The Astrophysical Journal, 358:28–35, 1990 July 20

CONSTRAINTS ON COLD DARK MATTER–DOMINATED UNIVERSES FROM COSMIC BACKGROUND RADIATION ANISOTROPIES

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Received 1989 October 30; accepted 1990 January 24

ABSTRACT

Detailed numerical calculations are made for the temperature fluctuations in the cosmic background radiation for universes dominated by cold dark matter. It is shown that the limit on temperature fluctuations measured by Readhead *et al.* at 7'.15 yields the strongest constraint on such universes. A low-density universe with $\Omega_0 \lesssim 0.3$ is allowed only when $\Omega_b \lesssim 0.03(0.001)$ for $H_0 \approx 100(50)$ km s^{-1} Mpc^{-1}, if galaxies trace the mass distribution.

Subject headings: cosmic background radiation — cosmology — dark matter

I. INTRODUCTION

The isotropy of the cosmic microwave background radiation (CBR) provides an important constraint on models of the formation of structure in the universe (Peebles and Yu 1970; Wilson and Silk 1981; Wilson 1983; see Kaiser and Silk 1986 for a review). It has been shown by Bond and Efstathiou (1984) and by Vittorio and Silk (1984) that in the presence of the cold dark matter one can develop a gravitational clustering theory in which the structure observed at a few megaparsec scale is consistent with the limit on small-scale temperature fluctuations in the CBR. They have shown at the same time that a large cosmological mass density ($\Omega_0 \gtrsim 0.3$) is necessary to drive the growth of perturbations sufficiently fast to make the observed structure consistently with the CBR anisotropy limit of Uson and Wilkinson (1984*a*, *b*) ($\delta T/T < 3 \times 10^{-5}$) at 4'.5. An even larger density $\Omega_0 \gtrsim 0.5$ is inferred from the null observation of Readhead *et al.* (1989) ($\delta T/T < 2.1 \times 10^{-5}$) at 7'.15 (Sugiyama 1989). Such a large mass density, however, is not supported by the analysis of the dynamics of galaxy clustering, which rather indicates $\Omega_0 \approx 0.1$–0.3.

It has been shown by Fukugita and Umemura (1989) that the limit on Ω_0 is correlated with the baryon mass density Ω_b, and a low-mass density universe is allowed if one assumes a sufficiently small Ω_b, while all previous authors assumed it to be a fixed value of $\Omega_b = 0.03$. The work by Fukigita and Umemura is based on the observation that the Silk damping scale that controls dominantly the evolution of fluctuations in baryons involves Ω_b and Ω_0 in a different way: $\lambda_S \sim 8$ Mpc $\Omega_b^{-1/2}\Omega_0^{-1/4}h^{-2/3}$ or $\theta_S \sim 5'\Omega_b^{-1/2}\Omega_0^{3/4}h^{-1/2}$ in terms of the angular scale with the Hubble constant h normalized by 100 km s^{-1} Mpc^{-1}. An approximation scheme was employed in their paper to calculate the CBR fluctuations using the transfer function formalism with its form taken from the simulation of cold dark matter–dominated universes.

In this paper we report the result of a quantitative analysis[1] for the constraint inferred from the limits on the small-scale anisotropy of the CBR in the cold dark matter–dominated universe. We calculate the growth of primordial density fluctuations by numerically integrating the evolution equation for baryons, photons, massless neutrinos, and the hypothetical cold dark matter. The CBR fluctuations predicted from the observed cosmic structure are then compared with the observational limits. In particular, we carry out an analysis with an extensive variety of parameter sets and attempt to derive constraints on the model parameters Ω_0, Ω_b, and the Hubble constant h. We consider only the adiabatic perturbations of the Harrison-Zeldovich type spectrum, the advantage of which has been discussed in the literature. For evolution equations we adopt the gauge-invariant formalism of Bardeen (1980) as further developed by Kodama and Sasaki (1984). This method was already used to calculate CBR anisotropies by Gouda and Sasaki (1986) and Gouda, Sasaki and Suto (1989) for baryon-dominated universes and extended by Sugiyama (1989) to the case for universes with the dark matter.

In § II we briefly summarize the formalism and present the method of calculations. Numerical results are given in § III, and constraints on cold dark matter universes are derived in § IV.

[1] See also Holtzman (1989), for a recent calculation of the anisotropy with extensive choices of model parameters.

II. CALCULATIONS

We write the perturbed Robertson-Walker metric in the form

$$ds^2 = -a^2(1 + 2AY)d\eta^2 - 2a^2BY_i\,d\eta\,dx^i + a^2(\gamma_{ij} + 2H_L\,Y\gamma_{ij} + 2H_T\,Y_{ij})dx^i\,dx^j\ , \quad (1)$$

where η is the conformal time and a is the cosmic scale factor; the harmonic function Y satisfies the equation

$$\gamma^{ij}Y_{|ij} = -k^2 Y\ , \quad (2)$$

and Y_i and Y_{ij} are given by

$$Y_i = -\frac{1}{k} Y_{|i} , \tag{3}$$

$$Y_{ij} = \frac{1}{k^2} Y_{|ij} + \frac{1}{3} \gamma_{ij} Y , \tag{4}$$

with "|" the covariant derivative, and their coefficients A, B, H_L, and H_T stand for the amplitudes of metric perturbations in the lapse function, the shift vector, and the spatial coordinates. There are two independent gauge-invariant combinations for the perturbations, the intrinsic spatial curvature Φ, and the gravitational potential Ψ as expressed by

$$\Phi \equiv H_L + \frac{1}{3} H_T + \frac{a'}{ak}\left(B - \frac{1}{k} H'_T\right) , \tag{5}$$

$$\Psi \equiv A + \frac{a'}{ak}\left(B - \frac{1}{k} H'_T\right) + \frac{1}{k}\left(B' - \frac{1}{k} H''_T\right) . \tag{6}$$

