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LUMINOSITY LIMIT FOR ALPHA-VISCOSITY ACCRETION DISKS

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ABSTRACT

We report the existence of a luminosity limit for α-viscosity physically thin accretion disks around black holes, using a new formulation of the radiation equation bridging optically thick and thin regimes. For α close to unity this limit can be lower than the Eddington limit. Physically this limit is due to the combined effects of gas and radiation pressure which become too large to satisfy vertical hydrostatic balance at intermediate optical depths for sufficiently high luminosities. This effect was overlooked in previous treatments using only the optically thin or thick limits of the radiative equation.

Subject headings: accretion — black holes — hydrodynamics

1. INTRODUCTION

For thin Keplerian disk models around black holes using the α-viscosity assumption (Shakura & Sunyaev 1973; Novikov & Thorne 1973), it is known that for most values of the accretion rate $\dot{M}$ and α, the steady state disk structure equations admit two distinct solutions in the inner region: one corresponding to the optically thick, radiation-pressure–dominated, low-temperature ($T \sim 10^5$ K) solution commonly used to model the UV–soft X-ray continuum of active galactic nuclei (AGNs) (e.g., Sun & Malkan 1986), and the other corresponding to the optically thin, gas-pressure–dominated, high-temperature ($T \gtrsim 10^9$ K) solution used to model the hard X-ray–γ-ray emissions of both AGNs and galactic black holes (Shapiro, Lightman, & Eardley 1976; Eardley et al. 1978; Liang & Thompson 1979). It is well known that the optically thick radiation-pressure–dominated solution is secularly unstable (Lightman & Eardley 1974), while the optically thin hot solution is stable (Piran 1978; Shakura & Sunyaev 1976). In the context of L (luminosity) versus Σ (vertical column density) plots, this is reflected in the fact that L increases with Σ for the hot solution, while L decreases with increasing Σ for the cool solution. It is then interesting to ask what happens when the two solutions cross each other at intermediate Σ (Fig. 1*a* [*inset in Fig. 1*]). This question cannot be answered in the conventional treatments because the radiative cooling equations used are valid only in the two opposite limits.

In this paper we propose a bridging formula for radiative cooling between the optically thick and thin regimes which are valid for *all optical depths*. Using this formula, we reconstruct the general solution to the thin α-disk equations. We find that the optically thin solution makes a smooth transition to the optically thick solution by turning over in the L-Σ plane, after reaching a maximum luminosity L_{max} (Fig. 1*a* [*inset in Fig. 1*]). Above this limiting L_{max} there is no solution, while below L_{max} there are two solutions—one optically thin and hot, the other optically thick and cool. As $\alpha \to 1$, L_{max} can be *substantially below the Eddington luminosity* L_{Edd}. Intuitively, the existence of this limit is due to the impossibility of the combined gas and radiation pressure to satisfy tidal hydrostatic balance at sufficiently high luminosity. We speculate, but have not proved, that above L_{max} the thin α-disk will admit only time-dependent solutions.

2. FORMULA FOR RADIATIVE COOLING AND DISK STRUCTURE EQUATIONS

To be completely general, we will assume in the optically thin limit that the disk cools via Comptonization of thermal bremsstrahlung plus other soft photons. This additional soft photon source will be left as a free parameter, so that it could be internal (e.g., synchrotron in an equipartition field) or external (e.g., blackbody photons from the outer disk; Shapiro et al. 1976). The interpolative formula that we propose here, valid for both the optically thin and thick regimes, is

$$F = \frac{4}{3}\frac{B(T)}{\tau_R}\bigg/\left[1 + (e^{\tau^*} - 1)^{-1}A^{-1}\tau_*^{-1}\left(\frac{F_{Cb}}{F_{Cb} + F_{Cs}}\right)\right], \quad B(T) \equiv \sigma T^4 , \quad (1)$$

where τ_R is the Rosseland mean optical depth ($= \tau_{es}$ in current regimes), F_{Cb} and F_{Cs} are the Comptonized bremsstrahlung and Comptonized soft photon flux, respectively, A is the Compton enhancement factor for thermal bremsstrahlung (Rybicki & Lightman 1979; Dermer, Liang, & Canfield 1991), T is the electron temperature, τ_{es} is the electron scattering depth, and τ_* is the effective absorption depth [$=(\tau_{ff}\,\tau_{es})^{1/2}$, where τ_{ff} is the *Planck mean* free-free optical depth; Rybicki & Lightman 1979]. This formula has three important properties: (i) in the optically thick limit ($\tau_* \gg 1$) it reduces to the diffusion form (Shakura & Sunyaev 1973):

