THE ASTROPHYSICAL JOURNAL, 357:493–501, 1990 July 10

BINARY PERIODS OF CLUSTER PULSARS

ROGER W. ROMANI
Department of Astronomy, University of California, Berkeley; and Institute for Advanced Study, Princeton
Received 1989 September 5; accepted 1989 December 29

ABSTRACT

Tidal capture products dominate the population of active binaries in globular clusters. Recycled radio pulsars will provide the first extensive sample of the period distribution of such binaries. We compute the expected orbital period distribution for the standard evolutionary scenarios involving mass transfer in binaries with captured stars near the present turn-off mass; comparison with the observed distribution suggests that this evolutionary picture requires significant modification. In agreement with studies of the population of recycled cluster pulsars, the orbital periods of the observed binaries indicate that these modifications should substantially shorten the mass transfer phases during which the pulsar progenitor is a bright X-ray source.

Subject headings: clusters: globular — pulsars — stars: binaries — X-rays: binaries

I. INTRODUCTION

Low-mass X-ray binaries have proved easy to find but difficult to study. While the X-ray sources are bright, optical counterparts are in general quite faint, and orbit parameters have often been impossible to determine. On the other hand, binary radio pulsars are faint and quite difficult to detect. Once discovered, however, pulse timing studies provide a wealth of detailed kinematic information. Recycled pulsars, i.e., the binary and millisecond radio pulsars (e.g., van den Heuvel 1987, 1989), are widely believed to represent the end products of the evolution of the bright X-ray sources. As such, these "fossils" of the bright mass transfer stage can lend much to studies investigating the identity and evolution of the parent population.

In the accepted picture of this evolution, the low-mass X-ray binaries (LMXB) involve transfer from a low-mass companion to an old neutron star with a low magnetic field. These systems are associated with the old (bulge and globular cluster) population and do not in general show X-ray pulsations. For an observational review see Bradt and McClintock (1983). The mass transfer is driven either by angular momentum losses from the system over a time scale $\gtrsim 10^9$ yr or by nuclear evolution of the companion, which generates a brighter, shorter lived source (see § II). As a consequence of this mass transfer, the old neutron star acquires substantial amounts of angular momentum and can be spun-up or "recycled" to spin periods as small as ~ 1 ms. There is good evidence that these old neutron stars will have residual persistent magnetic field components of $\gtrsim 10^8$ G. When recycled, such stars can reappear in the radio sky as long-lived fast radio pulsars.

Analysis of the birthrates of these pulsars seems, however, at some variance with this scenario. In a study of recycled pulsars in the Galactic disk Kulkarni and Narayan (1988) found that extrapolations from observed samples gives an estimated 10^5 recycled pulsars in the Galactic disk (i.e., $\sim 10^{-6}/M_\odot$ of Galactic matter). This is to be compared with the ~ 100 LMXB in the same volume. With lifetimes for the various systems inferred from the standard evolutionary picture, these authors find the LMBP birthrate exceeds that of the LMXB by a factor $\gtrsim 100$; in particular, the excess is strong for the short orbital period systems.

Globular clusters have long been known to be a preferred site for the formation of close, active binaries (Katz 1975); the ~ 10 LMXB found in cluster cores represent an incidence of $\gtrsim 10/(10^7\text{–}10^8)\ M_\odot$, i.e., $\sim 10^2\text{–}10^3$ times that of the Galactic disk. This is understood to be a consequence of the dense stellar environment in the cluster core where close binaries may be formed by tidal capture processes (Fabian, Pringle and Rees 1975; § 3). Thus clusters have been recognized as a profitable arena for the study of mass trasfer binaries such as LMXB and their products. This expectation has been realized with the discovery of millisecond radio pulsars in globular cluster cores. Despite the difficulty of their detection, surveys have found more than a dozen such objects to date; several authors have noted that such observations imply a large cluster population. Romani (1989) and Kulkarni, Narayan, and Romani (1990, hereafter KNR) have studied the cluster surveys and, in analogy to the analysis for the Galactic disk, have derived a quantitative estimate of the cluster pulsar population. The inferred $\gtrsim 10^4$ recycled pulsars represents an incidence of $\sim 10^{-3}$ to $10^{-4}/M_\odot$ of cluster core mass. This is a strong confirmation of the enhancement of their production due to tidal capture and, in fact, shows that the nominal birthrate for these systems exceeds that of the LMXB by $\gtrsim 100$, exactly as for the Galactic disk.

Thus the difficulties with the standard evolutionary scenario highlighted by the study of Kulkarni and Narayan (1988) extend to the globular clusters. Moreover, in deciphering this problem the cluster sample presents a significant advantage. Whereas for the Galactic disk the progenitor binaries leading to LMXB and LMBP are drawn from a largely unknown distribution of orbital separations and mass ratios (depending on poorly understood details of star formation), for the globular clusters the large incidence of such binaries per unit mass establishes these objects as products of tidal capture. Thus, with some assumptions about the cluster populations of various objects, the distribution of initial conditions for the binaries leading to LMXB and LMBP are known. It is therefore possible to calculate the orbital period distribution and other characteristics of the long-lived products, the LMBP, for specific evolutionary scenarios. Comparison with the observed distribution can then indicate revisions and refinements for our picture of progenitor population and its evolution.

In this paper, such a calculation is described. In § II, important elements of mass transfer binary evolution are reviewed, with particular attention to the low-mass systems dominant in the present cluster population. In § III, specific assumptions for the globular cluster populations are made and estimates of the distribution of tidal capture product properties are formed. As there is substantial uncertainty in the evolution of short orbital period systems, we concentrate on wider binaries ($P_b > 3^d$) whose mass donor has evolved by the present epoch. For these systems we estimate production rates and perform evolutionary calculations giving the final distribution of orbital periods for the resulting LMBP. The extension of these sums to $P_b \lesssim 1^d$ systems gives an estimate of the number of short-period binaries resulting from capture onto companions which eventually evolve, although the detailed period distribution in this range is probably sensitive to unmodeled effects. For the standard scenario, the tidal captures and subsequent evolution are effected by a neutron star. Alternatively, we consider the tidal capture of massive white dwarfs, as the subsequent binary evolution has been proposed to cause accretion-induced collapse (AIC) of the dwarf to a neutron star. Specific LMBP systems (e.g., Helfand, Ruderman, and Shaham 1983; van den Huevel and Taam 1984) have been argued to be the products of such evolution; moreover, AIC has been proposed as an alternative source of "recycled" pulsars sufficient to obviate difficulties with the standard model (e.g., Grindlay 1987; Bailyn and Grindlay 1990; KNR). While other authors have questioned the viability of an AIC origin for the recycled pulsars on theoretical (Narayan and Popham 1989) and statistical grounds (Verbunt, Lewin, and Van Paradijs 1989), the scenario has a number of attractive features, and it is desirable to have quantitative tests of its expected results.

Accordingly, in § IV the computed distributions are compared with our present sample of cluster millisecond pulsars. To do this, we must infer the orbital period at the end of mass transfer from the observed pulsar parameters. We argue that at least half of the detected pulsars fall in the long-period class; while the present statistics are too weak to make strong claims, it already appears from the large P_b objects that some amendments to the standard model of LMBP genesis are needed. In § V we discuss such modifications, including those applicable to the AIC picture. Thus the present sample of the binary period distribution, while quite limited, supports the implications of the lifetime studies—LMBP arise from systems with substantially accelerated evolution and shortened X-ray phases. As other cluster recycled pulsars are discovered and as pulse timing studies progress, more detailed statistical comparisons with specific distributions will be warranted.

