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THE DISSIPATION OF MAGNETOHYDRODYNAMIC TURBULENCE RESPONSIBLE FOR INTERSTELLAR SCINTILLATION AND THE HEATING OF THE INTERSTELLAR MEDIUM

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ABSTRACT

I reexamine the issue of heating of the interstellar medium via the damping of plasma irregularities responsible for interstellar scintillations of radio sources. The primary innovation of this paper is to consider the effects of recent observational results on the host plasma of the irregularities, as well as developments in the theory of MHD waves. I assume that the host plasma for the density irregularities causing interstellar scintillation is either extended envelopes of bright H II regions which have been revealed by low-frequency recombination lines, or the warm ionized medium of the McKee-Ostriker model. With the plasma characteristics approximate to these regions, one can calculate the rate at which the irregularities damp, and therefore the rate at which energy is input to the interstellar medium. The damping mechanisms I consider are linear Landau damping, ion-neutral collisional damping, nonlinear steepening of wave packets, a parametric decay instability, and nonlinear Landau damping. One of the principal parameters determining the heating rate is the outer scale of the power-law turbulence, a quantity which is subject to observational constraint. The heat input from all damping mechanisms can be accommodated by the region containing the irregularities (the "fluctiferous medium" or "fluctifer") if it has characteristics similar to the extended H II envelopes. Damping of hydromagnetic waves could quite possibly be the dominant heating mechanism for this part of the interstellar medium. Depending on the outer scale of the turbulence, the dominant heating mechanism of this sort in the H II region envelopes is ion-neutral collisional damping, linear Landau damping, or dissipation associated with a parametric decay instability of large-amplitude MHD waves. The damping of turbulence would provide excessive heat to the McKee-Ostriker warm ionized phase, unless the outer scale of the turbulence is greater than about 10^{17} cm. In any case wave dissipation would almost certainly be an important heating mechanism in this phase. The conclusion of this paper is that dissipation of the turbulence responsible for interstellar scintillation need not be a problem for our understanding of the interstellar medium, and that the fluctiferous medium is most plausibly identified with the extended envelopes of H II regions.

Subject headings: hydromagnetics — interstellar: matter — nebuale: H II regions — plasmas — turbulence

Nec quicquam tibi prodest Aerias temptasse—Horace

1. INTRODUCTION

Radio waves propagating through the interstellar medium are scattered and distorted by plasma density fluctuations in a phenomenon referred to as interstellar scintillation (ISS). The density fluctuations doubtlessly arise as a result of compressibility of plasma turbulence in the interstellar medium. This turbulence will dissipate, contributing to the heating of the interstellar medium.

This topic has been considered a number of times in the past, with somewhat surprising results. Previous authors have found that turbulence dissipation processes (1) should prevent a cascade of the density fluctuations to spatial scales of less than 10^{10} cm and (2) produce a heating rate which exceeds the possible cooling rate of the interstellar gas by several orders of magnitude. Both of these conclusions are in contradiction to observed properties of the interstellar medium.

In this paper I reconsider the role of wave dissipation in the heating of the interstellar medium. The motivation for doing so is the availability of new information, both on the turbulence responsible for the scintillation as well as the properties of density fluctuations in MHD turbulence. A major difference between this paper and previous ones (reviewed in § 2.1) is in the attitude toward the spectrum of density fluctuations. Previous authors have used their favorite turbulence model to discuss the existence of the broad spectrum of density fluctuations. I take the position that observations *prove* the existence of a power-law spectrum of density irregularities over a wide range of spatial scales, irrespective of our understanding of how this spectrum arose. We may then explore the consequences of the dissipation and damping of these irregularities. A tacit assumption here is that we better understand the mechanisms for damping of these irregularities than we do the whole process of development of a broad inertial subrange.

An explicit assumption of my analysis will be that the density fluctuations responsible for interstellar scintillations arise because of the compressibility of MHD waves. If this assumption is correct, then interstellar scintillations provide us with an observational handle on these waves, which are conjectured to have a variety of important functions such as the confinement and acceleration of the cosmic rays. In essence, then, I am modeling MHD turbulence as an ensemble of waves. Although this assumption has been made in most previous work (to be cited in § 2.1), it might not be correct. A comparison of turbulence models patterned after hydrodynamic turbulence theories and wave theories is discussed in § 3. In § 4 I nominate the various wave dissipation mechanisms which might be important in the interstellar medium and calculate their contribution to the heat input in the ISM. Both linear and nonlinear mechanisms are considered.

In § 5 I apply these results to the interstellar medium, specifically comparing the various wave heating rates to the cooling rate of interstellar gas. A result of this section is that damping of the irregularities responsible for interstellar scintillation does not constitute a thermodynamic problem if the waves are in the envelopes of H II regions. These regions, being dense, have adequate cooling capacity. However, the same conclusion does not necessarily apply to the warm ionized medium (WIM) of the McKee-Ostriker model. Unless special circumstances are invoked, the probable heat input from wave damping exceeds the cooling capacity of the medium.

2. SUMMARY OF RELEVANT WORK FROM THE LITERATURE

In this section I review published work relevant to the question of wave damping and the heating of the interstellar medium. There are three areas to be considered here, each of which is discussed in a subsection. In § 2.1 I present the customary review of previous work on the topic of ISS damping and the heating of the interstellar medium. Sections 2.2 and 2.3 present recent developments in the areas of interstellar scintillation and hydromagnetic wave properties, respectively, which have motivated the present paper.

2.1. *Previous Discussion of Turbulent Dissipation in the Interstellar Medium*

The two principal works on this subject are over 10 years old. McIvor (1977) discussed the circumstances under which a turbulent cascade could produce density fluctuations on spatial scales ranging from plausible "injection" scales to the short scales responsible for ISS. At that time there was no observational evidence to indicate which phase of the stellar medium might host the irregularities responsible for ISS, so McIvor considered a large number of possibilities. McIvor assumed, as I do, that the density perturbations arise from MHD waves such as Alfvén or magnetosonic waves and modeled the MHD turbulence in the interstellar medium as a collection of such waves. The specific mechanism for the cascade of wave power to smaller scales was a nonlinear, three-wave process. Weak turbulence theory was used to determine a time scale for the transfer of power from a given scale to a smaller one (essentially the growth rate for the three-wave decay process), which is naturally a function of the wavenumber of the parent wave. McIvor explored the plausibility of a broad inertial subrange by determining whether a variety of linear wave-damping processes could terminate the cascade, the condition for this being that the damping rate exceed the cascade rate.

A significant assumption employed in McIvor's (1977) work was that the turbulence consists of an isotropic wave distribution. Since wave-damping processes in general depend on the angle of propagation with respect to the magnetic field, this assumption could result in an overestimate of the wave damping, should it be the case that the waves are lightly damped due to quasi-parallel propagation. As a general result, McIvor found that damping processes tended to halt the cascade at spatial scales far greater than those indicated by ISS observations. It is interesting to note that the only phase of the interstellar medium seemingly capable of maintaining a cascade over a large range of wavenumbers is the hot intercloud phase. This has properties similar to the WIM of the McKee & Ostriker (1977) model of the interstellar medium, and also the diffuse halos of H II regions discussed by Anantharamaiah (1985, 1986). As will be discussed in § 2.2 these media seem the best candidates for the host plasma of the ISS irregularities.

Cesarsky (1980) reported work which independently reached the same conclusion as McIvor (1977), that is, that damping would seem to terminate a cascade on spatial scales far larger than those revealed by ISS. She also noted that Landau damping of the waves would seemingly cause a catastrophic heating of the ISM, in which the energy input rate exceeded the cooling rate by two orders of magnitude. It should be emphasized that this conclusion hinges on identifying the MHD waves causing the density perturbations as magnetosonic waves.

Recently, Ferriere, Zweibel, & Shull (1988) have considered the generation of interstellar turbulence and the irregularities responsible for interstellar scintillation. Their paper is mainly concerned with a theory for the generation of density fluctuations, in this case the production of sonic pulses when supernova remnants impinge on interstellar clouds. Exactly the same process has also been considered by Bykov & Toptygin (1985, 1987). Ferriere et al. consider the possibility that density perturbations arising in this way could produce interstellar scintillations. They come to the conclusion that ion-neutral damping, a process first discussed in this context by Kulsrud & Pearce (1969), and later investigated further by McIvor (1977), would cause the energy input to the interstellar medium to be three orders of magnitude higher than plausible cooling rates. They then proceed to calculate an irregularity spectrum *ab initio*, which is concentrated on very long spatial scales and does not contain such a thermal embarrassment. A subsequent paper by the same authors (Zweibel, Ferriere, & Shull 1988) further discussed the consequences of these calculations for the turbulence causing ISS. They concluded that this turbulence does not reside in the warm neutral phase of the interstellar medium, because wave heating would be too large.

Important contributions to this field have come from the work of Higdon (1984, 1986), who found that the dissipation of the turbulence is greatly reduced if the irregularities consist of highly elongated, field-oriented irregularities, or nonpropagating entropy structures, in analogy with phenomena observed in hydrodynamic turbulence. Higdon's theory differs from that contained in this paper and the previously mentioned works in that he envisions MHD turbulence as more closely resembling fluid turbulence than an ensemble of plasma waves. Further discussion of this matter will be given in § 3.

2.2. *Recent Progress in the Study of Interstellar Scintillation*

In the past decade there has been a great deal of theoretical and observational interest in the subject of interstellar scintillation. A thorough presentation of the state of knowledge as of 1988 is contained in Cordes et al. (1988). The subject of interstellar scintillation is crucial to the present topic because I am using these density perturbations as a "tracer" of the dynamically more important magnetic and velocity fields.

The recent results of most interest to this paper concern the form of the density spectrum and the inner and outer scales which truncate the spectrum. This information allows choice of a probable candidate for the region or phase of the interstellar medium which contains the density irregularities responsible for interstellar scintillation. I shall henceforth refer to the phase of the interstellar medium which contains the irregularities as the "fluctiferous medium," or "fluctifer" in more abbreviated form. The basis for believing that the irregularities responsible for at least heavy radio wave scattering are proper to a single phase of the ISM is the well-observed property that scattering varies markedly from one line of sight to another, indicating that the irregularities are spatially localized. Presumably these "clumps" of intense turbulence can be identified with a unique phase of the ISM.

As discussed in a number of articles in the monograph of Cordes et al. (1988), there is substantial evidence that the spatial power spectrum of the density fluctuations is a power law with index $\simeq 3.67$, the "Kolmogorov" value, that is,

$$P_{\delta n}(k) = C_N^2 k^{-\alpha} , \quad (1)$$

with $\alpha = 3.67$.

Very recent results have presented evidence for an inner scale to the power spectrum given in equation (1), that is, a maximum wavenumber k_i beyond which dissipation or other mechanisms truncate the spectrum. Spangler & Gwinn (1990) and Molnar et al. (1991) use radioastronomical interferometer measurements to show that $k_i \sim 2\pi(50\text{–}200 \text{ km})^{-1}$. Spangler & Gwinn (1990) suggest that this scale is associated with the ion-inertial length in the interstellar medium, defined as $l_i \simeq V_A/\Omega_i$, where V_A is the Alfvén speed and Ω_i is the ion gyro-frequency. The ion-inertial length is a scale on which wave damping and dispersion processes become pronounced.

