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RECONSTRUCTING THE PRIMORDIAL SPECTRUM OF FLUCTUATIONS OF THE UNIVERSE FROM THE OBSERVED NONLINEAR CLUSTERING OF GALAXIES

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ABSTRACT

We have discovered that the nonlinear evolution of the two point correlation function in N-body experiments of galaxy clustering with $\Omega = 1$ appears to be described to good approximation by a simple general formula. The underlying form of the formula is physically motivated, but its detailed representation is obtained empirically by fitting to N-body experiments. We present here this formula, along with an inverse formula which converts a final, nonlinear correlation function into the initial, linear correlation function. We apply the inverse formula to observational data from the CfA, *IRAS*, and APM galaxy surveys, and hence reconstruct the initial spectrum of fluctuations of the universe, if $\Omega = 1$.

Subject headings: cosmology — galaxies: clustering

A prominent goal of modern cosmology is to determine the primordial spectrum of fluctuations which gave rise to the observed clustering of matter in the universe. Determining the primordial power spectrum, or equivalently its Fourier transform the two-point correlation function, from observations of the clustering of galaxies has been hampered by complementary problems on large and small scales. On large scales, the correlation function should have retained its initial shape, but it requires surveying a goodly volume of the universe, and is small and difficult to measure. On small scales, the correlation function is large and relatively easy to measure, but its shape has been modified substantially by nonlinear gravitational clustering.

In this *Letter* we report the empirical finding that the nonlinear evolution of the two-point correlation function in N-body experiments with $\Omega = 1$ appears to follow, at least approximately, a simple general formula, equation (4). The inverse of this formula, equation (6), permits reconstruction of the initial correlation function from an observed nonlinear correlation function, if $\Omega = 1$.

We consider a flat ($\Omega = 1$), zero cosmological constant ($\Lambda = 0$), statistically homogeneous, isotropic universe with fluctuations described by two-point correlation function $\xi(r, a)$ at comoving separation r and epoch when the cosmic scale factor is a. We work in comoving coordinates throughout this paper. Let r_0 be a Lagrangian comoving coordinate for the conserved pair integral (Peebles 1980, § 71)

$$r_0^3 \equiv \int_0^r (1 + \xi) dr^3 = r^3(1 + \bar{\xi}) , \tag{1}$$

where $\bar{\xi} \equiv r^{-3} \int_0^r \xi \, dr^3$ is the mean correlation function interior to r. A common notation is $J_3 \equiv \int_0^r \xi r^2 \, dr$, in terms of which $r_0^3 = r^3 + 3J_3$. By definition, the average number of neighbors of a mass point (a galaxy) enclosed within a surface of constant r_0 remains constant in time. Initially, when fluctuations are vanishingly small, the physical coordinate r and Lagrangian coordinate r_0 of a conserved pair surface coincide, $r = r_0$. As the universe evolves, fluctuations grow by gravity, and r decreases below r_0. In the linear regime in a flat universe, $\Omega = 1$, the mean interior correlation function $\bar{\xi}$ grows as the square of the cosmic scale factor, $\bar{\xi} \propto a^2$ (linear). Becoming nonlinear, clusters turn around, collapse by a factor of 2, and virialize. In the highly nonlinear regime, clustering becomes stationary, so that $\bar{\xi}$ at fixed r_0 grows as the cube of the cosmic scale factor, $\bar{\xi} \propto a^3$ (nonlinear).

The above considerations motivate the hypothesis that the time evolution of the physical radius r of a conserved pair surface, $r_0 = \text{constant}$, in an $\Omega = 1$ universe is a universal function of cosmic scale factor a, appropriately scaled. Equivalently, we hypothesize that $\bar{\xi}$ is a universal function

$$\bar{\xi} = \text{function of } [a^2 \bar{\xi}_0(r_0)] , \tag{2}$$

where $\bar{\xi}_0$ is determined by initial conditions, $\bar{\xi} = a^2 \bar{\xi}_0(r_0)$ (linear).