II. WIDE BINARY EVOLUTION

Motivated in particular by the discovery of bright LMXB bulge sources and the wide orbit binary millisecond pulsars in the Galactic disk, recent theoretical work has resulted in a fairly cohesive picture of the binary evolution of low-mass accreting systems. Here we present a brief review of key elements that affect our present calculation, more complete descriptions of various evolutionary scenarios can be found in Webbink, Rappaport, and Savonije (1983), de Kool, van den Heuvel, and Rappaport (1986), Verbunt (1988), and references therein.

We take an approximate expression for the size of the Roche lobe of star 2 in terms of the orbital semimajor axis a, as given by Pacyński (1967)

$$R_L \approx 0.46a\left[\frac{M_2}{(M_1 + M_2)}\right]^{1/3} \qquad (1)$$

which is $\gtrsim 2\%$ accurate for $M_2/M_1 < 0.8$. For mass transfer systems in contact, the mean density $\bar{\rho}$ within the donor's Roche lobe is related to the binary period

$$P_b = \frac{2\pi a^{3/2}}{[G(M_1 + M_2)]^{1/2}} \approx 0.37\left(\frac{\rho_\odot}{\bar{\rho}}\right)^{1/2} \text{days} . \qquad (2)$$

Thus, when P_b is known, this allows identification of the mass donor. The angular momentum of the binary can be written $J = M_1 M_2 (Ga/M_T)^{1/2}$, with M_T the total system mass. The evolution of the semimajor axis can then be found from

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} + \frac{\dot{M}_T}{M_T} - \frac{2\dot{M}_1}{M_1} - \frac{2\dot{M}_2}{M_2} . \qquad (3)$$

There is a watershed in the period evolution of LMXB; systems with $P_b \lesssim 10$ hr evolve to ever shorter periods due to loss of angular momentum from the system. At long periods, mass transfer occurs due to nuclear evolution–driven expansion of the secondary (mass donor, M_2), which for $M_1 > M_2$ causes the orbital period to increase.

At short periods, the emission of gravitational radiation is a dominant source of orbital angular momentum loss

$$\frac{\dot{J}}{J} = \frac{-32G^3}{5c^5} \frac{M_1 M_2 M_T}{a^4} \qquad (4)$$

giving a orbital decay time of $\tau_{GW} \approx 3 \times 10^{11} (M_T^{1/3}/M_1 M_2) P_b^{8/3}$ yr with the binary period in days. For solar mass stars this is equal to the Hubble time for $P_b \sim 9$ hr. In addition, angular momentum loss from a magnetically coupled wind from the companion may also be effective, even for somewhat larger P_b. One parameterization of such magnetic braking (MB; Verbunt and Zwann 1981) gives

$$\frac{\dot{J}}{J} \approx -10^{-16} \frac{M_T}{M_1} \left(\frac{R_*}{R_\odot}\right)^\gamma \left(\frac{R_\odot}{a}\right)^2 P_*^{-2} , \qquad (5)$$

where R_* and P_* are the donor radius and spin period and $\gamma \sim 1$. For companions in corotation this gives an orbital evolution time $\tau_{MB} \sim 5 \times 10^9 M_1/M_T^{1/3} (R_*/R_\odot)^{-\gamma} P_b^{10/3}$ yr. Note that for solar masses and $a \sim R_\odot$, both these mechanisms lead to mass transfer phases lasting for much of a Hubble time and accordingly would not give rise to radio pulsars in clean systems without modifications to the evolution.

For periods longer than ~ 10 hr and low-mass companions, period evolution is effected by expansion and Roche-lobe overflow of the secondary. Since the size of the Roche lobe (and thus of star 2, when in contact) is a function of a and the masses: i.e., $R_L = f(M_1, M_2, a)$, we can relate the change of size of $R_2 = R_L$ to the orbital separation. For the formula (1) this gives

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{(M_1 \dot{M}_2 - M_2 \dot{M}_1)}{3M_2 M_T} . \qquad (6)$$

In some cases not all the mass-loss $\dot{M}_2$ will be retained by M_1.

For example the mass acceptance may be limited to the Eddington value ($\sim 10^{-8}\, M_\odot\, \text{yr}^{-1}$ for a neutron star) or nova flashes from a white dwarf accretor may cause mass loss from the system. Such nonconservative mass transfer should include orbital angular momentum loss from the system, typically with the specific angular momentum of the compact object [i.e., $\dot{J}/J = (M_2/M_1)\dot{M}_T/M_T$]. This, with equations (3)–(6), allows the secondary's growth to be written $\dot{R}_L/R_L = f(M_1, M_2, \dot{M}_2)$. Then a second relation between the secondary's mass and its radius allows a solution for the orbital parameters as function of time.

During the first part of a low-mass star's lifetime, its radius is proportional to the mass, while in the late stages of stellar evolution the envelope expansion is largely described by the growth of the compact stellar core. Thus results for the radius expansion of the mass donor are independent of the envelope lost in the transfer and can be derived from detailed evolutions of single stars. We maintain the main-sequence mass-radius relation until the stellar models form a well-defined core of $\sim 0.1\ M_\odot$; thereafter the envelope radius depends only on the core mass. Comparison with detailed models shows this to be a reasonable approximation. These expressions for R_2 as a function of time (through M_2 or M_c) allow computation of the mass transfer rate, the variation of component masses and the evolution of the orbital period (eqs. [3]–[6]). Evolution proceeds until the donor's envelope mass is $\lesssim 10^{-2}\ M_\odot$; at this point the system detaches, leaving a bare WD in orbit around a compact object. If the resulting compact object is a neutron star and has acquired sufficient angular momentum in the course of mass transfer, a recycled pulsar can result. In some cases the resulting LMBP may lose the remnant of its mass donor: energy output from a millisecond pulsar can apparently enhance mass loss by the companion and may even lead to its evaporation (e.g., Ruderman, Shaham, Tavani 1989; Phinney *et al.* 1988), while in a globular cluster a wide binary may be disrupted by stellar encounters.

III. PERIOD DISTRIBUTION IN GLOBULAR CLUSTERS

We will assume that the cluster initial mass function has a power-law form with an index close to the Salpeter value

$$dN \sim M^{-\alpha}\, dM\ , \quad \alpha \sim 2.5\ . \tag{7}$$

While observations of cluster main-sequence stars give a range of values for α (McClure *et al.* 1986) and for some clusters a break with a steeper IMF for the more massive stars seem to be indicated, the above is a reasonable approximation for the present stellar population of many clusters. We should also note that mass segregation in cluster cores will concentrate the high mass stars (Verbunt and Meylan 1988), leading to a flatter effective IMF in the dense regions of the cluster core. With these caveats we take a simple power law with the above index as an approximation to the cluster population. This allows us to estimate the initial production of tidal capture products and their subsequent evolution.

a) Tidal Capture and Initial Conditions

In the dense cluster core, close encounters of two stars are relatively frequent. These cause tides to be raised on a normal extended star; if the energy of the stars at infinity can be dissipated in these tides, capture into a close binary can occur (Fabian Pringle, and Rees 1975). The cross section is $\sigma_{\rm TC} = \pi(3R)^2[1 + 2G(M_1 + M_2)/(3Rv^2)]$ where the two stars have masses M_1 and M_2 and relative velocity v, and the star dissipating the tidal energy input has radius R. The maximum impact parameter for capture, $\sim 3R$, depends slightly on the detailed stellar structure (e.g., McMillan, McDermott, and Taam 1987). In globular clusters, the second term due to gravitational focusing is dominant, giving a lifetime to tidal capture for a given compact star of

$$\tau_{\rm TC} = (n_{\rm star}\,\sigma v)^{-1} \sim 5 \times 10^{10}\, \frac{v_6}{R_* M_{\rm Tot}\,\rho_5}\ {\rm yr}\ , \tag{8}$$

where the cluster velocity dispersion and central density are in units of 10^6 cm s^{-1} and $10^5\ M_\odot$ pc^{-3}, respectively, and the total binary mass and the noncompact stellar radius are in solar units.