As discussed in Spangler & Gwinn (1990), the numerical value of the ion-inertial length depends on the plasma properties of a medium; the value of 50–200 km indicates that the fluctiferous medium is either the WIM of the McKee & Ostriker (1977) model of the interstellar medium, or the low-density envelopes of bright H II regions reported by Anantharamaiah (1985, 1986). Evidence for the existence of these envelopes can be seen in the data of Lockman (1976), and Lockman (1980) specifically discussed the possibility of their existence. This result is incompatible with the coronal phase, or the dense, weakly ionized portions of the ISM, being the fluctifer. The relevant physical conditions for the candidate fluctiferous media are given in Table 1. This identification has allowed my discussion of wave damping to be considerably focused compared to that of McIvor (1977), who had to consider many possibilities for the fluctifer, with corresponding diversity in wave-damping properties.

The nature of the H II region envelopes is uncertain. One simple possibility, briefly referred to by Lockman (1980), would follow as a consequence if young stars form on the edges of molecular clouds. On the side facing the molecular cloud, the H II region would be ionization bounded a relatively short distance from the star. This region is identified as the H II region prominent in the optical or radio continuum. In the opposite hemisphere the star would face a medium of lower density, which would consequently be ionized to a greater distance. This latter region could appear to an observer as a "halo" associated with the H II region. This picture is supported by the work of Reynolds & Ogden (1979), who showed that an extensive system of diffuse H α emission and filaments is associated with the I Ori OB association and H II region. Figure 6 of Reynolds & Ogden shows a schematic representation of the spatial distribution of ionized hydrogen in Orion, which might serve as a good model for the physical nature of the "core-halo" H II region structure proposed by Anantharamaiah (1985, 1986).

TABLE 1

CANDIDATE REGIONS FOR THE FLUCTIFEROUS MEDIUM

Region	Density (cm^{-3})	Temperature (K)	Hydrogen Ionization Fraction
H II Region Envelopes	4	10^4	1.0
Warm Ionized Medium	0.25	8000	0.68

There has also been recent progress in determining the outer scale, corresponding to the minimum wavenumber k_0 for which equation (1) is valid. Cordes et al. (1990) have analyzed a number of observations of the pulsar 1937+214 and interpret the results as evidence for the spectrum (1) extending to an outer scale of at least 10^{14} cm. Lower limits only slightly smaller, and obtained by an independent technique, have been reported by Gwinn, Moran, & Reid (1988). Lazio, Spangler, & Cordes (1990) made Faraday rotation observations of several polarized extragalactic radio sources viewed through the Galactic plane in the Cygnus region where the magnitude of scattering is well measured. Their results may be interpreted as evidence that the Kolmogorov spectrum extends to scales of 10^{17}–10^{18} cm. This interpretation is not entirely without qualification, and the Cordes et al. results refers to one line of sight which is only lightly to moderately scattered, so the issue cannot be considered closed. However, these results provide at least an initial observational basis for constraining the outer scale of the turbulence.

A property of the interstellar turbulence which one would obviously like to extract from scintillation observations is its amplitude, parameterized by σ_n, the rms density fluctuation, the modulation index σ_n/n_0, where n_0 is the mean density, or of more dynamical importance, the amplitude of the wave fluctuations in $\boldsymbol{b}$ and $\boldsymbol{v}$, the magnetic field and plasma velocity. Unfortunately, in spite of the increasing number of lines of sight for which scattering measurements are available, we cannot unambiguously infer the amplitude of the turbulence.

The reason for the difficulty is as follows. Scintillation observations measure the "scattering measure" SM $\equiv C_N^2 Z$, where Z is the path length through the turbulent medium. Thus to infer C_N^2 we must have a good estimate of Z, which usually is unavailable. Furthermore, even C_N^2 does not directly give the amplitude of the turbulence; being the normalization constant of the density power spectrum (eq. [1]), it is related to σ_n via the outer scale of the turbulence, ℓ_0. As may be deduced from the preceding paragraphs, information on outer scales in the interstellar turbulence is recent, somewhat tentative, and confined to a few lines of sight. The following remarks will illustrate the limitations to our knowledge of the amplitude of ISM turbulence, as well as provide the best estimates currently available.

Spangler et al. (1986) give the following relation between the observed angular diameter due to interstellar scattering θ_1 and characteristics of the turbulence,

$$\theta_1 = \frac{8.0 \times 10^7 \sigma_n^{1.2} Z_{\rm pc}^{0.6}}{\ell_0^{0.4}} \text{ mas} , \quad (2)$$

where θ_1 is defined at a fiducial frequency of 1 GHz, σ_n is the rms density fluctuation in cm^{-3}, $Z_{\rm pc}$ is the path length through the turbulent medium in parsecs, and ℓ_0 is in cm. Equation (2) assumes a Kolmogorov spectrum.

Equation (2) illustrates the difficulty in determining the amplitude of the turbulence; the observable quantity θ_1 is determined by σ_n, Z_{pc}, and ℓ_0. The situation is rendered even more difficult if the modulation index σ_n/n_0 is considered the fundamental measure of turbulent amplitude, for in this case independent information is required for n_0.

Perhaps the best prospects for determining σ_n and σ_n/n_0 come from observations in the Cygnus region, where scattering measurements have been made for many lines of sight (Fey, Spangler, and Mutel 1989; Spangler & Cordes 1991). The scattered sources have a wide range of θ_1; a typical value of 50 mas may be assumed. The observations of Fey et al. (1989) and Spangler & Cordes (1991) show that the magnitude of the scattering changes drastically on angular scales of 1° or less. Identifying this as the angular scale of fluctiferous media, and assuming that these regions are associated with the Cygnus OB1 and OB2 associations at a distance of about 2 kpc, we obtain an estimate of $Z_{pc} \lesssim 30$ pc.

Finally, the Lazio (1990) Faraday rotation measurements refer to this region, so we adopt an outer scale $\ell_0 = 10^{18}$ cm. This leads to an estimate of $\sigma_n \gtrsim 1.2$ cm^{-3}. This value for σ_n is obviously a rough estimate; there is considerable variability in scattering, and the estimate for Z_{pc} is quite crude.

Referring to Table 1, and the estimates for the mean density of the fluctiferous media, it may be seen that σ_n exceeds our estimate for the mean density of the warm ionized medium, and $\sigma_n/n_0 \sim 0.25$ for the H II region envelopes. In either case, the amplitude of the density turbulence can be considered large. However, it should be emphasized that this large modulation of the density occurs on large spatial scales characteristic of the outer scale. On the much smaller spatial scales responsible for ISS, the fluctuations in density are considerably less than the mean value.

2.3. *The Density-Magnetic Field Relationship in Magnetohydrodynamic Waves*

The final topic to be discussed in this section is recent research which has increased our understanding of the relationship between density fluctuations and the magnetohydrodynamic wave fields producing these fluctuations. Interstellar scintillation is caused by density fluctuations which are essentially a "tracer" of the magnetic and velocity fields in the wave. The evolution of the waves is determined by the characteristics of the velocity $\boldsymbol{v}$ and magnetic field $\boldsymbol{b}$. To correctly describe the evolution of the waves, and their subsequent dissipation, we need to understand the processes of wave compressibility, so that we may properly "invert" the ISS data.

We have recently made a number of studies of the compressibility of MHD waves which might form the turbulence in the interstellar medium. Theoretical investigations (Spangler 1987, 1989) have discussed the types of compressive mechanisms and the relationship between wave magnetic field and density compression. We have also availed ourselves of the availability of in situ spacecraft observations of MHD waves near the Earth's bow shock (Spangler et al. 1988; Leckband & Spangler 1989, 1991. Based on these studies, we have a good idea of the relationship between wave characteristics and density compression, given the reasonable assumption that interstellar MHD waves have the same properties as those found in the solar wind. The mechanisms which produce density compression by MHD waves are described below.

2.3.1. *Oblique Propagation*

It is well known that obliquely propagating fast magnetosonic waves produce density fluctuations with a wavelength equal to that of the magnetic and velocity field of the wave. For small angles of propagation, the density compression is given by

$$\frac{\delta n}{n_0} = \tau \frac{b}{B_0} \sin \theta , \tag{3}$$

where δn is the density fluctuation, n_0 is the mean plasma density, b is the amplitude of the wave, B_0 is the magnitude of the large-scale field along which the wave propagates, and θ is the angle between the direction of propagation of the wave and the large-scale field. The coefficient τ is of order unity, determined by the properties of the unperturbed plasma. Associated with the density compression is Landau damping (Barnes 1966), the basis of the dissipation mechanism discussed by Cesarsky (1980).

This mechanism for density compression is generally seen in the waves near the Earth's bow shock, but it is not the dominant contributor to the density fluctuations. Furthermore, conditions near the bow shock are conductive to the generation of fast mode waves. This need not be the case in the ISM.

2.3.2. *Ponderomotive Density Perturbations*

If the MHD wave packets are spatially modulated, gradients in wave energy density produce ponderomotive forces which engender plasma flows and density compressions. These ponderomotive flows lie at the heart of the MHD wave nonlinearity which produces wave packet steepening, to be described below. The general relationship between wave packet energy density and plasma density, including plasma kinematic effects, is discussed by Spangler (1989). There will be enhanced damping associated with these density variations, but the rate, to be discussed in § 4.2, is far less than that of the linear Landau damping mentioned above.

2.3.3. *Parametric Decay Instability*

Nonlinear MHD waves are subject to a parametric decay instability, in which an Alfvén or fast mode wave transforms into a forward-propagating ion-acoustic wave and weak MHD wave, and a backward-propagating MHD wave. This mechanism will be considered further below, where it will be discussed as a damping and heating mechanism. For the moment I point out that it also provides a potential way of generating density fluctuations via the daughter ion-acoustic wave. Leckband & Spangler (1989) have found marginal evidence for this process as a sporadic phenomenon in the vicinity of the Earth's bow shock. However, it makes a negligible contribution to the wave-induced density fluctuations there.

2.3.4. *Transverse Modulation of Wave Packets*

Leckband & Spangler (1989, 1991) have found that the principal relationship between density fluctuations and wave field characteristics in the vicinity of the Earth's bow shock is between δn and b_x, the *fluctuating* component of the field in the direction of the mean field. Leckband and Spangler favor the interpretation that these density fluctuations arise as a result of transverse (to the static magnetic field) spatial modulation of the wave packets. The occurrence of density perturbations which are proportional to b_x arises directly from the theory of Mjølhus & Wyller (1988) for three-dimensional wave packets. The magnitude of these fluctuations indicates that the crossfield modulation scale for the waves upstream of the bow shock is about the same as that along the magnetic field.

A few other theoretically possible compression mechanisms are discussed in Spangler et al. (1988); none of them are observationally significant. The consequences of the above studies for my model of the interstellar medium are as follows. Large-amplitude magnetohydrodynamic waves produce density fluctuations on the same scale as the wavelength of the wave itself or the size of the wave packets. These density fluctuations can be of large amplitude; in the foreshock region where the wave magnetic field amplitude is typically between 35% and 100% of the static field, the standard deviation of the density ranges from 10% to 20% of the mean density. The functional proportionality between δn and b, the amplitude of the wave field, differs with the various mechanisms; for ponderomotive effects it is roughly $\delta n \propto b^2$, whereas for oblique propagation and transverse modulation $\delta n \propto b$. The case for the decay instability is not so clear but would probably be characterized by a quadratic dependence of the density perturbation on wave amplitude.

