

DISKS IN ELLIPTICAL GALAXIES

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ABSTRACT

We investigate the abundance and strength of disk components in elliptical galaxies by studying the photometric properties of models containing a spheroidal $r^{1/4}$ -law bulge and a weak exponential disk. For such idealized models the disk can be detected by a characteristic deviation of the isophotes from perfect ellipses (residuals of the form $\delta I = a_4 \cos 4\theta$ with $a_4 > 0$) or by deviations of the luminosity profile from a perfect $r^{1/4}$ law. The a_4 test is quite sensitive to weak disks if they are nearly edge-on. However, for random viewing angles, even disks containing 20% of the total light are expected to be too face-on for the a_4 signal to be detectable in more than half of the cases. For most such galaxies the residuals in the luminosity profile are also too small for an unambiguous detection of the disk. "Pointed" isophotes are observed in a substantial fraction of elliptical galaxies. If we interpret these isophote distortions in the framework of our models, then the statistics of observed samples suggest that *almost* all radio-weak (or "nonboxy") ellipticals could have disks containing roughly 20% of the light. We show the "E5" galaxy NGC 4660 to have the photometric signatures of a disk containing a third of the light.

Subject headings: galaxies: photometry — galaxies: structure — stars: stellar dynamics

1. INTRODUCTION

For a long time theoretical interest in elliptical galaxies concentrated on their high degree of symmetry and their apparently simple structure. These properties allowed a rigorous and even partly analytic treatment of their dynamics. Surprising observational findings, such as their low rotation speed (Bertola and Capaccioli 1975) and their isophote twists (e.g., Leach 1981), were explained elegantly in terms of velocity anisotropies and triaxial models (e.g., Binney 1978).

With the advent of CCDs the observed isophotes in elliptical galaxies were found to deviate systematically from ellipses at the percent level (Lauer 1985; Jedrzejewski 1987; Franx 1988; Bender, Döbereiner, and Möllenhoff 1988, hereafter BDM). One particular form of distortion is found frequently in otherwise regular galaxies; if the residual intensity variations along a best-fitting ellipse are expanded as a Fourier series in the azimuthal angle θ , the coefficient a_4 , associated with the $\cos 4\theta$ term, is found to have a relatively large amplitude. If a_4 is negative, the isophotes are *boxy*; if a_4 is positive the isophotes are *pointed*. (For an illustration, see Fig. 5 of Jedrzejewski 1987. Note that the term "pointed," used throughout the paper, is equivalent to the terms "disky" or "lemon-shaped" used by some other authors.) In their "almost statistical" sample of ellipticals, BDM found such distortions to be very common; roughly one-third of the objects showed pointed isophotes, and a comparable fraction showed boxy isophotes. The amplitudes of these distortions are typically about 1%.

In this paper we are interested in galaxies that show positive a_4 . There is no good physical reason why ellipticals should have perfectly elliptical isophotes. A strong population of particular orbital families could lead to nonelliptical isophotes: Binney and Petrou (1985) show orbits whose superposition can produce a boxy appearance, and Contopoulos and Grosbøl (1989), in an analysis of triaxial bar potentials, found an orbital family which can yield pointed isophotes. Here we investigate a simple, natural, but, as just mentioned, by no means exclusive, interpretation of ellipticals with pointed isophotes: We assume that these galaxies are composed of a flat disk component

superposed on a truly spheroidal bulge, a morphology similar to that of S0 galaxies, but with a more dominant spheroid. If such a disk is assumed to be nearly edge-on, a disk with about 1% of the total light can produce the observed amplitudes for a_4 . For example, Carter (1987) modeled NGC 4697, which has an a_4 parameter of +0.02, with a highly inclined disk which contributes only 2% of the total light. This assumption of a weak, nearly edge-on disk has been adopted by all authors who have tried to estimate such a disk contribution quantitatively (Carter 1987; Jedrzejewski, Davies, and Illingworth 1987; Capaccioli 1987; Capaccioli, Pionto, and Rampazzo 1988), and has led them all to conclude that disk components are very weak and dynamically unimportant.

In this present work we develop tools for assessing the importance of disks. We emphasize that our primary interest lies not in qualitative classification issues ("E" versus "S0") but rather in the clues that disks may give about the formation history and dynamics of these objects. Specifically, we investigate how observables such as the luminosity profile, $I(r)$, the ellipticity profile, $\epsilon(r)$, and $a_4(r)$ relate to physical parameters such as the spheroid-to-disk ratio, disk inclination, and the scale lengths of spheroid and disk. We choose a very simple model for the light distribution: an oblate spheroid of constant ellipticity and an exponential disk. With this model we explore the space of *physical* parameters, and "project" our models into the plane of *observable* parameters by creating pseudodata and performing surface photometry on them.

In the present paper we will use these models to answer the following questions: For which regions of parameter space do disks result in measurable isophote distortions? Are the observed isophote distortions predominantly caused by very weak, highly inclined disks? Can we estimate detection probabilities for such weak disks? What is the expected distribution of a_4 for random viewing angles? What fraction of "ellipticals" *might* have disks which are strong enough to be dynamically and kinematically important? These answers should be viewed as preliminary steps toward answering more fundamental questions regarding the structure and formation of early-type

galaxies, such as: Is there true continuity in this part of the Hubble sequence with the spheroid-to-disk ratio as the determining parameter? How different are the formation mechanisms for various types of “early” galaxies? If pointed ellipticals have a disk, did they form from mergers and acquire a weak disk through a later episode of gas accretion?

