# THE SURFACE BRIGHTNESS TEST FOR THE EXPANSION OF THE UNIVERSE. II. RADII, SURFACE BRIGHTNESS, AND ABSOLUTE MAGNITUDE CORRELATIONS FOR NEARBY E GALAXIES 

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#### Abstract

The correlation between absolute magnitude and surface brightness, averaged over a series of areas defined by metric (rather than isophotal) radii, is studied using E galaxies in the Virgo, Fornax, and Coma clusters, and in four additional samples of bright nearby field galaxies. The work is a calibration step in preparation for the Tolman $(1+z)^{4}$ surface brightness test for the reality of the expansion. As known before, and as is seen in the present data, the average surface brightness is strongly correlated with absolute brightness for giant galaxies. Galaxies brighter than $M_{B} \sim-20$ have fainter average surface brightness values as the total galaxy luminosity increases. The lack of a constant average surface brightness with changing absolute magnitude for bright E galaxies shows that Hubble's $m \sim-5 \log r^{\prime \prime}$ scaling law is not correct over the first few brightest magnitudes of the E luminosity function. Therefore measurements will be required of the $\langle\mathrm{SB}\rangle$ over a range of $\boldsymbol{M}_{\boldsymbol{T}}$ in distant clusters so as to reduce the $\langle\mathrm{SB}\rangle$ data to "standard conditions" before searching for the Tolman $(1+z)^{4}$ cosmological test signal. The purpose of the present paper is to calibrate the correlation between $M_{T}$ and $\langle\mathrm{SB}\rangle$ using local samples. In the samples studied here the dispersion in the $\langle\mathrm{SB}\rangle$ distributions, after reducing the data to absolute magnitude $M_{B}=-22$ is $\sim 0.5 \mathrm{mag}$. This value is smaller than the 1.8 mag Tolman $(1+z)^{4}$ factor, even at the modest redshift of $z=0.5$, showing that the Tolman test is feasible in practice as well as in principle.


Subject headings: cosmology — galaxies: clustering — galaxies: photometry

## I. INTRODUCTION

It was shown in Paper I of this series (Sandage and Perelmuter 1990, hereafter SP90) that the measurements of sizes and flux levels of galaxies at large redshifts required for the Tolman $(1930,1934)$ surface brightness (hereinafter SB) test for the reality of the expansion are feasible with the present technology using existing ground-based telescopes. However, the requirement that the test be practical depends on whether the dispersion in surface brightness of the galaxies at a particular absolute magnitude in a particular galaxy sample is small compared with the predicted $(1+z)^{4}$ shift in $\langle\mathrm{SB}\rangle$.

The purpose of the present paper is to determine the dispersion in $\langle\mathbf{S B}\rangle$ after the data have been reduced to a common absolute magnitude, thereby taking out the systematic variation of $\langle\mathrm{SB}\rangle$ with $M_{B(T)}$. To determine this systematic variation we analyze data for elliptical galaxies in the Virgo, Fornax, and Coma clusters and in the general field. The field samples are taken from the second edition of the Revised Shapley-Ames Catalog (Sandage and Tammann 1987, hereafter RSA2), from the list by Sadler (1984), and from the galaxies studied by Thomsen and Frandsen (1983) and by Djorgovski and Davis (1987). The data are discussed using measures of both the effective radius and the Petrosian $r(\eta)$ radii determined by the methods developed in Paper I. The

[^0]data in this and in Paper III to eventually follow have been taken from Oemler (1974, 1976), Fraser (1977), King (1978), Schombert (1986), Ichikawa, Wakamtsu, and Okamura (1986), Jedrzejewski (1987), Caldwell (1987), Vigroux et al. (1988), and Ferguson (1989).

## II. PROCEDURE

For each galaxy in our sample a measured intensity profile $I\left(r_{i}\right)$ exists in the literature, expressed as a series of surface brightness values at various radii $r_{i}$. The data are most often listed along the major axis, but sometimes along the "effective" axis $(a b)^{1 / 2}$. The measurements begin at some finite radius, typically $\sim 3^{\prime \prime}$ from the center. As described in Paper I, a fitting function from the Oemler (1976) family has been used to extend the measured $I\left(r_{i}\right)$ values to the center (where no data exist), permitting a complete calculation of the growth curve and the Petrosian $\eta(r)$ function to be made by numerical integration.

Various types of measurements have been made from these integrations, as shown in the example in Figure 1 where the SB profile in $B$ for NGC 4473 is shown as open circles from data by Vigroux et al. (1988). This profile is extrapolated inward by fitting an Oemler function to the listed data points that are closest to the center. The magnitude scale in $B$ for the calculated $m(r)$ growth curve is along the outer right-hand ordinate; that for the $\eta(r)$ function is along the inner right hand ordinate.

The $m(r)$ growth curves calculated in this way could be checked for many galaxies using literature values of aperture


Fig. 1.-Type of data obtained for each galaxy in our sample, using as an example the profile photometry for NGC 4473 from Vigroux et al. (1988). Curves show, respectively, the luminosity profile (in $B$ magnitudes $\operatorname{arcsec}^{-2}$ ) using the scale along the left ordinate, the $m_{B}(r)$ growth curve calculated from the profile, and the Petrosian $\eta(r)$ function calculated as described in Paper I (SP90). Directly observed values of the integrated magnitude through various apertures, taken from the literature, are shown along the growth curve.
photometry at a number of measured radii, as listed, for example, by Longo and de Vaucouleurs (1983, 1985). A sample of the aperture photometry for NGC 4473 at several measured aperture sizes are shown in Figure 1 as filled circles, plotted along the calculated $m(r)$ curve. This diagram shows the excellent agreement between the directly measured apertire photometry for NGC 4473 and the $m(r)$ curve that we have calculated from the $I(r)$ profile data.

Such agreement was not always obtained for some of the galaxies in our data set. Often there is a small systematic offset in the zero point of the magnitudes. Without doubt, the principal reason for the offset is the error made by integrating the $I(r)$ distribution using circular apertures, thereby neglecting the flattening for noncircular E galaxies. One should use appropriate elliptical apertures and the appropriate complete $I(r, \theta)$ distributions at all azimuth angles, such as done by Djorgovski and Davis (1987). But the correction is small. From the discussion in the RC2 (de Vaucouleurs, de Vaucouleurs, and Corwin 1977, § 3.3 f ) the correction to a $\langle\mathrm{SB}\rangle$ calculated with circular rather than elliptical apertures is shown to be $2.75(\log a / b)^{2}$, in magnitudes. For an E5 galaxy where $a / b=2$ (such large flattenings are rare in our sample) the correction amounts to only $\sim 0.3 \mathrm{mag}$, which, as we shall see in Figure 16 later, is negligible compared with the observed scatter in the data. We have neglected this ellipticity effect, but we have applied the observed zero-point offsets to the calculated $m(r)$ curves when appropriate and when the necessary photoelectric aperture data are available.

Note that the Petrosian $\eta(r)$ curves are unaffected by constant zero-point offsets in $m(r)$ at all $r$ because $\eta(r)$ is a ratio. Therefore, the effective radii $\left(r_{e}\right)$ as well as the $\eta$ radii, $r(\eta)$, are also unaffected by simply a change in the $m(r)$ zero point.

We have calculated the $B_{T}$ total magnitude for each galaxy in the sample by extrapolating the corrected $m(r)$ growth curve to $r=\infty$ by fitting one of the $m(r)$ template growth curves from

Paper I to the calculated $m(r)$ shape over the radii range covered by the data. Figure 1 for NGC 4473 shows $B_{T}=11.00$ mag from this extrapolation. The effective radius read from the $m(r)$ curve in Figure 1 at $B=11.75$ is $r_{e}=35^{\prime \prime}$, giving the surface brightness, averaged over this effective radius, to be $\langle\mathrm{SB}\rangle_{e}=20.74 B$ magnitudes per square arc second.

Various Petrosian radii can be read directly from the $\eta(r)$ curve in an obvious way. The radii at which $\eta$ has values of 1.3, 1.5, 1.7, 2.0, 2.5, and $3.0 \mathrm{mag}_{\mathrm{arcsec}^{-2}}$ for NGC 4473 in Figure 1 are $26^{\prime \prime}, 44^{\prime \prime}, 78^{\prime \prime}, 102^{\prime \prime}, 132^{\prime \prime}$, and $155^{\prime \prime}$, and as such are listed in Table 1 discussed later. The SB averaged over these $r(\eta)$ radii can then be calculated at various $r(\eta)$ radii for each of the galaxies we have used.

Systematic errors in the $\eta(r)$ curves, and hence in $r(\eta)$ and $\langle\mathrm{SB}\rangle_{n}$, exist due to the error which is made in the extrapolation of the SB profile into the center. Figure 2 shows the result of varying the extrapolated value of $B(0)$ by about 2 mag for four galaxies in the sample discussed in the following paper. The resulting variations in the $\eta(r)$ curve are shown in the right-hand panels. For small $\eta$ values the variation in the shape of the $\eta(r)$ curve is, of course, larger than for $\eta>1.5 \mathrm{mag}$ $\operatorname{arcsec}^{-2}$, because the error due to errors in $B(0)$ becomes diluted as the $\eta(r)$ integration proceeds outward due to the increasing numerical value of the integral. A typical error in $r(\eta)$ at $\eta=1.3 \mathrm{mag}_{\mathrm{arcsec}}{ }^{-2}$ is $\Delta \log r(1.3) \sim 0.2$ when there is a difference of 2 mag in the estimate of $B(0)$ at the center. By the time $\eta$ reaches $\sim 2 \mathrm{mag} \operatorname{arcsec}^{-2}$, the value of the integral giving $\langle\mathrm{SB}(r)\rangle$ is so large that the error in $\log r(\eta)$ is less than $\Delta \log r(\eta)=0.1$ even for the largest reasonable error we have made in $B(0)$ for most galaxies in our sample.

## III. DATA FOR E AND S0 GALAXIES FROM FIVE SEPARATE SAMPLES

Data on the intensity profiles, $I(r)$, needed to calculate $\eta(r)$ values [and thereby $r(\eta)$ and $\langle\mathrm{SB}\rangle_{\eta}$ ] have been obtained from the literature sources mentioned earlier for the three data samples comprising the Virgo, Fornax, and Coma clusters. Additional data on $r_{e}$ and $m_{B(e)}$ for the Sadler (1984) field galaxy sample are contained in her tables. Data on $r_{e}$ and $B_{e}$ for the E galaxies in the RSA2 (Sandage and Tammann 1987), constituting our fifth sample, have been adopted from the listing in the RC2 (de Vaucouleurs, de Vaucouleurs, and Corwin 1977).