There have also been substantial recent developments in understanding processes of evolution of large-amplitude magnetohydrodynamic waves. These are discussed in § 4.

3. A MODEL FOR THE INTERSTELLAR MHD TURBULENCE

The fluctiferous medium is a plasma, and the very existence of interstellar scintillations indicates that we are dealing with a turbulent plasma. Dissipation of the ISS irregularities is a basic dynamical process of the turbulence and requires a theoretical model of plasma turbulence. In this paper, when I use the term "plasma turbulence," it is to be understood that I am concerned with scales much larger than the ion-inertial length, that is, the MHD limit. I shall therefore also use the term "MHD turbulence."

Two viewpoints have arisen regarding the nature of plasma turbulence. According to one, the irregularities are modeled as a superposition of waves from linear theory, such as Alfvén and magnetosonic. Weak nonlinearities cause coupling between these waves, and phenomena such as cascades in wavenumber space occur (Nicholson 1983). Recent work in this area has incorporated large-amplitude versions of the linear waves as the basic theoretical units. This has led to descriptions of MHD turbulence as a "soliton gas" (Dawson & Fontan 1990).

An alternative viewpoint is that MHD turbulence can be considered a generalization of Kolmogorov-style fluid turbulence (Kraichnan & Montgomery 1980). Extreme exponents of this school contend that the waves from linear theory have little to do with fluctuations in real plasmas. An excellent and highly readable discussion of these points appears in an unfortunately obscure source, the report of a study group for a plasma turbulence explorer satellite (Montgomery 1980).

Guidance in this matter can be obtained by consultation of solar wind observations. I feel that strong evidence for *both* viewpoints can be found therein. Power spectra of quantities such as velocity, vector magnetic field, and density in the solar wind show power-law behavior over many decades, certainly suggestive of processes analogous to those operative in Kolmogorov-style fluid turbulence. On the other hand, there is absolutely no question that the fluctuations observed near planetary shocks are large-amplitude waves (e.g., Hoppe & Russell 1983). It has furthermore been known for some time that strong waves are seen near high-speed streams in the solar wind (Belcher & Davis 1971). A high correlation between the plasma velocity V and the magnetic field $\boldsymbol{B}$ is seen to persist up to large-spatial scales (e.g., Bruno, Bavassano, and Villante 1985), properties which suggest that the irregularities can be considered Alfvén waves. In summary, one can argue that the evidence favors aspects of both models in real solar wind turbulence.

The wave models allow consideration of one aspect of the turbulence which is almost certain to be important in the evolution of turbulence and is central to the topic of this paper, that is the nature of the damping processes. Fluid models for MHD turbulence have all dissipation occurring at the shortest spatial scales, and via an ad hoc viscous damping term. Dissipation in the ISM can also proceed through collisionless processes, and in general would occur throughout the spectral domain. An additional important point is that some of the most voracious dissipation mechanisms are likely to be ones involving wave-particle resonances, such as the Landau resonance. The incorporation of kinetic effects in plasma fluid phenomena is at an early stage, and to date hydroturbulence theories have not attempted such a description of kinetic effects. In summary, describing MHD turbulence in the interstellar medium in terms of large-amplitude waves, evolving subject to nonlinear as well as linear processes, is a worthwhile undertaking.

As mentioned above, ISS observations show that the interstellar density spectrum is of the Kolmogorov form given in equation (1). I now adopt a similar model for the interstellar magnetic power spectrum, describing the statistical properties of the wave magnetic fields:

$$P_B(k) = \frac{C_B^2}{[k_x^2 + \Lambda^2(k_y^2 + k_z^2)]^{[1+(s/2)]}} . \quad (4)$$

In equation (4) k_x, k_y, and k_z are spatial wavenumbers in the X-, Y-, and Z-directions, in which the X-axis is defined by the large-scale magnetic field. The quantity Λ is an anisotropy parameter. For $\Lambda > 1$ I am describing pancake-shaped wave packets propagating along the large-scale fields. If $\Lambda = 1$, one recovers a power spectrum very similar to (1), $P_B(k) = C_B^2 k^{-(s+2)}$. The coefficient C_B^2, in analogy with C_N^2 in equation (1), determines the intensity of the magnetic turbulence. Finally, values of s of 3/2 or 5/3 seem most likely on the basis of theory and solar wind observation. As in the case of equation (1) the spectrum (4) is truncated at the low wavenumber at $k_0 = 2\pi/\ell_0$, where ℓ_0 is the outer scale, and the high-wavenumber end at $k_i = 2\pi/\ell_i$, where ℓ_i is the inner scale. I assume these scales to be the same as those for the density turbulence.

I have chosen the form of the power spectrum (4) because I will be using the nearly parallel-propagating wave packets observed in the solar wind as an analogue for interstellar MHD turbulence and utilizing theoretical results for one-dimensional ($\Lambda \to \infty$) nonlinear wave packets. It seems reasonable to assume that these theoretical results will retain approximate quantitative validity for Λ greater than unity, but not necessarily by a large factor. Under this assumption, the only important component of the wavenumber is k_x, the wavenumber of the one-dimensional wave packets, which I take to be in the direction of the large-scale magnetic, or nearly so. In this case, the spectrum of the waves is described by the reduced power spectrum $\bar{P}_B(k_x)$, which is obtained by integrating equation (4) over k_y and k_z:

$$\bar{P}_B(k_x) = \frac{2\pi C_B^2}{s\Lambda^2} k_x^{-s} . \quad (5)$$

The core of this paper is the discussion, in § 4, of the damping rate γ_d for various wave-damping processes. For all dissipation processes this damping rate is a function of the wavenumber k_x and the properties of the plasma in which the wave is propagating. For nonlinear processes γ_d is also determined by wave properties such as the amplitude and modulation length. For linear processes γ_d is, as usual, the imaginary part of the wave frequency. For the nonlinear dissipation mechanisms, γ_d will be the reciprocal of a time scale for nonlinear development leading to dissipation, or the growth rate of a "modulational instability" in which a perturbed wave packet self-modulates. There is reason to believe that the outcome of this self-modulation will be dissipation. The details for each nonlinear mechanism will be discussed in § 4.

For all damping processes, linear and nonlinear, the energy dissipation rate, the quantity of interest, will be given by

$$\epsilon = \frac{1}{4\pi} \int_{k_0}^{k_i} dk\, 2\gamma_d \bar{P}_B(k) . \quad (6)$$

This energy dissipation rate is equal to the volumetric heating rate. Use of the reduced spectrum $\bar{P}_B(k)$ indicates that I am modeling the MHD turbulence as parallel-propagating or nearly parallel-propagating waves. The factor of $1/4\pi$ accounts for the conversion from magnetic variance to magnetic energy density, and a multiplicative factor of 2 to account for the fact that in a parallel-propagating MHD wave, half the energy is carried by plasma motion and half by the wave magnetic field.

The theory of the three nonlinear processes to be discussed in § 4, wave packet steepening, parametric decay instability, and nonlinear Landau damping, has largely been developed for narrow-band wave packets; computer simulations have studied waves with narrow spectra, and reference to observations has been mainly concerned with the quasi-periodic waves upstream of the Earth's bow shock. In contrast, the observational evidence indicates that magnetic irregularities in the interstellar medium have a power-law spectrum, as is the case for the interplanetary medium. The question then arises as to how we apply the theory for processes dependent on wave amplitude b_A, wavenumber k, and wave packet modulation scale l to a wave field whose power spectrum is given by equation (5).

My method of representation will be referred to as the *wave packet approximation*, and it is illustrated graphically in Figure 1. The top panel displays the reduced power spectrum of magnetic irregularities (eq. [5]), defined on the wavenumber interval between k_0 and k_i.

To make the connection between a power-law spectrum and an essemble of wave packets, take a portion of the spectrum with central wavenumber k and spectral width Δk, as indicated in Figure 1. In the spatial domain, this spectral segment may be represented by a set of wave packets, of mean amplitude b_A, wavelength λ, and modulation scale l, as shown in the lower portion of Figure 1. The evolution of these wave packets in isolation can be described by the linear and nonlinear mechanisms mentioned above and to be described in detail below. The essence of the wave packet approximation is that the entire power-law spectrum between wavenumber limits k_0 and k_i can be represented as a superposition of such sets of wave packets. It is assumed that the subsequent evolution of the turbulence is the result of the self-evolution of each wave packet, and that interactions between wave packets are ignored.

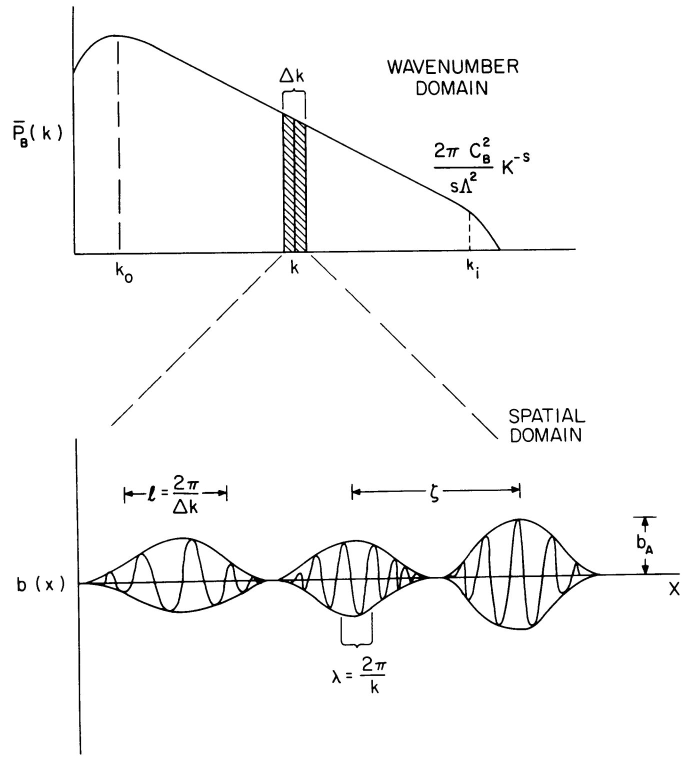

FIG. 1.—Graphical representation of the wave packet approximation of representing power-law magnetic irregularities in terms of a set of wave packets. Essentially each segment of the power-law spectrum is represented by a set of similar, amplitude-modulated wave packets. This model is not intended to provide a rigorous description of irregularities with a power-law spectrum, but rather to facilitate calculation of wave dissipation on a range of spatial scales.

The wave packet approximation is admittedly a crude representation of a power-law spectrum, and the approximation is deserving of some healthy skepticism. A realization of noise taken from a power-law spectrum like equation (5) does not resemble a train of distinct wave packets, nor does the modulation length l emerge naturally from such a spectrum; it must be imposed as an ad hoc parameter. A far better approach would be to undertake a study of MHD wave steepening and parametric decay for a power-law spectrum and to apply the results to the issue of ISM heating.

In the absence of such an investigation, I will utilize the wave packet approximation, for which the following supporting arguments can be made. First, the noise emergent from a power-law spectrum shows fluctuations and modulations on all length scales. Some of these fluctuations roughly resemble wave packets, for which steepening and decay might exist as important physical processes. Thus even though power-law noise does not strictly resemble the idealized wave packets shown in Figure 1, the net results as far as wave steepening and spectral transfer of wave energy might not be greatly different. This contention is weakly supported by numerical investigations of Spangler (1985), who found that band-limited noise evolved in the same manner as modulated wave packets.