Some of these questions have been addressed previously. For example, Michard (1984) and Capaccioli, Caon, and Rampazzo (1990) have drawn attention to photometric similarities between highly flattened E and S0 galaxies. Nieto (1988) and Capaccioli, Piotto, and Rampazzo (1988) have pointed out that a substantial fraction of galaxies classified as elliptical may have weak undetected disks. Similarly, van den Bergh (1990) has argued, from the deficiency of the RSA Catalog (Sandage and Tammann 1981) in face-on S0's, that many faint ellipticals are misclassified S0's. Furthermore, Bender (1988) has shown that the isophote shapes correlate with kinematical properties.

In the rest of this paper we give quantitative answers to some of the above questions. In § II we characterize our photometric models. Section III describes how disks of various strengths and inclinations are reflected in the isophote shapes. Section IV addresses the detection probability of disks. Alternative photometric means of detecting such disks are discussed in § V, where we also model the disk in NGC 4660. Section VI gives a summary and conclusions.

II. PHOTOMETRIC MODELING

To analyze the photometric effects of an exponential disk superposed on a spheroid, we create a sequence of “pseudo-CCD” frames and analyze them with surface photometry programs kindly provided by M. Franx (see Franx, Illingworth, and Heckman 1989). In this section we give details of our photometric models and our “observing” procedure.

The spheroidal components of our galaxies are represented by an oblate spheroid with a constant intrinsic ellipticity ϵ_0 and an $r^{1/4}$ law luminosity profile with effective radius r_{eff} and total luminosity L_B . The disks are taken to be exponential, with scale length r_{exp} and total light contribution L_D . The disk lies in the equatorial plane of the spheroid, and the whole galaxy is inclined with respect to the line of sight at an angle i . Thus the disk isophotes are ellipses with ellipticity $1 - \cos i$, and the spheroid isophotes are ellipses with ellipticity

$$\epsilon = 1 - \sqrt{\cos^2 i + (1 - \epsilon_0)^2 \sin^2 i}.$$

In addition to the overall scale parameters L_B and r_{eff} , this parameterization gives us the following four “shape” parameters: $\cos i$, $r_{\text{exp}}/r_{\text{eff}}$, L_D/L_B , and ϵ_0 . For comparison with observations we need to simulate the photometry over an appropriate radial range. We want our models to represent nearby bright ellipticals, as in the observational samples of Djorgovski (1985), Jedrzejewski (1987), BDM, and Franx (1988). BDM find $\langle r_{\text{eff}} \rangle \approx 20''$ for their sample, with a scatter of less than a factor of 2. Assuming that CCD photometry is conventionally done in the range $2''$ – $80''$, we perform “photometry” on our models over the range, $0.1r_{\text{eff}}$ to $4r_{\text{eff}}$. In practice, our data frame consists of a 512×512 array on which we placed a (in most cases noiseless) superposition of the spheroid and the disk. The photometric parameters we want to extract from these models are the luminosity profile, $I(r)$, the ellipticity profile, $\epsilon(r)$, and the profile of residual Fourier coefficients [e.g., $a_4(r)$]. These residual coefficients will be defined, and their significance will be discussed in the next section.

Before proceeding, we should state quite clearly the limi-

tations of this model. Although we assume the spheroid is oblate and has constant ellipticity, we know from measured isophote twists and other photometric evidence that neither assumption holds in many real ellipticals and S0's. Furthermore, an $r^{1/4}$ -law luminosity profile has little physical justification and is known to hold only approximately (within 0.1 mag) in most cases. However, we wanted to keep our modeling as straightforward as possible, because any additional freedom would cause a proliferation of free parameters and thus obfuscate the reader who has to wade through the results.

III. ISOPHOTE DISTORTIONS FROM DISK COMPONENTS

a) The Significance of Fitting Residuals

Let us summarize the isophote fitting procedure and the definition of the fitting residuals. To fit an isophote, the image intensity is first sampled along a trial ellipse. This intensity string, $I(\theta)$, is subsequently analyzed in a Fourier series:

$$I(\theta) = I_0 + \sum_{i=1}^N a_i \cos i\theta + b_i \sin i\theta,$$

where θ is the azimuthal angle as measured from the major axis and N determines the truncation of the Fourier expansion. In this paper we used $N = 6$. The first- and second-order expansion coefficients are used to improve the fitting parameters, the center of the ellipse, the ellipticity, and the position angle. Starting from these new parameters, the procedure is repeated until some convergence criterion is satisfied. For perfectly elliptical isophotes the intensity should then be constant along the sampled annulus, and all Fourier coefficients except I_0 should vanish. Since the fitting forces the first- and second-order coefficients to be very small, significant residuals can only be of order 3 or higher. A disk component will always have a higher apparent ellipticity than the spheroid, and will lead to excess intensity near the major axis. Thus it is expected to give residual Fourier components that have even symmetry and a maximum on the major axis ($\theta = 0$), i.e., $\cos 2n\theta$, with $n \geq 2$. Furthermore, the dominant coefficient, usually a_4 , must be positive. Indeed, most of the “power” of the disk signature in such an expansion is concentrated in a_4 . Any nonvanishing residuals with other symmetry cannot be explained by this model and must be attributed to tidal distortions, dust obscuration, or an intrinsically complex structure (Franx 1988). A positive a_4 parameter should be taken as indicating a disk only if it is the dominant residual; for the majority of galaxies with significant a_4 this appears to be the case (BDM), although there are exceptions. Franx, Illingworth, and Heckman (1989) have shown that b_4 is of comparable amplitude in some cases and have argued that this phenomenon could be explained by the superposition of a triaxial bulge and a disk.