Data for the three cluster samples are given in Tables 1-3. Listed in the heading for each galaxy are the total absolute magnitude $M_{B(T)}$, the effective angular radii, $r_{e}$, in arc seconds, the effective apparent magnitude $B_{e}=B_{T}+0.75$, and the average effective surface brightness (averaged over a circle of radius $r_{e}$ ). In the body of the tables are also listed the $r(\eta)$ radii (in arc seconds) obtained from the $\eta(r)$ curves read at our adopted discrete reading points, the $B$ magnitude inside radius $r(\eta)$, and the resulting calculated average surface brightness $\langle S B\rangle_{\eta}$ within these radii. Apparent magnitudes have been changed to absolute magnitudes by adopting distance moduli of $m-M=31.7$ for the Virgo cluster (Sandage and Tammann 1982a, b, 1985; Tammann and Sandage 1985, 1990), $m-M=31.9$ for Fornax (Sandage and Tammann 1985; Ferguson and Sandage 1988), and $m-M=35.5$ for Coma (Tammann and Sandage 1985, Table 5; Tammann 1986). Fornax cluster data, where only the effective parameters are available from either Caldwell (1987) or from the extensive catalog by Ferguson (1989), are set out in Table 2A. The first 31 entries of this table are due to Caldwell. The remaining


Fig. 2.-Illustration of the errors which are generated in the Pertrosian $\eta(r)$ radii caused by an error in extrapolating the profile data to the center to obtain $B(0)$ with which to begin the integration for $\eta(r)$. Left-hand panels show the observed intensity profiles (in magnitudes) as open circles, to which various extrapolations to the center are fitted; right-hand panels show the resulting families of $\eta(r)$ curves.
entries are due to Ferguson using his FCC (Fornax Cluster Catalog) numbers.

The data for the Coma cluster in Table 3 are based on measurements by Oemler (1976) for which he generously supplied his detailed data for the $I(r)$ profiles in digital form with which we performed the necessary integrations to obtain the equivalent of Figure 1 for each of his galaxies. Data on the absolute magnitudes and the effective photometric parameters for the galaxies measured by Sadler (1984) are set out in Table 4. Her listed $B_{T}$ total magnitudes have been changed to absolute magnitudes in column (3) using the adopted distance modulus values in column (2), calculated from the redshift corrected to the centroid of the Local Group, and then using a Hubble constant of $H_{0}=50 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$. Columns (4)-(6) give the effective $B_{e}$ apparent magnitudes, the effective radii in arc seconds, and the resulting effective average surface brightness $\langle\mathrm{SB}\rangle_{e}$.

Similar data for the E galaxies listed in the binning table in Part III of the RSA2 are listed in Table 5 using the $\langle\mathrm{SB}\rangle_{e}$ values set out in the RC2. Many of the Table 5 (mostly field) galaxies were also used by Binggeli, Sandage, and Tarenghi (1984, hereafter BST) to define the bright sample in their Figure 8 for the $\langle\mathrm{SB}\rangle$-absolute magnitude correlation. However, the photometric data used there were derived by BST from an independent evaluation of available literature data on aperture photometry rather than taking the $\langle\mathrm{SB}\rangle_{e}$ values from the RC2. The reductions by BST were, then, independent of the listings in the RC2 which we use here in Table 5. Hence, the data in Tables 1-5 are largely independent of those used by BST for their Figure 8. The decision to use different
data sets here to study the $\langle\mathrm{SB}\rangle, M$ correlation (Figs. 11 and 12 in later sections) was made to better determine the reliability of the $\langle\mathrm{SB}\rangle$-absolute magnitude correlation.
IV. VARIATION OF the effective radius and the petrosian RADII WITH ABSOLUTE MAGNITUDE FOR E GALAXIES

## a) Hubble's Scaling Law

The data from Table 1 are plotted in Figure 3 to show the variation of the linear radii, $R_{e}$, with $M_{B(T)}$ for Virgo cluster. The metric radii are in parsecs using $D=21.9 \mathrm{Mpc}$ ( $m-M=31.7$ ) for Virgo. The line of slope 5, zero-pointed at $\mathrm{SB}=19 \mathrm{mag} \mathrm{sec}{ }^{-2}$, is shown. The two branches of the $R_{e}$, $M_{B(T)}$ relation, meeting near $M_{B(T)}=-20$, each with a slope that differs from 5 , are evident.

The combined data for the Virgo, Fornax, and the Coma cluster, plus the two field samples from the RSA and from Sadler, are shown in Figure 4. The slope of the relation for galaxies brighter than $M_{B(T)}=-20$ is $d M / d \log R_{e} \sim-3$, showing that the $\langle\mathbf{S B}\rangle_{e}$ for these galaxies becomes fainter for increasing luminosity; the radius is increasing faster with brightness than the constant surface brightness condition. The opposite is true for galaxies fainter than $M_{B(T)} \sim-19$. The equivalent diagrams to Figure 4 that show the $\langle\mathrm{SB}\rangle$ correlation (rather than the radius correlation) with $M_{B(T)}$ are shown later as Figures 9-12. Figures 3 and 4 here are similar to Figure 7 of BST for their Virgo cluster data which they combined with other data from selected field galaxies.

Figure 4 shows that Hubble's (1926) scaling law of $m \sim-5$ $\log r^{\prime \prime}$ is valid only over the quite restricted magnitude range