The perturbed energy momentum tensor is given by

$$\begin{aligned} \tilde{T}_{(\alpha)}{}^0{}_0 &= -\rho_\alpha(1 + \delta_\alpha Y) , \\ \tilde{T}_{(\alpha)}{}^0{}_j &= (\rho_\alpha + p_\alpha)(v_\alpha - B) Y_j , \\ \tilde{T}_{(\alpha)}{}^i{}_0 &= -(\rho_\alpha + p_\alpha) v_\alpha Y^i , \\ \tilde{T}_{(\alpha)}{}^i{}_j &= p_\alpha(\delta^i{}_j + \pi_{L\alpha} Y \delta^i{}_j + \pi_{T\alpha} Y^i{}_j) , \end{aligned} \tag{7}$$

where ρ_α and p_α denote unperturbed energy density and pressure, and δ_α, v_α, $\pi_{L\alpha}$, and $\pi_{T\alpha}$ are perturbations with respect to energy density, velocity, and isotropic and anisotropic stresses with the subscript α referring to x (cold dark matter), b (baryons), r (photons), and v (massless neutrinos). We then introduce the gauge-invariant perturbation variables for the matter; for each component of α we define density perturbations Δ_α, shear velocity perturbations V_α, entropy perturbations Γ_α and anisotropic stress perturbations Π_α, all relative to the total-matter rest frame, to be

$$\Delta_\alpha \equiv \delta_\alpha + 3(1 + w_\alpha) \frac{a'}{ak} (v - B) , \tag{8}$$

$$V_\alpha \equiv v_\alpha - \frac{1}{k} H'_T , \tag{9}$$

$$\Gamma_\alpha \equiv \pi_{L\alpha} - \frac{c_\alpha^2}{w_\alpha} \delta_\alpha , \tag{10}$$

$$\Pi_\alpha \equiv \pi_{T\alpha} , \tag{11}$$

where v is the total-matter center of mass velocity, $w_\alpha \equiv p_\alpha/\rho_\alpha$, and $c_\alpha^2 \equiv p'_\alpha/\rho'_\alpha$. The gauge-invariant density perturbations and the shear-velocity perturbations for the total matter are

$$\Delta = \frac{\sum \rho_\alpha \Delta_\alpha}{\rho} , \tag{12}$$

$$V = \frac{\sum (\rho_\alpha + p_\alpha) V_\alpha}{\sum (\rho_\alpha + p_\alpha)} . \tag{13}$$

The initial data of perturbations are given at the epoch when the photon temperature is sufficiently high, $T = 10^8$ K in the present calculation. The spectrum of total density perturbations $\Delta(\eta_i, k)$ at the initial time η_i is taken to be the Harrison-Zeldovich form

$$|\Delta(\eta_i, k)|^2 \propto k^n \quad (n = 1) . \tag{14}$$

At that time the universe was radiation dominated, and hence for the adiabatic initial condition we take

$$\Delta(\eta_i, k) \propto a^2 . \tag{15}$$

For the amplitude of entropy perturbations

$$S_{br}(\eta_i) \equiv \Delta_b - \tfrac{3}{4}\Delta_r = 0 , \tag{16}$$

from the solution of the general growing mode for an early stage.

We carry out the numerical calculation of the evolution equation for each component of the perturbation variables. The detailed form of the equation is given in the paper by Sugiyama (1989; see also Kodama and Sasaki 1984 and Gouda and Sasaki 1986). The outline of our method is as follows. Until the photon temperature falls down to 6000 K, we treat baryons and photons as a single viscous fluid strongly coupled through Thomson scattering. For 6000 K $> T >$ 1000 K we treat baryons and photons separately. We follow Peebles (1968), and Jones and Wyse (1985) to calculate ionization. The collisional Boltzmann equation is then solved for baryons (treated as pressureless dust) and for the distribution function of photons $\tilde{f}(x^\mu, q^\mu)$ (x^μ and q^μ are the four-coordinate and four-momentum of photons). We define the frequency-averaged gauge-invariant brightness function by

$$\epsilon_r(\eta, x, \gamma) = \frac{4\pi}{\rho_r} \int dq(\tilde{f} - f) q^3 + \frac{4}{k} (v - B)\left(\frac{a'}{a} Y - k\gamma^i Y_i\right) , \tag{17}$$

where γ is the direction vector of the three-momentum, q the energy of the photons and f the blackbody distribution. Following Wilson (1983), we then carry out the multipole expansion for $\epsilon_r(\eta, x, \gamma)$:

$$\epsilon_r(\eta, x, \gamma) = \sum_{l=0}^{\infty} \epsilon_{r(l)}(\eta)(-k)^{-l} Y_{|i_1 \ldots i_l} P_{(l)}^{i_1 \cdots i_l}(x, \gamma) , \tag{18}$$

where $P_{(l)}^{i_1 \cdots i_l}$ are lth rank tensors defined by recursion equations

$$P_{(0)} \equiv 1 , \quad P_{(1)}^i \equiv \gamma^i ,$$

$$P_{(l+1)}^{i_1 \ldots i_{l+1}} \equiv \frac{2l + 1}{l + 1} \gamma^{(i_1} P_{(l)}^{i_2 \ldots i_{l+1})} - \frac{l}{l + 1} \gamma^{(i_1 i_2} P_{(l-1)}^{i_3 \ldots i_{l+1})} . \tag{19}$$

Parentheses for indices i indicate symmetrization. The photon perturbation variables are related with the harmonic components $\epsilon_{r(l)}$ as $\Delta_r = \epsilon_{r(0)}$, $V_r = (\frac{1}{4})\epsilon_{r(1)} + V$ and $\Pi_r = (\frac{3}{5})\epsilon_{r(2)}$. The evolution equation is solved by induction. The cold dark matter and massless neutrinos are treated as dust and a collisionless fluid, respectively.