Before we discuss our simulations, we would like to address one further point: The light that can be attributed to “disky” components is usually assessed by subtracting the best-fitting model of elliptical isophotes from the data and comparing the integrated residual light to the total light of the fitted model (see, e.g., Jedrzejewski 1987). This procedure results in disk-to-spheroid ratio estimates of at most a few percent. However, as we show below, it underestimates the disk contribution systematically. This error can be very substantial.

b) The a_4 Signature of Disks

In this section we will investigate how much $a_4(r)$ can tell us quantitatively about the presence of disks. Before carrying out

a systematic search of the parameter space, let us illustrate our method by analyzing the isophotes of a sequence of four models. The data we extract from these models are the ellipticity, $\epsilon(r)$, the isophote fitting residual, $a_4(r)$, and $\delta I(r)$, the residual from the best-fitting $r^{1/4}$ law over the range $0.1r_{\text{eff}} < r < 4r_{\text{eff}}$. Here r is the mean radius of the fitting ellipse, defined as the geometric mean of the major and minor axes, $r = (ab)^{1/2}$. The spheroid ellipticity, ϵ_0 , was set to be 0.35, as a fiducial value to represent a range of bulge ellipticities from 0.15 to 0.55. We varied the disk inclination, $\cos i$, from 0.1 to 0.4, and chose (by experiment) L_D/L_B to yield a maximum a_4 value, $a_{4,m}$, of about 2% in each case. The results are shown in Figure 1.

This sequence illustrates a number of points. First, even with our simple model the a_4 parameter does not yield unique information about the disk component. Even for fixed ϵ_0 and $r_{\text{exp}}/r_{\text{eff}}$, there is a tradeoff between disk mass and disk inclination that can result in very similar a_4 profiles. Second, the above-mentioned notion (e.g., Jedrzejewski 1987) that “ $n\%$ ” of a_4 translates into roughly an “ $n\%$ ” disk light contribution could lead to an underestimate of the disk fraction by an order of magnitude! Third, we find that the maximum of a_4 always occurs near the maximum disk light contribution along the major axis, $r_{md} \approx 2.4(r_{\text{exp}}/r_{\text{eff}})^{1/3}$. This formula can be derived by simply comparing the relative contributions of an $r^{1/4}$ law and an exponential profile of given scale lengths. For the above sequence $r_{\text{exp}}/r_{\text{eff}}$ was chosen so as to keep the radial coordi-

nate of $a_{4,m}$ constant, taking into account that this radial coordinate is the geometric mean of the apparent major and minor axes and thus depends on the apparent ellipticity of the system at this radius. Finally, we note that the profiles have qualitatively similar shapes, so that we can use $a_{4,m}$ to characterize the a_4 profile.

We now turn to a more systematic investigation of the dependence of a_4 on our model parameters, ϵ_0 , $\cos i$, L_D/L_B , and $r_{\text{exp}}/r_{\text{eff}}$. An exploration of parameter space for these four quantities shows that the dependence of a_4 on $r_{\text{exp}}/r_{\text{eff}}$ and ϵ_0 is substantially weaker than its dependence on L_D/L_B and $\cos i$. We considered parameters in the ranges $0.15 < \epsilon_0 < 0.55$ and $0.25 < r_{\text{exp}}/r_{\text{eff}} < 2$. The range of $r_{\text{exp}}/r_{\text{eff}}$ was chosen to allow comparison with the existing data samples. For the galaxies of interest these limits imply $5'' < r_{\text{exp}} < 40''$, which spans the range in which isophote shapes can be well measured without being affected by seeing. The variation in a_4 for both parameters is only about 50%, and we decided to use fiducial values of $\epsilon_0 = 0.35$ and $r_{\text{exp}}/r_{\text{eff}} = 0.75$.

We consider values for L_D/L_B ranging from 2% to unity. For $\cos i$ we choose limits of 0.05 and 0.95. We do not consider more edge-on disks because our models do not include any vertical disk structure. For random viewing angles, $\cos i$ is uniformly distributed between 0 and 1. We therefore exclude only a small fraction of cases by setting $\cos i_{\text{min}} = 0.05$. Figure 2 shows the dependence of a_4 on $\cos i$ for several values of L_D/L_B . We have only shown the range $0.5\% < a_4 < 10\%$, since