What elevates hypothesis (2) above the realm of the nebulous is the availability of N-body experiments to test it. The crucial experiments, for the present purpose, are the self-similar N-body experiments of Efstathiou et al. (1988, hereafter EFWD), which had $\Omega = 1$ and scale-free initial power spectra $\delta_k^2 \propto k^n$ with four different values for the index $n = -2, -1, 0, 1$. The advantage of self-similarity is that the entire history of the evolution of the correlation function is contained in its form at any one moment, and knowing the history is crucial to testing the hypothesis (2). In practice we were only able to use three of the self-similar experiments, with indexes $n = -1$, 0, and 1. The fourth experiment, $n = -2$, was not sufficiently self-similar (see Figs. 3, 4, and 5 of EFWD).

Figure 1*a* shows $\bar{\xi}$ as a function of $a^2 \bar{\xi}_0 = a^2 r_0^{-3-n}$ in the $n = -1, 0, 1$ self-similar experiments of EFWD. As can be seen, the data fall pretty much on top of each other, although the agreement is certainly not perfect. The agreement is general support for the hypothesis (2). In the experiments conserved pair surfaces collapse by about 1.8 on average after turnaround, rather than the virial factor of 2. At least part of this difference can be attributed to the finite softening length.

One consequence of hypothesis (2) which can be tested in non–self-similar as well as self-similar models, is that the

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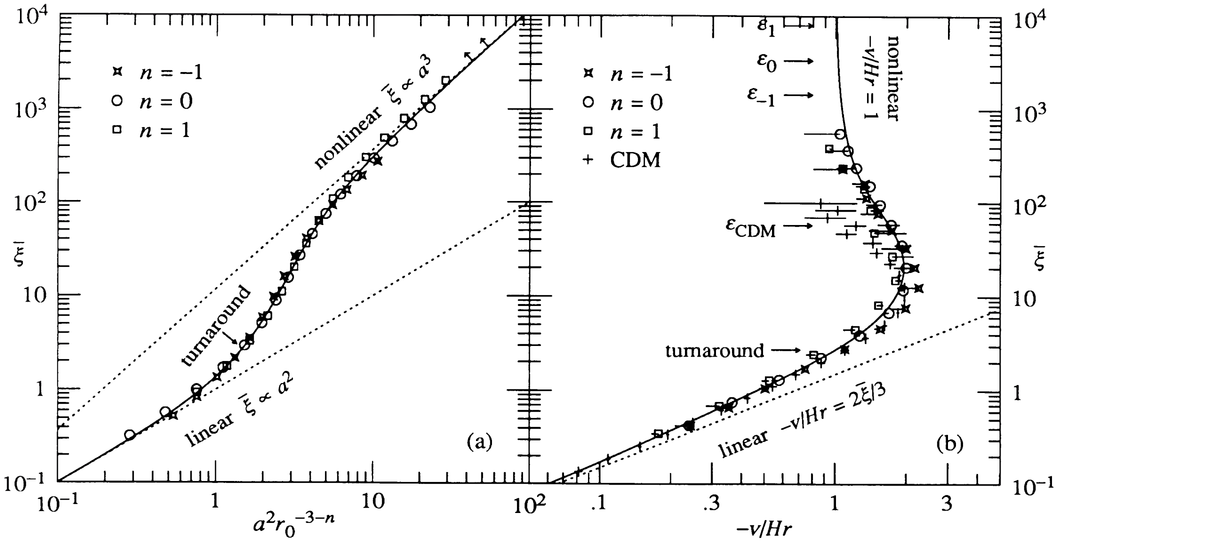