From equation (8) the tidal capture cross section is proportional to RM_T. For the low-mass stars which will dominate the total cross section, we have $R \sim M$. Since the stellar luminosity is $L_* \sim M^{3.5}$ and cluster star formation was presumably confined to an initial burst, the lifetime for which stars are available for tidal capture is $t \sim M/L \sim M^{-2.5}$. This gives the number of captures on stars above the turn-off mass $M_{\rm TO} \sim 0.8\ M_\odot$ as

$$N_{{\rm cap},\,>{\rm TO}} \sim \int_{M_{\rm TO}}^{\infty} \frac{\sigma t}{T_{\rm cl}}\, dN \sim \frac{N_{\rm CO}\, M_{\rm TO}^{-(\alpha+1/2)}}{(\alpha + 1/2)} + \frac{M_{\rm CO}^{-(\alpha-1/2)}}{(\alpha - 1/2)}\ , \tag{9}$$

where $M_{\rm CO}$ is the compact object mass and only captures on the main sequence have been considered. The efficiency of capture by post–MS stars is not fully known, but simple estimates suggest that the rate above will be increased by $\lesssim 10\%$. For an effective α in the cluster core of 2.5, this scaling shows that half of all such tidal captures will occur for stars between $M_{\rm TO}$ and $1.3M_{\rm TO}$, so products of nuclear evolution–driven binaries can be well characterized as having companion stars near the present turn-off mass. Lower mass stars will also suffer tidal capture, giving $N_{{\rm cap},\,<{\rm TO}} \sim \int_{M_c}^{M_{\rm TO}} \sigma\, dN \sim 2M_{\rm CO}(M_c^{-1/2} - M_{\rm TO}^{-1/2})$. For a cluster lower mass cut-off of $M_c = 0.15\ M_\odot$ and a turn-off mass $M_{\rm TO} = 0.8\ M_\odot$, stars evolving in a Hubble time $t_{\rm H}$ account for ~ 0.19 of the tidal captures. For $M_c = 0.3\ M_\odot$, the fraction is ~ 0.33.

Many of these main-sequence encounters should lead to an interacting phase; about one-third result in direct collisions between the stars. Moreover, the orbital captures will circularize at 2–6R_*, and so even low-mass companions that do not evolve can eventually come into contact due to angular momentum losses. If we take the gravity wave decay time derived from equation (4), consider the range of circularization radii and integrate over the IMF, we see that 21% (for $M_c = 0.3$) to 44% (for $M_c = 0.15$) of the main-sequence orbital captures will start mass transfer within the Hubble time. Magnetic braking may enhance this fraction. The number of these main-sequence interactions leading to recycled pulsars is not known. If we assume that no direct hits produce recycled pulsars and transfer from main-sequence stars results in pulsars of short orbital period, then with our above numbers the ratio of nuclear evolution driven products (which may have long orbital periods) to systems with $P_{\rm orb} < 1^{\rm d}$ will range from ~ 0.23 ($m_c = 0.15$) to ~ 0.52 ($m_c = 0.3$). However, even the widest tidal captures on pre–turn-off stars have orbital periods $< 1^{\rm d}$; the subsequent evolution is via angular momentum loss and so should lead to even shorter period systems. Thus, to compute the final distribution of the long-period radio pulsars we need consider only those captures which occur on stars more massive than $M_{\rm TO}$.

To derive the initial properties of the tidal capture products, we therefore consider the history of a near–turn-off star. To connect with earlier estimates of tidal capture probabilities (e.g., Verbunt 1988) calculations were made with masses and luminosities (giving core growth rates) interpolated from a number of $\lesssim M_\odot$, low-metallicity stellar evolution models (Rood 1972; Sweigert and Gross 1978). We present results from a 0.9 $M_\odot$, $z = 4 \times 10^{-4}$ star using the models of Swiegert and Gross (1978) and Mengel *et al.* (1979). The compact object involved in tidal capture was considered to be either a neutron star ($M = 1.4\ M_\odot$) or a massive white dwarf. We then follow the change in radius of the star over its lifetime, deriving the relative probability for tidal capture at each stage. The initial circularized orbital semimajor axes are distributed evenly between $2R_*$ and $6R_*$. Bailyn (1988) has argued that the decreased binding energy in a red giant envelope will reduce the maximum radius for tidal capture from the $3R_*$ in (8) by a factor $(M_{\rm env}/M_*)^{1/6}(R_*/R_{\rm ms})^{-1/6}$ where giant envelope has a mass $M_{\rm env}$ and radius much larger than the main sequence value $R_{\rm ms}$. We therefore also compute the distribution of successful tidal captures with this reduced efficiency.

b) *Binary Evolution and Orbital Period Distributions*

The above set of successful tidal capture orbits (with the corresponding evolutionary stage of the turn-off star companion) form our initial conditions for the binary evolution calculations. We follow the evolution of the binary semimajor axis a by integrating equation (3), allowing the orbit to shrink due to gravitational radiation (GR) and magnetic braking (MB) angular momentum losses. In addition, the companion star continues to evolve off the main sequence, expanding in radius. If the stellar radius exceeds that of the Roche lobe (eq. [1]) within the cluster lifetime ($\tau_H \sim 10^{10}$ yr) then mass transfer is initiated. For compact objects more massive than $\sim 1.1\ M_\odot$, this transfer should be stable (Verbunt and Rappaport 1988; Bailyn and Grindlay 1987).

After contact the mass transfer is computed from the equations in § II, which give the change in donor mass as a function of the stellar radius growth and angular momentum lost from the system. When the compact object is a neutron star, the mass accretion will be limited to the Eddington value, taken as $\dot{M}_{\rm Edd} = 1.5 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$. When the mass transfer rate exceeds this value, the additional mass is assumed to be lost from the system, carrying the specific angular momentum of the compact object, as appropriate for mass ejected as jets or a wind from the central part of the accretion disk. For very rapid mass loss, some orbital angular momentum may also be ejected through the secondary's wind (with the specific angular momentum of that object). We include such losses only for mass ejected in non-contact binaries at helium core flash, since winds will probably not dominate in the stable wide mass transfer binaries of principal interest here; in any case, the decrease in expected final P_b resulting from inclusion of such losses would only increase the observed discrepancies with the standard model (§ IV). For completeness there is also a loss of gravitational mass given by the binding energy radiated in settling on the neutron star; we take this to be $0.15M_{\rm acc}$. For phases during which the giant envelope temporarily shrinks, the system is evolved under angular momentum losses and core growth alone until contact is reestablished. Following this transfer until envelope exhaustion, we arrive at the final orbital period, core mass, and mass transferred for each system. This total transferred mass will determine the final state of the neutron star. For a neutron star moment of inertia $\sim 10^{45}$ g cm^{-2} and radius $\sim 10^6$ cm, the accretion of $0.1M_{-1}\ M_\odot$ can bring the neutron star to a spin period of $P \sim 2.3/M_{-1}$ ms. In order for the neutron star to reappear in the radio sky as a pulsar it is necessary that it spin up to a period less than that on the "death line" $P_d \approx 71\ B_9^{1/2}$ ms, where the neutron star dipole field has a residual value of $10^9 B_9$ G. This requires the accretion of $\delta M \gtrsim 4 \times 10^{-3} B_9^{-1/2}\ M_\odot$.

