SOUND AND THERMAL WAVES PROPAGATION IN A HYDROGEN PLASMA HEATED BY AN EXTERNAL RADIATION FIELD

MIGUEL H. IBÁÑEZ S.

Grupo de Astrofísica, Facultad de Ciencias, Universidad de Los Andes, Mérida, Venezuela

and

CÉSAR A. MENDOZA B.

Departamento de Física, Facultad Experimental de Ciencias, Universidad del Zulia, Maracaibo, Venezuela

(Received 11 July, 1989)

Abstract. The thermal equilibrium of a hydrogen plasma heated and ionized by an external radiation field, diluted by a factor W_* and defined by an effective temperature T_*, is studied. In addition, the problem of propagation of acoustic and thermal waves in the above plasma model is also analysed. It is found that an external radiation field has stabilizing effects against wave amplification. From the dispersion relation obtained, the phase velocity $\tilde{v}$ for sound and thermal waves and their respective scale-length of damping (or anti-damping) $\tilde{l}$ is calculated as a function of the frequency $\tilde{\omega}$ for representative values of the plasma temperature $\tilde{T}$ and the external radiation field T_*.

1. Introduction

A great many astrophysical problems can be worked out assuming either of the two well-known limit regimes concerning the energy transport: the optically thick or thin regime, which introduce drastic simplifications in the basic radiation gas dynamic equations. In particular, the problems of linear stability and wave propagation, which are very closely related problems, are simplified in such a way that they can be treated quasi-analytically.

In previous works, the propagation of small disturbances in a reacting-radiating (Ibañez, 1986) and in an ideal radiating (Ibañez and Pacheco, 1989) general fluid was studied by using the Eddington approximation to handle the radiation field. Also, applications to a pure hydrogen plasma were made. In another work (Ibañez and Mendoza, 1987) the above problem was solved but in the optically-thin limit; in particular, assuming a non-specified constant heating rate and the cooling function given by Hummer and Seaton (1963), and Hummer (1963) for a hydrogen gas. The local stability analysis of the above plasma has been studied by Ibañez and Parravano (1983) and Ibañez and Plachco (1989).

In several problems of interest in astrophysics, the plasma is ionized and heated by an external radiation field, for example: (1) in warm neutral (WNM) and ionized (WIM) regions of the interstellar medium (McKee and Ostriker, 1977; Cowie and McKee, 1977; McKee and Cowie, 1977; Parravano, 1987); (2) in planetary nebulae (Caprioti, 1971; Ferch and Salpeter, 1975); (3) in H II regions (Spitzer, 1978); (4) in Quasar clouds

Astrophysics and Space Science **164**: 193–211, 1990.

(Mathews, 1986); (5) in galactic and extragalactic jets (Bodo *et al.*, 1985; Blandford, 1985). Therefore, the study of the above physical situations is of particular importance.

In the present paper we will carry out the analysis of the propagation of linear waves in the plasma model studied by Ibañez and Mendoza (1987) (see also Ibañez and Parravano, 1983), but changing the constant heating rate by a heating due to an external radiation field.

2. Basic Equations

A gas mixture in which the general reaction

$$\sum M_i A_i = 0 \tag{2.1}$$

is in progress, A_i being the chemical symbols of the reagents and M_i being (positive or negative) integer numbers, is governed by the well-known equations of gas dynamics:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = + \rho \nabla \mathbf{v} = 0\,, \tag{2.2}$$

$$\frac{\mathrm{D}\xi}{\mathrm{D}t} + X(\rho, T, \xi) = 0\,, \tag{2.3}$$

$$\rho\,\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} + \nabla p = 0\,, \tag{2.4}$$

$$AR\,\frac{\mathrm{D}T}{\mathrm{D}t} - \frac{p}{\rho^2}\,\frac{\mathrm{D}\rho}{\mathrm{D}t} + BRT\,\frac{\mathrm{D}\xi}{\mathrm{D}t} + L(\rho, T, \xi) - \frac{1}{\rho}\,(\nabla \cdot \kappa \nabla T) = 0\,, \tag{2.5}$$