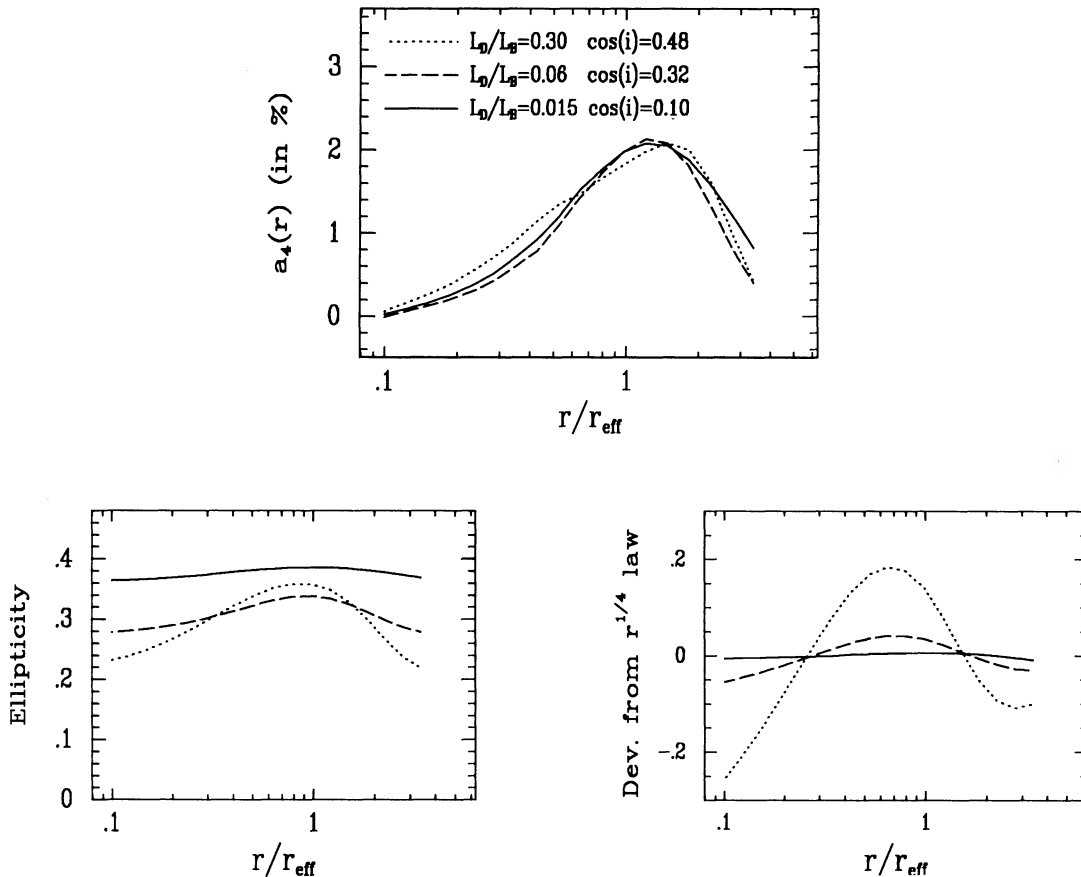


FIG. 1.—Top panel: The a_4 profile for three models. Although the disk light fraction L_D/L_B varies by a factor of 20 between these models, the a_4 profiles are very similar. This illustrates that the effects of disk strength and disk inclination on $a_4(r)$ are indistinguishable. Bottom panels: The run of the ellipticity (left) and the deviations of the luminosity profile from an $r^{1/4}$ law (right) for the same three models.

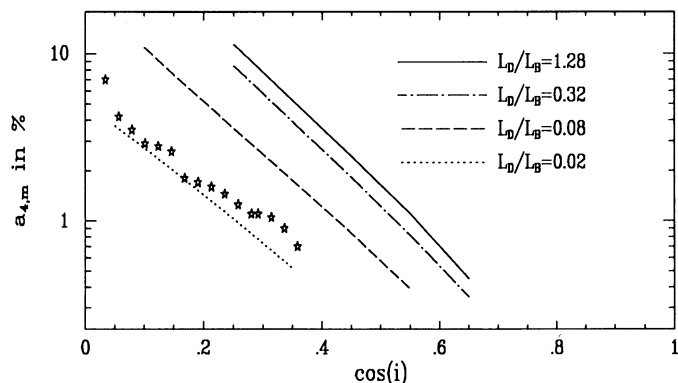


FIG. 2.—The four lines illustrate the dependence of $a_{4,m}$ on $\cos i$ for four different disk strengths, L_D/L_B . For any given L_D/L_B , the maximal a_4 declines exponentially toward more face-on viewing angles. Since for random viewing angles $\cos i$ is evenly distributed, any observed cumulative distribution of a_4 is expected to have this same functional form. The stars indicate the cumulative distribution of a_4 measured by BDM (1987), which, in its shape, is consistent with the model prediction. Note, however, that if any edge-on objects are missing in the BDM sample (because they were classified as S0's), the data points will shift to the right, toward higher disk fractions.

for bigger a_4 the disk is obvious to the eye, and smaller a_4 values are hard to measure observationally. We see that, for given L_D/L_B , $a_4(\cos i)$ is well approximated over this range by an exponential decline. In particular, we note that $a_{4,m}$ is small ($<1\%$) for $\cos i > 0.6$ for all L_D/L_B . In addition, Figure 2 shows that, for suitable inclination, a_4 is of the order of a few percent for a wide range of L_D/L_B . Finally we note that $a_{4,m}$ varies by large factors over our chosen parameter range for L_D/L_B and $\cos i$. This is an *a posteriori* justification for ignoring variations in $r_{\text{exp}}/r_{\text{eff}}$ and ϵ_0 ; these changed $a_{4,m}$ by factors of 1.5 or less over the range of interest.