$$F \simeq \tfrac{4}{3}B(T)/\tau_R \equiv F_b ; \quad (2)$$

(ii) in the optically thin limit ($\tau_* \ll 1$) it reduces to (recalling $F_{Cb} = F_b A\tau_*^2$; Wandel & Liang 1991):

$$F \simeq F_B/[1 + F_B/(F_{Cb} + F_{Cs})] \simeq F_{Cb} + F_{Cs} , \quad (3)$$

since F_{Cb}, $F_{Cs} \ll F_B$ for all relevant cases in this limit; hence it reproduces the correct form for the Comptonization of brems-

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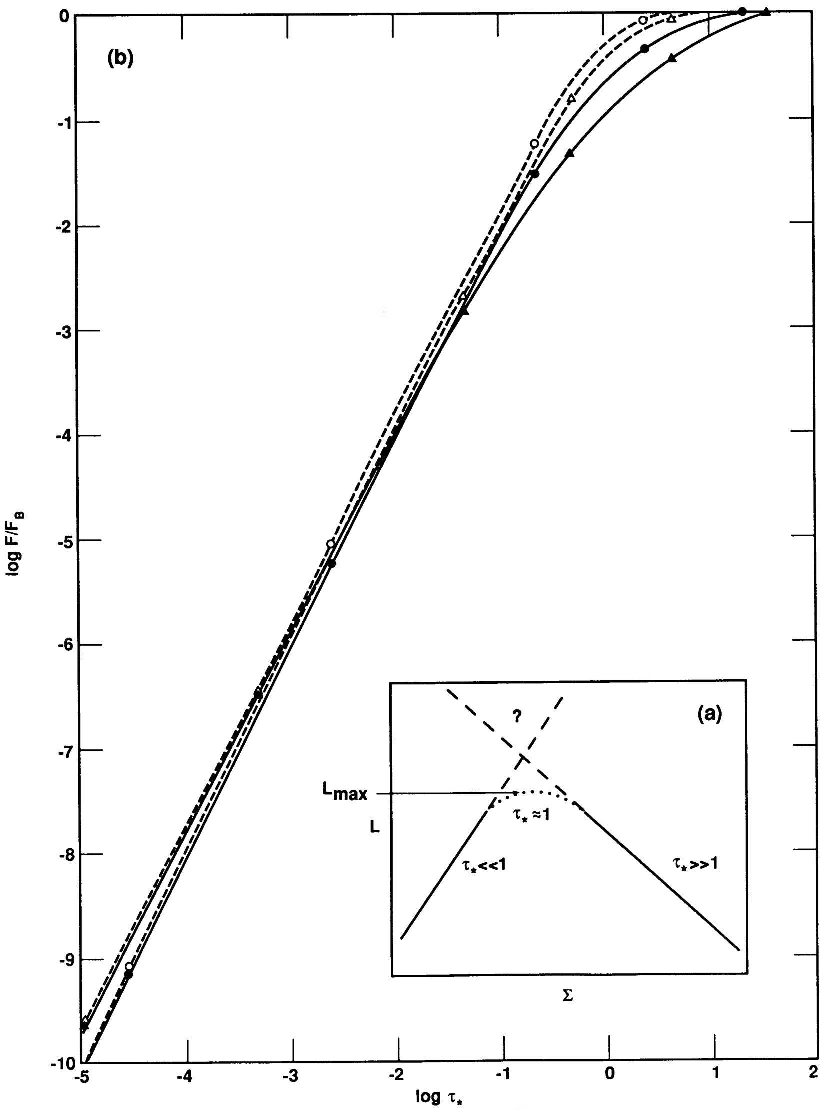

FIG. 1.—(*a* [*inset*]) Sketch of L vs. Σ for α-disk solutions. Dashed curves denote extrapolation of conventional optically thin and thick solutions. Dotted curve denotes correct solution with a turnover at L_{max}. (*b*) Comparison of eq. (1) (*dashed curves*) with sample Monte Carlo transport results (*solid curves*) for $T_e = 10^7$, $\tau_{es} = 1$ (*circles*) and $T_e = 10^8$, $\tau_{es} = 10$ (*triangles*) with $F_s = 0$.

strahlung and soft photons; (iii) it never violates the blackbody diffusion limit $F \le F_B$.