TABLE 1
Photometric Data for Galaxies in the Virgo Cluster


TABLE 1-Continued


TABLE 1-Continued

| Eta $r^{\prime \prime}$ B <SB> | Eta $r^{\prime \prime}$ B <SB> | Eta $r^{\prime \prime} \quad B \quad$ SB> | Eta $\mathrm{r}^{\prime \prime}$ B <SB> |
| :---: | :---: | :---: | :---: |
|  | V53 Ichikawa, etal | V57 Ichikawa, etal | V63 Ichikawa, etal. |
| V45 Ichikawa, etal | $\mathrm{MBT}=-13.75 \quad \mathrm{re}=6.9$ | $M B T=-14.91 \quad r e=10.5$ | MBT $=-16.53 \mathrm{re}=11.9$ |
| $\mathrm{MBT}=-16.28 \quad \mathrm{re}=19.4$ | $\mathrm{Be}=18.70 \quad\langle S B\rangle \mathrm{e}=24.1$ | $\mathrm{Be}=17.54 \quad\langle\mathrm{SB}\rangle \mathrm{e}=23.9$ | $\mathrm{Be}=15.92<\mathrm{SB}$ ¢e $=22.5$ |
| $B e=16.17 \quad\langle S B\rangle e=23.8$ | 1.0 12.6 18.20 <br> 124.9   | $\begin{array}{llll}1.0 & 19.0 & 17.06 & 24.7\end{array}$ | $\begin{array}{llllllll}1.0 & 12.6 & 15.89 & 22.6\end{array}$ |
| $\begin{array}{lllll}1.0 & 28.8 & 15.84 & 24.3\end{array}$ | $1.3 \quad 15.118 .07 \quad 25.2$ | $\begin{array}{llll}1.3 & 22.9 & 16.89 & 24.9\end{array}$ | $1.3 \quad 18.615 .61 \quad 23.2$ |
| $1.3 \quad 51.3 \quad 15.38 \quad 25.1$ | $1.5 \quad 15.918 .03 \quad 25.2$ | $\begin{array}{lll}1.5 & 24.0 & 16.89 \\ 25.0\end{array}$ | $\begin{array}{llll}1.5 & 26.3 & 15.42 & 23.7\end{array}$ |
| $\begin{array}{llll}1.5 & 56.2 & 15.33 & 25.3\end{array}$ | $1.7 \quad 16.218 .00 \quad 25.2$ | $\begin{array}{llll}1.7 & 25.1 & 16.86 & 25.1\end{array}$ | $1.7 \quad 33.9 \quad 15.28 \quad 24.1$ |
| $1.7 \quad 66.115 .28 \quad 25.6$ |  | $\begin{array}{llll}2.0 & 25.7 & 16.86 \quad 25.1\end{array}$ | $\begin{array}{lllll}2.0 & 40.7 & 15.18 & 24.4\end{array}$ |
| V46 Ichikawa, etal | V54 Ichikawa, etal MBT $=-15.89 \quad r e=8.7$ | V59 Ichikawa, etal | V64 Ichikawa,etal |
| MBT $=-14.17$ re= 9.1 | $\mathrm{Be}=16.56\langle S B>e=22.5$ | MBT $=-16.85 \quad \mathrm{re}=22.1$ | MBT $=-14.12 \quad \mathrm{re}=11.4$ |
| $\mathrm{Be}=18.28 \quad\langle\mathrm{SB}\rangle \mathrm{e}=24.3$ | $\begin{array}{lllll}1.0 & 11.8 & 16.36 & 22.9\end{array}$ | $\mathrm{Be}=15.60<\mathrm{SB}\rangle \mathrm{e}=23.5$ | $\mathrm{Be}=18.33 \quad\langle S B\rangle e=24.8$ |
| $\begin{array}{lllll}1.0 & 16.2 & 17.75 & 25.0\end{array}$ | $1.3 \quad 15.516 .15 \quad 23.3$ | $\begin{array}{llll}1.0 & 26.9 & 15.47 & 23.8\end{array}$ | $1.0 \quad 19.517 .76 \quad 25.4$ |
| $\begin{array}{llll}1.3 & 19.0 & 17.61 & 25.2\end{array}$ | $\begin{array}{llll}1.5 & 17.8 & 16.07 & 23.5\end{array}$ | $\begin{array}{llll}1.3 & 46.8 & 15.05 & 24.6\end{array}$ | $\begin{array}{llll}1.3 & 21.9 & 17.66 \quad 25.6\end{array}$ |
| 1.5 20.0 17.59 | 1.7 20.4 16.00 <br> 16.7   | $\begin{array}{llll}1.5 & 52.5 & 14.98 & 24.8\end{array}$ |  |
|  | $\begin{array}{llll}2.0 & 25.1 & 15.92 & 24.1\end{array}$ | $1.7 \quad 56.214 .94 \quad 24.9$ | V65 Ichikawa, etal. |
| V47 Ichikawa, etal | $2.5 \quad 28.215 .88 \quad 24.3$ | $2.0 \quad 58.9 \quad 14.93 \quad 25.0$ | MBT $=-14.57$ re= 11.0 |
| $\mathrm{MBT}=-15.23 \mathrm{re}=13.2$ |  |  | $\mathrm{Be}=17.88 \quad<\mathrm{SB}$ ¢e $=24.3$ |
| $\mathrm{Be}=17.22 \quad<\mathrm{SB}\rangle \mathrm{e}=24.0$ | V55 Ichikawa, etal | V60 Ichikawa, etal | $1.0 \quad 19.017 .38 \quad 25.0$ |
| $\begin{array}{llll}1.0 & 24.0 & 16.68 & 24.8\end{array}$ | $\mathrm{MBT}=-15.23 \mathrm{re}=8.1$ | $\mathrm{MBT}=-15.84 \mathrm{re}=12.7$ | $\begin{array}{llll}1.3 & 22.9 & 17.24 & 25.2\end{array}$ |
| $\begin{array}{llll}1.3 & 26.9 & 16.60 & 24.9\end{array}$ | $\mathrm{Be}=17.22<S B>e=23.0$ | $\mathrm{Be}=16.61 \quad\langle\mathrm{SB}\rangle \mathrm{e}=23.3$ | $\begin{array}{lll}1.5 & 24.0 & 17.20 \\ 25.3\end{array}$ |
| $1.5 \quad 30.216 .52 \quad 25.1$ | $1.0 \quad 9.8 \quad 17.10 \quad 23.2$ | $\begin{array}{llll}1.0 & 12.6 & 16.64 & 23.3\end{array}$ | 1.5 24.0 17.20 25.3 |
| $\begin{array}{llll}1.7 & 30.9 & 16.51 & 25.2\end{array}$ | $\begin{array}{llll}1.3 & 12.9 & 16.87 & 23.6\end{array}$ | $\begin{array}{llll}1.3 & 23.4 & 16.19 & 24.2\end{array}$ | V66 Ichikawa, etal |
| $\begin{array}{llll}2.0 & 31.6 & 16.50 & 25.2\end{array}$ | $\begin{array}{llll}1.5 & 19.0 & 16.65 & 24.2\end{array}$ | $1.5 \quad 38.0 \quad 15.91 \quad 25.0$ | $\mathrm{MBT}=-15.32 \mathrm{re}=10.3$ |
|  | $\begin{array}{lll}1.7 & 21.4 & 16.59 \\ 24.4\end{array}$ | $\begin{array}{llll}1.7 & 40.7 & 15.88 & 25.1\end{array}$ | $\mathrm{Be}=17.13 \quad<\mathrm{SB}\rangle \mathrm{e}=23.4$ |
| V48 Ichikawa, etal | 2.0 $24.0 \quad 16.55 \quad 24.6$ |  | $1.0 \quad 10.717 .14 \quad 23.5$ |
| $\mathrm{MBT}=-13.76 \quad \mathrm{re}=17.7$ |  | V62 Ichikawa, etal | $1.3 \quad 21.416 .60 \quad 24.4$ |
| $\mathrm{Be}=18.69 \quad<\mathrm{SB}>\mathrm{e}=24.3$ | V56 Ichikawa, etal | $\mathrm{MBT}=-14.94 \mathrm{re}=10.1$ | $\begin{array}{llll}1.5 & 25.7 & 16.52 & 24.8\end{array}$ |
| $1.0 \quad 15.118 .13125 .2$ | $\mathrm{MBT}=-14.72 \mathrm{re}=10.9$ | $\mathrm{Be}=17.51 \quad<\mathrm{SB}>\mathrm{e}=23.7$ | $\begin{array}{lllll}1.7 & 27.5 & 16.47 & 24.9\end{array}$ |
| $1.3 \quad 17.018 .04 \quad 25.4$ | $\mathrm{Be}=17.73 \quad<S B>e=24.1$ | $\begin{array}{llll}1.0 & 15.5 & 17.17 & 24.3\end{array}$ | $\begin{array}{llll}2.0 & 29.5 & 16.45 & 25.0\end{array}$ |
| $\begin{array}{llll}1.5 & 17.8 & 18.01 & 25.5\end{array}$ | $\begin{array}{llll}1.0 & 19.5 & 17.24 & 24.9\end{array}$ | $\begin{array}{lllll}1.3 & 21.4 & 16.92 & 24.8\end{array}$ |  |
|  | $\begin{array}{llll}1.3 & 22.9 & 17.10 & 25.1\end{array}$ | $\begin{array}{llll}1.5 & 23.4 & 16.85 & 24.9\end{array}$ | V69 Ichikawa, etal |
| V50 Ichikawa, etal. | $\begin{array}{llll}1.5 & 23.4 & 17.08 & 25.1\end{array}$ | $\begin{array}{llll}1.7 & 25.1 & 16.83 & 25.0\end{array}$ | MBT $=-16.48 \mathrm{re}=12.4$ |
| $\mathrm{MBT}=-14.85 \mathrm{re}=12.8$ | $\begin{array}{llll}1.7 & 25.1 & 17.04 & 25.2\end{array}$ | $\begin{array}{llll}2.0 & 25.7 & 16.82 & 25.1\end{array}$ | $\mathrm{Be}=15.97 \quad\langle\mathrm{SB}\rangle \mathrm{e}=22.6$ |
| $\mathrm{Be}=17.60 \quad<\mathrm{SB}>\mathrm{e}=24.3$ |  |  | $\begin{array}{llllll}1.0 & 11.8 & 16.05 & 22.6\end{array}$ |
| $\begin{array}{llll}1.0 & 24.0 & 16.99 & 25.1\end{array}$ |  |  | $1.318 .215 .72 \quad 23.2$ |
| $\begin{array}{llll}1.3 & 25.7 & 16.94 & 25.2\end{array}$ |  |  | $\begin{array}{llll}1.5 & 26.3 & 15.51 & 23.8\end{array}$ |
| $1.5 \quad 26.316 .9225 .2$ |  |  | $\begin{array}{llll}1.7 & 38.0 & 15.34 & 24.4\end{array}$ |
| $\begin{array}{llll}1.7 & 26.916 .90 \quad 25.2\end{array}$ |  |  | $\begin{array}{llll}2.0 & 42.7 \quad 15.30 \quad 24.6\end{array}$ |



Fig. 3.-Variation of the linear effective radii $R_{e}$ (in parsecs) with total blue absolute magnitude for Virgo cluster galaxies from the data in Table 1. The distance modulus of $m-M=31.7$ is adopted for the cluster. A line of constant effective surface brightness $\left(\langle\mathrm{SB}\rangle_{e}=19 \mathrm{mag}_{\operatorname{arcsec}}{ }^{-2}\right)$ is shown.
from -18.5 to $-21.5\left(H_{0}=50\right)$ (or $M_{T}$ from -13.5 to -16.5 on Hubble's 1936 distance scale). By the circumstances of how Hubble chose his sample (mostly field galaxies from Hardcastle's 1914 list of 400 bright galaxies known at the time from inspection of the Franklin-Adams charts), this is simply the range of absolute magnitude which Hubble had access to. Galaxies neither from the very bright end of the luminosity function more luminous than $M_{B(T)} \sim-21.5$, nor the faint dwarfs were in his lists. Figure 4 shows that it is in these magnitude regimes that the deviation from the constant $\langle\mathrm{SB}\rangle$ occurs, therefore unseen by Hubble.

Hubble's assumption that $m \sim-5 \log r^{\prime \prime}$ has dominated the literature for nearly half a century. The more recent discussions that the $d m / d \log r$ slope differs from -5 at the faint end began with Reaves (1956) and continued with Holmberg (1975), BST, Ichikawa, Wakamatsu, and Okamura (1986), Caldwell and Bothun (1987), Ferguson and Sandage (1988), Impey, Bothun, and Malin (1988), and undoubtedly by others.

From the first discussions by Oemler $(1974,1976)$ it became evident that $\langle\mathrm{SB}\rangle$ decreases with increasing luminosity for the brightest galaxies. This was confirmed and extended by Kormendy (1977, 1980, 1982, 1987), Strom and Strom (1978a, b, c), Michard (1979), Thomsen and Frandsen (1983), BST, Choloniewski (1985), Schombert (1986), Djorgovski and Davis (1987), and we suspect by others. Particularly striking illustrations of

TABLE 2
Photometric Data for Galaxies in the Fornax Cluster

the fainter SB for the brightest E galaxies are the photographs by Burbidge (1962) for NGC 6166 and its companions, and by Burbidge, Burbidge, and Crampin (1964) for the E pair of NGC 4782/83. These photographs gave one of the first indications of the larger effective radii for the brightest galaxies compared with the constant $\langle\mathrm{SB}\rangle$ case.

## b) Correlations Using Petrosian Parameters

We now inquire if the features in Figure 4 are also present in correlations that use the Petrosian metric radii listed in Tables $1-3$, rather than effective radii. Figure 5 shows the data for the Virgo cluster alone for $\eta$ values of $1.3,1.7,2.0$, and 2.5 mag