$$p = R\rho T \sum_i (M_i \xi + x_i^0)\,; \tag{2.6}$$

where ρ, $\mathbf{v}$, ξ, X, p, R, T, L, κ, x_i^0 are: mass density, velocity, chemical parameter, net generation rate, pressure, gas constant, gas temperatures, net cooling function, coefficient of thermal conductivity, and initial concentration of ith component, respectively. A and B are given by

$$A(\xi) = \sum_i \frac{M_i \xi + x_i^0}{\gamma_i - 1}\,; \qquad B(\xi, T) = \sum_i \left(\frac{M_i}{\gamma_i - 1} + \frac{M_i \varepsilon_1^0}{k_B T}\right); \tag{2.7}$$

r_i, ε_i^0 and k_B being the specific heat ratio, zero-point energy of the i-th component, and the Boltzmann constant, respectively (Ibañez and Parravano, 1983).

In the particular case of a pure hydrogen plasma heated and ionized by an external radiation field J_ν^*, the relations (2.1), (2.6), and (2.7) simplify to

$$H^+ + e^- - H^0 - (\chi) = 0\,, \tag{2.8}$$

$$p = R\rho T(1 + \xi)\,, \tag{2.9}$$

$$A(\xi) = \tfrac{3}{2}(1 + \xi)\,; \qquad B(T) = \frac{3}{2} + \frac{\chi}{k_B T}\,. \tag{2.10}$$

On the other hand, the coefficient of thermal conductivity is given by

$$\kappa(\xi, \rho, T) = 2.50 \times 10^3 (1 - \xi) T^{1/2} + 1.84 \times 10^{-5}\, \frac{\xi T^{5/2}}{\ln \Lambda}\,. \tag{2.11}$$

The first term on the right-hand side of Equation (2.11) corresponds to the heat conductivity by neutral atoms (Parker, 1953) and the second one by electrons (Spitzer, 1962). According to Spitzer (1962)

$$\ln \Lambda = \begin{cases} 23.24 + \ln \left[\dfrac{T_0^3}{n\xi} \left(\dfrac{\tilde{T}^3}{10^4} \right) \right]^{1/2}, & \tilde{T} < \dfrac{4.2 \times 10^5}{T_0}\,; \\[2ex] 29.71 + \ln \left[\dfrac{T_0}{(n\xi)^{1/2}} \left(\dfrac{\tilde{T}}{10^6} \right) \right], & \tilde{T} > \dfrac{4.2 \times 10^5}{T_0}\,; \end{cases}$$

n being the number density of atoms.

3. Equilibrium State

It is assumed that an external mean field J_ν^* diluted by a factor W_* is ionizing and heating the hydrogen plasma, J_ν^* being given by

$$J_\nu^* = \frac{2h_\nu^3}{c^2}\, \frac{W_*}{e^{h\nu/kT_*} - 1}\,, \tag{3.1}$$

where h, ν, c, and T_* are Planck's constant, frequency of radiation, light velocity, and temperature of the ionizing radiation, respectively. Therefore, the net rate function X becomes

$$X(\rho, T, \xi) = \alpha_B(T)\xi^2 n - [s_1 F_1 (1 - \xi) + q(T) n \xi (1 - \xi)]\,, \tag{3.2}$$

where the coefficients $\alpha_B(T)$ and $q(T)$ are given by Hummer and Seaton (1963) and Hummer (1963), and

$$F_1(T_*, W_*) = W_* \int_{y_1}^{\infty} \frac{\mathrm{d}y}{y(e^y - 1)}\,, \tag{3.3}$$

$$s_1 = \frac{8\pi \nu_1^3 \sigma_0}{c^2}\,, \qquad y_1 = \frac{h\nu_1}{k_B T_*}\,, \qquad y = \frac{h\nu}{k_B T_*}\,, \tag{3.4}$$

ν_1 being the frequency of the Lyman limit and $\sigma_0 = 6.30 \times 10^{-18}\ \mathrm{cm}^2$.