However, this study was concerned with noise possessing a Gaussian spatial power spectrum, so the results cannot be directly applied to a current situation of a power-law spectrum. Second, it is possible that, although the density and magnetic field spectra are power laws for some global average, locally the waves are more narrow band and thus better represented as wave packets. In this case, the existing theory for nonlinear evolution of wave packets would be entirely adequate for calculating the local heating of the interstellar medium. A disadvantage of this hypothesis is that it proposes a situation which is not similar to that in the solar wind, so one would not expect it to be appropriate for the interstellar medium. However, locally narrow-band waves do occur near the Earth's bow shock.

Third and finally, it seems likely that there are places in the interstellar medium where large-amplitude waves are generated on spatial scales much smaller than those discussed by Cordes et al. (1990) and Lazio et al. (1990). The power spectra in this case need not be power-law, and the wave packet approximation might be quite good. It is of interest to know the local heating rate in such hypothesized regions.

A final comment which should be made in reference to the wave packet model is that it is used exclusively for the calculation of the nonlinear dissipation processes. This is because these mechanisms (or more accurately the present theoretical descriptions of them) depend on the amplitude and modulation scale of waves with wavenumber k. The wave packet approximation is a method of extracting estimates of these quantities, given the basic assumption of a power-law spectrum for the magnetic irregularities.

With this introduction giving a description of the physical content of the wave packet approximation, and a recognition of its shortcomings, I will now develop its mathematical framework. The goal is to obtain expressions for the modulation scale l and wave packet amplitude b_A given a power-law spectrum. By the definition of a spatial power spectrum, an integral over wavenumber equals an expectation value of a square of a wave magnetic field,

$$\left\langle b^2\left(l = \frac{2\pi}{k}\right)\right\rangle = \frac{1}{L}\int_0^L dx\, b^2(x) = \int_{k-\Delta k/2}^{k+\Delta k/2} dk\, \bar{P}_B(k) , \quad (7)$$

where it is to be understood that k signifies k_x. In terms of wave packets, we have

$$\frac{1}{L}\int_0^L dx\, b^2(x) = \frac{N}{L}\int_{\text{packet}} dx\, b^2(x) ,$$

where the integral is over a wave packet, and N is the number of wave packets in an interval of length L. The number of such wave packets is $N = L/\xi$, where ξ is the mean spacing between wave packets as indicated in Figure 1.

For models of the wave packets, I adopt a simple circularly polarized wave with a Gaussian modulating function,

$$b^2(x) = b_A^2 \exp\left(\frac{-2x^2}{l^2}\right) , \quad (8)$$

where l is the modulation length and b_A is the amplitude of the wave packet. Using equations (7) and (8), we then have

$$\left\langle b^2\left(l = \frac{2\pi}{k}\right)\right\rangle = \sqrt{\frac{\pi}{2}}\left(\frac{l}{\xi}\right) b_A^2 . \quad (9)$$

Finally, this expression is equated with that obtained from the power spectrum of irregularities. Substitute equation (5) into equation (7), and we obtain

$$\left\langle b^2\left(l = \frac{2\pi}{k}\right)\right\rangle = \frac{2\pi C_B^2}{s(s-1)\Lambda^2}\, \delta(s, \eta) k^{-(s-1)} , \quad (10)$$

where η is the fractional bandwidth of the wave packet $\eta = \Delta k/k$, and

$$\delta(s, \eta) \equiv \frac{1}{[1 - (\eta/2)]^{s-1}} - \frac{1}{[1 + (\eta/2)]^{s-1}} . \quad (11)$$

Equating equations (9) and (10), we have a relation between the model wave packet intensity and properties of the power-law turbulence,

$$b_A^2 = \frac{\delta(s, \eta)}{R_l}\, b_T^2 \left(\frac{k}{k_0}\right)^{-(s-1)} , \quad (12)$$

where $(\pi/2)^{1/2}(l/\xi)$ is assumed to be independent of the wavenumber k, and b_T^2 is the turbulent magnetic field variance, obtained by integrating the spectrum (4) over all wavenumbers. Note that the anisotropy parameter Λ no longer appears in the expression.

Equation (12) (together with the identification of modulation scale l with $2\pi/\Delta k$) provides the basis of the wave packet approximation. Equation (12) clearly demonstrates how the wave packet amplitudes decline with decreasing scale size, given that a power-law magnetic irregularity spectrum is adopted.

4. MHD WAVE DISSIPATION PROCESSES

In this section I discuss five wave dissipation processes which I feel are most likely to be of importance in the WIM or H II region envelopes, regions which are most probably the fluctiferous media. For each of the mechanisms I will describe the basic physical process at work, discuss the conditions under which this process will be operative, and then give a simple estimate of the heating rate under interstellar conditions. I separately discuss linear and nonlinear damping processes. The expository discussion for the linear process will be abbreviated, since these mechanisms have been discussed at length in the previous literature. By contrast, the nonlinear processes discussed in § 4.2 have never, to my knowledge, been considered in the context of the interstellar medium.

It will become apparent in the discussions that each of the damping mechanisms is determined by one or more variables which are poorly known in the interstellar medium. In general, one can specify these heating rates to logarithmic accuracy only.

4.1. *Linear Processes for Wave Damping*

The previous work by McIvor (1977), Cesarsky (1980), and others has identified two processes which are of primary importance in the interstellar medium, linear Landau damping and ion-neutral collisional damping. McIvor also summarizes a number of other essentially collisional processes, such as damping due to viscosity and thermal conductivity. For conditions appropriate to the envelopes of H II regions or the WIM, these dissipation mechanisms are weaker than, or at most comparable to, ion-neutral collisional damping.

4.1.1. *Linear Landau Damping*

An obliquely propagating fast magnetosonic wave will produce density perturbations, as discussed in § 2.3. Corresponding to these density perturbations is an electric field with which ions can resonantly interact. The resonant interaction leads to damping of the wave with a damping rate (Ginzburg 1961)

$$\gamma_d = \sqrt{\frac{\pi}{8}}\, V_A k \frac{\sin^2\theta}{\cos\theta} \times \left[\left(\frac{v_{\theta i}}{V_A}\right)^2\left(\frac{V_A}{v_{\theta e}}\right) + 5\left(\frac{v_{\theta i}}{V_A}\right)\exp\left(-\frac{V_A^2}{2v_{\theta i}^2\cos^2\theta}\right)\right]. \quad (13)$$

In equation (13), V_A is the Alfvén speed, $v_{\theta i}$ is the ion-thermal speed, $v_{\theta e}$ is the electron thermal speed, and θ is the angle between the direction of wave propagation and the static magnetic field. The linear Landau damping rate is most crucially dependent on θ and the ratio $V_A/v_{\theta i}$, which is directly related to the standard plasma β.

Defining $F(\beta)$ as the expression in square brackets in equation (13), and making the assumption that $\theta \ll 1$, equation (13) simplifies to

$$\gamma_d = \sqrt{\frac{\pi}{8}}\, V_A F(\beta)\theta^2 k\ . \quad (14)$$

Substituting equation (14) into equation (6), we have

$$\epsilon_{LLD} = 1.2 \times 10^{-20}\theta^2 F(\beta) V'_A b_T'^2 (\ell'_i \ell_0'^2)^{-1/3} \text{ ergs s}^{-1}\text{ cm}^{-3} \quad (15)$$

for $s = 5/3$. For $s = 3/2$, the numerical coefficient is about an order of magnitude higher, and the dependence on the normalized inner and outer scale is proportional to $(\ell'_i \ell'_0)^{-1/2}$. Certain of the variables introduced in equation (15) will be similarly defined in the remainder of the paper. The dimensionless inner and outer scales ℓ'_i and ℓ'_0 have been normalized by 10^8 and 10^{17} cm, respectively. The normalized Alfvén speed V'_A is in units of 10^6 cm s^{-1}, and the normalized rms magnetic field fluctuation b'_T is in units of 10^{-6} G. In considering the amplitude of the magnetic turbulence, the rms (turbulent) field should be compared with the large-scale interstellar field B_0. I adopt a value of 3×10^{-6} G for B_0, and this adoption affects certain of the normalization constants to appear below.

The dependence of the linear Landau damping rate on θ and $F(\beta)$ should be particularly noted. If the interstellar MHD waves are either Alfvén waves or parallel-propagating fast mode waves, the linear Landau damping will be identically zero. The damping rate (15) also depends strongly on the value of the plasma β. In a low-β plasma linear Landau damping is greatly diminished. In conclusion, one can easily imagine circumstances under which the heating rate (15) could be lowered by orders of magnitude.

4.1.2. *Ion-Neutral Collisional Damping*

This wave-damping mechanism was discussed by Kulsrud & Pearce (1969) and McIvor (1977). MHD waves are plasma modes, so neutral atoms are unaffected by the wave electric and magnetic fields. There is a plasma motion associated with the wave, so the ionized component flows through the neutral component of the plasma. Collisions between the ions and the neutral atoms will transfer wave energy to neutral thermal energy and thus damp the wave. This form of damping was considered by Kulsrud and Pearce, who found that the damping rate depends on the ion-neutral collision frequency ν_0.

If the wave frequency is considerably in excess of the collision frequency, then the damping rate is

$$\gamma_d = \frac{\nu_0}{2}\ . \quad (16)$$

If, however, the wave frequency is much less than the collision frequency, the expression is (McIvor 1977, in a modification to the results of Kulsrud & Pearce 1969)

$$\gamma_d = \frac{V_A^2 k^2}{2\nu_0(1+g)^2}\ , \quad (17)$$

where $g \equiv \rho_i/\rho_n$, where $\rho_{i,n}$ represents the mass density of the ionized and neutral species, respectively.

Corresponding to the collision frequency is a critical wavenumber $k_c \equiv (1+g)\nu_0/V_A$; it is the wavenumber of an Alfvén wave with a frequency roughly equal to the collision frequency. Substituting equations (16) and (17) into equation (6), we have the following expression for the heating due to ion-neutral damping

$$\epsilon = \frac{1}{4\pi}\int_{k_0}^{k_c} dk\, \frac{V_A^2 k^2}{\nu_0(1+g)^2}\, \bar{P}_B(k) + \frac{\nu_0}{4\pi}\int_{k_c}^{k_i} dk\, \bar{P}_B(k)\ . \quad (18)$$

The two terms in the sum represent damping below and above the critical wavenumber, respectively. The expression (18) assumes $k_0 < k_c$; if this is not the case, only the second term remains. Equation (18) is an approximation in that the damping rates (16) and (17) are employed as valid in the wavenumber ranges $k \geq k_c$ and $k \leq k_c$, respectively, whereas these formulae are actually asymptotic expressions, valid for $k \gg k_c$ and $k \ll k_c$.

Evaluation of equation (18) yields the following expression for the energy dissipation rate

$$\epsilon \simeq \left(\frac{1}{4\pi}\right)\nu_0 b_T^2 \left(\frac{k_0}{k_c}\right)^{s-1} \quad (19)$$

if $k_0 < k_c$, and

$$\epsilon = \left(\frac{1}{4\pi}\right)\nu_0 b_T^2$$

if $k_0 \geq k_c$.

The energy input rate for this process is therefore directly proportional to the ion-neutral collision frequency.