Additional Photometric Data for Galaxies in the Fornax Cluster

| Name (1) | Be <br> (2) | MBT <br> (3) | $\begin{aligned} & \text { re" } \\ & (4) \end{aligned}$ | $\langle S B\rangle e$ (5) | Name (1) | Be <br> (2) | MBT <br> (3) | $\begin{aligned} & \text { re" } \\ & (4) \end{aligned}$ | $\begin{gathered} \langle S B\rangle e \\ (5) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N1419 | 14.35 | $-18.3$ | 13.8 | 21.3 | FCC90 | 15.75 | -16.9 | 9.6 | 21.9 |
| NG3 | 16.85 | -15.8 | 15.1 | 24.0 | FCC93 | 20.15 | -12.5 | 8.7 | 26.1 |
| NG5 | 17.55 | -15.1 | 18.2 | 25.1 | FCC94 | 20.05 | -12.6 | 7.1 | 25.5 |
| NG6 | 17.35 | -15.3 | 12.6 | 24.1 | FCC97 | 19.75 | -12.9 | 9.4 | 25.9 |
| NG7 | 16.25 | -16.4 | 15.5 | 23.4 | FCC99 | 17.85 | -14.8 | 10.5 | 24.2 |
| NG8 | 17.05 | -15.6 | 14.5 | 24.1 | FCC100 | 16.25 | -16.4 | 16.8 | 23.6 |
| NG9 | 17.65 | -15.0 | 12.6 | 24.4 | FCC103 | 20.05 | -12.6 | 7.2 | 25.6 |
| NG10 | 16.45 | -16.2 | 20.4 | 24.2 | FCC104 | 19.25 | -13.4 | 8.3 | 25.1 |
| NG11 | 16.95 | -15.7 | 20.9 | 24.8 | FCC105 | 18.05 | -14.6 | 14.1 | 25.0 |
| NG25 | 17.25 | -15.4 | 14.1 | 24.2 | FCC107 | 17.75 | -14.9 | 9.3 | 23.8 |
| NG29 | 18.15 | -14.5 | 11.5 | 24.7 | FCC108 | 19.85 | -12.8 | 7.1 | 25.4 |
| NG38 | 19.15 | -13.5 | 7.9 | 24.9 | FCC109 | 19.05 | -13.6 | 8.8 | 25.0 |
| NG44 | 17.35 | -15.3 | 13.2 | 24.2 | FCC110 | 17.55 | -15.1 | 15.4 | 24.7 |
| NG61 | 17.95 | -14.7 | 10.7 | 24.3 | FCC111 | 17.55 | -15.1 | 13.0 | 24.4 |
| NG69 | 15.75 | -16.9 | 17.4 | 23.2 | FCC114 | 20.45 | -12.2 | 6.3 | 25.7 |
| NG77 | 19.05 | -13.6 | 6.9 | 24.5 | FCC116 | 16.85 | -15.8 | 13.4 | 23.7 |
| NG80 | 16.95 | -15.7 | 7.1 | 22.4 | FCC117 | 18.95 | -13.7 | 15.7 | 26.2 |
| NG83 | 19.35 | -13.3 | 10.0 | 25.6 | FCC118 | 18.35 | -14.3 | 9.1 | 24.4 |
| NG102 | 20.45 | -12.2 | 5.0 | 25.2 | FCC100 | 16.25 | -16.4 | 16.8 | 23.6 |
| G8 | 14.75 | -17.9 | 15.1 | 21.9 | FCC125 | 19.45 | -13.2 | 0.9 | 20.5 |
| G26 | 16.21 | -16.4 | 11.2 | 22.7 | FCC126 | 21.15 | -11.5 | 3.7 | 25.2 |
| G43 | 16.65 | -16.0 | 10.7 | 23.0 | FCC127 | 20.55 | -12.1 | 5.3 | 25.4 |
| G75 | 15.95 | -16.7 | 12.6 | 22.7 | FCC131 | 21.05 | -11.6 | 4.2 | 25.4 |
| G79 | 15.95 | -16.7 | 7.6 | 21.6 | FCC132 | 19.25 | -13.4 | 5.0 | 24.0 |
| G84 | 15.15 | -17.5 | 6.5 | 20.4 | FCC133 | 18.25 | -14.4 | 12.0 | 24.9 |
| G91 | 15.25 | -17.4 | 12.9 | 22.0 | FCC134 | 18.35 | -14.3 | 6.0 | 23.5 |
| G92 | 16.65 | -16.0 | 9.3 | 22.7 | FCC136 | 15.55 | -17.1 | 16.3 | 22.9 |
| G103 | 13.95 | -18.7 | 24.5 | 22.1 | FCC137 | 17.65 | -15.0 | 13.4 | 24.5 |
| G104 | 16.45 | -16.2 | 7.2 | 22.0 | FCC138 | 19.35 | -13.3 | 4.5 | 23.9 |
| G118 | 15.65 | -17.0 | 6.8 | 21.0 | FCC140 | 19.75 | -12.9 | 6.2 | 25.0 |
| G142 | 16.25 | -16.4 | 20.0 | 24.0 | FCC141 | 20.45 | -12.2 | 2.1 | 23.3 |
| FCC25 | 18.45 | -14.2 | 8.6 | 24.4 | FCC142 | 19.25 | -13.4 | 9.5 | 25.4 |
| FCC27 | 20.05 | -12. 6 | 6.8 | 25.5 | FCC143 | 15.05 | -17.6 | 11.8 | 21.6 |
| FCC30 | 19.65 | -13.0 | 12.5 | 26.4 | FCC144 | 19.95 | -12.7 | 6.6 | 25.3 |
| FCC31 | 17.55 | -15.1 | 8.5 | 23.4 | FCC145 | 20.35 | -12.3 | 4.4 | 24.8 |
| FCC32 | 16.15 | -16.5 | 8.2 | 22.0 | FCC146 | 20.25 | -12.4 | 4.7 | 24.9 |
| FCC34 | 19.15 | -13.5 | 8.5 | 25.0 | FCC149 | 20.65 | -12.0 | 5.6 | 25.6 |
| FCC36 | 16.65 | -16.0 | 6.7 | 22.0 | FCC150 | 16.45 | -16.2 | 6.5 | 21.8 |
| FCC38 | 18.25 | -14.4 | 7.9 | 24.0 | FCC151 | 18.75 | -13. 9 | 12.3 | 25.4 |
| FCC40 | 18.35 | -14.3 | 7.9 | 24.1 | FCC154 | 19.95 | -12.7 | 7.9 | 25.7 |
| FCC42 | 19.05 | $-13.6$ | 13.1 | 25.9 | FCC155 | 19.05 | -13.6 | 6.7 | 24.4 |
| FCC46 | 16.35 | -16.3 | 5.8 | 21.4 | FCC156 | 17.95 | -14.7 | 12.1 | 24.6 |
| FCC48 | 17.85 | -14.8 | 9.1 | 23.9 | FCC157 | 19.05 | -13. 6 | 11.5 | 25.6 |
| FCC49 | 19.95 | -12.7 | 6.5 | 25.3 | FCC158 | 17.55 | -15.1 | 16.6 | 24.9 |
| FCC50 | 17.35 | -15.3 | 19.9 | 25.1 | FCC159 | 18.85 | -13.8 | 11.8 | 25.5 |
| FCC51 | 19.25 | -13.4 | 7.4 | 24.8 | FCC160 | 18.45 | -14.2 | 10.6 | 24.8 |
| FCC52 | 19.25 | -13.4 | 7.8 | 25.0 | FCC163 | 20.55 | -12.1 | 7.3 | 26.1 |
| FCC54 | 18.75 | -13.9 | 8.7 | 24.7 | FCC165 | 18.25 | -14.4 | 11.1 | 24.7 |
| FCC56 | 18.25 | -14.4 | 10.4 | 24.6 | FCC166 | 18.85 | -13.8 | 8.3 | 24.7 |
| FCC57 | 18.55 | -14.1 | 6.6 | 23.9 | FCC168 | 19.45 | -13.2 | 8.3 | 25.3 |
| FCC58 | 18.75 | -13.9 | 7.2 | 24.3 | FCC171 | 18.65 | -14.0 | 18.0 | 26.2 |
| FCC59 | 20.15 | -12.5 | 8.1 | 25.9 | FCC172 | 21.15 | -11.5 | 3.9 | 25.4 |
| FCC61 | 18.75 | -13.9 | 6.3 | 24.0 | FCC174 | 17.45 | -15.2 | 7.2 | 23.0 |
| FCC63 | 13.45 | -19.2 | 18.0 | 21.0 | FCC175 | 19.55 | -13.1 | 17.7 | 27.0 |
| FCC65 | 18.25 | -14.4 | 7.4 | 23.8 | FCC178 | 17.95 | -14.7 | 10.1 | 24.2 |
| FCC66 | 17.65 | -15.0 | 13.6 | 24.6 | FCC181 | 17.95 | -14.7 | 9.0 | 24.0 |
| FCC68 | 17.25 | -15.4 | 6.9 | 22.7 | FCC185 | 20.25 | -12.4 | 5.6 | 25.2 |
| FCC70 | 19.25 | -13.4 | 6.7 | 24.6 | FCC187 | 18.25 | -14.4 | 12.3 | 24.9 |
| FCC71 | 21.25 | -11.4 | 4.1 | 25.6 | FCC188 | 16.85 | -15.8 | 12.1 | 23.5 |
| FCC73 | 20.15 | -12.5 | 6.2 | 25.4 | FCC189 | 19.55 | -13.1 | 5.0 | 24.3 |
| FCC74 | 18.45 | -14.2 | 4.2 | 22.8 | FCC191 | 20.05 | -12.6 | 6.3 | 25.3 |
| FCC75 | 20.55 | -12.1 | 7. 1 | 26.0 | FCC192 | 20.35 | -12.3 | 8.3 | 26.2 |
| FCC77 | 18.95 | $-13.7$ | 11.9 | 25.6 | FCC194 | 18.95 | -13.7 | 6.4 | 24.2 |
| FCC79 | 19.45 | -13.2 | 8.4 | 25.3 | FCC195 | 17.45 | -15.2 | 11.4 | 24.0 |
| FCC80 | 16.35 | -16.3 | 10.6 | 22.7 | FCC196 | 18.45 | -14.2 | 10.2 | 24.7 |
| FCC81 | 17.85 | -14.8 | 7.5 | 23.5 | FCC197 | 19.85 | -12.8 | 6.3 | 25.1 |
| FCC82 | 17.15 | -15.5 | 12.2 | 23.8 | FCC198 | 18.65 | -14.0 | 7.9 | 24.4 |
| FCC84 | 20. 25 | -12.4 | 5.6 | 25.2 | FCC200 | 18.15 | -14.5 | 8.5 | 24.0 |
| FCC85 | 17.05 | -15.6 | 13.6 | 24.0 | FCC201 | 17.45 | -15.2 | 12.9 | 24.3 |
| FCC86 | 18.25 | -14.4 | 9.7 | 24.4 | FCC202 | 16.05 | -16.6 | 9.8 | 22.3 |
| FCC87 | 19.35 | $-13.3$ | 10.7 | 25.7 | FCC203 | 16.25 | -16.4 | 13.0 | 23.1. |
| FCC89 | 18.75 | -13.9 | 10.2 | 25.0 | FCC206 | 16.55 | -16.1 | 18.6 | 24.1 |