Below $T =$ 1000 K we use the analytic growing mode solution to calculate the evolution of matter (Weinberg 1972). For the photon propagation we solve the free-streaming equation for an optically thin universe,

$$\frac{d}{d\lambda}\left(\frac{1}{4} \epsilon_r + \Psi - \frac{a'}{ka} V - \mu V\right) = \frac{\partial}{\partial \eta} (\Psi - \Phi) , \tag{20}$$

with λ the conformal affine parameter along the photon trajectory. We finally obtain the brightness function at the present epoch η_0 measured in the rest frame of the matter (see, e.g., Gouda, Sasaki, and Suto 1989):

$$\Theta_m(\eta_0, \gamma) \equiv \left[\frac{1}{4}\,\epsilon_r(\eta_1, \gamma) + \Psi(\eta_1) - \frac{a'}{ka}\,V(\eta_1) - \mu V(\eta_1)\right] \times e^{-ik\mu\lambda'} - \Psi(\eta_0) + \frac{a'}{ka}\,V(\eta_0) + \mu V(\eta_0) \quad (21)$$

where $\mu \equiv \boldsymbol{k} \cdot \gamma/k$ and η_1 is the conformal time at some epoch arbitrarily chosen after recombination, at which numerical solution is switched into the free-streaming formula. We choose this epoch to be $T = 1000$ K. For a flat universe, $\lambda' \equiv \eta_0 - \eta_1$ and for an open universe

$$\lambda' \simeq (-K)^{-1/2} \sinh\,[(-K)^{1/2}(\eta_0 - \eta_1)]$$

for a small angular separation with K given by $K = -a^2H(1 - \Omega)$.

In calculating the anisotropy of CBR we note that the monopole component of Θ_m does not contribute to the anisotropy and the dipole component contributes only to the bulk motion of the total matter relative to the CBR. We then subtract these components from Θ_m, and define

$$\Theta \equiv \left\{\Theta_m(\eta_0, \gamma) - \left[-\Psi(\eta_0) + \frac{a'}{ka}\,V(\eta_0) + \mu V(\eta_0)\right]\right\} e^{ik\mu\lambda'} = \frac{1}{4}\,\epsilon_r(\eta_1, \gamma) + \Psi(\eta_1) - \frac{a'}{ka}\,V(\eta_1) - \mu V(\eta_1)\,, \quad (22)$$

where the second and third terms represent the Sachs-Wolfe (1967) effect. For a small angular separation we calculate the intrinsic temperature correlation function as

$$C(\theta) = \left\langle \frac{\delta T}{T}(\gamma)\,\frac{\delta T}{T}(\gamma')\right\rangle = 4\pi \int dk k^2 \,|\,\Theta(k)\,|^2_{\rm rms}\, j_0(\lambda k\theta)\,, \quad (23)$$

where $\delta T/T(\gamma)$ is the photon temperature fluctuation brought from the last scattering surface and $\theta = |\,\gamma - \gamma'\,|$.

To compare the prediction with actual observations, we have to take account of the effect of the antenna beam width σ (as defined by FWHM). The effective correlation function $C(\theta, \sigma)$ is expressed by (Wilson and Silk 1981)

$$C(\theta, \sigma) = \frac{2}{(\sigma/1.2)^2} \int d\theta'\theta' C(\theta') \times \exp\left[-\frac{(\theta^2 + \theta'^2)}{(\sigma/1.2)^2}\right] I_0\left[\frac{2\theta\theta'}{(\sigma/1.2)^2}\right], \quad (24)$$

with I_0 the modified Bessel function. The mean square temperature anisotropy given by a triple beam (double switching) measurement reads

$$\left\langle \left|\frac{\delta T}{T}(\theta, \sigma)\right|^2 \right\rangle = \frac{3}{2}\,C(0, \sigma) - 2C(\theta, \sigma) + \frac{1}{2}\,C(2\theta, \sigma)\,. \quad (25)$$

We fix the normalization of perturbations by assuming the second moment of the two-point mass correlation function,

$$J_3(R) \equiv \int_0^R dr r^2 \xi(r) = 4\pi \int dk k^2 \,|\,\Delta(\eta_0, k)\,|^2 \int_0^R dr r^2 j_0(kr)\,, \quad (26)$$

at the present epoch to agree with that for the galaxy-galaxy correlation. We take $J_3(R = 10h^{-1}\ \mathrm{Mpc}) \simeq 280h^{-3}\ \mathrm{Mpc}^3$ in agreement with the data from the CfA redshift survey (Davis and Peebles 1983*a*). Alternatively, we try to normalize the perturbation by

$$\left\langle \left|\frac{\delta M}{M}(R = 8h^{-1}\ \mathrm{Mpc})\right|^2 \right\rangle \equiv 4\pi \int dk k^2 W(kR)\,|\,\Delta(\eta_0, k)\,|^2 \simeq 1\,, \quad (27)$$

with the assumption that light traces mass, where $W(x) = [3j_1(x)/x]^2$. We found that the difference in the result between two normalizations is small; it is typically 10%–20% in $\delta T/T$.