The value of ν_0 depends on the chemical composition and ionization structure of the fluctiferous medium. First, consider the envelopes of the H II regions as the fluctifer, and assume totally ionized hydrogen and completely neutral helium. In this case, ν_0 is the proton-neutral helium collision frequency. Dalgarno & Dickinson (1968) present theoretical calculations of ν_{12}, the H^+–He collision frequency per unit density of helium. Although they present values for this quantity for temperatures only up to 3000 K, far less than the probable temperature of the fluctifer, ν_{12} shows a weak dependence on temperature in the range. I adopt a value of $\nu_{12} = 1.6 \times 10^{-9}$ cm^3 s^{-1}. To obtain ν_0, ν_{12} is multiplied by the density of helium. Assuming standard cosmic abundance of helium, the collision frequency is

$$\nu_0 = 1.3 \times 10^{-10} n \text{ s}^{-1}\ , \quad (20)$$

where n is the number density of hydrogen in the fluctifer. Substitution of equation (20) into (19) gives

$$\epsilon_{\rm IN} = 1.0 \times 10^{-23} n b_T'^2 \left(\frac{k_0}{k_c}\right)^{s-1} \quad k_0 < k_c$$
$$= 1.0 \times 10^{-23} n b_T'^2 \, , \quad k_0 > k_c \quad \text{ergs s}^{-1}\ \text{cm}^{-3} \, . \quad (21)$$

For the case of the WIM, the McKee-Ostriker model posits only partial hydrogen ionization, so proton-hydrogen collisions are the dominant collisional process. With characteristics of the WIM as presented by McKee & Ostriker (1977), that is, temperature of 8000 K, hydrogen density of 0.25, and fractional ionization of 0.68, equation (8) of McIvor (1977) may be used to calculate the H^+–H I collision frequency, which is $\nu_0 = 1.7 \times 10^{-9}\ s^{-1}$. Substitution of this collision frequency into equation (19) gives

$$\epsilon_{\rm IN} = 1.3 \times 10^{-22} b_T'^2 \left(\frac{k_0}{k_c}\right)^{s-1} \quad k_0 < k_c$$
$$= 1.3 \times 10^{-22} b_T'^2 \, , \quad k_0 > k_c \quad \text{ergs s}^{-1}\ \text{cm}^{-3} \, . \quad (22)$$

Comparison of equations (21) and (22) shows little difference (~factor of 3) in the ion-neutral collisional heating of the WIM and H II envelopes when allowance is made for the difference in density of the two regions. This difference is rendered even less significant by the realization that the adopted values of density for the WIM and H II envelopes are approximate. This conclusion is clearly predicted on my assumption that the hydrogen in the H II region envelopes is nearly totally ionized. If a significant fraction is neutral, ν_0 would be higher, with a corresponding increase in $\epsilon_{\rm IN}$. For both media, the scale corresponding to the critical wavenumber k_c is about 10^{15} cm.

4.2. *Nonlinear Dissipation Processes*

The linear mechanisms discussed above have been considered by previous authors, albeit without an identification of the fluctiferous medium which we now possess. By way of contrast, the processes to be described now have not been previously discussed in the context of the interstellar medium. An understanding of these processes has emerged as the result of analytic and numerical studies of nonlinear MHD waves over the past decade, and support of those theories by observations of MHD waves in the solar wind.

Nonlinear processes can broadly be defined as those in which the damping rate γ_d depends on the amplitude of the wave, b_A, as well as on the modulation scale l. The importance of nonlinear processes is that in some instances they can cause transformation of a wave field from an undamped state to a state where damping is heavy. In other cases there are innately nonlinear damping mechanisms. The net effect is that the true damping of an MHD wave field can greatly exceed that calculated from linear theory alone. This point is well illustrated in the work of Machida, Spangler, & Goertz (1987), who used a hybrid code computer simulation to study the evolution of Alfvénic wave packets. In what follows I discuss two of the principal mechanisms identified by Machida et al. as important in the damping of MHD waves, and an additional, intrinsically nonlinear damping mechanism.

4.2.1. *Wave Packet Steepening*

This mechanism has been identified on the basis of extensive studies of the Derivative Nonlinear Schrödinger equation, a model for the evolution of nonlinear Alfvén and fast mode wave packets. This equation and its solutions are discussed in Spangler, Sheerin, & Payne (1985) and Spangler (1985), as well as further in this section. An example of the evolution of a large-amplitude circularly polarized Alfvén wave packet is shown in Figure 2 (taken from Spangler 1985). The wave packet intensity as a function of position at three times is shown in the top set of panels, and one component of the wave field as a function of position appears in the bottom three panels. It is seen that the wave packet steepens, becoming more intense and of smaller spatial extent. By the second panel the wave packet has developed a sharp, spatially confined pulse.

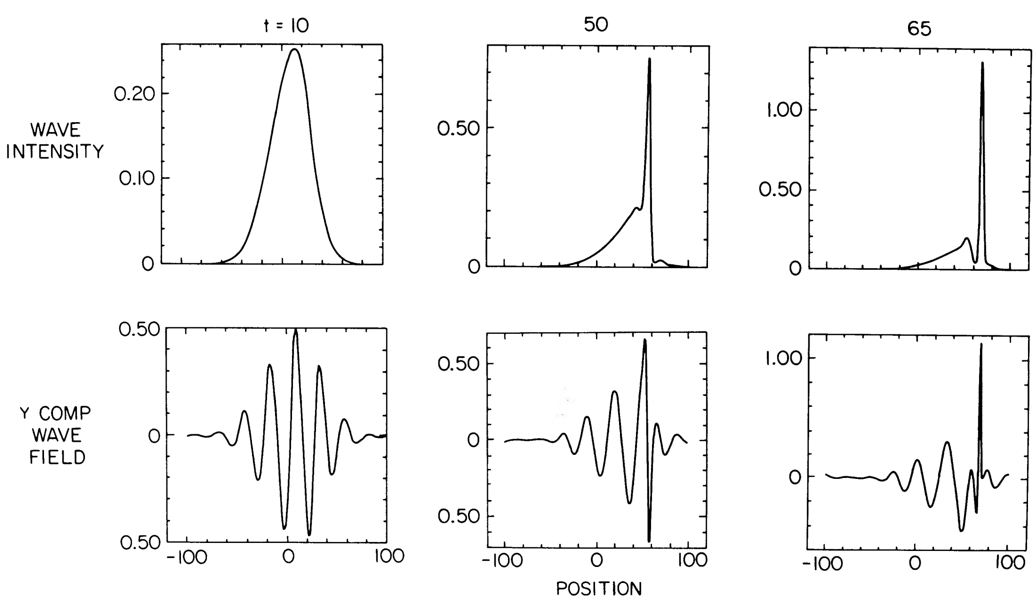

FIG. 2.—Evolution of a large-amplitude, left circularly polarized Alfvénic wave packet. Shown are the wave packet intensity (*top panels*) and one component of the wave field (*lower panels*) as a function of spatial position at three times. All physical quantities are dimensionless according to the prescription of Spangler (1985), from which the calculations are taken. Note that the scale of the ordinate changes from one time frame to the next.

Such steep spatial gradients will lead to enhanced dissipation of the wave packets. Another way of appreciating this is via the plots in the bottom panels; the nonlinear evolution produces wave energy at higher wavenumbers where damping is much more effective. The result of previous work (Spangler 1985, 1986) is that the sharp gradients and contracted wave packets are formed on the scale of the ion-inertial length V_A/Ω_i, irrespective of the initial size of the wave packet. Wave dissipation should be strong on such scales. Spangler (1986) investigated a combination of nonlinear evolution, dispersion, and dissipation at high-spatial wavenumbers and found that nonlinear spectral transfer of wave energy could substantially enhance wave dissipation.

The time scale nonlinear steepening of a wave packet in the manner shown in Figure 2 depends on the initial wave packet characteristics in the following fashion (Spangler 1985):

$$\tau_{\rm NL} = \frac{l}{2N_1 V_A}\left(\frac{B_0}{b_A}\right)^2 , \tag{23}$$

where all parameters have been defined except N_1, which is a dimensionless coefficient, dependent on the plasma β and the ion-to-electron temperature ratio. For most circumstances, this coefficient will be of order unity (Spangler 1990). It should be noted that equation (23) differs slightly from that in Spangler (1985) in that N_1 replaces $1/2(1-\beta)$. This modification results from recent studies of kinetic effects on nonlinear Alfvén waves (Spangler 1989, 1990). A brief description of this topic is given below; the few interested readers are referred to the aforementioned papers.

I have assumed here that the modulation scale l is the scale of the circularly polarized wave packets, as indicated in Figures 1 and 2. The motivation for the choice is that recent theoretical work on MHD wave steeping has concentrated on circularly polarized waves, and the waves near the Earth's bow shock (a plausible model for waves in the ISM) are elliptically to circularly polarized. However, if the interstellar waves are linearly polarized, the relevant modulation scale is half the wavelength, so the nonlinear steepening time would be correspondingly shorter and the heating rate larger.

Given equation (23), an expression for the wave dissipation rate ϵ may be derived. First, a nonlinear steeping rate is obtained as the reciprocal of equation (23),

$$\gamma_s = 2N_1 V_A\left(\frac{b_A}{B_0}\right)^2 l^{-1} . \tag{24}$$

This is converted to a rate dependent on wavenumber by recalling the properties of the wave packet model illustrated in Figure 1,

$$l^{-1} = \frac{\Delta k}{2\pi} = \frac{\eta k}{2\pi} .$$

Next, I relate this steepening rate to overall properties of the power-law turbulence by substituting equation (12) in equation (24), obtaining

$$\gamma_s(k) = \frac{2N_1 V_A \eta}{B_0^2} \frac{\delta(s, \eta)}{R_l s(s-1)\Lambda^2} C_B^2 k^{2-s}$$

or

$$\gamma_s(k) = \frac{N_1 V_A}{\pi B_0^2} \frac{\eta\delta(s, \eta)}{R_l} b_T^2 k_0^{s-1} k^{(2-s)} . \tag{25}$$

The rate given by equation (25) is a steepening rate, as opposed to an actual energy dissipation rate. The two may be formally related by $\gamma_d = \kappa\gamma_s$, where κ is the fraction of the wave packet energy which is actually dissipated as plasma heating; the remainder $1-\kappa$ is converted into wave energy which is undamped.

An unfortunate feature of our present state of knowledge is that κ must remain an ad hoc parameter. I will assume that κ is greater than a few percent but less than unity. With this recipe for obtaining the dissipation rate, substituting equation (25) into (6), ϵ is given by

$$\epsilon = \Sigma\left(\frac{V_A}{B_0^2}\right) b_T^4 k_0 , \tag{26}$$

where k_0 is the wavenumber corresponding to the outer scale of the turbulence, and Σ is a parameter depending on characteristics of the spectrum and other features of the model

$$\Sigma\left(s = \frac{3}{2}\right) = \frac{N_1 \kappa\eta\delta(3/2, \eta)}{(2\pi)^2 R_l} \ln\left(\frac{k_i}{k_0}\right)$$

and

$$\Sigma\left(s = \frac{5}{3}\right) = \frac{N_1 \kappa\eta\delta(5/3, \eta)}{\pi^2 R_l} .$$

Note that $\Sigma(3/2)$ depends on both the inner and outer scale. However, since the dependence is a logarithmic one, it will not greatly affect the numerical value of Σ.

To obtain a numerical estimate of the heating rate due to wave packet steepening, values must be prescribed for the parameters which appear in the coefficient Σ. This task is rendered difficult by the fact that many of these parameters can be termed "imponderables," that is, poorly constrained by observations.