IV. DETECTION PROBABILITIES

As mentioned above, the $\cos i$ axis can be considered a probability axis. We now exploit this fact to make statistical arguments about the possible abundance and strength of disks in observed “ellipticals.” From Figure 2 we see that for given L_D/L_B , $\log a_{4,m}$ is roughly a linear function of $\cos i$ over the range plotted. For random viewing angles there is an equal probability of measuring $a_{4,m}$ in each interval, $\Delta \log a_{4,m}$, for $0.5\% < a_{4,m} < a_{4,m,0}$, where $a_{4,m,0}$ is the value of $a_{4,m}$ for edge-on disks. Since this holds true for each L_D/L_B , it also holds approximately for any distribution of L_D/L_B as long as the measured values of $a_{4,m}$ are less than $a_{4,m,0}$ for most of the population. Thus, treating the $\cos i$ axis as a probability axis, the cumulative distribution of observed a_4 values is expected to have the same functional form in this diagram as our theoretical lines. This then allows us to estimate a “typical” value for L_D/L_B by comparing, say, the number of objects with $1\% < a_{4,m} < 5\%$ to the number of objects with $a_{4,m} < 1\%$ (including all objects without detectable a_4).

Only BDM’s “nearly statistical” sample contains a large enough number of galaxies for such statistical tests. We extracted $a_{4,m}$ values (from BDM’s figures) for all objects in which a_4 was the dominant (positive) residual. Assuming that these a_4 values indeed do arise from a disk component, we sorted the a_4 ’s, normalized their abscissa values by the total number of sample members, and plotted them as asterisks in Figure 2. The distribution of observed a_4 values is, at least qualitatively, consistent with the hypothesis that they arise

from disk components seen at random angles. Note that this construction assumes that *no* edge-on galaxies are missing because they were classified as S0’s, a situation which is quite improbable if our basic disk hypothesis about galaxies with pointed isophotes is correct. Also, van den Bergh’s (1990) result shows that the Hubble type classification of early galaxies is probably viewing-angle-dependent. Unfortunately, there is also some ambiguity as to which of the elliptical galaxies should be included in the total sample. All galaxies were included in Figure 2, but Bender (1988) argues that the radio-loud ellipticals should be considered as qualitatively different objects from the other galaxies. His argument is based on the very good correlation between radio luminosity and isophote shape (Bender *et al.* 1989): the majority of the boxy ellipticals have $L_R > 10^{21} \text{ W Hz}^{-1}$ (at 1.4 GHz), while *none* of the significantly pointed galaxies have $L_R > 10^{21} \text{ W Hz}^{-1}$. It thus seems that strong radio emission indicates a structurally distinct population of galaxies. Note that if we were to exclude all galaxies in BDM’s sample which show significant boxiness at all radii (eight objects), we would obtain virtually the same results from a statistical analysis as we obtain by excluding all objects with strong radio emission. Thus it does not matter in this context which criterion is chosen to distinguish the potential two classes of ellipticals.

If we include all of Bender’s objects in our disk fraction estimate outlined above (i.e., 12 objects with $1\% < a_{4,m} < 5\%$ out of a total of 47 objects), we obtain $L_D/L_B \approx 0.05$; if we exclude the nine radio-loud objects ($L_R > 10^{21} \text{ W Hz}^{-1}$), then 12 out of 38 objects have large a_4 , and we obtain $L_D/L_B \approx 0.30$. Purely statistical errors in these fractions also produce large uncertainties in L_D/L_B , reflecting the relatively weak sensitivity of a_4 to L_D/L_B . The main conclusion of this exercise would seem to be that disks containing of the order of 20% of the light could be present in most ellipticals without contradicting the a_4 data. Furthermore, “typical” disks of this order are actually suggested by the rather substantial fraction of radio-weak ellipticals which have $a_{4,m} > 1\%$.

Given these uncertainties, let us address a more general problem: For any given L_D/L_B , what is the probability that such a disk can be detected photometrically? As we have seen, for $\cos i > 0.6$ the isophotes shapes yield hardly any information even for strong disks. In such cases the major-axis profile might be used as an alternative indicator of a disk. Although we have no secure *a priori* knowledge of what the underlying luminosity profile (without disk) might be, it appears that an $r^{1/4}$ law approximates $I(r)$ within 0.1 mag for the majority of giant ellipticals (Burstein *et al.* 1987), when fitted over a factor of 20 or so in radius.

For our models we evaluate the rms deviation, $\overline{\delta I}$, of the *major-axis profile* from an $r^{1/4}$ law (where the influence of a disk component is maximized), fitted over a factor of 40 in radius, and we use it as a second observable signature of the presence of disks. Although in real galaxies there is no rigorous way to decide which deviations are significant and which ones are not, it seems reasonable to use the luminosity profile to put upper limits on L_D/L_B , because it is unlikely that an underlying spheroid profile will conspire with an exponential disk to mimic an $r^{1/4}$ law. With these two disk detection criteria, we can estimate the fraction of disk systems which still eludes recognition. Figure 3 shows contours in the $(L_D/L_B, \cos i)$ -plane both for $a_{4,m}$ and for $\overline{\delta I}$. The solid contours present essentially the same information as in Figure 2. They again demonstrate that a_4 detects disks very efficiently when $\cos$