FIG. 1.—Mean interior correlation function $\bar{\xi}$ plotted against (*a*) cosmic scale factor squared times the initial mean interior correlation function, $a^2\bar{\xi}_0$, and (*b*) minus the dimensionless mean peculiar pair separation velocity, $-v/Hr$. Solid lines represent the rational function fit (4) to the data. Open symbols are data from the $n = -1, 0, 1$ self-similar experiments of EFWD, for which $\bar{\xi}_0 = r_0^{-3-n}$. Self-similar data represent the third from last output time graphed in Figs. 4 and 5 of EFWD; error bars on velocities represent the range over the adjacent two output times, i.e., the second and fourth from last output times. Crosses on the right-hand graph represent the unbiased $\Omega = 1$ CDM experiment of DEFW (their Figs. 4*a* and 7*a*) at the expansion factor of 1.8 when $\xi \propto r^{-1.8}$. Arrows labeled with an ϵ mark the softening lengths of each experiment. Arrows labeled turnaround mark the point of maximum proper expansion ($-v/Hr = 1$) of conserved pair surfaces. Dotted lines show linear and nonlinear asymptotes. Arrows on the nonlinear asymptote in Fig 1*a* show where this asymptote would be shifted if conserved pair surfaces collapsed by the virial factor 2 instead of the experimental value of 1.8.

dimensionless peculiar pair separation velocity v/Hr, where H is the Hubble constant,

$$\frac{v}{Hr} \equiv \frac{d\ln r}{d\ln a}\bigg|_{r_0} = -\frac{\partial \ln(1+\bar{\xi})}{3\,\partial \ln a}\bigg|_{r_0}, \tag{3}$$

should be a universal function of $\bar{\xi}$. Figure 1*b* shows $-v/Hr$ plotted against $\bar{\xi}$ for the $n = -1, 0, 1$ self-similar experiments of EFWD, and also for unbiased $\Omega = 1$ cold dark matter (CDM) from Davis et al. (1985, hereafter DEFW). Again, the data for the three self-similar models seem to line up more or less on top of each other. CDM does not agree so well, the CDM velocity falling below the self-similar velocity on scales less than about twice the CDM softening length $\epsilon_{\rm CDM}$. The relatively large softening length of the CDM experiment makes it difficult to assess the reality of this discrepancy.

We have fit the data shown in Figure 1*a* to a rational function:

$$\bar{\xi} = \frac{x + 0.358x^3 + 0.0236x^6}{1 + 0.0134x^3 + 0.00202x^{9/2}}, \quad x \equiv a^2\bar{\xi}_0(r_0). \tag{4}$$

The maximum deviation between the fit (4) and the experimental data shown in Figure 1*a* is 18%. Given an initial mean interior correlation function $\bar{\xi}_0$ in an $\Omega = 1$, $\Lambda = 0$ universe, formula (4) yields the evolved, nonlinear $\bar{\xi}$ at cosmic scale factor a as a function of the Lagrangian conserved pair coordinate r_0, and hence implicitly as a function of pair separation r through equation (1), $r^3 = r_0^3/(1+\bar{\xi})$. It is straightforward to derive the two-point correlation function ξ by differentiating, $\xi = \partial r^3\bar{\xi}/\partial r^3|_a$, and the peculiar pair separation velocity v through equation (3). Figure 1*b* shows as a solid line the predictions of formula (4) for v/Hr.

Figure 2*a* shows the predicted two-point correlation function ξ along with the experimental data for each of the four self-similar experiments of EFWD. The single formula (4) for the most part does a fairly decent job of reproducing the various bumps and wiggles in ξ, except on the smallest and (notably for $n = -2$) largest scales, where the softening length and boundary effects respectively depress the experimental ξ.