The coefficient N_1 is relatively easy to fix; an examination of Figure 3 of Spangler (1990) shows that if the plasma β and ion-to-electron temperature ratio are not extreme, a value of $N_1 \simeq 0.75$ is reasonable. As mentioned above, the fraction of energy in the steepened wave packet which is converted to heat, κ, is difficult to specify. I adopt a value of $\kappa = 0.3$, with the recognition that there is considerable attendant uncertainty. The quantity R_l is a measure of the proximity of wave packets. I adopt a value of 0.4 for R_l, indicating close-packed wave packets. The wave steepening rate for $s = 3/2$ is slightly dependent on the inner scale, as indicated in the expression for $\Sigma(3/2)$. I assume $k_i = 10^8 k_0$. The parameter η determines the "coherence" of the wave packets forming the model of turbulence. A very small value would describe highly coherent wave packets with many wave cycles within a packet; as discussed above this seems unlikely to be the case in broad-band, power-law turbulence. At the same time, a value of unity (or greater) would not be consistent with my assumption of a wave packet nature of the turbulence. I adopt a value of $\eta = 0.3$. The value of δ depends on both s and η. For the value of η chosen, $\delta = 0.15$ is representative for s in the range 3/2 to 5/3.

With these choices, the energy dissipation rate is

$$\epsilon \simeq 0.05 \frac{V_A}{B_0^2} \frac{b_T^4}{\ell_0} , \tag{27}$$

where the numerical coefficient is a rough mean of the values of Σ for $s = 5/3$ and 3/2. Expressing equation (27) in terms of dimensionless quantities B_0', V_A', b_T', and ℓ_0', (in units of

3×10^{-6} G, 10 km s^{-1}, 10^{-6} G, and 10^{17} cm, respectively), the heating rate due to steepening is given by

$$\epsilon_{\rm WS} \simeq 5.6 \times 10^{-26} V'_{\rm A} b_T'^4 B_0'^{-2} \ell_0'^{-1} \text{ ergs s}^{-1} \text{ cm}^{-3} . \quad (28)$$

Note that the dissipation rate is inversely proportional to the outer scale. The smaller the outer scale, the more intense is the turbulence contained in wave packets of small spatial extent. As may be seen in equation (23), wave packets of small spatial extent steepen quickly, and thus lead to fast energy dissipation.

A point should be made regarding the credibility of equation (28), given the previously discussed uncertainty concerning the wave packet approximation on which it is based. There are legitimate questions about how well defined is the idea of wave packet modulation for wave numbers in the "inertial subrange" between k_0 and k_i. however, there can be no doubt that, virtually by definition, modulations will occur in the wave amplitude on scales of the outer scale $\ell_0 = 2\pi/k_0$. On such large scales the wave amplitude is comparable to b_T. Given these two quantities, one can repeat the arguments in equations (23)–(28), albeit more simply, to obtain an expression for the mean heating rate due to steepening of waves on the largest scales. Such an expression should give a less precise but more robust heating rate due to wave packet steepening. An exercise like this was carried out and yielded an expression that was identical to equation (28) within a factor of 2 in the numerical coefficient. This supports the validity of my estimate for the heating rate due to wave steepening, and also for the nonlinear Landau damping to be discussed in § 4.2.3.

In assessing the role of wave packet steepening in the interstellar medium, it should be kept in mind that there are certain "selection rules" for the occurrence of this steepening. These are the conditions for the modulational instability of MHD waves. Fluid theories predict that such steepening will occur for right-hand polarized waves if the plasma β is greater than unity, and left-hand polarized waves if the β is less than unity. Recent work by Mjølhus & Wyller (1988), Spangler (1990), and Flå, Mjølhus, & Wyller (1989), including kinetic effects on the evolution of Alfvén waves, indicates that the conditions for modulational instability might be somewhat different. Briefly, if the electron and ion temperature are approximately the same, it seems probable that right-hand polarized waves will be modulationally stable (will not steepen), regardless of the plasma β, and left-hand polarized waves will steepen, again regardless of the value of β. Since we are uncertain of the generation mechanism of these interstellar waves, we cannot be certain of their polarization and hence their proclivity for steepening.

Furthermore, we cannot be sure that the fluctiferous medium is not characterized by an extreme ion-to-electron temperature ratio; the interplanetary medium has an electron temperature several times the ion temperature, and it is possible that the "fluid-like" selection rules are valid. Finally, the kinetic calculations discussed above were made assuming Maxwellian distribution functions with equal parallel and perpendicular temperatures. It is possible that more realistic distribution functions might be capable of inducing modulational instability for waves in the interstellar medium which might be thought stable.

4.2.2. *Parametric Decay Instability*

Another nonlinear MHD wave process which can potentially lead to dissipation is the parametric decay instability. This process was first discussed by Galeev & Oraevskii (1963) and has been considered in a number of subsequent theoretical works, both analytic and numerical (Goldstein 1978; Longtin & Sonnerup 1986; Wong & Goldstein 1986; Terasawa et al. 1986; Machida et al. 1987). In this mechanism, a parent Alfvén or fast magnetosonic wave decays into a backward-propagating MHD wave and a forward-propagating ion-acoustic wave. There is also an energetically negligible, forward-propagating MHD wave. My interest in this mechanism is the fact that in a plasma with β approximately unity, the ion-acoustic wave will be heavily damped, thus furnishing a channel for conversion of MHD wave energy into thermal plasma energy.

In spite of the extensive theoretical investigation of this instability, observational supporting evidence remains slim or nonexistent. Leckband & Spangler (1989, 1991) in a study of spacecraft data near the Earth's bow shock have occasionally detected a Fourier component in the plasma density power spectrum which is consistent with a "decay line." However, Hoppe & Russell (1983) did not detect MHD waves propagating back toward the bow shock, which would seem to be a severe observational argument against an important role for this process in the dynamics of MHD waves. A full observational search for the decay instability remains an interesting topic for future research.

For the present purposes, I will assume that the decay instability proceeds according to the aforementioned theoretical descriptions, and I will calculate the heating of the interstellar medium via damping of the daughter ion-acoustic wave. The growth rate of this instability, that is, the rate at which the parent wave is transformed into the daughter waves, is given by (Machida et al.),

$$\gamma_g = \frac{1}{\sqrt{8}\,\beta^{1/4}} \left(\frac{b_A}{B_0}\right) V_{\rm A} k . \quad (29)$$

This growth rate emerges from a simplified treatment of the decay instability, but it is in fairly good agreement with the results of more sophisticated calculations. Note the dependence of the growth rate (29) on the wave packet amplitude.

Equation (29) predicts a very rapid decay of waves with large amplitude (~ 3 wave periods for $(b_A/B_0) \sim 1$). Nonetheless, Goldstein (1978) showed that the equations of MHD predict that this instability should proceed for $(b_A/B_0) \sim 1$. In the present application, two factors may serve to ameliorate this nearly evanescent quality of the waves. First, it is possible that $b_A \ll B_0$, in which case the decay time will be many wave periods. In the application of the theoretical results to be presented in § 5, I will assume $(b_T/B_0) = 0.1$. There is no guarantee, of course, that the wave amplitude will be sufficiently small as to permit the wave to persist for many wave periods, but by the same token $b_A/B_0 \ll 1$ on the scales of interest is completely compatible with observations.

The second consideration is to invoke equation (12) which shows that for power-law turbulence, the wave amplitude decreases with decreasing wavelength. Thus even if the turbulence is sufficiently strong so that $b_T \simeq B_0$, the wave amplitude b_A will be comparable to B_0 only for the largest wavelengths comparable to the outer scale. For waves with $k \gg k_0$ the decay period will greatly exceed the wave period. As will be seen below, the form of the decay instability growth rate is such that waves throughout the spectrum, and not just at the largest scales, make contributions to the heating rate. Thus even if suppression of the instability due to large wave amplitude were to occur for the largest wavelengths, waves of smaller wavelength would not be so affected, and the heating rate for the entire spectrum, calculated using equation (29), would presumably not be greatly affected.

Equation (29) expresses the rate at which the parent MHD wave transforms itself into an ion-acoustic wave and a daughter MHD wave. I equate this to an energy input rate because if the plasma β is approximately unity, the ion-acoustic wave should be damped at a rate which is much faster than the instability growth rate. The validity of this statement is better for smaller amplitude parent waves. I reintroduce the coefficient κ to denote the fraction of parent wave energy going to the ion-acoustic wave rather than the undamped daughter MHD wave.

Once again I relate the growth rate to overall properties of the turbulence spectrum by substituting equation (12) into equation (29). Substituting the resultant expression into equation (6) produces

$$\epsilon = \frac{\kappa(s-1)\delta^{1/2}}{2\pi\sqrt{8}\,R_l^{1/2}}\,\beta^{-1/4}\left(\frac{V_A}{B_0}\right)b_T^3\,k_0^{(3/2)(s-1)}\int_{k_0}^{k_i} dk\,k^{(3/2)(1-s)} . \tag{30}$$

A comment is in order regarding the integral in equation (30), which yields different results depending on the value of s. If $s = 3/2$, the integral is $4k_i^{1/4}$, while if $s = 5/3$, it is $\ln(k_i/k_0)$. In either case the dissipation rate depends on the value of the inner scale. However, the final expression for ϵ is only weakly dependent on the ratio k_i/k_0; as before, I choose a value of $k_i = 10^8 k_0$ for purposes of calculation. A value of $\kappa = 0.5$ is assumed for this damping process. The resultant expression for $s = 5/3$ is

$$\epsilon_{DI} = 5.6 \times 10^{-24} V_A' B_0'^{-1} b_T'^3 \ell_0'^{-1}\ \text{ergs s}^{-1}\ \text{cm}^{-3} . \tag{31}$$

The heating rate for an $s = 3/2$ spectrum is quite comparable to equation (31) if the outer scale is small, of order 10^{13} cm. If the outer scale is much larger, 10^{17} cm, and all other parameters are as assumed above, then the $s = 3/2$ heating rate exceeds (31) by about an order of magnitude. The occurrence of the decay instability is also governed by "selection rules." Fluid theory indicates that the conditions for the decay instability to proceed are in fact the complement of those for wave packet steepening; right-hand waves are subject to the decay if β is less than unity, and left-hand waves are susceptible if β exceeds unity. The consequences of our ignorance of the polarization of interstellar waves apply here as well as in the case of wave packet steepening. However, fluid theory would indicate that *either* the decay instability *or* wave packet steepening will occur regardless of polarization and the β of the interstellar plasma.

Even this last comment, however, is subject to some reservation. Inhester (1990) has recently carried out a study of kinematic effects on the decay instability. He finds that the growth rate tends to be lowered, and that a larger range of modulational wavenumbers tends to be unstable, relative to the fluid description. If this result is applicable to the interstellar medium, it would mean that the heating rate given in equation (31) is an overestimate. However, Inhester also finds that for certain realistic distribution functions, the growth rate is enhanced relative to its fluid value.

4.2.3. *Nonlinear Landau Damping*

Earlier in this section I discussed the steepening of MHD wave packets as a source of wave dissipation. This steepening was described by the Derivative Nonlinear Schrödinger equation (DNLS), a model for the evolution of large-amplitude Alfvén and fast mode waves. Recently Mjølhus & Wyller (1986, 1988) and Spangler (1990) have derived a generalization of the DNLS which incorporates a kinetic description of the plasma particles rather than a fluid one. As pointed out by Mjølhus & Wyller (1986), this approach allows one to describe more accurately the effects of resonant particles, which otherwise are poorly described by fluid theories.