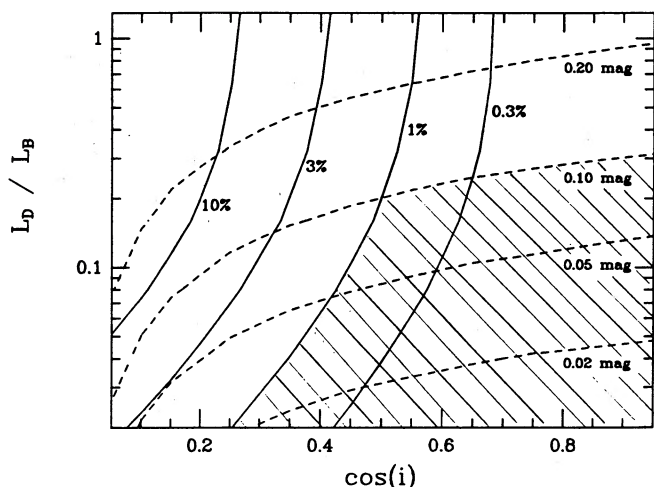


FIG. 3.—Impact of a disk component on the isophote shapes and the radial profile in the $(L_D/L_B, \cos i)$ parameter plane. Loci of constant $a_{4,m}$ (solid lines) and constant rms deviation from an $r^{1/4}$ law, δI (dashed lines), are indicated. It is apparent that for highly inclined disks ($\cos i < 0.40$), a_4 is a sensitive diagnostic, while for low inclinations ($\cos i > 0.6$), there is virtually no a_4 signature, independent of L_D/L_B . For nearly face-on disks the luminosity profile must be employed to detect any disk component. If we adopt $a_4 > 1\%$ and $\delta I > 0.1$ mag as our detection criteria, then disks in the shaded area of the parameter plane are undetectable by photometric means. Since the $\cos i$ axis can be interpreted as a probability axis, this implies that 50% of all disks with $L_D/L_B < 0.25$ cannot be detected by photometric means.

$i < 0.4$, but very inefficiently when $\cos i > 0.6$. The dashed lines are contours of δI . Requiring $\delta I > 0.1$ mag to detect a disk, we see that, for $L_D/L_B < 0.25$, a_4 is a more sensitive indicator of disks than the luminosity profile. The shaded area indicates the region of parameter space in which disks cannot be detected photometrically, adopting $a_{4,m} > 1\%$ and $\delta I > 0.1$ mag as detection criteria.

Again we recall that the $\cos i$ axis can be regarded as a probability axis. Figure 3 then shows that, for random viewing angles and for disk-to-spheroid ratios of less than $\frac{1}{4}$, fewer than 50% of the disks can be detected photometrically. Consequently, there should be many undetected systems with disks as strong as those found so far. Using BDM's sample, and adopting Bender's (1988) view that the ellipticals, which have strong radio emission (or are significantly boxy at all radii), are fundamentally different objects from those which do not, we find that somewhat less than half of these "quiet" objects show a significantly positive a_4 parameter. If these pointed isophotes are due to the presence of a disk, this again implies that almost all the other "quiet" ellipticals should also have disk components. Figure 3 also shows that there is no reason to suppose that most of these disks are extremely weak.

It seems that disk components may not be small "flaws" of a perfectly ellipsoidal model but rather might turn out to be substantial components, which have important consequences for the kinematics and dynamics.

V. OTHER PHOTOMETRIC SIGNATURES OF DISKS

In the last section we were forced to retreat to the somewhat swampy ground of statistics because the knowledge of a_4 alone was insufficient to determine both L_D/L_B and $\cos i$. We now discuss whether additional photometric information can overcome this ambiguity.

a) Luminosity and Ellipticity Profiles

As seen in the bottom panels of Figure 1, for a given a_4 both $\epsilon(r)$ and $\delta I(r)$ are, in principle, capable of differentiating between various $(L_D/L_B, \cos i)$ combinations. The obvious problem is that this is true only if the underlying structure of the subcomponents is well described by our model. Nevertheless, there are two regimes in which $\epsilon(r)$ and $\delta I(r)$ can provide useful constraints. First, they can yield upper limits on L_D/L_B by assuming that the observed profile should not be flatter or smoother than predicted by the model. Second, our models predict both $\epsilon(r)$ and $\delta I(r)$ to have a very characteristic shape in the presence of a disk. For example, the ellipticity shows a maximum at the radius of the maximum disk light contribution, as originally pointed out by Michard (1984). Therefore, if the observed variations in, say, $\epsilon(r)$ match the predictions of the models, then more significance can be attributed to the fit. We will illustrate this point in § Vc.

b) Higher Order Residuals

As mentioned before, a_4 is not the only Fourier residual with the appropriate symmetry to show the presence of a disk; the next higher order one is a_6 . We implemented the calculation of a_6 in the isophote analysis software to test its usefulness in untangling $\cos i$ and L_D/L_B . We found that in all cases a_6 is smaller than a_4 , but that for high inclinations ($\cos i < 0.2$) the two coefficients are of comparable size. The relation of a_6 to a_4 as a function of $\cos i$ is shown in Figure 4, for several values of L_D/L_B . For inclinations lower than $\cos i = 0.40$, a_6 is almost negligible.

How feasible is it to measure a_6 ? To answer this, we added various amounts of Poisson noise to the "data" frames before analyzing the isophotes. We find, both from the "smoothness" of $a_4(r)$ and $a_6(r)$ and from the formal error calculation, that a_6 is as robust as a_4 with respect to Poisson noise. Therefore, a_6 should be measurable to an accuracy level comparable to that for a_4 . At present the only published data on a_6 (Michard and Simien 1988) are not of sufficient quality to test this idea. Nevertheless, a_6 seems like a good diagnostic for highly inclined disks. One caveat is that a boxy bulge in conjunction