TABLE 2-Continued

| Name (1) | Be <br> (2) | MBT <br> (3) | $\begin{aligned} & \text { re" } \\ & (4) \end{aligned}$ | $\begin{gathered} \langle S B\rangle e \\ (5) \end{gathered}$ | Name (1) | Be <br> (2) | MBT <br> (3) | $\begin{aligned} & \text { re" } \\ & (4) \end{aligned}$ | $\underset{(5)}{\langle S B>e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FCC207 | 16.65 | -16.0 | 8.5 | 22.5 | FCC271 | 19.45 | -13.2 | 7.5 | 25.1 |
| FCC208 | 18.05 | -14.6 | 11.7 | 24.6 | FCC272 | 19.95 | -12.7 | 9.9 | 26.2 |
| FCC209 | 19.45 | -13.2 | 6.7 | 24.8 | FCC273 | 19.55 | -13.1 | 7.1 | 25.0 |
| FCC210 | 19.45 | -13.2 | 7.9 | 25.2 | FCC274 | 17.25 | -15.4 | 12.0 | 23.9 |
| FCC211 | 17.05 | -15.6 | 5.6 | 22.0 | FCC275 | 20.05 | -12.6 | 7.7 | 25.7 |
| FCC212 | 18.35 | -14.3 | 15.2 | 25.5 | FCC277 | 14.55 | -18.1 | 10.1 | 20.8 |
| FCC214 | 19.75 | -12.9 | 6.2 | 25.0 | FCC278 | 17.55 | -15.1 | 6.0 | 22.7 |
| FCC215 | 19.95 | -12.7 | 7.4 | 25.5 | FCC279 | 17.45 | -15.2 | 14.0 | 24.4 |
| FCC216 | 19.95 | -12.7 | 6.9 | 25.4 | FCC280 | 19.35 | -13.3 | 8.3 | 25.2 |
| FCC218 | 19.25 | -13.4 | 7.6 | 24.9 | FCC281 | 18.65 | -14.0 | 10.4 | 25.0 |
| FCC220 | 20.25 | -12.4 | 5.6 | 25.2 | FCC284 | 19.75 | -12.9 | 10.4 | 26.1 |
| FCC221 | 18.45 | -14.2 | 6.0 | 23.6 | FCC286 | 18.85 | -13.8 | 6.3 | 24.1 |
| FCC222 | 16.35 | -16.3 | 14.5 | 23.4 | FCC287 | 18.75 | -13.9 | 9.2 | 24.8 |
| FCC223 | 16.95 | -15.7 | 16.6 | 24.3 | FCC289 | 19.45 | -13.2 | 7.3 | 25.0 |
| FCC225 | 20.35 | -12.3 | 5.7 | 25.4 | FCC291 | 20.15 | -12.5 | 7.5 | 25.8 |
| FCC226 | 18.55 | -14.1 | 18.5 | 26.1 | FCC292 | 17.85 | -14.8 | 10.0 | 24.1 |
| FCC227 | 20.05 | -12. 6 | 7.3 | 25.6 | FCC293 | 18.35 | -14.3 | 10.4 | 24.7 |
| FCC228 | 19.55 | -13.1 | 9.2 | 25.6 | FCC294 | 17.95 | -14.7 | 13.1 | 24.8 |
| FCC229 | 19.85 | -12.8 | 6.8 | 25.3 | FCC295 | 19.65 | -13.0 | 7.1 | 25.1 |
| FCC230 | 17.95 | -14.7 | 8.2 | 23.8 | FCC296 | 17.05 | -15. 6 | 10.8 | 23.5 |
| FCC231 | 19.15 | -13.5 | 8.3 | 25.0 | FCC297 | 18.55 | -14.1 | 13.3 | 25.4 |
| FCC232 | 18.15 | -14.5 | 9.0 | 24.2 | FCC298 | 17.35 | -15.3 | 7.6 | 23.0 |
| FCC234 | 17.95 | -14.7 | 13.3 | 24.8 | FCC300 | 16.85 | -15.8 | 16.4 | 24.2 |
| FCC236 | 19.95 | -12.7 | 5.7 | 25.0 | FCC301 | 14.95 | -17.7 | 12.4 | 21.7 |
| FCC237 | 18.55 | -14.1 | 8.9 | 24.5 | FCC303 | 16.25 | -16.4 | 13.5 | 23.1 |
| FCC238 | 19.45 | -13.2 | 8.9 | 25.4 | FCC304 | 19.55 | -13.1 | 9.7 | 25.7 |
| FCC239 | 19.65 | -13.0 | 5.4 | 24.5 | FCC307 | 18.45 | -14.2 | 15.3 | 25.6 |
| FCC241 | 17.35 | -15.3 | 15.9 | 24.6 | FCC309 | 18.25 | -14.4 | 19.1 | 25.9 |
| FCC242 | 18.55 | -14.1 | 11.0 | 25.0 | FCC314 | 17.25 | -15.4 | 14.7 | 24.3 |
| FCC243 | 17.25 | -15.4 | 11.7 | 23.8 | FCC316 | 17.05 | -15.6 | 15.4 | 24.2 |
| FCC245 | 16.75 | -15.9 | 12.9 | 23.5 | FCC317 | 18.45 | -14.2 | 8.9 | 24.4 |
| FCC246 | 19.85 | -12.8 | 8.5 | 25.7 | FCC318 | 16.85 | -15.8 | 15.8 | 24.1 |
| FCC248 | 19.45 | -13.2 | 4.9 | 24.1 | FCC319 | 17.15 | -15.5 | 14.8 | 24.2 |
| FCC250 | 17.85 | -14.8 | 9.4 | 24.0 | FCC320 | 18.05 | -14.6 | 6.6 | 23.4 |
| FCC251 | 19.75 | -12.9 | 8.6 | 25.7 | FCC321 | 19.55 | -13.1 | 12.2 | 26.2 |
| FCC252 | 16.75 | -15.9 | 11.1 | 23.2 | FCC325 | 18.95 | -13.7 | 7.8 | 24.6 |
| FCC253 | 17.05 | -15.6 | 10.8 | 23.5 | FCC326 | 19.35 | -13.3 | 7.4 | 24.9 |
| FCC254 | 18.35 | -14:3 | 11.8 | 25.0 | FCC327 | 18.15 | -14.5 | 11.7 | 24.7 |
| FCC256 | 20.85 | -11.8 | 4.5 | 25.4 | FCC328 | 19.15 | -13.5 | 10.9 | 25.6 |
| FCC257 | 18.85 | -13.8 | 6.7 | 24.2 | FCC329 | 18.95 | -13.7 | 5.5 | 23.9 |
| FCC258 | 20.35 | -12.3 | 6.8 | 25.8 | FCC330 | 20.15 | -12.5 | 4.6 | 24.7 |
| FCC259 | 18.55 | -14.1 | 13.6 | 25.5 | FCC331 | 17.85 | -14.8 | 19.2 | 25.5 |
| FCC260 | 17.75 | -14.9 | 14.9 | 24.9 | FCC332 | 16.45 | $-16.2$ | 8.3 | 22.3 |
| FCC262 | 18.55 | -14.1 | 9.2 | 24.6 | FCC333 | 17.55 | -15.1 | 6.0 | 22.7 |
| FCC253 | 18.85 | -13.8 | 13.2 | 25.7 | FCC334 | 19.35 | -13.3 | 6.8 | 24.8 |
| FCC266 | 16.65 | -16.0 | 5.6 | 21.6 | FCC335 | 14.95 | -17.7 | 12.9 | 21.8 |
| FCC268 | 19.05 | -13.6 | 7.1 | 24.5 | FCC336 | 18.35 | -14.3 | 7.8 | 24.0 |
| FCC269 | 19.05 | -13.6 | 12.2 | 25.7 | FCC339 | 18.25 | -14.4 | 10.4 | 24.6 |
| FCC270 | 20.45 | -12.2 | 5.4 | 25.4 | FCC340 | 18.95 | -13.7 | 5.7 | 24.0 |

$\operatorname{arcsec}^{-2}$. The absolute magnitudes on the abscissae refer to the values using $m-M=31.7$ for the distance modulus.

Similar data from Oemler's (1976) photometry for the Coma cluster are shown in Figure 6. The correlation of the effective radius versus total absolute blue magnitude is in the upper left panel. These data are to be compared with Figure 3 for Virgo. The Coma cluster is between 3.8 and 4.0 mag more distant than Virgo (Sandage and Tammann 1984; Dresseler 1984; Tammann and Sandage 1985; Tammann 1986); hence, only the brighter galaxies in the cluster are available from these data, in constrast to the deeper grasp into the Virgo cluster in Figure 3. The other three panels in Figure 6 show the correlation of Petrosian $r(\eta)$ radii with $M_{B(\eta)}$ for $\eta=1.7,2.5$, and 3.0 mag arcsec ${ }^{-2}$, again for Coma galaxies. The steeper slope than -5 for $d M / d \log R$ is evident in all four panels of Figure 6.

A combination of the data for Virgo, Fornax, and Coma from Tables 1-5 is shown in Figure 7. Effective radii are used in
the upper left panel. Petrosian $r(\eta)$ radii, listed at discrete values of $\eta$ bteween 1.3 and $2.5 \mathrm{mag} \mathrm{arcsec}^{-2}$, are used in the remaining five panels. The division into two branches with different slopes is again evident in each diagram.

As discussed in Paper I of this series, the purpose of using $r(\eta)$ radii rather than the effective radius has been to eliminate the necessity to extrapolate the $m(r)$ growth curve to $r=\infty$ that is needed to define $B_{T}$, and thereby to obtain $r_{e}$. A second reason is that the $r(\eta)$ radii are larger than $r_{e}$ by the important factor of $\sim 5$ for $\eta=3.0 \mathrm{mag} \operatorname{arcsec}^{-2}$ (see SP90, Table 1 for the factors for other $\eta$ values). Measurements of metric radii to large angular distances from the center are, of course, necessary if galaxies are to be observed to large enough redshifts to make the Tolman test practical by correcting for the seeing profile.

The data for the Virgo and Coma clusters are combined in Figure 8 which shows how well the various $r(\eta)$ radii are cor-

TABLE 3
Photometric Data for Galaxies in the Coma Cluster


TABLE 3-Continued

related with $r_{e}$ and also how the zero point of the correlation is offset relative to $r_{e}$ for each $\eta$ value. The agreement of the correlation between the clusters is excellent. The scatter of the correlations about the mean line is $\Delta(\log R) \lesssim 0.1$ dex. The factors by which $r_{e}$ must be multiplied to obtain $r(\eta)$ are approximately $2,2.5,3.6$, and 5.3 for $\eta=1.5,2.0,2.5$, and 3.0 , similar to the factors calculated for the various formal templates in the first paper of this series (SP90, Table 1). The factors for the de Vaucouleurs $r^{1 / 4}$ fitting template alone was discussed previously by Thomsen and Frandsen (1983, their Table V).

The difference in the slope of the data from that of the line of equality in Figure 8 for $\eta>2$ shows that a common template $I(r)$ profile does not, in fact, fit E galaxies of all absolute magnitudes at large distances from the center. Galaxies with larger intrinsic radii (and therefore brighter absolute magnitudes) have more extended outer regions [in their ratio of $r(\eta)$ to $r_{e}$ ]
than do fainter E galaxies. It is the same effect as discussed by Oemler (1974, 1976), Kormendy (1977), Malumuth (1983), and by Schomert (1987, 1988). Their statement that the more luminous galaxies have extended envelopes is equivalent to stating that the Oemler $\alpha / \beta$ ratios and/or the King $r_{t} / r_{c}$ ratios increase as the absolute magnitudes brighten, giving thereby the change of slope from 1 in the data points in the lower two panels of Figure 8.