In Figure 1*b* we compare equations (1) with *exact Monte Carlo* results for sample T and τ_{es} (and $F_{Cs} = 0$) over the entire range of τ_*. The worst error in adopting equation (1) appears to be approximately a factor of 4 or less at intermediate τ_*. This translates into errors in the solution for T of less than 40% and negligible effect on L_{max}. Hence, we are confident of the general validity of our conclusions. (When $F_{Cs} = 0$, equation [1] reduces to the formula used in Wandel & Liang 1991.)

There is no simple analytic formula for F_{Cs} valid for all T, τ_{es}, and ν_s (the soft photon frequency). However, for most subrelativistic regimes we can use the formula of Dermer et al. (1991) with an error of ~45% or less:

$$\frac{F_{Cs}}{F_s} = 1 + \frac{1}{\alpha - 1}\left[1 - \left(\frac{h\nu_s}{3kT}\right)^{\alpha - 1}\right], \tag{4}$$

where $\alpha = (9/4 + 4/y)^{1/2} - 3/2$; $y \equiv 4T_* \tau_{es}(\tau_{es} + 2/3)12/\pi^2$ (Sunyaev & Titarchuk 1980), F_s is the soft photon flux, and $T_* \equiv kT/mc^2$. Following popular conventions, we introduce the parameter $\eta \equiv F/F_s$ and use η as a free parameter instead of F_s. Thus, assuming that the soft photons form a blackbody, we can write

$$\frac{h\nu_s}{k} \sim 3T_s = 3\left(\frac{F_s}{\sigma}\right)^{1/4} = 3\left(\frac{F}{\eta\sigma}\right)^{1/4}.$$

To equation (1) we add the other thin disk structure equations:

a) Angular momentum conservation:

$$4\pi h \pi^r_\phi \equiv \dot{M}\omega J, \quad \omega \equiv \left(\frac{GM}{r^3}\right)^{1/2}, \quad J = 1 - \left(\frac{6}{r_*}\right)^{1/2}, \tag{5}$$

where $r_* \equiv rc^2/GM$, M is the mass of the cenral black hole, and $\dot{M}$ is the accretion rate.

b) α-Viscosity model:

$$\pi_\phi^r = \alpha p , \tag{6}$$

where $p = p_{\rm gas} + p_{\rm rad} = \rho k(T_i + T)/m_p + \tau_{\rm es} F/c$; m_p is the proton mass and ρ is the density.

c) Hydrostatic balance:

$$p = \rho h^2 \omega^2 . \tag{7}$$

d) Energy conservation:

$$F = \frac{3}{8\pi} \omega^2 \dot{M} J . \tag{8}$$

e) Electron-ion Coulomb coupling:

$$F = \tfrac{3}{2} \nu_{\rm ei} h \rho k(T_i - T)(1 + T_*^{1/2}) m_p^{-1} , \tag{9}$$

where $\nu_{\rm ei} = 2.4 \times 10^{21} \rho T^{3/2} \ln \Lambda$ is the electron-ion coupling frequency ($\ln \Lambda \approx 20$). In the high-Σ limit, equation (9) automatically drives T_i toward T, and we regain the $T_i = T$ radiation-pressure–dominated solution.