To recapitulate, we find the distribution of initial binary parameters by computing the relative number of captures during each phase of a turn-off star's lifetime. We then evolve the semimajor axis of these binaries by integrating equation (3), where the angular momentum loss is given by equations (4) and/or (5). Once the system reaches contact we use the secondary radius–mass or radius–time relations given above, equations (3)–(6) and a prescription for the mass and angular momentum loss from the system during nonconservative stages (if any) to solve implicitly for the secondary mass loss rate, $\dot{m}_2$. This allows solution for $\dot{m}_1$, $\dot{J}$, and continued integration of equation (3) until the companion envelope is exhausted and a bare white dwarf core remains. During these integrations, the time steps were controlled to assure high accuracy in computation of the evolution rates and accurate account of the mass and angular momentum losses. The distribution of resultant P_b may then be found for a variety of compact objects and varied details of the mass transfer. Some 1.5×10^4 evolutions were computed for each distribution.

In Figure 1, we show distributions of successful orbital tidal captures of the 0.9 $M_\odot$ star when the compact object is a neutron star. The heavy line and right-hatched histogram are the distributions resulting with and without the effects of magnetic braking, respectively. The left-hatched histogram shows the decrease in very long period products resulting if the red giant capture cross section is reduced as discussed by Baliyn (1988). We see that the final distribution of orbital periods depends strongly on the details of the binary evolution. In particular, without MB losses, the eventual evolution of the captured companion drives the majority of systems to oribtal periods of 3–20$^{\rm d}$. If magnetic braking as specified in § II augments the angular momentum loss, then these systems will be kept at orbital periods $\lesssim 1^{\rm d}$, and many will shrink to $P_b \ll 1^{\rm d}$. The distribution of periods $\lesssim 0\overset{\rm d}{.}01$. shown here is probably inaccurate due to the approximations made in the evolutionary calculations, and in fact acceleration of the evolution at these short periods may in some cases eliminate the companions altogether. It is also clear that if the reduction of giant capture efficiency is as severe as suggested above, there will be very few capture products reaching periods $\gtrsim 100^{\rm d}$. Initial binaries and early capture of more massive stars may, however, produce a few such systems.

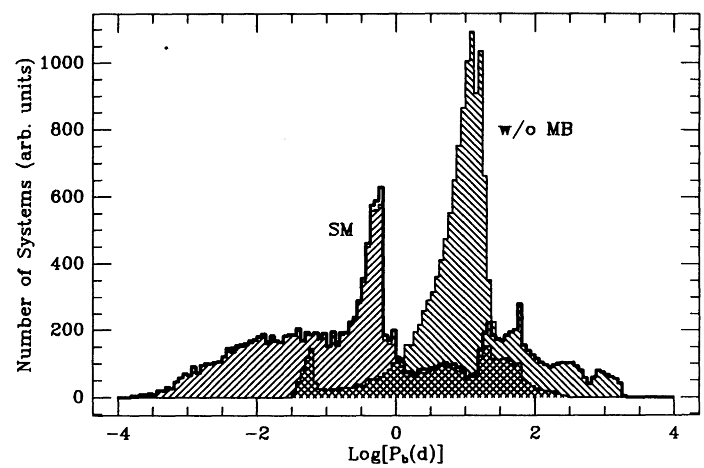

FIG. 1.—Orbital period distribution for neutron star capture of a 0.9 $M_\odot$ star. Curves show the number of systems (arbitrary units) as a function of final period for the standard evolutionary model (SM, *heavy* line), evolution without magnetic braking (*right-hatched histogram*), and standard evolution with a reduced capture cross section for extended giants (*left-hatched histogram*).

We also consider the fate of a tidally captured white dwarf. Here the response to accreted matter is more complicated. For some ranges of $\dot{M}$, $M_{\rm wd}$, surface detonations will result in nova-like activity and substantial mass ejection; it is unlikely that mass growth can occur under these conditions. From Nomoto (1987) mass growth of a C-O white dwarf will not occur when $\dot{M} < \dot{M}_{\rm det}$, $M_{\rm wd} < 1.13\ M_\odot$; $10^{-9}\ M_\odot\ {\rm yr}^{-1} < \dot{M} < \dot{M}_{\rm det}$, $M_{\rm wd} > 1.13\ M_\odot$; $\dot{M}_{\rm det} < \dot{M} < 2.7 \times 10^{-6}\ M_\odot\ {\rm yr}^{-1}$ $M_{\rm wd} < 1.2\ M_\odot$, where $\dot{M}_{\rm det} = 4 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$ for accretion of Population II material (see Fig. 2). In addition the mass acceptance rate is limited to the Eddington value, $\dot{M}_{\rm Edd} \approx 1.5 \times 10^{-5} M_{\rm wd}^{-1/3}\ M_\odot\ {\rm yr}^{-1}$, where the white dwarf mass is in solar units. For O-Ne-Mg white dwarfs, which are more massive than $\sim 1.1\ M_\odot$, mass ejection is estimated to occur for $10^{-9}\ M_\odot\ {\rm yr}^{-1} < \dot{M} < \dot{M}_{\rm det}$, or $\dot{M} < \dot{M}_{\rm det}$, $M_{\rm wd} < 1.2\ M_\odot$. In our calculations, we follow white dwarfs of each type as the binaries evolve through the ($\dot{M}$, $M_{\rm wd}$) plane; during epochs of nova-like activity, we assume that all accreted mass is ejected, carrying away the specific angular momentum of the compact object. Examples of the tracks of several binaries are shown in Figure 2.

In Figure 3 the orbital period distribution resulting from a 1.2 $M_\odot$ CO white dwarf primary is shown as the left-hatched histogram, with magnetic braking contributions included. For orbital periods $8^{\rm d} \lesssim P_b \lesssim 160^{\rm d}$, the slow mass transfer of incipient giant evolution does not allow growth of the white dwarf, and few recycled pulsar systems result. Also the necessity of accreting several tenths of a solar mass precludes collapse in systems with the longest orbital periods and in the high $\dot{M}$ systems of period $\lesssim 1$ hr. Evolutions with white dwarfs of less than 1.1 $M_\odot$ were found to almost never result in core collapse and a millisecond pulsar. Finally the O-Ne-Mg white dwarfs, which are presumably quite rare, when starting from masses within a few percent of the critical mass for collapse were found to produce an orbital period distribution quite similar to that of capturing neutron stars, reproduced in Figure 3 for comparison. Thus we see that a relative deficit in the numbers of pulsar binaries between $\sim 10^{\rm d}$ and $\sim 100^{\rm d}$ would be a strong indicator of an AIC origin for the pulsar population.