III. NUMERICAL RESULTS

We have made the calculation for varieties of the parameters Ω_0, Ω_b, and h as shown in Table 1. The result for $\delta T/T \equiv \langle|\,\delta T/T(\theta, \sigma)\,|^2\rangle^{1/2}$ is presented in Figure 1 for sample choices of parameters for the beam width $\sigma = 1'.5$(*a*) and $\sigma = 8°$ (*b*). The observational limits of Uson and Wilkinson (1984*a*, *b*) ($\delta T/T < 3 \times 10^{-5}$, $\theta = 4'.5$, $\sigma = 1'.5$) and of Readhead *et al.* (1989) ($\delta T/T < 2.1 \times 10^{-5}$, $\theta = 7'.15$, $\sigma = 1'.8$) are marked in Figure 1*a*, and the value reported by Davies *et al.* (1987) ($\delta T/T = 3.7 \times 10^{-5}$, $\theta = 8°$, $\sigma = 8°$) in Figure 1*b*. We found an agreement between our result and that of Bond and Efstathiou (1984; hereafter BE) to an accuracy of 30%–40% with our result of $\delta T/T$ always larger. We consider the origin of this discrepancy due to the neglect of Sachs-Wolfe terms and the dipole term at $\eta = \eta_1$ (see eq. [22]) in BE as clear from their equation (2b). To demonstrate the contribution of these terms we show in Figure 2 the angular correlation function $C(\theta)$ and $C'(\theta)$ obtained by subtracting the Sachs-Wolfe and dipole terms for the model $\Omega_0 = 1.0$, $\Omega_b = 0.03$, and $h = 0.5$. While the Sachs-Wolfe effect is known to be important at large scales, it modifies substantially $C(\theta)$ also at small scales and increases $\delta T/T$ by an appreciable amount. We found that our correlation function shows a very good agreement with that of BE, once the Sachs-Wolfe and the dipole terms are subtracted from $C(\theta)$.

For convenience we made a fitting in the form

$$C(\theta) = \frac{1}{1 + (\theta/\theta_c)^m}\,C(0) \quad (28)$$

and present resulting parameters in Table 1.

We also compared our calculation with that of Vittorio and Silk (1984). We found that our result agrees with theirs within 20%–30%, their $\delta T/T$ being always larger. We did not attempt to locate the origin of this discrepancy further, for there is no sufficient detail given in their paper. Our $C(0)$ agrees with the value recently reported by Holtzman (1989) to an accuracy of 10%. This slight discrepancy is mostly due to the different normalization for the two-point mass correlation function.

Let us make a comment on the difference seen in the present result and that by the simple treatment of Fukugita and Umemura (1989). In their work the brightness function is assumed essentially to take the form

$$\Theta(k) = Ak^{1/2}e^{-(k/k_S)^2}T(\eta_0, k)\,, \quad (29)$$

with k_S the wave number corresponding to the Silk damping scale and T the transfer function for the cold dark matter. We found in our calculation, however, that $\Theta(k)$ does not damp as fast as $\exp\,[-(k/k_S)^2]$ but $\sim\exp\,[-(k/k_S)^{1.2}]$ for the range of

TABLE 1

MODEL PARAMETERS AND THE RESULT OF THE CALCULATION

		$\delta T/T \times 10^5$				$C(\theta)$		
Ω_0	Ω_b	4'.5	7'.5	10'	8°	$C(0) \times 10^{10}$	$\theta_c(')$	m
				$h = 1.0$				
1.0......	0.3	0.91	1.9	2.9	0.48	44	15	2.6
1.0......	0.1	0.41	0.76	1.1	0.34	7.4	21	1.8
1.0......	0.06	0.33	0.59	0.80	0.31	3.8	23	1.6
1.0......	0.03	0.26	0.47	0.63	0.28	2.0	28	1.3
1.0......	0.01	0.17	0.34	0.48	0.27	1.3	38	1.0
0.8......	0.03	0.35	0.60	0.77	0.32	2.6	25	1.3
0.6......	0.18	1.2	2.1	3.0	0.63	44	15	2.4
0.6......	0.03	0.53	0.86	1.1	0.38	3.8	21	1.3
0.6......	0.01	0.38	0.66	0.85	0.34	2.4	24	1.1
0.4......	0.2	2.9	5.0	7.2	1.4	220	13	2.7
0.4......	0.1	1.5	2.4	3.0	0.76	37	15	2.1
0.4......	0.06	1.2	1.8	2.1	0.61	16	16	1.7
0.4......	0.03	0.98	1.5	1.7	0.51	8.2	16	1.4
0.4......	0.01	0.73	1.2	1.4	0.45	5.1	17	1.2
0.3......	0.1	2.6	4.0	4.9	1.1	90	13	2.2
0.3......	0.02	1.4	2.1	2.3	0.60	13	13	1.3
0.2......	0.03	3.2	4.4	4.6	1.1	51	11	1.5
0.2......	0.01	2.4	3.4	3.7	0.84	26	10	1.3
0.2......	0.006	2.1	3.0	3.4	0.77	22	11	1.4
0.2......	0.003	1.6	2.5	3.0	0.75	18	12	1.4
0.1......	0.002	3.8	6.6	8.0	1.8	120	9.6	1.3
				$h = 0.75$				
1.0......	0.03	0.35	0.64	0.85	0.35	3.0	28	1.1
0.4......	0.03	1.5	2.3	2.6	0.65	16	13	1.4
0.4......	0.01	1.1	1.8	2.2	0.63	11	15	1.3
0.3......	0.03	2.6	3.6	4.0	0.86	34	11	1.5
0.2......	0.03	5.4	7.2	7.7	1.4	110	8.7	1.5
0.2......	0.003	2.5	4.0	4.8	1.1	48	11	1.5
				$h = 0.5$				
1.0......	0.1	0.85	1.4	1.8	0.60	12	22	1.3
1.0......	0.06	0.73	1.3	1.7	0.55	9.6	22	1.3
1.0......	0.03	0.55	1.0	1.4	0.50	7.3	20	1.4
1.0......	0.01	0.32	0.65	0.98	0.50	5.9	28	1.2
0.9......	0.2	1.2	2.0	2.5	0.79	29	21	1.7
0.8......	0.08	1.2	1.9	2.3	0.69	17	19	1.3
0.8......	0.03	0.81	1.5	1.9	0.62	11	19	1.3
0.6......	0.2	2.8	4.3	5.1	1.3	95	16	1.7
0.6......	0.08	1.9	3.0	3.5	0.91	33	15	1.4
0.6......	0.03	1.4	2.3	2.8	0.74	21	15	1.4
0.6......	0.01	0.84	1.6	2.1	0.74	16	19	1.4
0.6......	0.008	0.75	1.4	2.0	0.74	15	19	1.4
0.5......	0.1	2.9	4.4	5.0	1.2	66	14	1.5
0.5......	0.006	0.93	1.8	2.4	0.82	21	18	1.4
0.4......	0.03	2.9	4.4	5.1	1.1	55	11	1.5
0.3......	0.09	8.0	11	12	2.4	310	9.8	1.6
0.2......	0.03	11	15	16	2.5	430	7.5	1.6
0.2......	0.01	7.4	11	12	2.1	260	8.6	1.7
0.2......	0.003	4.7	7.7	9.7	1.6	180	10	1.8