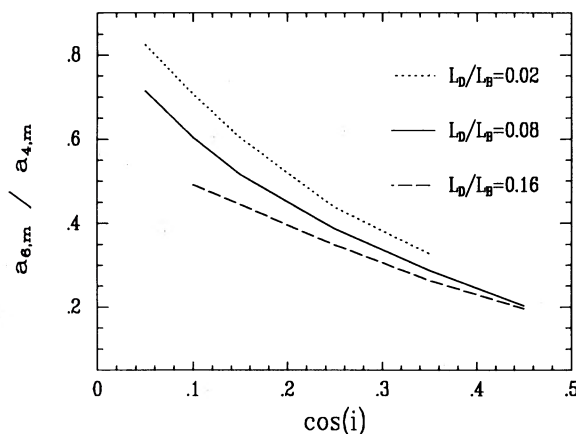


FIG. 4.—The importance of higher order residuals is shown as a function of disk inclination. While for nearly edge-on disks the higher order coefficients (here a_6 compared with a_4) are of comparable importance, they are small compared with a_4 for less highly inclined disks. This should allow an observational test of the hypothesis that most observed disk components are very weak and nearly edge-on.

with a disk might give misleading results for all these parameters.

c) The Disk in NGC 4660

As an example, we apply the techniques discussed above to determine the disk parameters of NGC 4660 (classified as E5 in the RC2 [de Vaucouleurs and de Vaucouleurs 1976]). This galaxy is one of the best examples of pointed isophotes in BDM's sample. The data for the ellipticity and the a_4 profile were taken from BDM; the luminosity profile was taken from Djorgovski's (1985) thesis. The open squares in Figure 5 show the data. Superposed are the predictions of two models that roughly approximate the observed a_4 profile. The dashed lines are for a model with $L_D/L_B = 3\%$ and $\cos i = 0.1$, and the solid lines are for a model with $L_D/L_B = 48\%$ and $\cos i = 0.40$. The model with the more luminous disk is clearly a better fit to the data. The observed stronger "curvature" in both the ellipticity and the luminosity residual profile are probably due to a disk that cuts off more steeply than an exponential disk. To avoid the introduction of additional parameters, we did not try to model this.

So far we have used $\epsilon(r)$ and $\delta I(r)$ to untangle L_D/L_B and $\cos i$. The bottom right-hand panel of Figure 5 shows the a_6 profiles predicted by our two models. Although we do not have data on a_6 , we note that the models make very different predictions for it. Based on the other profiles, we would predict a_6 to be undetectable over the main body of this galaxy.

NGC 4660 is a rapidly rotating elliptical: $v_{\max} = 150 \text{ km s}^{-1}$ (Bender 1988). Bender linked this to the isophote shapes, arguing that ellipticals with pointed isophotes are oblate rotators. In light of our modeling, another explanation seems viable: between $4''$ and $50''$ the disk in our model contributes more than 50% of the light on the major axis. Assuming that

the disk is kinematically "colder" than the spheroid, it is likely that the rotation curve measurement is dominated by the disk rotation. The deprojected disk velocity (assuming $\cos i = 0.4$) would be at least 200 km s^{-1} . The observed velocity dispersion of 150 km s^{-1} implies, assuming an isotropic velocity distribution, an expected rotation velocity of $\approx 2^{1/2} \sigma = \text{km s}^{-1}$. Taking into account that in both the velocity measurement and the dispersion measurement a possible multicomponent structure of the velocity distribution has been neglected, the two estimates for the rotation velocity appear consistent.

The simultaneous presence of these two kinematically distinct components is expected to cause characteristic asymmetric absorption-line profiles, as described, for example, in Wagner, Dettmar, and Bender (1989). Those line profiles can be used to confirm the presence of a disk kinematically, as has been done for NGC 4621 by Bender (1990). Detailed interpretation of such line profiles would require careful modeling both of the structure of NGC 4621 and of the observational procedures employed to obtain them.

VI. SUMMARY AND CONCLUSION

We have set out to investigate the photometric signatures of disk components in early-type galaxies, concentrating on their isophote shapes and modeling them by an $r^{1/4}$ -law spheroid of constant ellipticity and a flat exponential disk. For these simple photometric models we have found the following results:

1. Deviations of the isophotes from perfect ellipses, as characterized by the parameter a_4 , depend predominantly on the spheroid-to-disk ratio and the galaxy's inclination. Dependences on the scale lengths of the luminosity profiles and on the intrinsic spheroid ellipticity are substantially weaker for the parameter range of interest.