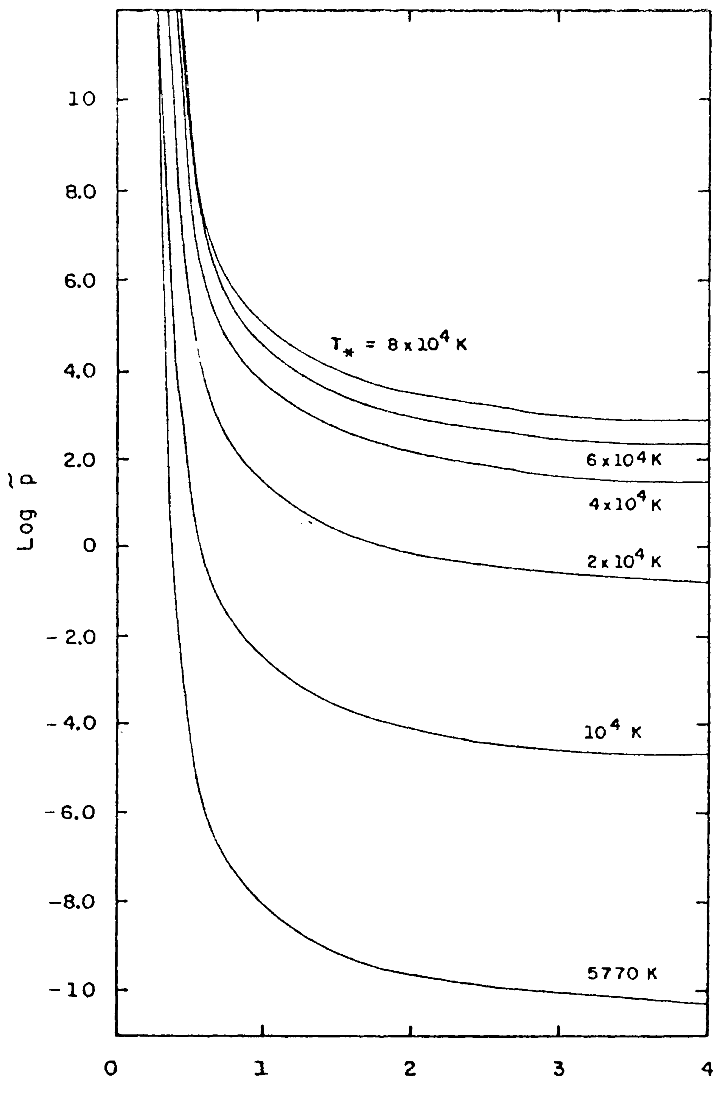

Fig. 1. The equilibrium dimensionless pressure $\tilde{p}$ as a function of the dimensionless temperature $\tilde{T}$ for several values of the temperature of the external radiation field T_*.

On the other hand, the net cooling rate becomes

$$L(\rho, T, \xi) = N_0 R \rho \xi^2 T \beta_D(T) + N_0^2 \chi \rho \xi (1 - \xi) [q(T) + \phi(T)] - \\ - s_1 R F_2 (1 - \xi) , \tag{3.5}$$

N_0 and χ being the Avogadro number, and the hydrogen-ionization potential, respectively. The coefficients $\beta_B(T)$ and $\phi(T)$ are those given by Hummer and Seaton (1963) and Hummer (1963), and F_2 is given by the expression

$$F_2(T_*, W_*) = T_* W_* \int_{y_1}^{\infty} \frac{\mathrm{d}y}{e^y - 1} - T_* y_1 F_1(T_*, W_*) . \tag{3.6}$$