Together these form a closed system of equations for the five unknowns: ρ, T_i, T, h, and F. It is more convenient to use Σ ($= h\rho$) instead of h as an independent variable. After some algebraic manipulations, the disk structure equations can be cast into two coupled equations for the two unknowns T and Σ, which we solve numerically:

$$2T + 2 \times 10^{-3} r_*^{-9/4} L_*^{3/2} \alpha^{-1/2} \Sigma^{-5/2} T^{3/2} (1 + T_*^{1/2})^{-1} = 4.5 \times 10^{14} r_*^{-3/2} L_* \alpha^{-1} \Sigma^{-1} (1 - 3.9 r_*^{-3/4} L_*^{1/2} \alpha^{1/2} \Sigma^{1/2}) , \tag{10}$$

$$1.7 \times 10^{-24} M_8 r_*^3 L_*^{-1} \Sigma^{-1} T^4 = \left[1 + (e^{\tau_*} - 1)^{-1} A^{-1} \tau_*^{-1} \left(\frac{F_{\rm Cb}}{F_{\rm Cb} + F_{\rm Cs}} \right) \right] \tag{11}$$

where

$$\tau_* = 1.7 \times 10^5 M_8^{-1/2} r_*^{-3/8} L_*^{1/4} \alpha^{1/4} \Sigma^{7/4} T^{-7/4} ,$$

$$L_* = JL/10^{46} M_8 , \quad M_8 = M/10^8 \, M_\odot .$$

3. NUMERICAL SOLUTIONS

In Figure 2 we plot T and L against Σ for solutions of three sample values of (α, r) when $F_{\rm Cs} = 0$ (pure Comptonized bremsstrahlung). We see that for $\alpha = 1$ and $r_* = 10\frac{2}{3}$ the upper limit for L is minimum and less than $L_{\rm Edd}$. In Figure 3 we give the solutions for the opposite case, $F_{\rm Cs} \gg F_{\rm Cb}$ (Comptonization of copious soft photons) for $\alpha = 1$, $r_* = 10\frac{2}{3}$, and various values of η. Again we see that there exists an upper limit to L which is less than $L_{\rm Edd}$. These solutions clearly demonstrate that the optically thick and thin solutions connect smoothly at an intermediate Σ and a limiting luminosity $L_{\rm max}$, above which there is no steady thin α-disk solution.

By studying the dependence of $L_{\rm max}$ on α, r (and η in the soft photon case), we see the following:

1. $L_{\rm max}$ is minimum at $r_* = 10\frac{2}{3}$ independent of α.
2. For fixed r, $L_{\rm max}$ is minimum when $\alpha \to 1$.
3. $L_{\rm max}$ decreases with increasing η in the soft photon case.

4. DISCUSSION

The existence of a limiting luminosity $L_{\rm max}$ is obviously related to the α-viscosity assumption. As discussed in Wandel & Liang (1991), there is no such limit in β-viscosity models, in which the azimuthal stress is proportional to gas pressure instead of total pressure. In the context of L versus Σ plots, both the optically thick and the optically thin β-solutions have positive slopes (secularly stable) and do not intersect for all relevant parameters. There are always two distinct solutions.

The α-viscosity law has always been suspect because of its secular instability for radiation-pressure–dominated solutions. It was thought that the way to rescue it was to make a transition to the optically thin hot solution, which is gas-pressure–

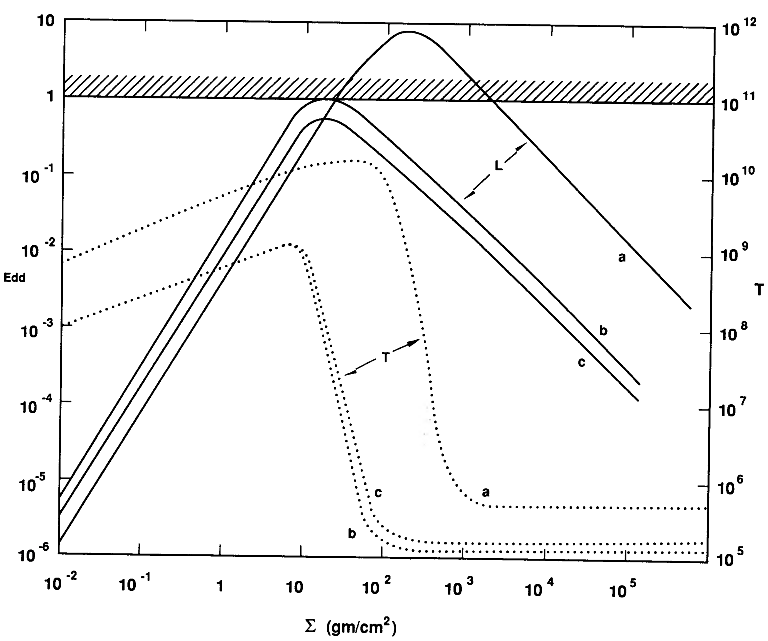

FIG. 2.—Numerical solutions of α-disks for $10^8\, M_\odot$ black holes with $F_s = 0$ for (*a*) $\alpha = 10^{-2}, r_* = 10\frac{2}{3}$; (*b*) $\alpha = 1, r_* = 20$; (*c*) $\alpha = 1, r_* = 10\frac{2}{3}$. Solid curves are for $L/L_{\rm Edd}$ (*left-hand scale*). Dashed curves are for T_e (*right-hand scale*).