As another example, Figure 2*b* shows the two-point correlation function ξ for unbiased $\Omega = 1$ CDM at several redshifts, as derived by inserting into formula (4) the following initial mean interior correlation function $\bar{\xi}_{0,\rm CDM}$, which gives an excellent fit to the spectrum of Bardeen et al. (1986, eq. [G3]):

$$\bar{\xi}_{0,\rm CDM}(r_0) = 0.51\left[\ln\left(1+\frac{5}{r_0}\right)\right]^3 \frac{\ln(1+r_0/12)}{r_0/12} \times \frac{1 + 0.394r_0 + 0.00316r_0^2}{1 + 0.142r_0 + 0.00129r_0^2} \quad (r_0 \text{ in } \Omega^{-1}h^{-2}\text{ Mpc}) \tag{5}$$

where $h \equiv H_0/100$ km s^{-1} Mpc^{-1} is the present Hubble constant. Equation (5) is normalized so that the resulting nonlinear two-point correlation function most closely approximates $\xi \propto r^{-1.8}$ when $a = 1$. Also shown in Figure 2*b* are the results of DEFW's $\Omega = 1$ CDM *N*-body experiment at the expansion factor 1.8 when $\xi \propto r^{-1.8}$ (DEFW's CDM spectrum is not the same as that of Bardeen et al. 1986, but it is virtually indistinguishable over the range of the experiment). The predicted and experimental ξ seem in satisfactory agreement, notwithstanding the discrepancy in peculiar velocities previously remarked in Figure 1*b*.

Just as formula (4) predicts how the two-point correlation function evolves from given initial conditions, so its inverse can postdict the initial correlation function from the observed nonlinear correlation function at any subsequent time, like now, provided $\Omega = 1$ and $\Lambda = 0$. The following minimax fit to the inverse of formula (4) is accurate to better than 1% over its entire range:

$$a^2\bar{\xi}_0 = \bar{\xi}\left(\frac{1 + 0.0158\bar{\xi}^2 + 0.000115\bar{\xi}^3}{1 + 0.926\bar{\xi}^2 - 0.0743\bar{\xi}^3 + 0.0156\bar{\xi}^4}\right)^{1/3}. \tag{6}$$

