lessens the energy density needed in the jet itself. This in turn reduces the demanded outflow from a purported black hole environment, and hence lowers the black hole mass needed to explain the overall jet phenomena (Baker et al. 1988). Since estimated hole masses seem quite large (Rees 1984), increased radiative efficiency may be an attractive possibility.

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