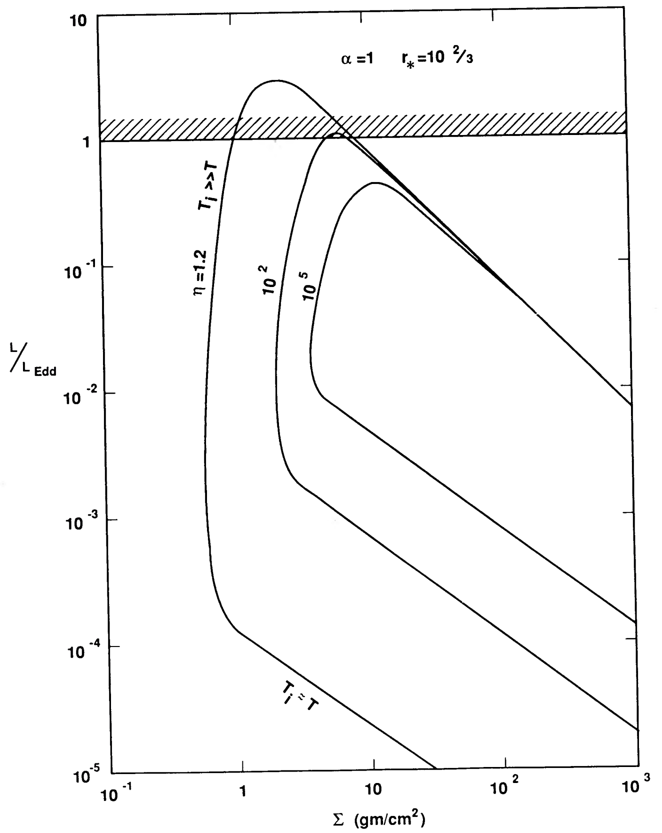

FIG. 3.—Same as Fig. 2, with $F_{Cs} \gg F_{Cb}$ (soft photon case) for $\alpha = 1$, $r_* = 10^{2/3}$, and $\eta = 1.2$, 10^2, and 10^5

dominated and stable (Thorne & Price 1975; Shapiro et al. 1976). But the result of this paper shows clearly that the optically thin hot solution does not exist for sufficiently high accretion rates and high α-values because it requires a column density so high that it automatically turns optically thick and radiation-pressure–dominated, making the original assumption self-inconsistent. What is interesting is that this limit could become sub-Eddington. From the observational point of view, one way to rule out thin α-viscosity Keplerian disks is the confirmation of the existence of steady accretion disks with $L > L_{max}$ (here defined to be the L_{max} evaluated at $r_* = 10^{2/3}$), assuming that we have an independent estimate of the values of α and η, say from temperature and optical depth estimates based on spectral fitting.

Another interesting question is how the optically thick unstable solution evolves into the optically thin solution. Such evolution must pass through a regime of intermediate optical depth. Equation (1) proposed here would be of value in a time-dependent analyses, which we will undertake in a future paper.

Since $T > 10^8$ K near L_{max}, we should ask whether pairs (Liang 1977; Lightman & Band 1981; Svensson 1982) might alter our results. However, recent work on quasi-spherical pair clouds (Liang 1991) finds similar turnover transition between the radiaton-pressure– and gas-pressure–dominated regimes. This suggests that the existence of L_{max} is not sensitive to either the thin Keplerian disk assumption or neglect of pairs. Alternatively, for pair-balanced hot thin disks Kusunose & Takahara (1988) and White & Lightman (1989) find that the pair-deficient branch is little different from the no-pair solution. Hence we believe that inclusion of pair effects will not alter our basic finding here.

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