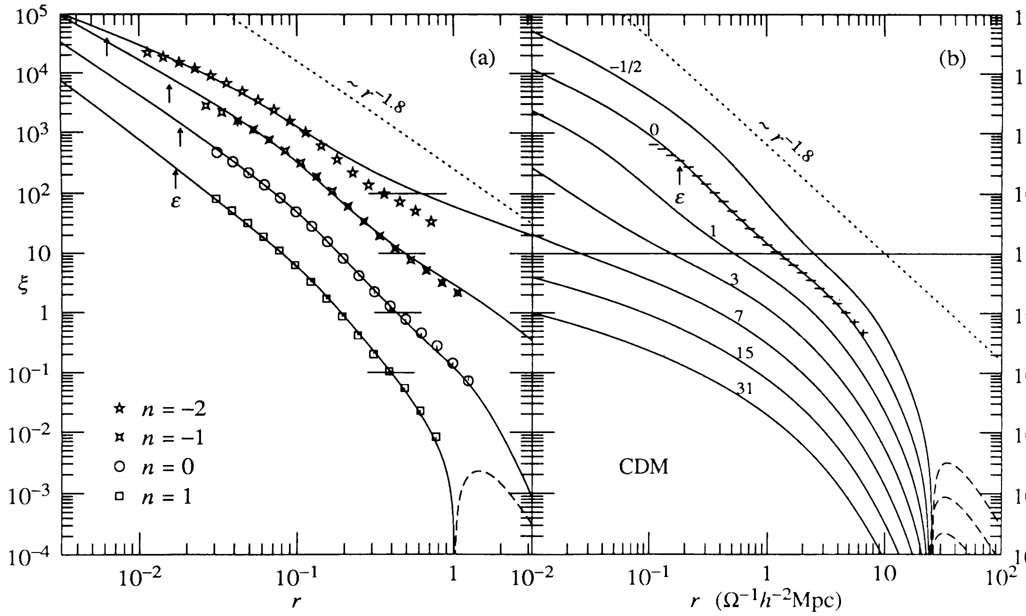

FIG. 2.—Two-point correlation function ξ as a function of separation r in (a) the $n = -2, -1, 0, 1$ self-similar solutions, and (b) unbiased $\Omega = 1$ CDM at several redshifts, as marked. Solid lines show the predictions of formula (4), with broken lines showing $-\xi$ when ξ is negative. For clarity, self-similar ξ are shifted vertically by 10^{-n}; horizontal lines mark where $\xi = 1$. Self-similar data are from Fig. 4 of EFWD, third from last output time, with error bars (omitted for $n = -2$) representing the range over the adjacent two output times, appropriately scaled to the third from last output time. The scale of r is such that $\bar{\xi}$ asymptotes to $\bar{\xi} = r^{-3-n}$ on large scales. CDM data and error bars are from Fig. 4a of DEFW, expansion factor 1.8. Vertical arrows mark the softening length in each experiment. Dotted lines illustrate, for reference, the classic $r^{-1.8}$ shape ascribed to the observed galaxy correlation function.

The mean interior correlation function $\bar{\xi}$, equation (1), can be extracted directly from observations as a function of separation r, by counting the average number of neighbors within distance r of a galaxy. Given an observed $\bar{\xi}$, formula (6) yields the initial mean interior correlation function $\bar{\xi}_0$ as a function of separation r, and hence implicitly it yields $\bar{\xi}_0$ as a function of the initial comoving separation r_0 through equation (1), $r_0^3 = r^3(1 + \bar{\xi})$.

We have applied formula (6) to reconstruct the initial correlation function of the universe from observational data, namely the CfA redshift survey (Huchra et al. 1983), the APM two-dimensional galaxy survey (Maddox et al. 1990b), and the *IRAS* redshift survey (Strauss et al. 1990). For the CfA survey on scales $\geq 2h^{-1}$ Mpc, we took J_3 from Table 2 of Davis & Peebles (1983). On smaller scales, the CfA redshift correlation function is significantly affected by peculiar velocities, so there we used the correlation function derived by Davis & Peebles (1983), their Figure 3, from the projected correlation function. For the APM survey, we converted the angular correlation function of Maddox et al. (1990a, Fig. 1) to a