From the chemical ($X = 0$), and thermal ($L = 0$) equilibrium conditions, one obtains the degree of ionization ξ and the number density of atoms n at the equilibrium state,

$$\xi = \frac{\tilde{q}\left(1 + \frac{T_*}{T_1}\,\psi\right) + \tilde{\phi}}{\tilde{q}\left(1 + \frac{T_*}{T}\,\psi\right) + \tilde{\phi} + \frac{T_*}{T_1}\,\psi\tilde{\alpha}_B - \frac{T_0}{T_1}\,\tilde{T}\tilde{\beta}_B}\,, \tag{3.7}$$

$$\frac{n}{n_0} \equiv \tilde{n} = \frac{\frac{T_*}{T_1}\,\psi}{\left[\frac{T_0}{T}\,\frac{\zeta}{1-\xi}\,\tilde{T}\tilde{\beta}_B + \tilde{q} + \tilde{\phi}\right]\xi}\left[\frac{F_1(T_*)}{F_1(10^4)}\right]; \tag{3.8}$$

where $T_1 = \chi/k_B$, $T_0 = 15\,062.0$ K, $n_0 = s_1 F_1/\beta_B(T_0)$, $\tilde{T} = T/T_0$, $\tilde{\alpha}_B = \alpha_B/\beta_B(T_0)$, $\tilde{\beta}_B = \beta_B/\beta_B(T_0)$, $\tilde{q} = q/\beta_B(T_0)$, $\tilde{\phi} = \phi/\beta_B(T_0)$, and $\psi = F_2/T_* F_1$.

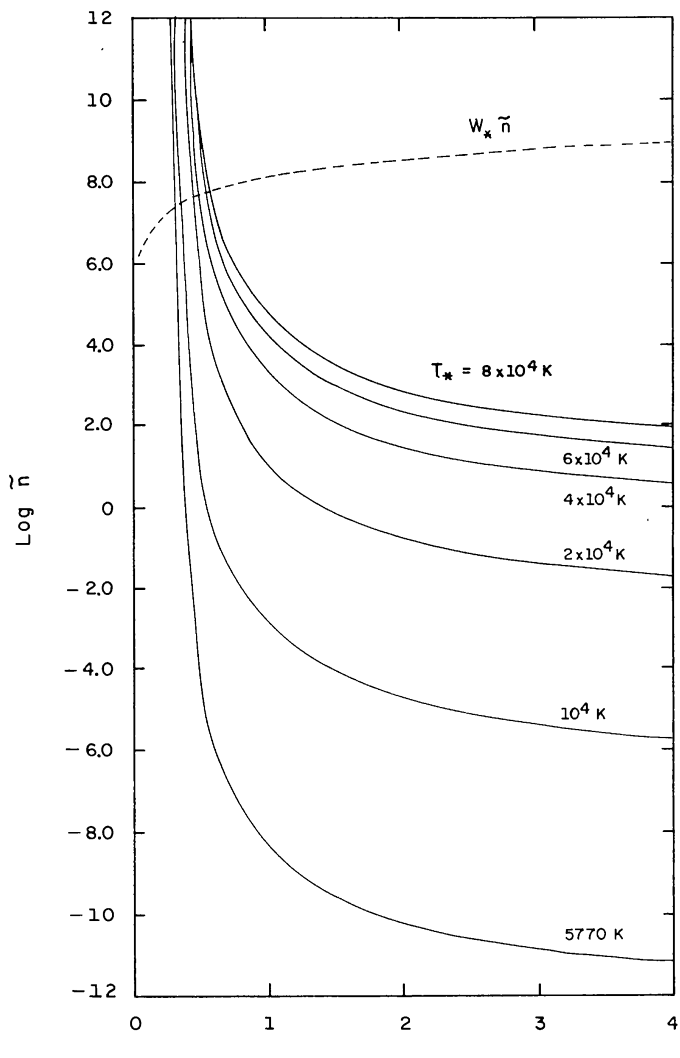

Fig. 2. The equilibrium dimensionless particle density $\tilde{n}$ as a function of the dimensionless temperature $\tilde{T}$ for several values of T_* (continuous lines). The limit value of $W_*\tilde{n}$ for which the plasma becomes a non-relativistic degenerate Fermi gas is also shown (dashed line).

Due to the fact that ψ becomes independent of W_*, the values of ξ and $\tilde{n}$ for the equilibrium state do not depend on W_*. Instead, they depend only on T_* and the gas temperature T.