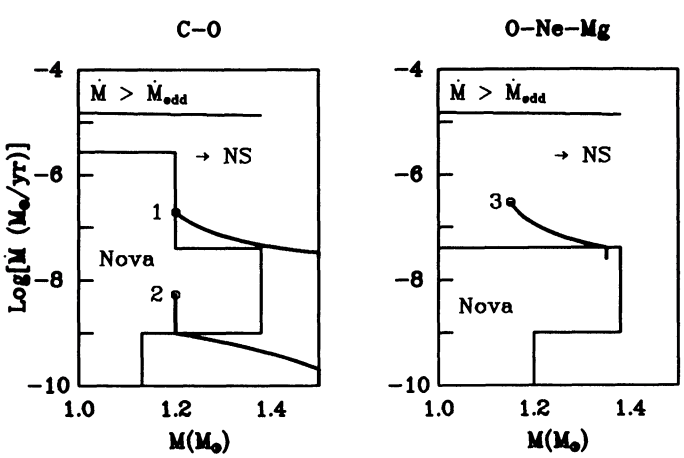

FIG. 2.—Evolution of accreting white dwarfs in the (M, $\dot{M}$) plane. *Left*: In the region marked "Nova", no growth of the dwarf will occur; thick tracks show the evolution of a 1.2 $M_\odot$ white dwarf; the binaries have initial orbital periods $53^{\rm d}$ (1) and $0^{\rm d}\!.60$ (2) and final periods $196^{\rm d}$ (1) and $0^{\rm d}\!.54$ (2). *Right*: Growth of an O-Ne-Mg white dwarf (see text). The binary (3) has initial period $47^{\rm d}$ and final period $230^{\rm d}$.

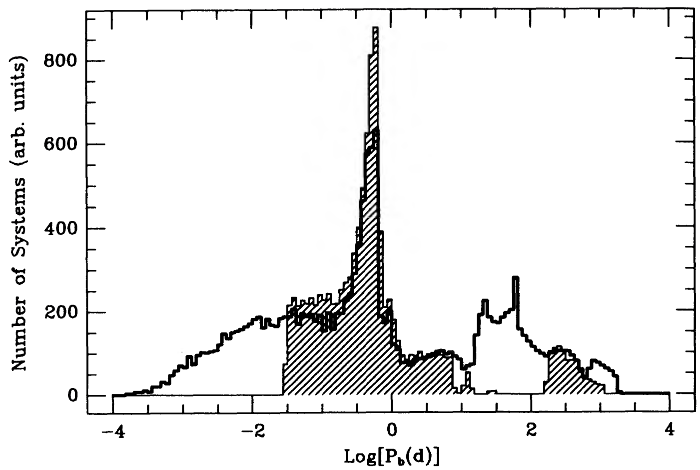

FIG. 3.—Orbital period distribution for 1.2 $M_\odot$ white dwarf capture of a 0.9 $M_\odot$ star. The right-hatched histogram gives the number of successful AIC products as a function of P_b. For an O-Ne-Mg white dwarf close to the Chandrasekhar mass the final period distribution is quite close to that of the neutron star capture of Fig. 1, reproduced for comparison (*heavy line*).

IV. COMPARISON WITH OBSERVATIONS

We now wish to compare the predicted orbital period distributions with that of the presently reported sample of 13 cluster pulsars. These objects are listed along with the parent cluster, the observed spin period, estimates of the orbital period and the *IAU Circular* number reporting their detection. Of these, two pulsars (PSR 0021−72A, B in 47 Tuc) have reported dispersion measures inconsistent with subsequent observations (Manchester *et al.*, 1989) and are possibly spurious. We exclude them from further discussion, although their inclusion would not strongly affect our results.

Of the 11 remaining pulsars three are presently in wide orbits; PSR 1620−26 in the cluster M4 has a binary period of $191^{\rm d}$, and preliminary data on PSR 1516+02B in M5 and PSR 1310+18 in M53 indicate orbital periods of 7 and >100 days, respectively (S. Kulkarni, private communication). In addition, PSR 1516+02A appears to be in a wide ($P_b > 100^{\rm d}$) binary, although this value is presently quite uncertain. One object (PSR 2127+11C in M15) has rather high eccentricity and short $0^{\rm d}\!.3$ orbital period (S. Kulkarni, private communication). The origin of this system is quite difficult to understand unless the pulsar is an interloper, released from another birth site and introduced via an exchange interaction into a second tidal capture binary. Although some inference can be made about is birth site, the peculiarity of the present binary makes evolutionary scenarios unclear, and, in any case, the very short orbital period allows us to leave this pulsar out of the long P_b sample.

The six other globular cluster pulsars appear to be single, although PSR 0021−72C may be in a very long period binary. Since these objects are presumed to have undergone a period of mass accretion, we must consider mechanisms for the removal of their former companions. One possibility is that in a wide binary system, weak encounters with other cluster members can either cause complete disruption of the system (ionization) or replacement of the recycled pulsar with the incoming cluster star (exchange) (Romani, Kulkarni, and Blandford 1987; Rappaport, Putney, and Verbunt 1989). Rappaport *et al.* have done extensive simulations of such processes and determined the rate of liberation as a function of

TABLE 1

CLUSTER BINARY PERIODS

Pulsar (1)	Cluster (2)	P_p (ms) (3)	P_b (days) (4)	P_{dis} (days) (5)	P_{pred} (days) (6)	P_{fid} (days) (7)	IAUC Number (8)
0021−72A[a]	47 Tuc	4.79	0.02	1.4×10^3	<180	...	4602
0021−72B[a]	47 Tuc	6.12	7–95	1.4×10^3	<180	...	4602
0021−72C	47 Tuc	5.76	?	1.4×10^3	<180	...	4892
1310+18	M53	33.	>100	2.5×10^5	250	250	4853
1516+02A	M5	5.5	>100?	8.3×10^3	<180	100	4880
1516+02B	M5	7.9	7	8.3×10^3	<180	7	4880
1620−26	M4	11.1	191	7.8×10^3	210	191	4470
1640+36	M13	10.	...	5.0×10^5	$\lesssim$200	...	4819
1745−20	N6440	288	...	66	470	470	4905
1821−24	M28	3.05	...	1.6×10^3	<180	...	4401
2127+11A	M15	110	...	170	350	350	4552
2127+11B	M15	53.	...	170	280	280	4762
2127+11C	M15	30.	0.33	170	240	...	4772

Questionable sources.
COL. (1).—Pulsar name.
COL. (2).—Cluster site.
COL. (3).—Spin period.
COL. (4).—Observed value of binary period.
COL. (5).—Binary period at which a pulsar would be extracted from the binary in $\lesssim 10^{9.5}$ yr (see § IV).
COL. (6).—Orbital period predicted from field decay arguments (see § IV).
COL. (7).—Fiducial value for the binary period at the end of mass transfer, as derived from these estimates.
COL. (8).—*IAU Circ.* number which reports the pulsar's discovery.

radius. Scaling the cross sections in this paper to values appropriate to the unmodeled clusters, using parameters from the Webbink (1985) catalog, we can estimate the minimum orbital period allowing extraction of the recycled pulsar in a reasonable fraction of a Hubble time, say $10^{9.5}$ yr^{-1}. This period is listed for the clusters in column (5) of Table 1. For the clusters M15 and NGC 6440 the minimum period for extraction is quite short, and we would expect many long period binaries to be disrupted. In fact, P_{dis} in these clusters can be taken as a lower limit to the original binary period of a presently single pulsar, when other means of extraction are not viable. The other clusters have rather large minimum P_b for extraction.

Indeed, for the pulsars in M28 and M13 (and those M5 and 47 Tuc, if single), very large orbital periods would be required for reasonable probability of extraction from a wide binary. However, for all these objects the pulsar spin period is $\lesssim 10$ ms; such objects have spin energies $\sim 10^2$–10^3 times those of the single M15 pulsars. In analogy with the observed accelerated evolution and mass loss from the eclipsing, ablating binary pulsar PSR 1957+20, these single pulsars may have evaporated their former companions. This mechanism suggests that their initial separations should not have been too large, which would allow a substantial fraction of the pulsar energy to be tapped. We can then conservatively assign PSR1821−24, PSR 1640+36 and PSR 0021−72C to the short binary period group. There is significant support for this picture in that slow, low spin energy pulsars are only found to be single in clusters where stellar encounters *should* have caused disruptions. We also note, following Rappaport *et al.*, that in intermediate cases encounters may weakly perturb the orbits seen. For example, for PSR 1620−26 $P_b/P_{dis} \sim 0.02$, and this is of order the eccentricity induced to this binary orbit. We similarly confirm the other P_b seen today are much less than the corresponding P_{dis} and thus provide fairly accurate measures of the period at the end of mass transfer.