## V. VARIATION OF SURFACE BRIGHTNESS AVERAGED OVER VARIOUS METRIC RADII FOR GALAXIES OF DIFFERENT ABSOLUTE MAGNITUDES

The data in the previous sections are now combined to determine how average SB values, suitably defined, vary with absolute magnitude. We use here both the effective and the Petrosian $r(\eta)$ radii to define the various $\langle\mathrm{SB}\rangle$ measures. Both types of radii give metric rather than isophotal sizes; the dis-

TABLE 4
Photometric Data for Galaxies in the Sadler Field

| Name | $m-M$ | MBT | Be | re | ＜SB＞e | Name | $m-M$ | MBT | Be | re | $<S B>e$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N312 | 35.97 | －22．47 | 14.25 | 22.8 | 22.28 | N7117 | 35.22 | －21．32 | 14.65 | 16.5 | 21.98 |
| N323 | 35.95 | －22．45 | 14.25 | 18.1 | 21.78 | N7118 | 35.05 | －21．35 | 14.45 | 18.1 | 21.98 |
| N641 | 35.47 | －22．37 | 13.85 | 23.8 | 21.98 | N7192 | 33.71 | －21．21 | 13.25 | 20.8 | 21.08 |
| N745 | 35.32 | －21．52 | 14.55 | 18.1 | 22.08 | N7216 | 34.12 | －20．62 | 14.25 | 15.7 | 21.48 |
| N1419 | 32.18 | －18．68 | 14.25 | 17.3 | 21.68 | N7676 | 34.08 | －20．68 | 14.15 | 12.5 | 20.88 |
| N1930 | 34.54 | －21．54 | 13.75 | 27.4 | 22.18 | N7796 | 34.08 | －21．48 | 13.35 | 18.1 | 20.88 |
| N2305 | 34.04 | －21．34 | 13.45 | 22.8 | 21.48 | IC1459 | 32.64 | －21．64 | 11.75 | 32.9 | 20.58 |
| N2663 | 32.93 | －21．23 | 12.45 | 72.0 | 22.98 | IC1864 | 34.73 | －21．23 | 14.25 | 16.5 | 21.58 |
| N2887 | 33.58 | －20．78 | 13.55 | 22.8 | 21.58 | IC3370 | 33.67 | －21．67 | 12.75 | 36.1 | 21.78 |
| N3087 | 33.40 | －20．80 | 13.35 | 18.9 | 20.98 | IC4421 | 34.20 | －20．40 | 14.55 | 9.1 | 20.58 |
| N3136B | 32.39 | －18．89 | 14.25 | 15.0 | 21.38 | IC4451 | 34.40 | －21． 50 | 13.65 | 31.4 | 22.38 |
| N3224 | 33.65 | －20．65 | 13.75 | 18.1 | 21.28 | IC4889 | 33.43 | －21．23 | 12.95 | 18.9 | 20.58 |
| N5090 | 34.01 | －20．81 | 13.95 | 15.7 | 21.18 | IC4943 | 33.75 | －19．85 | 14.65 | 9.1 | 20.68 |
| N6483 | 34.90 | －21．60 | 14.05 | 15.7 | 21.28 | IC5105 | 35.16 | －22．26 | 13.65 | 18.9 | 21.28 |
| N6730 | 34.53 | －21．23 | 14.05 | 14.4 | 21.08 | 107－GO | 34.02 | －21．12 | 13.65 | 19.8 | 21.38 |
| N6758 | 34.08 | －21．48 | 13.35 | 21.7 | 21.28 | 182－GO | 34.95 | －21．05 | 14.65 | 8.7 | 20.58 |
| N6849 | 35． 38 | －21．98 | 14.15 | 22.8 | 22.18 | 208－G2 | 31.05 | －18．85 | 12.95 | 25.0 | 21.18 |
| N6861 | 33.72 | －21．52 | 12.95 | 14.4 | 19.98 | 235－G4 | 34.90 | －21．30 | 14.35 | 15.7 | 21.58 |
| N6868 | 33.73 | －22．03 | 12.45 | 34.4 | 21.38 | 235－G8 | 33.26 | －19．16 | 14.85 | 9．9＇ | 21.08 |
| N6909 | 33.63 | －21．03 | 13.35 | 27.4 | 21.78 | 286－G4 | 35.06 | －20．96 | 14.85 | 9.9 | 21.08 |
| N6958 | 33.69 | －21．29 | 13.15 | 19.8 | 20.88 | 286－G5 | 33.60 | －20．00 | 14.35 | 16.5 | 21.68 |
| N7002 | 35.86 | －22．56 | 14.05 | 32.9 | 22.88 | 318－G2 | 34.79 | －21．09 | 14.45 | 16.5 | 21.78 |
| N7029 | 33.72 | －21．22 | 13.25 | 16.5 | 20.58 | 323－G3 | 34.51 | －22．01 | 13.25 | 21.7 | 21.18 |
| N7097 | 33.46 | －20．76 | 13.45 | 16.5 | 20.78 |  |  |  |  |  |  |

tinction is highly important for the Tolman test（Sandage 1972）．

Figure 9 shows the $\langle\mathrm{SB}\rangle_{e}, M_{B(T)}$ relation for the Virgo cluster data in Table 1．The dwarf branch，defined as compris－ ing galaxies fainter than $M_{B} \sim-19$ ，is well formed and shows decreasing 〈SB〉 with decreasing luminosity，again，as in BST


Fig．4．－Same as Fig．3，but with the data for the Fornax and Coma clusters added from Tables 2 and 3，combined with the data for the field galaxies from Tables 4 and 5．The Fornax data are calculated using the dis－ tance modulus of $m-M=31.9$ ；these for Coma with $m-M=35.5$ ，and those for the field data with the Hubble constant of $50 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$ ．Open circles are for the Virgo cluster，filled circles are for Fornax，filled triangles are for Coma，asterisks are galaxies studied by Sadler，and filled squares are the RSA2 field galaxy sample．
（1984）and in Caldwell and Bothun（1987）．The giant branch is ill defined using Virgo data alone because there are so few bright ellipticals in Virgo．
The sample is increased in Figure 10 by showing each of the data sets in turn for the Virgo plus the Fornax data from Tables 1 and 2 and 2A，for the Coma data from Table 3， and for the RSA2 galaxies from Table 5 and the Sadler field galaxy data from Table 4．The combined date are shown in Figure $11 .{ }^{2}$

[^1]TABLE 5
Photometric Data for Galaxies in the RSA + RC2 Sample

| Name | $m-M$ | MBT | Be | re | <SB>e | Name | $m-M$ | MBT | Be | re | <SB>e |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N 147 | 24.32 | -14.36 | 10.71 | 190.8 | 23.8 | N3818 | 32.05 | -19.26 | 13.54 | 18.0 | 21.1 |
| N 185 | 24.32 | -14.59 | 10.48 | 169.2 | 23.2 | N3872 | 33.88 | -21.08 | 13.55 | 18.0 | 21.0 |
| N 221 | 24.32 | -15.53 | 9.54 | 32.4 | 18.7 | N3904 | 32.13 | -20.30 | 12.58 | 18.0 | 20.3 |
| N 533 | 35.27 | -22.52 | 13.50 | 43.2 | 22.6 | N4168 | 31.70 | -19.49 | 12.96 | 28.8 | 21.6 |
| N 596 | 33.03 | -21.15 | 12.63 | 28.8 | 21.2 | N4261 | 33.09 | -21.71 | 12.13 | 43.2 | 21.4 |
| N 720 | 32.74 | -21.59 | 11.90 | 36.0 | 21.0 | N4278 | 30.37 | -19.24 | 11.88 | 28.8 | 20.5 |
| N 741 | 35.29 | -22.75 | 13.29 | 43.2 | 22.4 | N4283 | 31.62 | -18.50 | 13.87 | 14.4 | 20. 8 |
| N 750 | 35.16 | -21.99 | 13.92 | 18.0 | 22.2 | N4365 | 31.70 | -21.10 | 11.35 | 54.0 | 21.3 |
| N 821 | 32.90 | -21.01 | 12.64 | 43.2 | 21.9 | N4374 | 31.70 | -21.47 | 10.98 | 54.0 | 20. 9 |
| N1199 | 33.65 | -21.23 | 13.17 | 21.6 | 21.2 | N4373 | 34.00 | -22.14 | 12.61 | 25.2 | 21.7 |
| N1209 | 33.62 | -21.36 | 13.01 | 21.6 | 20.4 | N4473 | 31.70 | -20.63 | 11.82 | 36.0 | 20.8 |
| N1275 | 35.18 | -23.27 | 12.66 | 25.2 | 21.5 | N4476 | 31.70 | -18.62 | 13.83 | 14.4 | 21.0 |
| N1399 | 3240 | -21.61 | 11.54 | 28.8 | 20.1 | N4478 | 31.70 | -19.55 | 12.90 | 14.4 | 20.0 |
| N1404 | 32.40 | -21.34 | 11.81 | 14.4 | 19.1 | N4486 | 31.70 | -22.08 | 10.37 | 93.6 | 21.4 |
| N1407 | 32.74 | -21.81 | 11.68 | 36.0 | 20.7 | N4494 | 32.01 | -21.27 | 11.49 | 43.2 | 20.9 |
| N1426 | 32.24 | -19.87 | 13.12 | 25.2 | 21.2 | N4564 | 31.70 | -19.83 | 12.62 | 21.6 | 20.5 |
| N1453 | 3450 | -21.91 | 13.34 | 21.6 | 21.2 | N4621 | 31.70 | -21.03 | 11.42 | 43.2 | 20.9 |
| N1521 | 34.61 | -22 03 | 13.33 | 32.4 | 22.0 | N4660 | 31.70 | -19.83 | 12.62 | 21.6 | 20.5 |
| N1549 | 3149 | -20.79 | 11.45 | 32.4 | 20.4 | N4696 | 33.66 | -22.30 | 12.11 | 43.2 | 22.0 |
| N1600 | 34.88 | -22.87 | 12.76 | 32.4 | 21.7 | N4782 | 34.39 | -21.64 | 13.50 | 14.4 | 20.9 |
| N1700 | 34.48 | -22.67 | 12.56 | 25.2 | 21.0 | N4783 | 34.74 | -21.94 | 13.55 | 21.6 | 21.4 |
| N2325 | 33.06 | -21.28 | 12.53 | 32.4 | 21.7 | N4889 | 35.56 | -22.99 | 13.32 | 36.0 | 22.2 |
| N2314 | 34.56 | -21.73 | 13.58 | 18.0 | 21.2 | N4936 | 33.93 | -21.65 | 13.03 | 18.0 | 20.6 |
| N2434 | 31.75 | -19.68 | 12.82 | 25.2 | 21.4 | N5044 | 33.48 | -21.61 | 12.62 | 32.4 | 21.7 |
| N2672 | 34.46 | -21.95 | 13.26 | 25.2 | 21.5 | N5077 | 33.55 | -21.03 | 13.27 | 14.4 | 20.5 |
| N2693 | 35.02 | -22.32 | 13.45 | 25.2 | 21.7 | N5322 | 32.95 | -22.04 | 11.66 | 61.2 | 21.7 |
| N2749 | 34.59 | -21.56 | 13.78 | 21.6 | 21.7 | N5557 | 34.09 | -22.08 | 12.76 | 25.2 | 20.9 |
| N2832 | 35.71 | -23.32 | 13.14 | 36.0 | 22.3 | N5576 | 32.27 | -20.51 | 12.51 | 21.6 | 20.3 |
| N2865 | 33.46 | -21.37 | 12.84 | 18.0 | 20.5 | N5638 | 32.51 | -20.31 | 12.95 | 25.2 | 21.1 |
| N2974 | 32.80 | -21.12 | 12.43 | 32.4 | 21.3 | N5813 | 32.78 | -21.21 | 12.32 | 43.2 | 21. 9 |
| N2986 | 33.16 | -21.32 | 12.59 | 25.2 | 20.8 | N5831 | 32.53 | -20.07 | 13.21 | 25.2 | 21.6 |
| N3078 | 33.23 | -21.31 | 12.67 | 25.2 | 21.0 | N5982 | 33.92 | -21.89 | 12.78 | 21.6 | 20.8 |
| N3136 | 32.24 | -20.82 | 12.17 | 25.2 | 21.0 | N6721 | 34.68 | -21.75 | 13.68 | 21.6 | 21.7 |
| N3158 | 35.74 | -22.84 | 13.65 | 28.8 | 22.0 | N7014 | 34.88 | -21.60 | 14.03 | 21.6 | 21.9 |
| N3193 | 32.09 | -20.26 | 12.58 | 25.2 | 21.0 | N7144 | 33.07 | -21.32 | 12.50 | 43.2 | 21.8 |
| N3250 | 33.55 | -21.76 | 12.54 | 32.4 | 21.7 | N7145 | 32.86 | -20.73 | 12.88 | 28.8 | 21.5 |
| N3258 | 33.54 | -21.06 | 13.23 | 18.0 | 21.2 | N7507 | 32.53 | -21.10 | 12.18 | 28.8 | 20.7 |
| N3268 | 33.50 | -20.93 | 13.32 | 21.6 | 21.5 | N7619 | 34.53 | -22.36 | 12.92 | 28.8 | 21.5 |
| N3309 | 34.41 | -21.76 | 13.40 | 18.0 | 21.3 | N7626 | 34.30 | -22.13 | 12.92 | 32.4 | 21.9 |
| N3348 | 33.91 | -21.83 | 12.83 | 25.2 | 21.2 | N7785 | 34.54 | -21.87 | 13.42 | 21.6 | 21.3 |
| N3377 | 30.36 | -19.26 | 11.85 | 43.2 | 21.2 | I1459 | 32.57 | -21.61 | 11.71 | 36.0 | 20.7 |
| N3379 | 30.91 | -20.58 | 11.08 | 57.6 | 20.9 | 13370 | 33.65 | -21.74 | 12.66 | 28.8 | 21.5 |
| N3557 | 33.78 | -22.55 | 11.98 | 43.2 | 21.5 | I4296 | 34.15 | -22.72 | 12.18 | 46.8 | 22.0 |
| N3608 | 31.72 | -19.84 | 12.63 | 25.2 | 21.0 | I5105 | 35.14 | -22.53 | 13.36 | 25.2 | 21.7 |
| N3640 | 31.86 | -20.60 | 12.01 | 36.0 | 21.1 | HA85-2 | 33.60 | -21.16 | 13.19 | 10.8 | 20.2 |