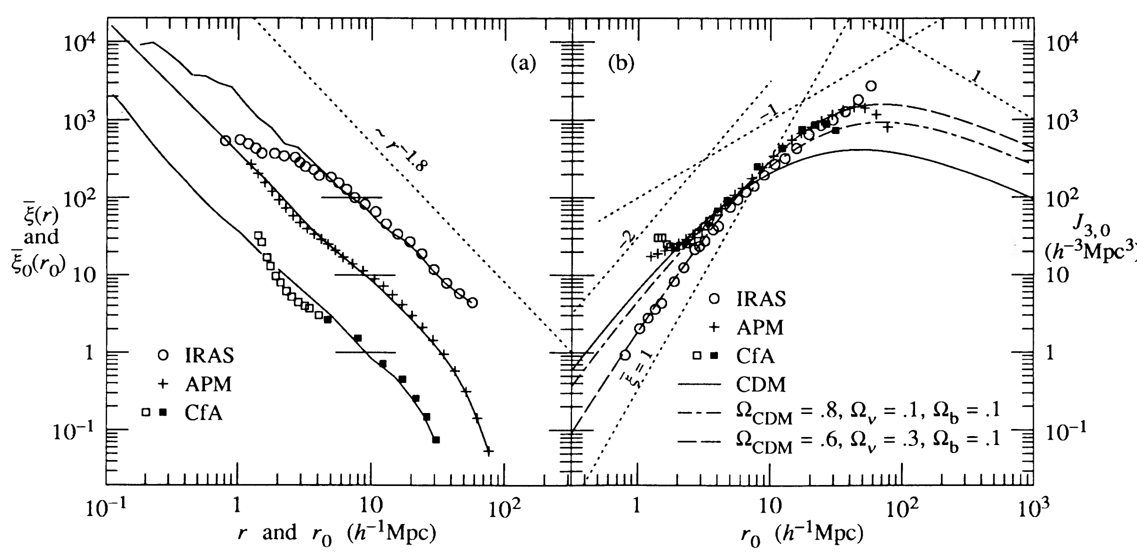

FIG. 3.—(a) Observed mean interior correlation function $\bar{\xi}(r)$ (*solid lines*), along with reconstructed initial mean interior correlation function $\bar{\xi}_0(r_0)$ (*symbols*), for each of the CfA, APM, and *IRAS* galaxy surveys. For the CfA, the open and filled squares are based on the projected and redshift correlation functions, respectively (Davis & Peebles 1983). For clarity, APM and *IRAS* correlation functions have been shifted upward by factors of 10 and 100; horizontal lines mark where $\bar{\xi} = 1$. Dotted line shows, for reference, the classic $r^{-1.8}$ shape. (b) Reconstructed initial $J_{3,0} \equiv \int_0^{r_0} \xi_0 r_0^2 dr_0 = r_0^3 \bar{\xi}_0/3$ (*symbols*) for each of the CfA, APM, and *IRAS* galaxy surveys. Solid line shows standard scale-invariant CDM with $\Omega = 1$ and $h = 0.5$, eq. (5), normalized upward by 2.5. Broken lines show two hybrid models from Holtzman (1989), again with total $\Omega = 1$, $h = 0.5$, and a scale-invariant ($n = 1$) primordial spectrum, but with matter comprising CDM, one species of heavy neutrino, and a seasoning of baryons, with Ω values as indicated. Dotted lines illustrate power laws with various power spectrum indexes n, as marked.

three-dimensional correlation function ξ using the z-dependent luminosity function suggested by Maddox et al. (1990a) (after correcting the misprint), and taking into account the evolution of ξ in the linear approximation. Our inferred correlation function gave an excellent fit to the angular correlation function of Maddox et al. (1990a). For the *IRAS* survey, we took a weighted average of the redshift correlation functions graphed by Davis et al. (1988, Fig. 1*b*). *IRAS* galaxies may be less affected than optical samples by peculiar velocities, since they avoid rich clusters and show no "finger of god" effect (Yahil 1988).

Figure 3*a* shows the resulting observed nonlinear mean interior correlation function $\bar{\xi}(r)$, along with the reconstructed initial mean interior correlation function $\bar{\xi}_0(r_0)$, for each of the CfA, APM, and *IRAS* galaxy surveys. The lower correlation function of the *IRAS* galaxies on scales $\lesssim 2h^{-1}$ Mpc presumably results at least in part from peculiar velocities.

Figure 3*b* shows the reconstructed initial $J_{3,0} \equiv \int_0^{r_0} \xi_0 r_0^2 dr_0$ from the three surveys, compared to three models. The solid line is standard scale-invariant CDM, equation (5), with $\Omega = 1$ and $h = 0.5$, normalized upward by a factor of 2.5. As widely advertized, standard CDM does not have enough power on large scales. However, almost anything one can think of adds large-scale power to CDM, without compromising Ω and h—for example, baryons, neutrinoes, a cosmological constant, isocurvature instead of adiabatic fluctuations, or a flatter than $n = 1$ primordial spectrum. Figure 3*b* illustrates two relatively unradical models which do a better job of fitting the observations. The models, from Holtzman (1989), have $\Omega = 1$, $h = 0.5$, and a scale-invariant primordial spectrum, but in addition to CDM they include one species of heavy neutrino with $\Omega_\nu = 0.1$ and 0.3, respectively (neutrino mass 2 eV and 7 eV), plus $\Omega_b = 0.1$ in baryons thrown in for good measure.

We conclude with the caveat that in reconstructing the primordial correlation function from observations of galaxy clustering we have assumed that galaxies are an unbiased tracer of mass. But that is another interesting matter.

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