NOTE.—Root mean square fluctuations $\delta T/T$ at 4'.5 (σ = 1'.5), 7'.15 (σ = 1'.8), 10' (σ = 3') and 8° (σ = 8°) are shown in cols. (4)–(7). Cols. (8)–(10) are fitting parameters for the two-point correlation function (eq. [28]).

parameters concerning our interest. This behavior contrasts to the case of baryon-dominated universe, where the $\exp[-(k/k_S)^2]$ behavior is clearly observed (Gouda and Sasaki 1986). Therefore, we suppose that this moderate damping is a characteristic of cold dark matter–dominated universes. Quantitatively the result of Fukugita and Umemura is correct up to a factor of 2 as remarked in their paper. A typical perturbation spectrum at the epoch of $T = 1000$ K is exhibited in Figure 3. We observe the Silk damping in Θ on the short-wavelength side. On the other hand, the perturbations in baryons Δ_b have already caught up with those in the cold dark matter (Δ_x).

IV. CONSTRAINTS ON COLD DARK MATTER–DOMINATED UNIVERSES

Constraints are derived by comparing the prediction of $\delta T/T$ with the limits by Uson and Wilkinson (1984*a*, *b*) and by Readhead *et al.* (1989). The result is displayed on the $\Omega_0 - \Omega_b$

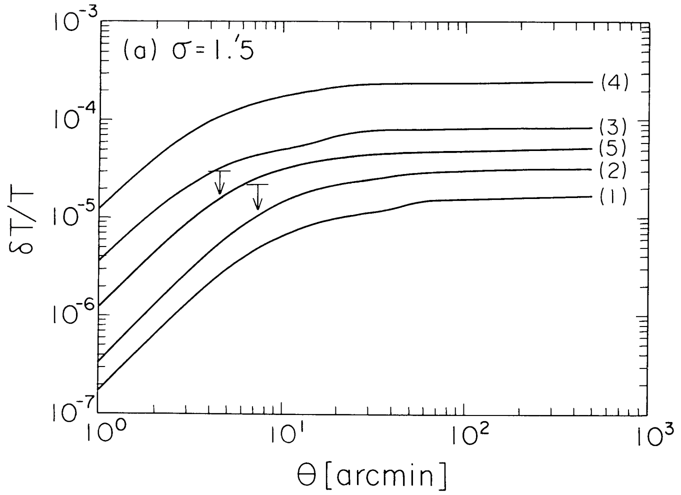

FIG. 1a

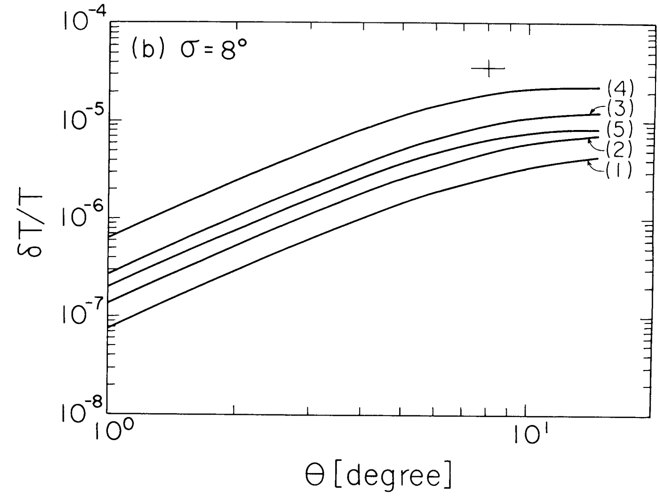

FIG. 1b

FIG. 1.—Root mean square temperature fluctuations $\delta T/T \equiv \langle(\delta T/T)^2\rangle^{1/2}$ as a function of θ. The beam width is assumed to be $\sigma = 1.'5$ (FWHM) for (*a*) and 8° for (*b*). The models are (1) $\Omega_0 = 1.0$, $\Omega_b = 0.03$, $h = 1.0$; (2) $\Omega_0 = 1.0$, $\Omega_b = 0.03$, $h = 0.5$; (3) $\Omega_0 = 0.2$, $\Omega_b = 0.03$, $h = 1.0$; (4) $\Omega_0 = 0.2$, $\Omega_b = 0.03$, $h = 0.5$; (5) $\Omega_0 = 0.2$, $\Omega_b = 0.003$, $h = 1.0$. The upper limits by Uson and Wilkinson (1984*a*, *b*) at $\theta = 4.'5$ and by Readhead *et al.* (1989) at $\theta = 7.'15$ are also shown in (*a*) and the value reported by Davies *et al.* (1987) at $\theta = 8°$ in (*b*).

plane in Figure 4*a* for $h = 1.0$ and in Figure 4*b* for $h = 0.5$. We also added a line corresponding to the fluctuations $\delta T/T = 1.0 \times 10^{-5}$ at $\theta = 10.'0$ and $\sigma = 3.'0$. The most stringent limit is placed by the observation of Readhead *et al.* (1989). The constraint is particularly strong for low-density universes; for $\Omega_0 \lesssim 0.3$, for example, we are led to $\Omega_b \lesssim 0.03$ for $h = 1$ and $\Omega_b \lesssim 0.001$ for $h = 0.5$.