Mjølhus & Wyller (1986, 1988) found that under most circumstances the more general nonlinear wave equation will differ significantly from the fluid-derived DNLS. In this section I briefly summarized these results and discuss their implications for heating of the interstellar medium. Mjølhus and Wyller derived the following equation describing the evolution of nonlinear MHD waves

$$i\frac{\partial\phi}{\partial t} + i\frac{1}{2}\frac{\partial}{\partial x}\left\{\phi\left[N_1|\phi|^2 + N_2 \fint \frac{|\phi|^2(x')}{(x'-x)}dx'\right]\right\} \pm \mu\frac{\partial^2\phi}{\partial x^2} = 0 , \tag{32}$$

where $\phi \equiv b_y + ib_z$ and $\fint$ denotes the Cauchy principal value. The direction of wave propagation is x, and the parameter μ is a measure of the strength of wave dispersive effects. Equation (32) has been dedimensionalized so that ϕ is normalized by B_0, the spatial coordinate x by V_A/Ω_i, and the time by the inverse ion-cyclotron frequency Ω_i^{-1}. If $N_2 = 0$, equation (32) is exactly the DNLS. Mjølhus & Wyller (1986, 1988) noted two differences between equation (32) and the DNLS as derived from fluid theories. The first is the origin of the functionally novel term proportional to N_2, which Mjølhus and Wyller refer to as the "nonlocal term." The parameter N_2 is a function of the plasma β and the ratio of the electron temperature to ion temperature. For low β, N_2 is small, and the nonlocal term can be ignored. Second, the coefficient N_1 of the DNLS nonlinearity (previously introduced in eq. [23]) in general differs from that derived in fluid theory. Kinetic theory shows that unless the electron temperature greatly exceeds the ion temperature, N_1 is a quite gradual function of the plasma β, whereas fluid theory has $N_1 = 1/2(1-\beta)$.

The DNLS is a so-called completely integrable equation, meaning that it has an infinite number of constants of the motion, the first of which corresponds physically to the total magnetic energy in the waves. A consequence of this complete integrability is that an arbitrary initial wave field will transform into a set of solitons (Dawson & Fontan 1990). Flå et al. (1989) studied numerical solutions to equation (32) and found that the solitons damped, whereas the pure DNLS has these objects persist indefinitely. The nonlocal term may therefore be identified as a nonlinear damping term, and as such of interest to this study.

I interpret this nonlinear damping as follows. As mentioned in § 2.3, ponderomotive effects cause a perturbation in the plasma density, with an associated magnetic field–aligned electric field. Thermal ions can undergo a Landau resonance with this electric field, causing a strong coupling between the MHD wave and the thermal plasma and thus damping. For this reason this dissipation mechanism may be referred to as nonlinear Landau damping.

I now proceed to obtain a rough but adequate estimate of this nonlinear damping for the wave packet model for plasma turbulence. The method used is very simple and relies on the existence of the constants of the motion of the DNLS. Consider the first constant of the motion of the DNLS, which is $C_0 \equiv \int_{-\infty}^{\infty} dx\, \phi\phi^*$. The equation for this quantity is obtained by taking equation (32) and the equivalent equation for the complex conjugate ϕ^*. Equation (32) is multiplied by $\phi^* dx$ and integrated over all space. The complex conjugate of equation (32) is multiplied by ϕ and similarly manipulated. The two equations are then added thereby, obtaining an expression for $d/dt \int_{-\infty}^{\infty} dx\, \phi\phi^*$. This is nothing other than the time rate of change of the total wave magnetic energy. This decline in wave energy must be going to heat the plasma.

This method of computation is simple because the contribution of all terms other than that proportional to N_2 are zero, a consequence of the fact that $\int_{-\infty}^{\infty} dx\, \phi\phi^*$ is a constant of the motion of the DNLS. The nonvanishing nature of the contribution of the nonlocal term indicates that kinetic effects contribute to the dissipation of the wave packets.

Instead of the time derivative of C_0 being zero as in the case of the DNLS, equation (32) engenders the following

$$\frac{d}{dt}\int_{-\infty}^{\infty} dx\, \phi\phi^* = -\frac{1}{2} N_2 \int_{-\infty}^{\infty} dx\, (\phi\phi^*) \frac{d}{dx} \mathscr{S}(\phi\phi^*) , \quad (33)$$

where

$$\mathscr{S}(\phi^2) \equiv \oint \frac{|\phi|^2(x')}{(x'-x)}\, dx' .$$

In general, equation (33) gives an integro-differential equation for the time rate of change of wave packet energy. However, for my purposes it will suffice to obtain a rough estimate of the time scale for dissipation of a wave packet. I therefore *prescribe* the functional form of the wave packet to be a Lorentzian, for reasons of computational convenience:

$$|\phi|^2(x) = \frac{b_A^2 l^2}{x^2 + l^2} . \quad (34)$$

Substituting equation (34) into equation (33), we have

$$\frac{dC_0}{dt} = \frac{\pi^2}{8} N_2 b_A^4 . \quad (35)$$

It is worth noting that for all cases of plasma distribution functions discussed by Spangler (1990), the coefficient N_2 was negative, so equation (35) insures that a damping of wave energy occurs.

Approximating (dC_0/dt) by the term $2\gamma_d C_0$ and using the form of C_0 appropriate to a Lorentzian,

$$\gamma_d = \frac{\pi}{4} N_2 V_A \left(\frac{b_A}{B_0}\right)^2 l^{-1} , \quad (36)$$

where the dimensions have been restored. Comparison with equation (24) shows that equation (36) is of the same form as that obtained for the steepening of nonlinear waves discussed above. The only difference, aside from a small difference in multiplicative constants of order unity, is that nonlinear Landau damping is determined by the coefficient N_2, whereas the celerity of wave packet steepening is determined by N_1. An important distinction between the two processes is that for wave packet steepening, dissipation of the wave energy had to be introduced in an ad hoc manner through the coefficient κ, whereas the nonlinear Landau damping process is bona fide wave dissipation which can be directly equated to plasma heating.

Another important distinction between the two processes arises as a result of the different magnitudes of the parameters N_1 and N_2; these parameters are shown in Figure 3 of Spangler (1990). For purposes of calculation we choose $N_2 = 0.075$, which is a good representative value for $\beta \sim 1$. It should be kept in mind that N_2 will be drastically smaller if β is much smaller than unity, and much larger than we have assumed if the fluctifer is a high β plasma. It will be noticed that for $\beta \leq 1$, the parameter $N_2 \ll N_1$, tending to make nonlinear Landau damping a slower heating process than wave packet steepening.

The similar functional form of the damping rates of the two processes allows the heating rate for nonlinear Landau damping to be immediately written as

$$\epsilon_{\rm NLD} \simeq 7.2 \times 10^{-27} B_0'^{-2} V_A' b_T'^4 \ell_0'^{-1} \text{ ergs s}^{-1} \text{ cm}^{-3} . \quad (37)$$

Unlike the other nonlinear heating mechanisms mentioned in this section, nonlinear Landau damping will occur regardless of the value of plasma β or sense of wave polarization, as discussed by Flå et al. (1989) and Spangler (1990).

5. APPLICATION OF RESULTS TO THE INTERSTELLAR MEDIUM

The main results of the paper to this point are contained in the expressions for the heating rates for the five wave-damping processes we have considered, equations (15), (21), (22), (28), (31), and (37). It will be noticed that all of the heating rates are dependent on the outer scale of the power-law turbulence, ℓ_0. For all save ion-neutral damping, there is a monotonic increase in the heating rate with decreasing outer scale. These facts emphasize the importance of the observational constraints on the outer scale by Cordes et al. (1990) and Lazio et al. (1990).

The natural comparison to be made is between these heating rates and the expected radiative cooling rates for the candidate fluctiferous media. Obviously, in equilibrium the energy dissipation rate due to wave damping or any other input cannot exceed the cooling rate in the fluctifer. In the remainder of this section I will therefore make a comparison between the energy dissipation rates obtained in § 4 and estimated cooling rates for the media discussed in § 2.2. If the wave dissipation rate is much less than the estimated cooling rate, one would conclude that wave damping is an unimportant process in the thermodynamics of the fluctifer. If the two rates are equal, wave damping is a promising candidate for the heating mechanism of the medium. A dissipation rate considerably in excess of the cooling capacity of the host media clearly indicates that either the hypothesized damping process does not occur as described, or the relevant medium has been misidentified.

Both candidate fluctiferous media may be described as H II regions, and so the cooling curve appropriate to H II regions may be used. The cooling function has recently been discussed by Reynolds (1990), who describes the radiative cooling as

$$L = p(T) n_e^2 \text{ ergs s}^{-1} \text{ cm}^{-3} , \quad (38)$$

where n_e is the electron density, and $p(T)$ is a function of the temperature T. Figure 1 of Reynolds displays the function $p(T)$; it exhibits a minimum of $\sim 10^{-23}$ ergs s^{-1} cm^{-3} for a temperature of $\sim 10^4$ K. Strictly speaking, equation (38) describes the power required to maintain ionized hydrogen as a function of electron temperature, and includes the recombination energy responsible for the increase in $p(T)$ for temperatures less than 10^4 K. If wave dissipation is the dominant energization and ionization source for the fluctiferous medium, the function $p(T)$ in Figure 1 of Reynolds should be used in our comparison. However, if some other agency, such as ultraviolet light from hot stars provides hydrogen ionization in the fluctifer, and we are interested in radiative removal of the incremental power from wave dissipation, a smaller value of p is applicable.

In this case, we may consult a diagram such as Figure 3.2 of Osterbrock (1974). The function $p(T)$ is then dominated by collisional excitation of ionized oxygen; a value of p of 2–4×10^{-24} ergs cm^{-3} s^{-1} is appropriate. For purposes of cooling calculations, I will choose a plausible value of $p \equiv 5 \times 10^{-24}$ ergs cm^{-3} s^{-1} as a compromise between the values given by the expressions of Reynolds (1990) and Osterbrock (1974); we cannot be certain that wave dissipation is not providing the source of energy for maintaining the ionization of the fluctifer, although this seems unlikely. This value could be an underestimate of the cooling capacity if the fluctifer is a region of enhanced heavy elements.

It is worth emphasizing that the corresponding cooling rates for the most part greatly exceed the standard number of $L \sim 10^{-27}$ ergs s^{-1} cm^{-3} quoted in previous works on this subject, for example, Ferriere et al. (1988). While this latter number is appropriate to a predominantly neutral gas with a density of less than 1 cm^{-3} and a temperature of a few thousand degrees, it is a substantial underestimate of the cooling capacity of the more dense, fully ionized gas which observations seem to indicate are characteristic of the fluctiferous medium.

A comparison between the wave dissipation heating rates and the cooling rates of the various media is made in graphical form in Figure 3. The ordinate is the heating rate due to wave dissipation or the cooling rate of the host plasma. The abscissa is the outer scale to the turbulence, which as we have seen is an important parameter in determining the wave heating rate. The horizontal flagged lines represent the cooling rates for the two candidate host plasmas. The solid lines indicate the heating rates for the five wave-damping processes discussed in § 4. I have set $F(\beta) = 1$ in equation (15), which is roughly appropriate for $\beta \simeq 1$. For ion-neutral collisional damping, I have taken the mean of the expressions (21) and (22) for this process in the WIM and the envelopes of H II regions. It is worth recalling that the observational limits on the outer scale provided by Cordes (1990) and Lazio et al. (1990) would correspond to the vertical lines at 10^{14} cm (*lower limit*) and in the range 10^{17}–10^{18} cm (*upper limit*).