The Tolman test will be the search for the displacement faintward in the ordinate of Figure 11 by either $2.5 \log (1+z)^{4}$ mag or $2.5 \log (1+z)$ mag as the redshift increases, after the measured $\langle\mathrm{SB}\rangle$ values of the distant galaxies have been corrected for the total K-term (i.e., the selective plus the $1+z$ band width nonselective term; see Humason, Mayall, and Sandage 1956, Appendix A; Oke and Sandage 1968).

The turnover in the diagram near $M_{B(T)} \sim-20$ is a potentially valuable feature of Figure 11 because if distant cluster data can be obtained over the first five magnitudes (say) of the luminosity function, this turnover could be found observationally, and the abscissa values for each cluster need not be calculated from a cosmological model. Therefore, the test could be made independent of $q_{0}$, and of even the form of the Hubble diagram. But if the turnover cannot be found due to an insufficient flux grasp, the test would not be quite as clean in principle; $M_{B(T)}$ must be calculated for each ( $m, z$ ) cluster data set, from (say) equation (33) of Sandage (1988), so as to find the
abscissa $M_{B(T)}$ value in Figure 11. The result would then depend, but only very slightly, on the cosmology through $q_{0}$, despite the fact that the $(1+z)^{4}$ factor is, itself, model independent (Sandage 1972). However, it needs to be pointed out that this dependence of the abscissa value in Figure 11 on $q_{0}$ is very small compared with the $(1+z)^{4}$ factor, shown as follows.

Between the two limiting cosmological cases of $q_{0}=0$ and $1 / 2$ ( $\Omega$ between 0 and 1 ), the absolute magnitude difference calculated from the equation for the Hubble diagram in the standard cosmological model is 0.27 mag (see Sandage 1988, eqs. [41] and [42]). This uncertainty in the abscissa in Figure 11 for a distant cluster would be read as a displacement in the ordinate if the correlation line is moved horizontally in Figure 11. But as the slope of the correlation line is at most 1 (see Table 6 below), the uncertainty in $\langle\mathrm{SB}\rangle$ caused by this lack of knowledge of $M_{B(T)}$ is at most $\sim 0.3$ mag. Note that there is no additional uncertainty in the $\langle\mathrm{SB}\rangle$ ordinate because the calculation of 〈SB〉 will use only the directly observed apparent


Fig. 5.-Correlation of the Petrosian $R(\eta)$ linear radii with the absolute magnitudes inside these respective radii for galaxies in the Virgo cluster from the data in Table 1, adopting $m-M=31.7$ for the distance modulus. Data for four different values are shown.


Fig. 6.-Same as Fig. 6, but for the Coma cluster for three values of $\eta$ (using $M_{B(\eta)}$ for the abscissa), and for the effective radii (using total $B$ magnitude for the abscissa), adopting $m-M=35.5$ for the distance modulus. Lines of constant average surface brightness, using the appropriate $M_{B}$ values are shown.


Fig. 7.-Similar to Figs. 5 and 6 but for the combined data for the Virgo, Fornax, and Coma clusters. Symbols have the same meaning as in Fig. 4. The straight line in each panel is the constant surface brightness condition.


Fig. 9.-Correlation between the surface brightness averaged over the effective radius and the total blue absolute magnitude for Virgo cluster galaxies from Table 1.


Fig. 8.-The relations between the effective linear radii and the Petrosian $R(\eta)$ radii for galaxies in the Virgo, Fornax, and Coma clusters for $\eta$ values of $1.5,2.0$, 2.5 , and $3.0 \mathrm{mag}^{\operatorname{arcsec}}{ }^{-2}$. Same symbols as in Figs. 4 and 7. Data are from Tables 1 and 3. The drawn lines are for equality between ordinate and abscissa.


Fig. 10.-Same as Fig. 9, but for the separate data sets of Virgo plus Fornax, Coma alone, the RSA2 sample from Table 5, and the Sadler sample from Table 4.
magnitudes ( $K$-corrected) and angular radii, both of which are measured, either at some Petrosian $\eta$ radius or at the half-light radius. Hence, the uncertainty in $M_{T}$ due to uncertainty in $q_{0}$ is small compared to the Tolman $(1+z)^{4}$ factor of 1.8 mag at $z=0.5$. Furthermore, as the redshift increases the Tolman factor increases relative to the model-dependent uncertainty in


Fig. 11.- Correlation of effective surface brightness and absolute total $B$ magnitude for the combined data from Tables 1-5. Symbols are the same as in Figure 4, 7, and 8. Note that these data are for E and dE galaxies only, with no contamination from later Hubble types which, if included, would produce a scatter diagram (N.B. footnote 2).


Fig. 12.-Same as Fig. 11, for Coma, Virgo, and Fornax, but using Petrosian parameters calculated for $\eta$ values of $1.5,1.7,2.0$, and $2.5 \mathrm{mag} \mathrm{arcsec}^{-2}$.
the abscissa due to an uncertain $q_{0}$ value. And even if the universe does not expand, and therefore if the standard model does not apply, our test would simply be to show that the Tolman factor for the expanding model could not be found.

Finally, we set out in Figure 12 the $\langle\mathrm{SB}\rangle_{\eta}$, magnitude relation for four different Petrosian $r(\eta)$ radii using the combined data from Tables 1-3. The abscissa are $M_{B(\eta)}$ rather than "total" magnitude, circumventing the problem of extrapolating the growth curve to $r=\infty$. The second advantage of using Petrosian parameters is, again, the greater angular resolution they provide. However, the disadvantage is that the sampling at such large $\eta$ values works in the regime of flux starvation at large redshifts.

## VI. INDEPENDENT CONFIRMATION OF THE 〈SB〉 DECREASE FOR

 INCREASING LUMINOSITY OF GIANT E GALAXIESBecause of the importance of Figure 11 and 12 for the Tolman test, we summarize here two modern independent studies which show that the sense of the bright-end slope of the $\langle\mathrm{SB}\rangle, M$ correlation in Figures 11 and 12 is correct.
The essence of the correlation had already been found in the prior works by Oemler (1974, 1976), by Kormendy (1977, 1980), and by Strom and Strom (1978a, b, c). They each have set out their data as a correlation between $\langle\mathrm{SB}\rangle$ and $R_{e}$, not showing the absolute magnitude relation explicitly as in Figure 11, but, of course, implicitly because $\langle S B\rangle$ is derived from $M$ and $R$ alone. A review of much of this early work is given by Thomsen and Frandsen (1983) in the discussion of their photometry of the brightest E galaxies in two nearby galaxy groups and three clusters.

These authors fitted a de Vaucouleurs $r^{1 / 4}$ template to each
of their measured profiles. From the results listed in their Table II for the de Vaucouleurs $r_{e}$ fitting parameter, it is possible to calculate $\langle\mathrm{SB}\rangle_{e}$. (Thomsen and Frandsen list the SB at $r_{e}$. Their Table V, as well as Table 1 of SP90, shows that $\eta=1.39$ mag $\operatorname{arcsec}^{-2}$ at $r_{e}$, which, when added to the SB at $r_{e}$ gives $\langle\mathrm{SB}\rangle_{e}$ averaged over the circular area of radius $r_{e}$.) We also calculate $M_{T}$ from the redshift data, using $H_{0}=50$. Combining the $\langle\mathrm{SB}\rangle$ and $M$ data gives a diagram that is equivalent to Figure 11 here, but using only their data. The sense of fainter $\langle\mathrm{SB}\rangle_{e}$ for brighter $M_{T}$ is clearly seen in the reduced data of Thomsen and Frandsen. The slope, scatter, and zero-point closely resembles Figure 11. The data are of high weight, having been obtained and reduced digitally using a linear electronic image camera.