The pressure $\tilde{p}$ in units of $k_B n_0 T_0$, the density $\tilde{n}$, and the degree of ionization ξ at equilibrium ($X = 0$ and $L = 0$) as functions of the temperature $\tilde{T}$ have been plotted in

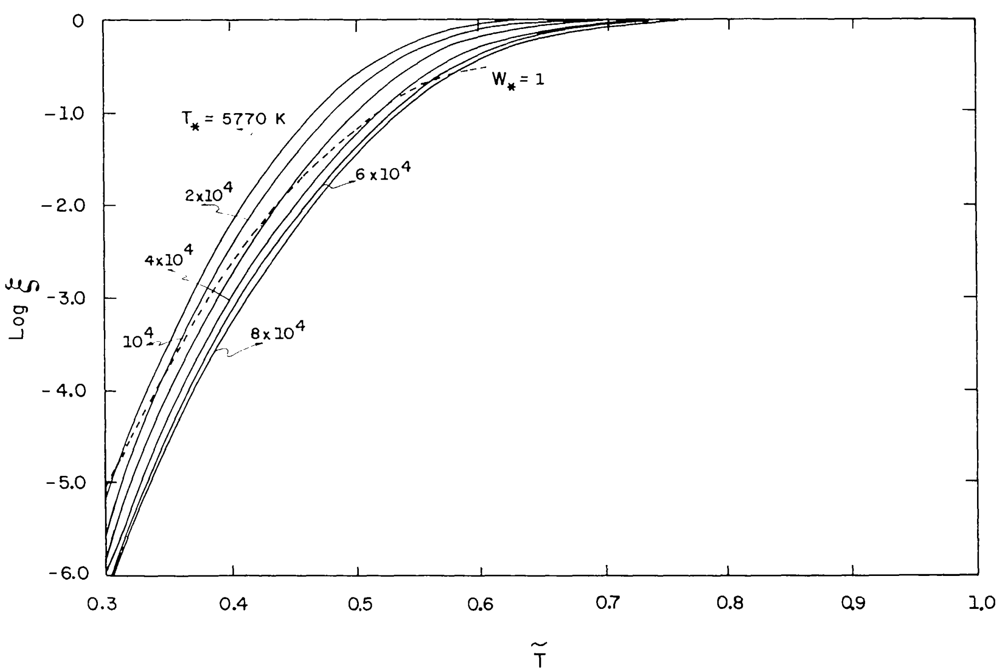

Fig. 3. The equilibrium of ionization ξ as a function of the dimensionless temperature $\tilde{T}$ for several values of T_* (continuous lines). The limit values of ξ for which the plasma becomes a non-relativistic degenerated Fermi gas when $W_* = 1$ is also plotted (dashed line).

Figures 1, 2, and 3, respectively, for different values of the temperature T_* of the external radiation field. For a given equilibrium temperature $\tilde{T}$, the gas pressure and the density are increasing functions of T_* while the degree of ionization ξ becomes a decreasing function of $\tilde{T}_*$ (see Figure 3). This last result is due to the fact that for the same plasma temperature T, an increase of T_* has to be compensated with a strong increase of density $\tilde{n}$. Therefore, the recombinations become more effective.

On the other hand, for a given temperature T_* plasmas with very low density may reach very high equilibrium temperature $\tilde{T} > T_*/T_0$ and very high ionization because the cooling becomes quite ineffective as well as the recombination, as it is expected on physical grounds, because the collisional nature of the above two physical processes.

Figure 4 is a plot of the dimensionless pressure $\tilde{p}$ as a function of the dimensionless density $\tilde{n}$ for several values of T_*. There are minima values of $\tilde{p}$ and $\tilde{n}$, which are increasing functions of T_*, such that plasmas at equilibrium cannot exist below such values of pressure and density.

In addition, because an ideal gas becomes a non-relativistic degenerate Fermi gas for $n > 1.58 \times 10^{16} T^{3/2}$ (Schatzman, 1970), the above results have a physical meaning only for $W_* \tilde{n} < 1.38 \times 10^8 \tilde{T}^{3/2}$. Also, in Figure 2, the upper limit of $W_* \tilde{n}$ can be noticed. The corresponding limit values for the degree of ionization ξ are indicated in Figure 3 for $W_* = 1$.