For convenience we present empirical laws for $\delta T/T$, which are valid approximately for $\Omega_b \ll \Omega_0$:

$$\frac{\delta T}{T} = 5.9 \times 10^{-6}\Omega_0^{-1.8}\Omega_b^{0.31}h^{-1.5}, \quad \theta = 4.'5, \quad \sigma = 1.'5 , \quad (30)$$

$$\frac{\delta T}{T} = 8.0 \times 10^{-6}\Omega_0^{-1.6}\Omega_b^{0.24}h^{-1.6}, \quad \theta = 7.'15, \quad \sigma = 1.'8 , \quad (31)$$

$$\frac{\delta T}{T} = 8.8 \times 10^{-6}\Omega_0^{-1.6}\Omega_b^{0.23}h^{-1.7}, \quad \theta = 10.'0, \quad \sigma = 3.'0 . \quad (32)$$

A substantial deviation from this law is found for $\Omega_b \gtrsim 0.1\Omega_0$ for $h = 1.0$ as guessed from Figure 4. We remark that the Ω_b dependence is weaker than that inferred by Fukugita and Umemura. We also obtain

$$\frac{\delta T}{T} = 3.1 \times 10^{-6}\Omega_0^{-0.81}\Omega_b^{0.082}h^{-1.2}, \quad \theta = 8°, \quad \sigma = 8° . \quad (33)$$

for the angle relevant to the observation by Davies *et al.* (1987). When fitting formula (31) is combined with the limit by Readhead *et al.* (1989), we obtain a constraint on the mass density parameter:

$$\Omega_0 \gtrsim 0.55\Omega_b^{0.15}h^{-1} , \quad (34)$$

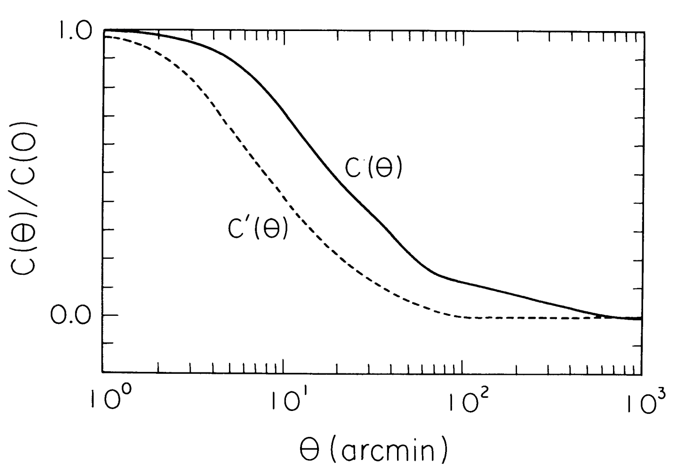

FIG. 2.—The normalized temperature correlation function $C(\theta)/C(0)$ vs. θ. The dashed line [$C'(\theta)$] represents the correlation function with the Sachs-Wolfe and dipole terms subtracted from $C(\theta)$ [normalized by $C(0)$]. The model parameters are $\Omega_0 = 1.0$, $\Omega_b = 0.03$ and $h = 0.5$.

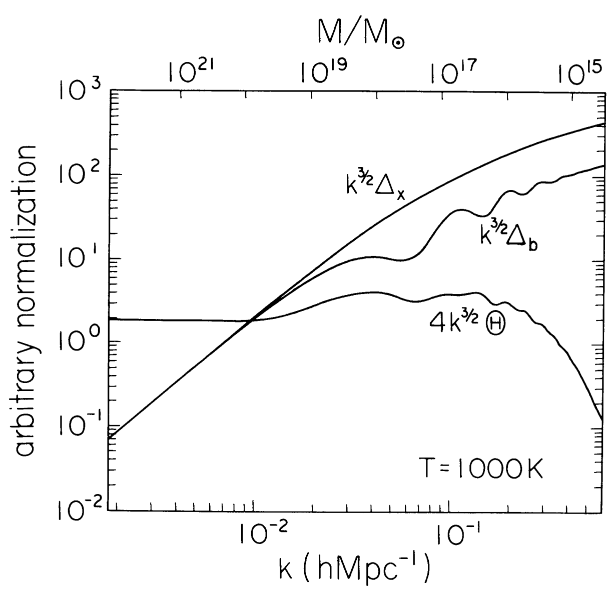

FIG. 3.—Perturbation spectra $k^{3/2}\Delta_x$ (cold dark matter), $k^{3/2}\Delta_b$ (baryons), and $k^{3/2}\Theta_{rms}$ (photons) as a function of k at $T = 1000$ K for the same model as Fig. 2. The normalization is arbitrary.