In a CCD study, Djorgovski and Davis (1987, their Table 1) give reduced data for a number of field E galaxies, also fitted with a de Vaucouleurs $r^{1 / 4}$ template, therefore giving, as in Thomsen and Frandsen, the fitting parameter $r_{e}$. Figure 13, shows their linear effective (not half-light) radius correlated with their listed absolute magnitude (reduced to $H_{0}=50$ ) in the $R$ band. The line for the constant $\langle\mathrm{SB}\rangle$ scaling law of $M \sim-5 \log R$ is shown. As in Figure 4 for our data, the brightest galaxies in the DD sample have fainter $\langle\mathrm{SB}\rangle$ than those that are fainter; the radius increases more rapidly with luminosity than the constant SB case.

Figure 14 shows the data in Tables $1-5$ in the $\langle\mathrm{SB}\rangle, \log R_{e}$ plane in which Oemler, Kormendy, and Strom and Strom originally showed their data. The line, which is put with the slope found by DD from their data, clearly satisfies the Tables 1-5 data as well. The new feature of Figure 14., not seen in the original studies cited earlier, is the dwarf branch in this representation, showing again the reverse slope from the giant branch, breaking at $\langle\mathrm{SB}\rangle_{e} \sim 20.5$ in $B$, and $\log R_{e}(\mathrm{psc}) \sim 3.5$, consistent with the two-branch relations in Figures 4, 7, and 10-12.

These tests with independent data obtained with linear elec-


Fig. 13.-Independent data from Djorgovski and Davis (1987) for a sample of giant field E galaxies using their derived parameters (reduced to $H_{0}=50$ ). A line of constant surface brightness (slope $d M / d \log R=5$ ) is shown. The four galaxies of high SB below the line are the compact (stripped?) galaxies M32, NGC 4486B, NGC 5845, and NGC 5846B.


Fig. 14.-The $\langle\mathrm{SB}\rangle, \log R_{e}$ diagram from the data used in the present paper from Tables $1-5$. The line has the slope determined by Djorgovski and Davis from their independent data for giant galaxies.
tronic detectors are satisfactory in showing that the trends in the data plotted in Figures 11 and 12, which we analyze in the next section, are reliable.

## VII. THE INTRINSIC SCATTER IN $\langle\mathrm{SB}\rangle_{e}$ AND $\langle\mathrm{SB}\rangle_{\eta}$ FOR GIANT E GALAXIES AT FIXED ABSOLUTE MAGNITUDE

We now inquire into the intrinsic dispersion of the correlation of $\langle\mathrm{SB}\rangle$ and absolute magnitude. We envisage that the Tolman test will be made by measuring the $\langle\mathrm{SB}\rangle$ of many galaxies in a given cluster of galaxies, sampling as deep as possible into the cluster's luminosity function. This is necessary because use of only the first-ranked member is inadequate since there is a spread of $\sim 2$ mag for known first-ranked galaxies (Sandage and Perelmuter 1991, in Paper III of this series), meaning that the first ranked in one cluster is not equivalent to the first ranked in any other to within this spread. We must therefore map segments of $\langle\mathrm{SB}\rangle, M$ diagram for each cluster, giving individual correlations similar to Figures 11 and 12 but displaced faintward by the Tolman effect by an amount dependending on the redshift. ${ }^{3}$

[^2]

FIG. 15.-Distribution of the effective average surface brightness values for three data samples, after removing the correlation of $\langle\mathrm{SB}\rangle$ with absolute magnitude that is tabulated in Table 6, and reducing the $\langle\mathrm{SB}\rangle$ data to the absolute magnitude of $M=-22\left(H_{0}=50\right)$.


Fig. 16.-Same as Fig. 15, for Virgo, Coma, and Fornax, using Petrosian radii over which to average the surface brightnesses, again reduced to $M=-22$ for the absolute magnitude inside the particular $r(\eta)$ radius, by using the linear correlations set out in Table 6.

TABLE 6
Linear Regressions of the Form $\langle\mathrm{SB}\rangle=A\left(M_{B}\right)+C$ for the Correlation of Surface Brightness and Absolute Magnitude of Giant Galaxies for the Samples Shown in Figures 11 and 12

| Radius Type <br> (1) | Sample <br> (2) | $\begin{gathered} n \\ (3) \end{gathered}$ | $A$ (slope) <br> (4) | $\begin{gathered} C \\ (5) \end{gathered}$ | Galaxies for $M_{B}<x$ <br> (6) | $\underset{(\mathrm{mag})}{\mathrm{sd}}$ <br> (7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Effective | $\mathrm{cl}+\mathrm{fld}$ | 162 | -0.48 | 11.02 | -20.00 | 0.49 |
| Effective | cl only | 46 | -0.46 | 11.82 | -20.00 | 0.73 |
| Effective | fid only | 116 | -0.37 | 13.27 | -20.00 | 0.31 |
| $\mathrm{Eta}=1.5$ | $V+F+C$ | 35 | -0.43 | 13.34 | -19.75 | 0.45 |
| $\mathrm{Eta}=1.7$ | $V+F+C$ | 34 | -0.44 | 13.61 | -19.75 | 0.51 |
| $\mathrm{Eta}=2.0$ | $V+F+C$ | 37 | -0.46 | 13.90 | -19.75 | 0.80 |
| $\mathrm{Eta}=2.5$ | $V+F+C$ | 39 | -0.45 | 14.55 | -19.25 | 0.55 |

Only the giant branches of each of the clusters are expected to be sampled in the test. The purpose of the present section is to determine the ratio of the intrinsic dispersion to the expected Tolman SB shift faintward along the ordinates in Figures 11 and 12 for these giant branches.
To this end, the mean correlation for the giant branches in Figures 11 and 12 have been calculated by least squares, with the results given in Table 6 for data using both Petrosian parameters and the effective $\langle\mathrm{SB}\rangle$ and $M_{B}$ values. These linear regressions permit the $\langle\mathrm{SB}\rangle$ data to be reduced to a fixed absolute magnitude. The data in Tables 1-5 for galaxies brighter than the $M$ values listed in column (6) of Table 6 have been reduced to $M=-22$ to take out the systematic variation of $\langle\mathrm{SB}\rangle$ with absolute magnitude. The standard deviations have
then been calculated for each case from the residuals about mean $\langle\mathrm{SB}\rangle$ values, so reduced.

The results are shown in Figure 15 for the cluster and the field samples in Tables $1-5$ using the effective radii as the measure of metric size. Similar results for the combined Virgo, Fornax, and Coma data are presented in Figure 16 using various Petrosian radii. Although there are significant differences in the calculated standard deviations (for reasons related to stochastic variations which are due to small sample size and to our inclusion in the calculation of a few highly deviant points), even the largest standard deviation of 0.80 mag is small compared to the expected Tolman effect at quite moderate redshifts. For example, at $z=0.5$ the Tolman $(1+z)^{4}$ factor is 1.8 mag . The effect is so large relative to the spread shown in Figures 15 and 16 that the Tolman effect should easily be found if it exists, even considering passive evolution in the look-back times.
If the large $(1+z)^{4}$ shifts are not found when the highredshift data become available, we shall then have a choice between denying the standard model or again invoking evolution (in this case it must be active rather than passive) as an adjustable parameter to save the theory. However, that decision is yet some distance ahead; in these first papers of the series our aim is simply to prepare the way by calibrating the local galaxies so as to form the base before venturing into the regime of large redshifts. The additional preparation that is required beyond that given here and in Paper I is the subject of Paper III on the calibration of the $\langle\mathrm{SB}\rangle$ parameters for the first few ranked galaxies in many nearby clusters directly to provide an additional foundation upon which to compare the data expected in distant rich clusters.

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[^1]:    ${ }^{2}$ Davies，Phillipps，and Disney（1989，and earlier references cited therein， hereafter DPD）state that the correlation in Fig． 11 does not exist．They believe that whatever correlation appears in diagrams similar to Fig． 11 is a selection effect．Because Fig． 11 is so central to applying the Tolman test，a comment is necessary here．

    The data in Fig． 11 are for E and dE galaxy types alone．Adding spirals or background galaxies to the sample would degrade the correlation into a scatter diagram because，for different Hubble types，the 〈SB〉 becomes pro－ gressively fainter at a given absolute magnitude along the Hubble sequence （Sandage 1983）．The effect on Fig． 11 of this variation of $\langle\mathrm{SB}\rangle$ with Hubble type has been demonstrated by removing known background galaxies from a previous sample in the Fornax cluster used by Davies et al．（1988）．Many galaxies considered by DPD as Fornax cluster members had been identified by Ferguson and Sandage（1988，hereafter FS88）as background spirals and brightest E glaxies in distant clusters．Their assessment was bsed on large－ scale，high－resolution plates taken with the du Pont reflector at Las Campanas． Removing the contaminating spirals leaves only dE and dS0 galaxies，all con－ sidered by FS to be cluster members．These form a tight $\langle\mathrm{SB}\rangle, M$ diagram （FS88，Fig．4）similar to Fig．11．In addition，the galaxies which FS considered as background on the basis of morphology alone are uniformly spread over the area，whereas those which they assigned to the Fornax cluster on the same basis are concentrated toward the cluster center．

    The galaxy sample which we are discussing in Fig． 11 is complete for E and dE galaxies in the Virgo and Fornax clusters within the $\langle\mathrm{SB}\rangle$ range shown in the diagram．There are no low $\langle\mathrm{SB}\rangle \mathrm{E}$ and dE galaxies in the relevant absolute magnitude range．Galaxies with $M_{T}$ between -24 and -17 with $\langle\mathrm{SB}\rangle$ values between 23 and $26 \mathrm{mag}_{\mathrm{arcsec}}{ }^{-2}$ would have been found in the catalog surveys of Virgo（Binggeli，Sandage，and Tammann 1985）and of Fornax（FS88，Fergu－ son 1989）．What does not exist cannot lead to a noncorrelation in Fig． 11.
    In their lists，Davies et al．（1988）and Davies，Phillipps，and Disney（1989） make no morphological distinction，nor do they have information on cluster membership or relative distances in their noncluster fields，clouding their claim that Fig． 11 for a complete sample would be a scatter diagram．

[^2]:    ${ }^{3}$ In comparing the data of one cluster to those of others at different redshifts, we suppose that the abscissa (i.e., the absolute magnitude, corrected for $K$ dimming due to redshift) can be determined. If the universe expands, eq. (33) of Sandage (1988) can be used. If it does not, we can simply use the observed tight relation between fully corrected apparent magnitude and redshift and assume a linear redshift-" photometric distance" relation at large redshifts, justified by its proof at small redshifts (Sandage and Hardy 1973, Fig. 7). If, on the other hand, the observations can be pushed faint enough to reach beyond the turnover in Fig. 11, the data can simply be compared by horizontal shifts of the apparent magnitude scale (again after $K$-correction) until the turnover parts of the separate diagrams are made to coincide. This approximation neglects the evolutinary effects discussed in Paper I, but which were estimated there to be small.