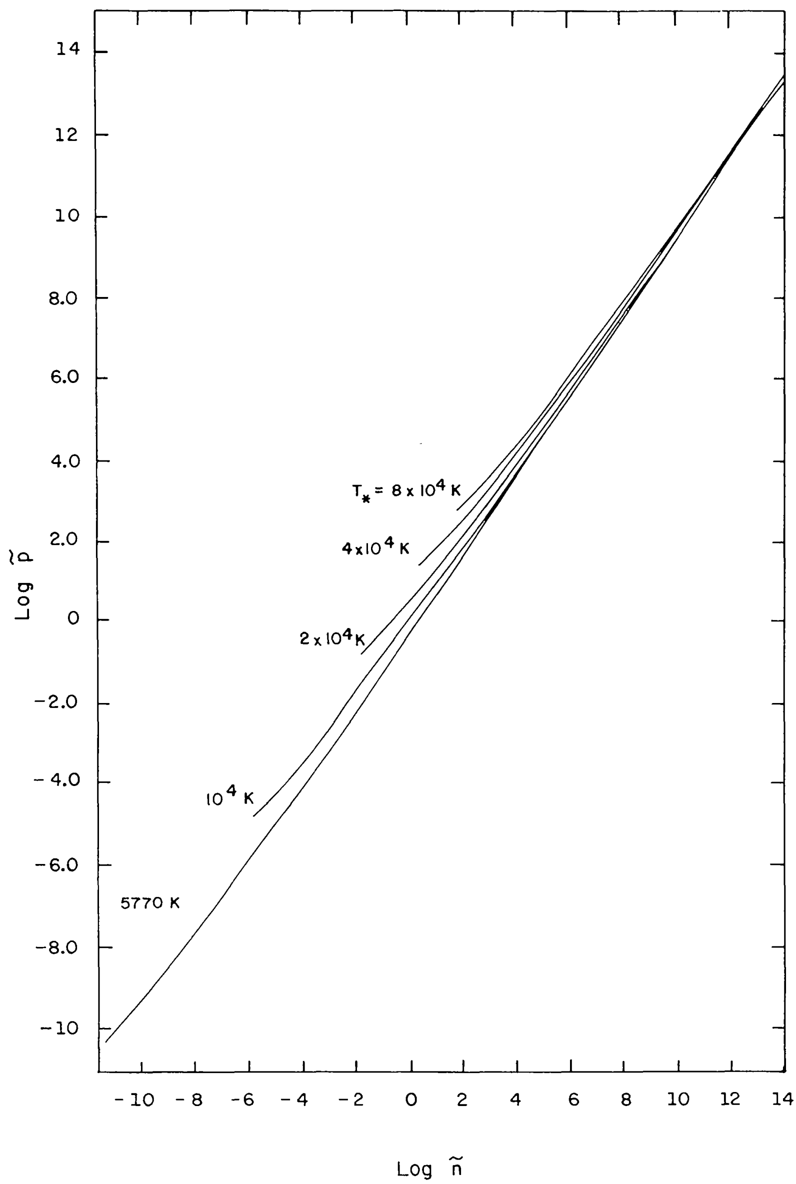

Fig. 4. The equilibrium dimensionless pressure $\tilde{p}$ as a function of the dimensionless particle density $\tilde{n}$ for several values of T_*.

Figure 4 shows a sharp difference between a plasma model in which the thermal effects of the external radiation fields are neglected and when the above effects are taken into account. In the first case, the pressure is a many-valuated function of density (Ibañez and Parravano, 1983; Figure 2) and in the second one $\tilde{p}$ becomes a monotonic increasing function of $\tilde{n}$.

The study of the local stability of the above equilibrium states for the hydrogen plasma in question has been carried out by Ibañez and Mendoza (1989). The next section will be dedicated to analyse the propagation of the linear waves in such plasmas.