Evolutionary arguments lend additional support to the idea that the single pulsars in M15 and NGC 6440 originated in wide binaries. Empirically, the widest disk binary, PSR 0820+02, also has the largest spin period of any clearly recycled pulsar. This object has long been considered a likely AIC product, since weak orbital binding energy makes retention of the low-mass dwarf through a supernova event rather unlikely. Moreover, as P_b becomes larger, the mass transfer time becomes increasingly short. If AIC results in a neutron star with a normal (few $\times 10^{12}$ G) magnetic field and if the mass transfer continues after collapse, then magnetic field decay will cause the field at the end of mass transfer, and hence the final equilibrium spin period reached, to vary inversely with the binary orbital period. Subsequent to the end of mass transfer, the pulsar may continue to spin down, but allowing for this evolution, we see that the pulsar spin period is a relic of the magnetic field at the end of the mass transfer phase. Thus for orbital periods long enough that during transfer the field remains above its residual value ($P_{b*} \lesssim 150^d$), the present spin of the pulsar affords an estimate of the binary orbital period (see de Kool and van Paradijs 1987). Alternatively, in some magnetic field evolution theories (Romani and Hernquist, 1990) field decay to asymptotic $\sim 10^8$ G levels can occur only during phases of steady accretion; these scenarios would give a similar spin period $-P_b$ relation. Several authors (Chanmugam and Brecher 1987; Grindlay and Bailyn 1989) have argued that AIC should result directly in a low-field, short P_p pulsar. When this occurs, the $P_b - P_p$ relationship above would not be expected. We note, however, that the observed wide binaries, and in particular the high field of PSR 0820+02, show that collapse to high-field object should often occur. Moreover, theoretical arguments (Narayan and Popham 1989) also suggest that AIC is more likely to produce highly magnetized neutron stars.

Taking an initial magnetic field of 2×10^{12} G which decays exponentially with a time constant of $\tau_d = 10^{6.5}$ yr, we use the duration of the mass transfer phase in our model calculations to estimate the spin period at pulsar rebirth as a function of final orbital period of the parent binary. In some cases we find that the limiting P_p is determined by the total mass transferred rather than the time for field decay. In Figure 4 these results

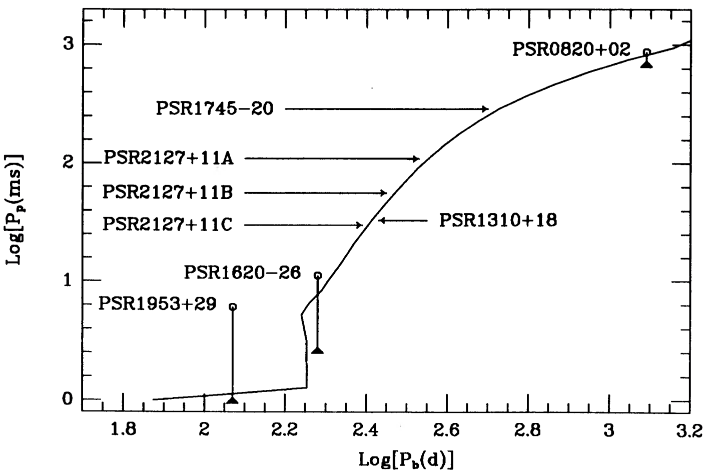

FIG. 4.—Orbital period *vs.* pulsar spin period for wide binary pulsars. Circles give the present spin periods for three pulsars; extrapolation over the estimated lifetimes give original spins on the lines bounded by triangles. The curve represents the initial spin of a AIC pulsar formed in a binary with a final period P_b given by our calculations; a postcollapse field of $B = 2 \times 10^{12}$ G and a field decay time scale of $\tau_d = 10^{6.5}$ yr. The present spin periods of the M15, M53 and NGC 6440 pulsars are also indicated.

are presented along with values for three wide binary pulsars with well-measured periods; circles represent the present spin period and the line below gives the range of rebirth periods allowed after spindown over the estimated age of the pulsar. Note that with the above field decay rate, for orbital periods shorter than $\sim 150^d$ the field can reach a residual value $\lesssim 10^9$ G, and the pulsars can then spin up to a period of order 1 ms, which is the dynamical limit. For shorter orbital periods the present pulsar spin will then no longer correlate with the binary orbit. The pulsars in M15, M53, and NGC 6440 have spin periods lying well within the range where correlation is expected and are shown in Figure 4, as well. The P_b values predicted from this relation are listed in the table. For small P_p objects this provides only an upper limit. The exact values will of course depend on the field evolution, the particular initial binary parameters, and how much spindown has occurred since rebirth, but it is encouraging that these values are comfortably above the required periods for orbit dissolution in the cores of M15 and NGC 6440. We therefore adopt these estimates for the final orbital periods of their original binaries, $P_{\rm fid}$ (Table 1). We note that these sums also provide a prediction of P_b for the pulsar in M53 ($\sim 250^d$) and for the parent binary of PSR 2127+11C ($\sim 240^d$), if the present system indeed arose from an exchange encounter. To be conservative, we exclude this later estimate from our fiducial list $P_{\rm fid}$.

Our sample thus has at least two, and arguably more than six, pulsars with parent binary orbital periods $\gtrsim 100^d$. This relatively large number of long periods is somewhat surprising and is to be compared with the distribution functions computed above. However, we must worry about the completeness of our orbital period sample. As discussed in Kulkarni, Narayan, and Romani (1989), present cluster surveys rely on long integrations to achieve the requisite sensitivity, and thus Fourier pulse search methods suffer from Doppler smearing due to the varying line-of-sight velocities in short-period binaries. If we require the Doppler broadening of the mth pulse harmonic of a pulsar of period P_p over an observation period of $10^3\ T_3$ s to be less than the spectral resolution of the transform, then with a companion mass M_2 in units of 0.3 $M_\odot$ and a total system mass of 1.8 $M_\odot$, we find that survey sensitivities will be reduced for orbital periods shorter than

$$P_b \sim 1.7\, d[M_2(m/4)(10\ {\rm ms}/P_p)]^{3/4} \times T_3^{3/2}(M_T/1.8\ M_\odot)^{-1/2}\ , \tag{10}$$

where we have averaged over orbital inclinations and binary phases. Thus present surveys, which use observations as long as ~ 1 hr, will have reduced sensitivity to pulsars in orbits of $\lesssim 10^d$. This means that a substantial fraction β of the binary orbits will not be detectable with direct Fourier surveys. Recent efforts to include variable accelerations in the pulse searches have met with substantial success, revealing the 8 hr binary pulsar 2127+11C in M15 (Anderson *et al.* 1989). Although some of the most favorable clusters have begun to be searched with these techniques, our present sample is probably still substantially incomplete for $P_b \lesssim 3^d$ due to accelerations alone (other sensitivity issues, of course, cause more general incompleteness; KNR). KNR estimated the factor β from the limited observations to be ~ 1.5.