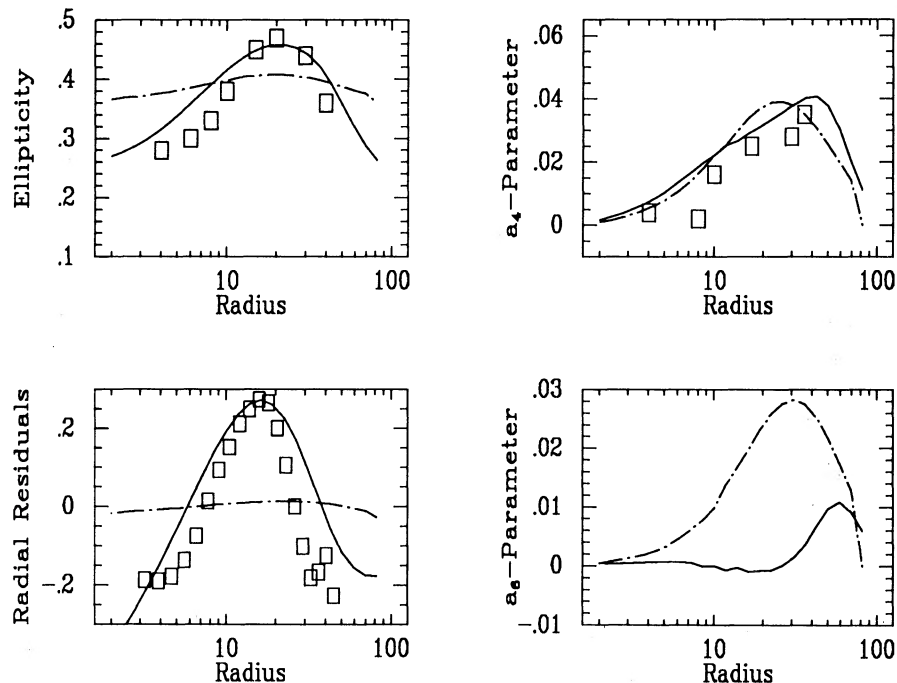


FIG. 5.—Squares represent photometric parameters for NGC 4660, taken from BDM and Djorgovski (1985). We try to determine the strength and inclination of the disk component in this galaxy, using the profiles of a_4 , the ellipticity, and the deviations of the radial profile from an $r^{1/4}$ law. The lines indicate the predictions of two of our models: the dashed line represents a model with $L_D/L_B = 0.03$ and $\cos i = 0.10$, while the solid line shows a model with $L_D/L_B = 0.48$ and $\cos i = 0.40$. It is obvious that the latter model provides an acceptable fit, while the model with the weak disk must be rejected. Despite its classification as an elliptical galaxy, this object has a bulge-to-disk of only 2:1. As a future check of this interpretation we show the predicted a_6 profiles.

2. Similar a_4 profiles can be produced by a wide range of spheroid-to-disk ratio and inclination combinations. In particular, there is little reason to assume that most observed elliptical galaxies with pointed isophotes have weak, nearly edge-on disks. Higher order Fourier coefficients (in particular a_6) should provide a good diagnostic for nearly edge-on disks.

3. The expected shape of the probability distribution of a_4 parameters is consistent with the observed distribution of Bender, Döbereiner, and Möllenhoff (1988). Surveying a wide range of spheroid-to-disk ratios and inclinations, we find that for many interesting parameter combinations the disks are practically undetectable. In particular, we find that for $\cos i > 0.6$, there is no detectable a_4 signature in the isophote shapes for any L_D/L_B . Deviations from a perfect $r^{1/4}$ law are also difficult to detect for most galaxies with $L_D/L_B < 0.25$.

4. The detection probability for disks in current observational samples is less than 50%. Applying this correction to BDM's sample, we find that, if the isophote distortions are indeed caused by disk components, almost all radio-weak (or nonboxy) "ellipticals" have disks, many of which may well be substantial and thus dynamically important. Statistical estimates of "typical" values for L_D/L_B are of the order of 25% for radio-weak galaxies. The presence of substantial disks might also explain Bender's (1988) observation that almost all pointed ellipticals show strong rotation.

5. We applied our modeling to NGC 4660 and found that it has a spheroid-to-disk ratio of about 2:1 and an inclination corresponding to $\cos i = 0.4$. A disk of this strength could dominate the measured rotation speed of this object over a wide range of radii, independent of the rotation of the spheroid. NGC 4660 should "properly" be classified as an S0. Such classification issues seriously confuse the problem of estimating the frequency and strength of disks in early-type galaxies, because visual classifications have been used to define the samples of "elliptical" and "disk" galaxies without a full understanding of their limitations. We will try to remedy this problem by obtaining photometry of the S0 galaxies required to complement BDM's sample of "ellipticals."

We stress that our analysis depends critically, at least in its quantitative aspects, on the assumption that most galaxies are well modeled as a superposition of an $r^{1/4}$ law spheroid and an exponential disk. While there is no known *a priori* reason for

this to be true, and there is considerable evidence that at least some bulges are triaxial (e.g., Gerhard, Vetri, and Kent 1989), there is corroborating evidence that our interpretation may hold approximately in the majority of cases: We know that "S0's" with small values of L_D/L_B exist (e.g., NGC 3115: $L_D/L_B = 0.14$ in the r band; da Silva *et al.* 1989), and we know that such disks are readily detectable only for nearly edge-on viewing angles. As stressed by Capaccioli, Caon, and Rampazzo (1990) and van den Bergh (1990), analogous objects seen more face-on would reveal only much more subtle signs of a disk and would probably be termed "ellipticals." Furthermore, our model implies a correlation between the apparent ellipticity of the galaxy and the measured value of a_4 , in the sense that flatter objects should have a larger (positive) a_4 . Such a correlation is indeed found for pointed ellipticals (Bender *et al.* 1989).

Analysis of the observed absorption-line profiles, which should clearly reflect the two-component nature of the velocity distribution, would appear to offer a powerful test for the presence of disks which is quite independent of the assumptions of our photometric models.

The photometric modeling we have presented supports the view that many "ellipticals" are two-component systems, smoothly extending the Hubble sequence beyond the classical S0's. Two somewhat different formation scenarios appear possible. The spheroid could have formed from the merger of two bona fide galaxies, with the disk condensing from residual gas either immediately or during a later episode accretion. Alternatively, a substantial amount of subclumping during the formation of the galaxy could have converted most of the gas into stars before the final collapse (Katz 1989). Only relatively little gas would have been left to settle into a disk. Both these pictures can be tested by suitable simulations, once more information is available on the scale lengths and surface densities of the disks. An interesting constraint on star formation theory will come from the fact that almost all the gas in these relatively weak disks is now in the form of stars.

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