4. Wave Propagation

If one-dimensional plane disturbances, $\sim \exp[i(kx - \omega t)]$, are imposed on the equilibrium state studied in Section 3 above, the set of Equations (2.2)–(2.6) can be linearized so that, one finds the dispersion relation

$$\hat{k}^4 - (\beta_1 + i\beta_2)\hat{k}^2 + (\varepsilon_1 + i\varepsilon_2) = 0\,, \tag{5.1}$$

where $\hat{k} = c_s k/\omega$, c_s being the isentropic sound speed, and the coefficients β_j and ε_j are defined in the Appendix.

The dispersion relation (5.1) is a quadratic equation in $\hat{k}^2$. Therefore, there are two roots that correspond to the sound $\hat{k}_s^2$ and the thermal $\hat{k}_T^2$ waves, respectively. Each of these roots involves two identical roots of opposite signs, which correspond to waves that are propagating in opposite directions. Therefore, the roots of Equation (5.1) can be written as

$$\hat{k}_\alpha = \hat{k}_{1\alpha} + i\hat{k}_{2\alpha}\,, \quad \alpha = s, T\,. \tag{5.2}$$

The above wave numbers define the scale lengths l_α for damping ($l_\alpha > 0$) or amplification ($l_\alpha < 0$) of sound and thermal waves,

$$\tilde{l}_\alpha = \frac{l_\alpha}{\lambda_\alpha} = \frac{1}{2\pi}\frac{\hat{k}_{1\alpha}}{\hat{k}_{2\alpha}}\,, \quad \alpha = s, T\,; \tag{5.3}$$

λ_α being the corresponding wavelength. In addition, to the above wave modes corresponds the dimensionless phase velocity

$$\tilde{v}_\alpha = \frac{v_\alpha}{c_s} = \frac{1}{\hat{k}_{1\alpha}}\,, \quad \alpha = s, T\,. \tag{5.4}$$

The values of $\tilde{v}_s$ as a function of the dimensionless frequency $\tilde{\omega}$, for representative values of $\tilde{T}$, have been plotted in Figures 5(a), 6(a), 7(a), and 8(a) for $T_* = 5770$ K, 10^4 K, 2×10^4 K, and 8×10^4 K, respectively. Also, the respective values of $\tilde{l}_s$ have been plotted as a function of $\tilde{\omega}$ in Figures 5(b), 6(b), 7(b), and 8(b).

At any equilibrium temperature $\tilde{T}$, the maximum damping (minimum value of $\tilde{l}_s$) of sound waves occurs at the values of frequency $\tilde{\omega}$ where the phase velocity $\tilde{v}_s$ changes from the isentropic to the isothermal value, or *vice versa*. For very large values of $\tilde{\omega}$, the sound waves propagate as weakly damped isothermal waves. But, for very small values of $\tilde{\omega}$, $\tilde{v}_s$ can depart from the isentropic value depending on the values of $\tilde{T}$ and T_* (see Figures 5(a)–8(a)). The values of the frequency $\tilde{\omega}$ at which the strongest damping occurs, depend on the value of the equilibrium temperature $\tilde{T}$ and the effective temperature T_* of the external radiation field. In Figures 5(b)–8(b) we can