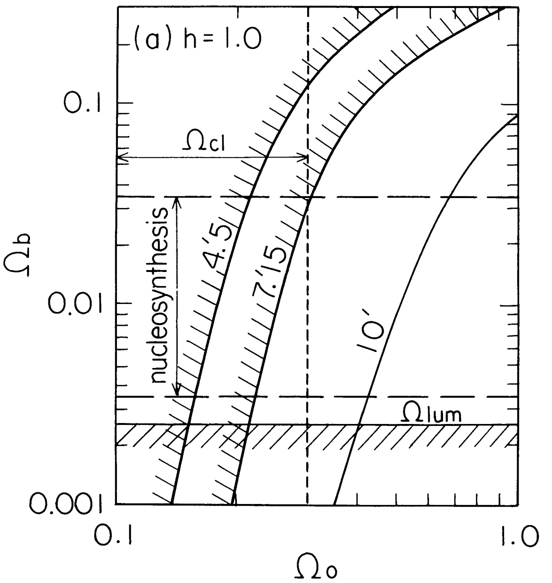

FIG. 4*a*

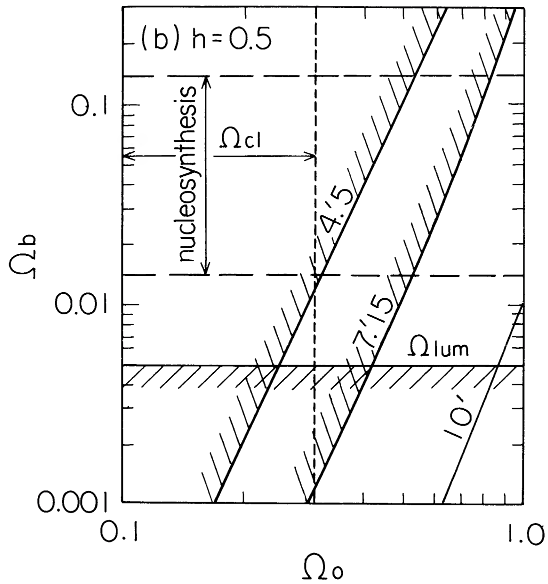

FIG. 4*b*

FIG. 4.—Constraints on cold dark matter–dominated universes for (*a*) $h = 1.0$ and (*b*) $h = 0.5$. The limits set by the observation of Uson and Wilkinson (1984*a*, *b*) at 4'.5 and that of Readhead *et al.* (1989) at 7'.15 are displayed together with a curve for the case of $\delta T/T = 1.0 \times 10^{-5}$ at $\theta = 10'$ ($\sigma = 3'.0$). The limits on Ω_b and Ω_0 from various observations are also plotted.

which yields a fair approximation for the curve shown in Figure 4. We also comment that the value of $\delta T/T$ reported by Davies *et al.* is significantly larger than is expected in cold dark matter–dominated universes if $n \gtrsim 1$ (see Fig. 1*b*).

Let us now discuss the implication of the limit. The analyses for the cosmological mass density mostly support the low-density universe. The observed Virgocentric infall velocity of ~ 300 km s^{-1} provides $0.1 \lesssim \Omega_0 \lesssim 0.3$ with the aid of the spherical perturbation theory (Davis and Peebles 1983*a*; Yahil 1983). (Davis and Huchra 1982 derived a larger value $0.4 < \Omega_0 < 0.5$ from an analysis of the local peculiar gravity, however.) Using the cosmic virial theorem Davis and Peebles (1983*b*) and Bean *et al.* (1983) obtained $\Omega_0 = 0.2e^{\pm 0.4}$ from the observed peculiar velocity of correlated galaxy pairs. On the other hand, Yahil, Walker, and Rowan-Robinson (1986) and Villumsen and Strauss (1987) suggested a nearly flat cosmological model by an analysis of the *IRAS* dipole moment. Fukugita and Ichikawa (1989) criticized, however, that the dipole moment constructed from the IRAS sample underestimates the dipole moment, and a correction for such an effect would also lead to a result in support for a low-mass density universe.

There are several arguments on the baryonic matter density. The most familiar is the one that is derived from primordial nucleosynthesis (Yang *et al.* 1984). Under the assumption of homogeneous early universe the baryon to photon number ratio is restricted conservatively in the range of $10^{-10} \lesssim n_B/n_\gamma \lesssim 10^{-9}$. This leads to the baryonic matter density:

$$0.0035h^{-2} \lesssim \Omega_b \lesssim 0.035h^{-2} . \tag{35}$$

A reliable constraint on Ω_b is that estimated from the abundance of the observed luminous matter in the universe. The luminosity density is estimated using the number of observed galaxies correcting for unseen ones with the aid of the luminosity function, which leads to $\sim(1\text{–}2) \times 10^8 h\, L_\odot$ Mpc^{-3} in the blue band (Davis and Huchra 1982; Kirshner *et al.* 1983; Felten 1985). This luminosity density is converted to the luminous mass density using the average luminous mass-to-luminosity ($M_{\rm lum}/L_B$) ratio of each galaxy. If we take this to be $M_{\rm lum}/L_B \simeq 3$ from the value for the disk of the Galaxy in the solar neighborhood (Faber and Gallagher 1979), we obtain $\Omega_b \simeq 0.0025h^{-1}$ when combined with the luminosity density. We may take this value as a firm lower bound on the baryonic matter density in the universe.

The last argument for Ω_b is based on the ionized intergalactic matter using the Gunn-Peterson test. The optical depth of the intergalactic neutral hydrogen is given by $\tau_{\rm H\,I} \propto \Omega_b^2/J_{\rm UV}$, where $J_{\rm UV}$ is the UV background flux which is estimated from the proximity effect of the QSO Lyα absorption system near the Lyα emission line (Bajtlik, Duncan, and Ostriker 1988). The constraint on $\tau_{\rm H\,I}$ gives an upper bound on the baryonic matter density to be $\Omega_b < 0.1h^{-1}$ (Ostriker and Ikeuchi 1983).

Now, we discuss how a cosmological model of cold dark matter universes is constrained from the isotropy of CBR, when we use the mass density bounds given above (the limits are indicated in Fig. 4). The lower bound on Ω_b from the luminosity density and $M_{\rm lum}/L_B$ gives a lower bound on the mass density of the universe $\Omega_0 \gtrsim 0.22$ if $h = 1$ and $\Omega_0 \gtrsim 0.42$ if $h = 0.5$. The lower bound on Ω_b from primordial nucleosynthesis leads to a similar constraint $\Omega_0 \gtrsim 0.23$ if $h = 1$ and $\Omega_0 \gtrsim 0.54$ if $h = 0.5$. We are left with a narrow range for low-density universes, $0.2 \lesssim \Omega_0 \lesssim 0.3$ if the Hubble constant is close to 100 km s^{-1} Mpc^{-1}. On the other hand, we have no allowed region for $H_0 = 50$ km s^{-1} Mpc^{-1}.