Accordingly, we consider several of our models and take the cumulative distribution functions for $P_b > 3^d$; the results for a 1.4 $M_\odot$ neutron star (with and without angular momentum loss due to magnetic braking) and a 1.2 $M_\odot$ CO white dwarf are shown in Figure 5, along with the stepped histogram of the seven "fiducial" period values for the $P_b > 3^d$ cluster pulsars (Table 1, col. [7]). The large excess of $P_b \lesssim 20^d$ predicted in the absence of magnetic braking is in fairly strong conflict with the observations; a Kolmogorov-Smirnov test gives a probability 1.1×10^{-3} that this is the correct model. The present limited data are insufficient to discriminate between the other distributions, although a K-S test marginally prefers the prediction for the accreting white dwarf (Prob ~ 0.35 vs. Prob ~ 0.04). In fact the fraction of $P \gtrsim 200^d$ systems (5/7) seems even larger than that expected for the accreting dwarfs; we suggests further amendments to the evolution below. Note that if we only use the three objects which are *definitely* binaries at present, the data are consistent with all three computed distributions at the $\gtrsim 10\%$ level. Despite the present lack of discrimination, the

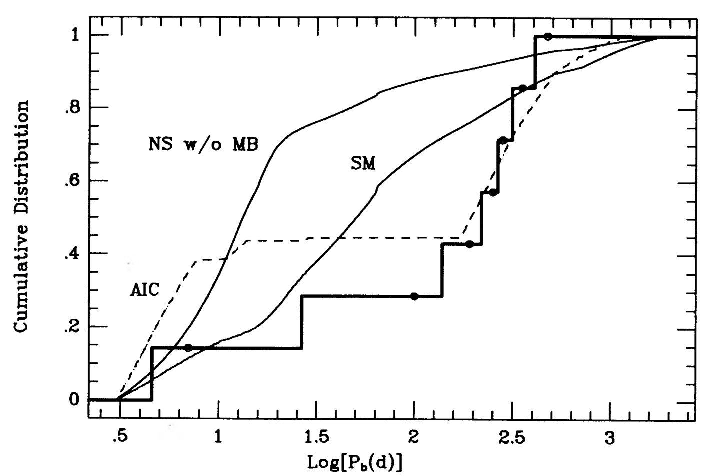

FIG. 5.—Cumulative fraction of $P_b > 3^d$ systems as function of orbital period. Solid curves show the predictions for the standard model (SM) and neutron star tidal capture when magnetic braking (MB) is not important. The dashed curve gives the prediction for AIC of 1.2 $M_\odot$ WD systems. The stepped histogram corresponds to the seven observed cluster pulsars believed to be from wide binaries; fiducial periods (Table 1) are marked by the circles.

large differences between the computed distributions show that with $\gtrsim 10$ good P_n estimates for cluster binaries, as should accrue shortly, more refined comparisons can be made.

V. DISCUSSION AND CONCLUSIONS

Although the present sample is quite limited and observational selection effects complicate the interpretation, it appears that very long period binaries are substantially overrepresented in the recycled cluster pulsars compared to the distribution expected for the standard model, i.e., when evolving stars are tidally captured by cluster neutron stars. Our evolutions show that a competing model, the AIC of massive white dwarfs, does indeed increase the proportion of wide orbits in the recycled pulsar population but possibly not as strongly as in the observed sample.

There are several ways in which this continued discrepancy might be resolved. More observations may of course bring the detections into better agreement with the expected distributions. However, if we have *severely* underestimated the selection effects of our present insensitivity to accelerations, the observed sample might be restricted to pulsars with $P_{\rm orb} \gtrsim 100^{\rm d}$. Alternatively, there may be an additional effect which restricts successful recycling to the large P_b class. For example, if the maximum stable mass of a neutron star is as small as 1.8 $M_\odot$ (as would be implied by the submillisecond pulsations from SN 1987A, if these represent neutron star rotation; Kristian *et al.* 1989), then systems with final periods $P_b \lesssim 120^{\rm d}$ will force too much mass accretion and thus contain a low-mass black hole instead of a radio pulsar. Similarly, our evolutions show that very few systems with $P_b \lesssim 10^{\rm d}$ survive as neutron stars, even if the maximum stable mass is as large as 2.0 $M_\odot$. Note however, that all these solutions select out a *subset* of the original wide binaries to represent our present sample. Accordingly, for any such solution, the number of required progenitors (which is already surprisingly large; see KNR) must be substantially increased.

In fact we can use our evolutions to estimate the correction factor β for the binaries too highly accelerated for detection at present. For the standard neutron star and the 1.2 $M_\odot$ CO white dwarf AIC models including the effects of magnetic braking, the fractions of tidal captures resulting in binaries with $P_b > 3^{\rm d}$ are 0.29 and 0.095, respectively. These should be reasonable approximations for all orbital captures on $t_{\rm ev} < t_{\rm H}$ stars. Then, as discussed in § III, we must compare these fractions to the expected number of products formed with lower mass stars and direct collisions to find the correction factor β by which we must multiply the numbers of pulsars inferred from our sample of wide binaries. If direct collisions never give rise to recycled pulsars, then for the standard model we predict $\beta \sim 8$ ($M_c = 0.15\ M_\odot, \alpha = 2.5$) and $\beta \sim 3.6$ ($M_c = 0.3$). For the AIC case, the correction factors are $\beta \sim 24$ ($M_c = 0.15$) and $\beta \sim 10$ ($M_c = 0.3$). If direct hits can contribute to the recycled pulsar populations, these factors will be even larger. Further restricting the fraction of binaries resulting in successful recycling, as with a low maximum neutron star mass, will increase β and the number of required progenitors yet further.

In some fraction of the collisions and/or short period $P_b < 1^{\rm d}$ systems, it appears that the accretion energy onto the neutron star can be tapped in some way, accelerating the mass loss from the companion or even leading to ablation (Fruchter, Stinebring, and Taylor 1988; Eichler and Ko 1988; Ruderman, Shaham, and Tavani 1989). For tidal capture products, for which such short period systems comprise $\sim 2/3$ of the systems with $t_{\rm ev} < t_{\rm H}$ stars and 80%–90% of all capture products, modification of the binary evolution can dramatically affect the population of accreting progenitors and the final distribution of orbital periods $P_b \lesssim 3^{\rm d}$. This accelerated evolution can help explain the deficit of short-period LMXB progenitors noted by Kulkarni and Narayan (1988), by dramatically shortening the lifetimes of the accreting sources. This can also apply to the cluster population, leading to many presently unobservable short period binaries and (via evaporation) some single millisecond pulsars. For wider orbits, where our calculations can predict the shape of the final period distribution, there may also be some evidence for such shortened X-ray lifetimes; the apparent deficit of $3^{\rm d} \lesssim P_b \lesssim 20^{\rm d}$ systems compared to predictions of models without magnetic braking or comparable enhanced mass/angular momentum loss suggests that such processes remain important out to quite large P_b.

At even longer periods, the shape of the distribution may also help explain the deficit of LMXB progenitors. Our calculations suggest that very long periods are overrepresented in the population of wide binaries. These wide systems have short X-ray lifetimes, decreasing the ratio of LMXB to LMBP. Moreover, if AIC does in fact dominate in the wide systems, then we have found that successful collapse will seldom occur in the range $8^{\rm d} \lesssim P_b \lesssim 200^{\rm d}$. Systems in this period range will remain faint in the X-ray, persisting as UV bright, long-period cataclysmic variables. Proposed *ROSAT* and HST observations should help find these binaries; at long periods the P_b distributions can be predicted by scaling our calculations above by the system lifetimes (R. W. Romani, in preparation). The longest period systems, that do lead to successful AIC, may form millisecond pulsars directly if the initial spin period and dipole magnetic field are small; in this case the spindown luminosity of the newly born pulsar may be sufficient to strip the weakly bound envelope of the very extended giant, allowing one to pass directly from a cataclysmic variable to a MSP in analogy to the scenario discussed by Grindlay and Bailyn (1989). However, if (as appears more likely from the observed systems) the initial magnetic field is large, then the newly formed neutron star continues accretion and field decay during the final expansion of the giant. The X-ray phase in this case is also quite short-lived. As discussed above, the pulsar spin period then provides a measure of this X-ray lifetime.