The wave damping rates in Figure 3 are calculated for a value of $b_T = 0.3$ μG, that is 10% of the value assumed for the systematic field. If the true value for b_T is larger, all heating curves would be moved up by an amount which is dependent on the process. Since nonlinear heating processes are proportional to a higher power of b_T than are linear ones, their contribution to ISM heating would be more important with a higher value of b_T. It is difficult to say if the true value of b_T is higher than the value I have assumed. A recent analysis of Faraday rotation measurements by Rand & Kulkarni (1989) indicates that the fluctuating component of the Galactic magnetic field is comparable to or even larger than the systematic

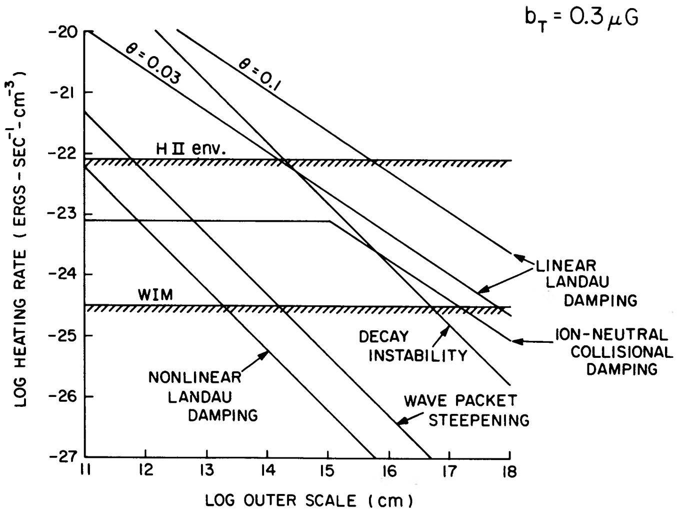

FIG. 3.—Comparison between energy dissipation due to MHD wave damping and cooling of different media. Solid lines indicate heating due to the various wave-damping processes, which are functions of the outer scale to the turbulence. The horizontal lines represent the cooling capacities of the candidate media. The calculations are carried out for an assumed rms turbulent magnetic field (b_T) of 0.3 μG. The linear Landau damping heating rates are dependent on the angle θ (in radians) between the direction of wave propagation and that of the large-scale magnetic field.

component, but the magnitude of this random field cannot be directly equated to b_T for two reasons. First, the scale length of the random field in the model of Rand and Kulkarni is 55 pc, far greater than the scales of 10^{17}–10^{18} cm that I have considered. One might expect the amplitude of fluctuations on subparsec scales to be smaller than that of fluctuations on scales of tens of parsces, but we are largely ignorant of the spectrum of interstellar turbulence in this wavenumber range. Second, the analysis of Rand and Kulkarni used sources from the large part of the sky, and thus refers to the whole ISM and not just the fluctiferous medium.

An argument in favor of larger values of b_T relative to B_0 is the result of § 2.2 where I argued, subject to disclaimers, that density fluctuations on scales of the order of the outer scale were of the order of 25% of the mean density. If the turbulence responsible for ISS resembles MHD waves (the basic premise of this paper), then such density fluctuations could not exist without magnetic field fluctuations comparable in amplitude to the mean field. The MHD waves upstream of the Earth's bow shock can be cited as a plasma in which the small-scale wave magnetic field approaches the magnitude of the large-scale field.

The linear Landau damping contribution in Figure 3 assumes all waves are propagating with small angle θ with respect to the magnetic field. This assumption is consistent with the model for interstellar MHD waves being explored in this paper, that is, that of quasi–one-dimensional, parallel-propagating (or nearly so) waves described by equations (4) and (5). The true distribution of wave energy with θ will depend on the important and damping processes. There are two ways in which a field-aligned distribution with a narrow range of θ could be generated. First, the wave production mechanism itself could be highly anisotropic and field directed. For example, the growth rate of MHD waves due to a field-aligned streaming ion distribution is a maximum for parallel propagation. Second, it is possible that the production mechanism is isotropic, but the waves are mainly parallel propagating as a consequence of the strong propagation-angle dependence of linear Landau damping (eqs. [13] and [14]). In general there will be a substantial difference in the plasma heating in these two cases; heating rates comparable to equation (15) result when waves are generated with small propagation angles. For isotropic wave generation or generation over a large range of propagation angles, the linear Landau damping rate would be substantially higher than that given in equation (15) and plotted in Figure 3.

A brief justification for these last statements is as follows. In a steady state, the heating rate equals the integral (over wavenumber space) of the spectral wave production rate. In the first case above of parallel wave generation, this integral includes only a cone of angles centered on the large-scale magnetic field. In the case of isotropic wave generation, this integral extends over the large volume of wavenumber space containing highly damped oblique modes which contribute little to the total wave energy density. As a result, one would expect the heating rate to be higher in the case of isotropic wave generation. Finally, it should be kept in mind that consideration of linear Landau damping is relevant only if the MHD waves responsible for ISS are magnetosonic waves, rather than the linearly incompressible Alfvén mode which is not subject to this damping mechanism (Hollweg 1975). Alfvén waves could produce the density fluctuations responsible for scintillations through ponderomotive effects or transverse modulation, as discussed in § 2.3.

The conclusions to be drawn from Figure 3 are as follows. First, these results reproduce those of earlier writers on the subject; the linear processes of linear Landau damping and ion-neutral collisional damping provide energy input which is orders of magnitude larger than the cooling capability of largely neutral gas with a temperature of a few thousand degrees kelvin and a density of order 1 cm^{-3}. This statement is rendered more emphatic by the substantial heating contribution of the parametric decay instability, a process not previously recognized in the context. This result does not constitute an argument against the existence of regions in the interstellar medium with such characteristics (essentially the Warm Neutral Medium [WNM] of the McKee-Ostriker model), only that the plasma waves responsible for interstellar scintillation do not reside there.

However, wave damping does not produce a thermodynamic embarrassment if the envelopes of H II regions are the fluctiferous media. If the outer scale to the turbulence is of order 10^{17}–10^{18} cm, as indicated by the observations of Lazio et al. (1990), none of the damping processes considered is within an order of magnitude of the cooling capacity of this medium. Even if the outer scale is as small as the Cordes et al. (1990) limit of 10^{14} cm, the energy input due to linear Landau damping (given our assumption of quasi-parallel propagation) and the parametric decay instability would only slightly exceed our estimates for the cooling rate of the fluctifer. Both of these mechanisms would be eliminated if the MHD turbulence consists of parallel-propagating waves which are not subject to the decay instability. The estimated contributions of ion-neutral collisional damping, wave packet steepening, and nonlinear Landau damping are apparently a minor energetic input to the H II region envelopes.

The effect of wave dissipation may not be so negligible if a medium similar to the WIM is the fluctifer. Even for an outer scale as large as that indicated by the Lazio et al. (1990) observations, the energy input due to linear Landau damping appears to exceed the cooling capability. Even if these mechanisms are removed by invoking parallel-propagating waves which are not subject to the decay instability, the power input from ion-neutral collisional damping is roughly equal to the cooling capacity.

The feasibility of the WIM as the fluctiferous medium is rather sensitively dependent on the outer scale being as large as indicated in the observations of Lazio et al. (1990). If the outer scale of the turbulence is substantially smaller than the Lazio et al. (1990) limit, wave dissipation via several mechanisms vastly exceeds the cooling capability of the WIM, and a thermodynamic embarrassment exists.

Of the nonlinear mechanisms, the parametric decay instability would seem to be the most important. Depending on the outer scale of the turbulence (and other parameters considered), the energy input from the process can be comparable to or exceed that of the linear processes. It would seem that the other two nonlinear processes, wave packet steepening and nonlinear Landau damping, are unimportant as regards heating of the interstellar medium. For these mechansims to play an important role, a number of conditions would have to be fulfilled. First, the fluctiferous medium must have a density comparable to the WIM. Second, the outer scale of the turbulence would have to be comparable to or smaller than the Cordes et al. (1990) lower limits. Third, the other three processes discussed would have to be inoperative. While I have discussed quite plausible circumstances under which linear Landau damping and the decay instability can be suppressed, it is difficult to imagine reasonable conditions under which ion-neutral collisional damping would not occur. For this damping process to be less important than wave packet steepening and nonlinear Landau damping, it would be necessary for helium to be ionized, a requirement which seems extreme.

These two nonlinear processes could be important if there are places in the interstellar medium where waves are generated on small spatial scales (i.e., a very small value of ℓ_0), perhaps via cosmic-ray instabilities (Spangler, Fey, & Cordes 1987). However, at the present time there is no observational evidence for such regions.

A final caveat to be made is that a great number of assumptions were made in my analysis about poorly known or unknown physical quantities. An example is the value of the rms magnetic field fluctuation b_T. The conclusions presented above were predicated on a value of $b_T = 0.3$ μG. If the true value is substantially higher, and the fluctuations on such large scales behave as MHD waves, heating by wave dissipation could exceed the cooling capacities of the model fluctifers in Table 1. By the same token, the values for the mean plasma densities in Table 1 could be underestimates, in which case I have underestimated the cooling capacity of the fluctiferous media. Observational projects currently in progress should improve our knowledge of plasma parameters in fluctiferous media along lines of sight to heavily scattered sources. Such data should permit a more refined version of these calculations in the near future.

6. CONCLUSIONS

In this paper I have reexamined the issue of heating of the interstellar medium by dissipation of the turbulence responsible for interstellar scintillation. I have utilized recent progress in our understanding of scintillation, as well as developments in the theory of nonlinear MHD waves. My conclusions are as follows.

1. If the regions containing the irregularities responsible for enhanced interstellar scintillation (referred to in this paper as the fluctiferous media or fluctifers) are the extended envelopes of H II regions, then the radiative cooling capacity exceeds the estimated power input from wave dissipation. There is then no thermodynamic enbarrassment associated with a broad spectrum of density irregularities. This conclusion regarding the thermodynamic consequences of interstellar scintillation differs from that of previous authors on this subject in that a different plasma is considered for the fluctiferous medium. The envelopes of H II regions, being fully ionized and relatively dense, have a much larger radiative cooling capacity than some of the ISM phases previously discussed.

2. The Warm Ionized Medium of the McKee-Ostriker model of the interstellar medium would have more difficulty accommodating the power input from wave damping. For the WIM to be the fluctifer, the MHD waves must be Alfvén waves or parallel-propagating fast mode waves, and the outer scale of the turbulence must be close to the observational estimate of Lazio et al. (1990).

3. Linear Landau damping is the mechanism with potentially the greatest heat input to the ISM, but under quite plausible circumstances it could be largely suppressed.

4. Of the three nonlinear damping mechanisms considered, the parametric decay instability seems to be most important for heating of the interstellar medium. Under circumstances considered here, it can be comparable to or in excess of linear Landau damping and ion-neutral collisional damping.

5. The other two nonlinear processes considered, wave packet steepening and nonlinear Landau damping, do not seem to play an important role in the heating of the interstellar medium. These mechanisms could be important if there are regions of space with intense waves on small spatial scales.

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