This leads to the condition that the low-density universe with $\Omega_0 \lesssim 0.3$ is viable only when the Hubble constant takes a value 80–100 km s^{-1} Mpc. It is a matter of great significance whether such a value of the Hubble constant is consistent with the age inferred from globular clusters and nucleochronology,

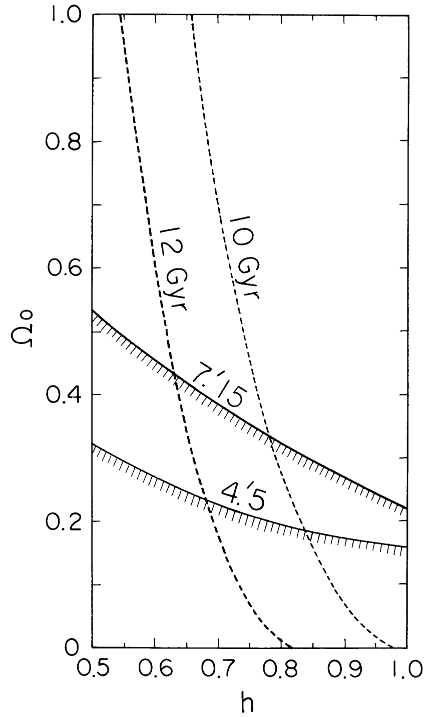

FIG. 5.—Constraints on the $h - \Omega_0$ plane. The constraints shown are derived from the combined use of the limits on CBR temperature fluctuations and the lower limit on Ω_b from nucleosynthesis. Curves corresponding to the age $t_0 = 12$ and 10 Gyr are also shown.

$t_0 \gtrsim 12$ Gyr. Figure 5 shows the constraints on Ω_0 derived using the limit on Ω_b from nucleosynthesis. The restriction from the age of the universe is also plotted on the $h - \Omega_0$ plane, when the cosmological constant vanishes. The Hubble constant is restricted to be $h \lesssim 0.65$ from $t_0 \gtrsim 12$ Gyr. This means that $\Omega_0 \gtrsim 0.4$ and hence the low density universe is marginal.

Such a stringent constraint, however, can be relaxed if galaxies are formed with biasing. This is based on the idea that galaxies are formed preferentially in high density regions having the amplitude of peak density fluctuations $\delta \geq \nu\sigma$, with σ the rms value of the fluctuation amplitude (Kaiser 1984; Silk 1985). In Figure 6 the constraint on the $\Omega_0 - \Omega_b$ plane is shown for the biasing parameter $\nu = 2$–3. Some N-body experiments have shown that the observed properties on large-scale distributions of galaxies can be accounted for with the biasing parameter $\nu = 2$–3 (Davis *et al.* 1985). For $\nu = 3$, say, the constraint on Ω_0 becomes as weak as $\Omega_0 \gtrsim 0.12h^{-1.3}$ for the lower bound on Ω_b given from nucleosynthesis. We finally remark that the strong constraints from the age of the universe and from the isotropy of CBR are both relaxed if the cosmological constant is non-vanishing.

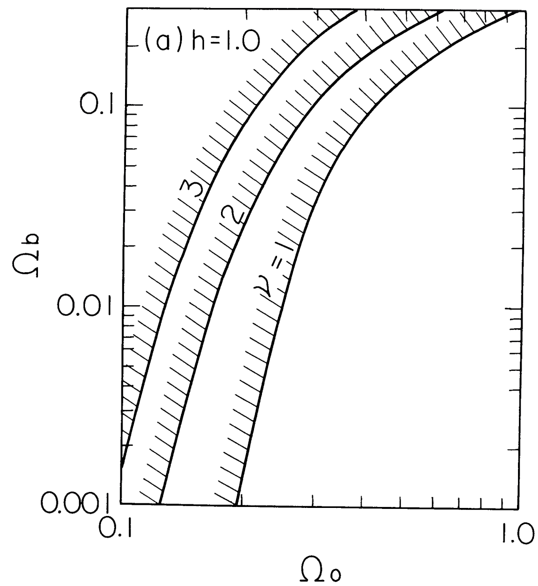

FIG. 6a

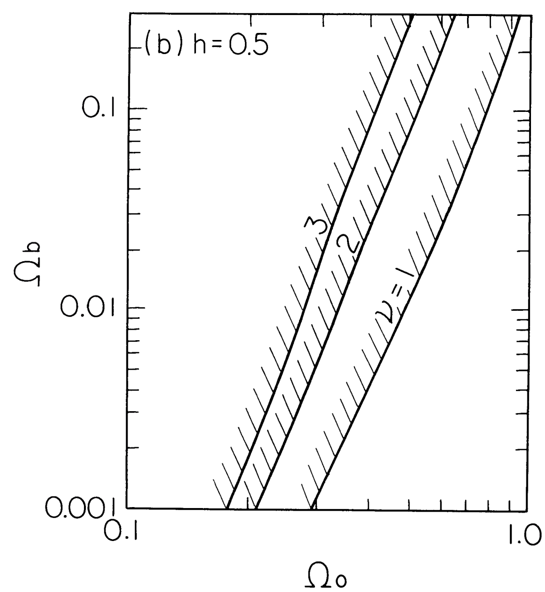

FIG. 6b

FIG. 6.—Constraints given by the observation of Readhead *et al.* (1989) are shown for various biasing parameters, $\nu = 3$, 2, and 1 (no biasing), (*a*) $h = 1.0$ (*b*) $h = 0.5$.

We should like to thank Naoteru Gouda and Misao Sasaki for valuable discussions. One of us (N. S.) is supported by the Soryuushi Shogakkai foundation. The work is also supported in part by the Grant-in-Aid of the Ministry of Education (No. 01629507).

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