Thus accelerated evolution at very short periods, possibly abetted by AIC at longer orbital periods, can explain the surprisingly large LMBP/LMXB ratio implied by recent globular cluster observations. These cluster results will be an important clue to understanding related discrepancies in the Galactic disk. Moreover, the calculations described in this paper show that these two processes will have large effects on the distribution of final orbital periods for the long-lived recycled pulsars, making observations of the P_b distribution an important diagnostic. In particular, accelerated mass and angular momentum loss at short periods (such as that due to magnetic braking) removes the large peak of binaries at periods of $\sim 3^{\rm d}$–$20^{\rm d}$ and produces many very short period systems or even single millisecond pulsars. The absence of substantial numbers of LMBP with orbital period in the range $8^{\rm d} \lesssim P_b \lesssim 200^{\rm d}$ was found to be a strong indication of an AIC origin of these pulsars. Further, the incidence of very wide systems can give some clues to the tidal capture efficiency of highly evolved giants. The present limited sample of cluster pulsars suggest that AIC and accelerated evolution are important; a larger sample of binary periods should make more refined discrimination possible shortly. Finally, while the modifications to the standard scenario for the LMBP evolution suggested by the P_b distributions can help explain the high LMBP/LMXB ratio found by KNR, these modifications substantiate, and even increase, the large number of recycled pulsars expected. The correspondingly large number of compact objects needed as capture sites, whether low-velocity neutron stars retained in the cluster or massive white dwarfs capable of AIC, presents substantial constraints on the cluster upper main seqence population.

It is a pleasure to thank S. Kulkarni, R. Narayan, G. Raffelt, and A. Wolszczan for helpful discussions. Support for this work was provided by NSF grants ZST 86-15816 (UCB) and PHY86-20226 (IAS) and a grant from the Corning Glass Works.

REFERENCES

Anderson, S., Gorham, P., Kulkarni, S., Prince, T., and Wolszczan, A. 1989, *IAU Circ.*, No. 4772.
Bailyn, C. 1988, *Nature*, **332**, 330.
Bailyn, C., and Grindlay, J. 1987, *Ap. J.*, **316**, L25.
———. 1990, *Ap. J.*, **353**, 159.
Bradt, H. V. D., and McClintock, J. E. 1983, *Ann. Rev. Astr. Ap.*, **21**, 13.
Chanmugam, G., and Brecher, K. 1987, *Nature*, **329**, 696.
de Kool, M., van den Huevel, E. P. J., and Rappaport, S. 1986, *Astr. Ap.*, **164**, 73.
de Kool, M., and van Paradijs, J. 1987, *Astr. Ap.*, **173**, 279.
Eichler, D., and Ko, Y.-K. 1988, *Ap. J.*, **328**, 179.
Fabian, A. C., Pringle, J. E., and Rees, M. J. 1975, *M.N.R.A.S.*, **172**, 15P.
Fruchter, A. S., Stinebring, D. R., and Taylor, J. H. 1988, *Nature*, **333**, 236.
Grindlay, J. 1987, in *The Origin and Evolution of Neutron Stars*, ed. D. J. Helfand and J. H. Huang (Dordrecht: Reidel), p. 173.
Grindlay, J., and Bailyn, C. 1989, *Nature*, **336**, 48.
Helfand, D., Ruderman, M., and Shaham, J. 1983, *Nature*, **304**, 423.
Katz, J. I. 1975, *Nature*, **253**, 698.
Kristian, J., *et al.* 1989, *Nature*, **338**, 234.
Kulkarni, S., and Narayan, R. 1988, *Ap. J.*, **335**, 755.
Kulkarni, S., Narayan, R., and Romani, R. W. 1990, *Ap. J.*, **356**, 174 (KNR).
Manchester, R. N., Lyne, A. G., Johnstone, S., D'Amico, N., Lim, J., and Kniffen, R. Z. 1989, *IAU Circ.*, No. 4892.
McClure, R. D., *et al.* 1986, *Ap. J. (Letters)*, **307**, L49.
McMillan, S. L. W., McDermott, P. N., and Taam, R. E. 1987, *Ap. J.*, **318**, 261.
Mengel, J., Sweigart, A., Demarque, P., and Gross, P. 1979, *Ap. J. Suppl.*, **40**, 733.
Narayan, R., and Popham, R. 1989, *Ap. J. (Letters)*, **346**, L25.
Nomoto, K. 1987, in *The Origin and Evolution of Neutron Stars*, ed. D. J. Helfand, and J. H. Huang (Dordrecht: Reidel), p. 281.
Paczyński, B. 1967, *Acta Astr.*, **17**, 287.
Phinney, E. S., Evans, C., Blandford, R. D., and Kulkarni, S. 1988, *Nature*, **333**, 832.
Rappaport, S., Putney, A., and Verbunt, F. 1989, *Ap. J.*, **345**, 210.
Romani, R. W. 1989, in *Proc. of Goa Workshop on SN and Stellar Evolution*, ed. A. Ray (in press).
Romani, R. W., and Hěrngurst, L. 1990, in preparation.
Romani, R. W., Kulkarni, S., and Blandford, R. 1987, *Nature*, **329**, 309.
Rood, R. T. 1972, *Ap. J.*, **177**, 681.
Ruderman, M., Shaham, J., and Tavani, M. 1989, *Ap. J.*, **336**, 507.
Sweigart, A., and Gross, P. 1978, *Ap. J. Suppl.*, **36**, 405.
van den Huevel, E. P. J. 1987, in *The Origin and Evolution of Neutron Stars*, ed. D. J. Helfand and J. H. Huang (Dordrecht: Reidel), p. 393.
———. 1989, in *Timing Neutron Stars*, ed. H. Ogelmann and E. P. J. van den Huevel (Dordrecht: Reidel), p. 593.
van den Huevel, E. P. J., and Taam, R. E. 1984, *Nature*, **309**, 235.
Verbunt, F. 1988, in *Physics of Neutron Stars and Black Holes*, ed. Y. Tanaka (Tokyo: ISAS), p. 159.
Verbunt, F., Lewin, W. H. G., and van Paradijs, J. 1989, *M.N.R.A.S.*, **241**, 51.
Verbunt, F., and Meylan, G. 1988, *Astr. Ap.*, **203**, 297.
Verbunt, F., and Rappaport, S. 1988, *Ap. J.*, **332**, 193.
Verbunt, F., and Zwann, C. 1981, *Astr. Ap.*, **100**, L7.
Webbink, R. 1985, in *IAU Symposium 113, Dynamics of Star Clusters*, ed. J. Goodman and P. Hut (Dordrecht: Reidel), p. 541.
Webbink, R., Rappaport, S., and Savonije, G. 1983, *Ap. J.*, **270**, 678.

Note added in proof—Recent observations have shown that PSR 1310+18 in M53 has an orbital period $P_b \sim 270^d$, in good agreement with the above prediction (S. Anderson, private communication).

ROGER W. ROMANI: Institute for Advanced Study, School of Natural Sciences, Princeton, NJ 08540