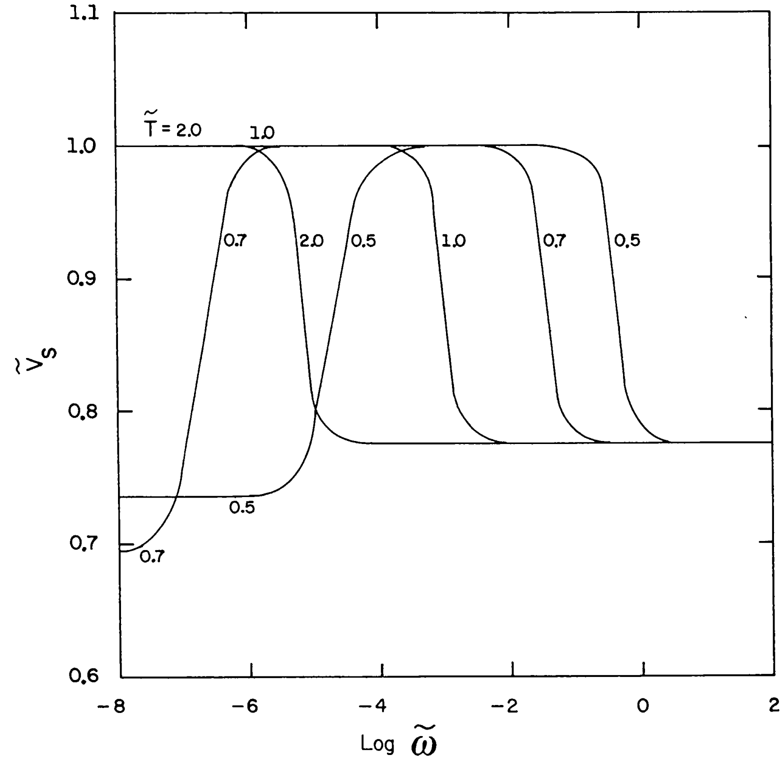

Fig. 5a. The dimensionless phase velocity of the acoustic waves $\tilde{v}_s$ as a function of the dimensionless frequency $\tilde{\omega}$ for several values of $\tilde{T}$ and $T_* = 5770$ K.

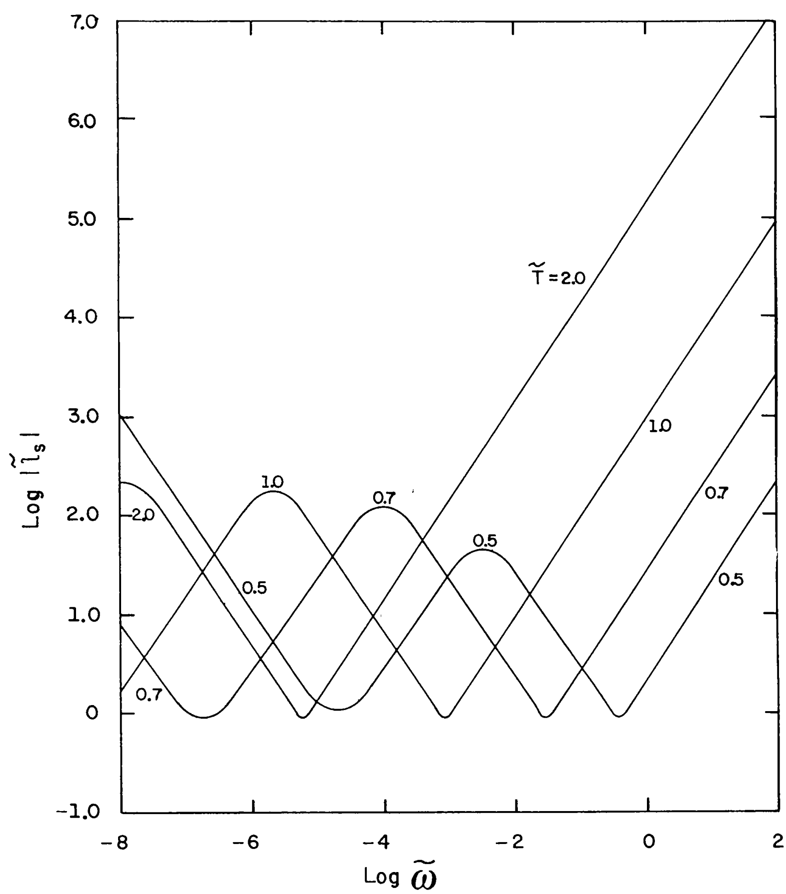

Fig. 5b. The dimensionless scale-length for damping (continuous lines) or anti-damping (dashed lines) for sound waves $\tilde{l}_s$ as a function of the dimensionless frequency $\tilde{\omega}$ for several values of $\tilde{T}$ and $T_* = 5770$ K.

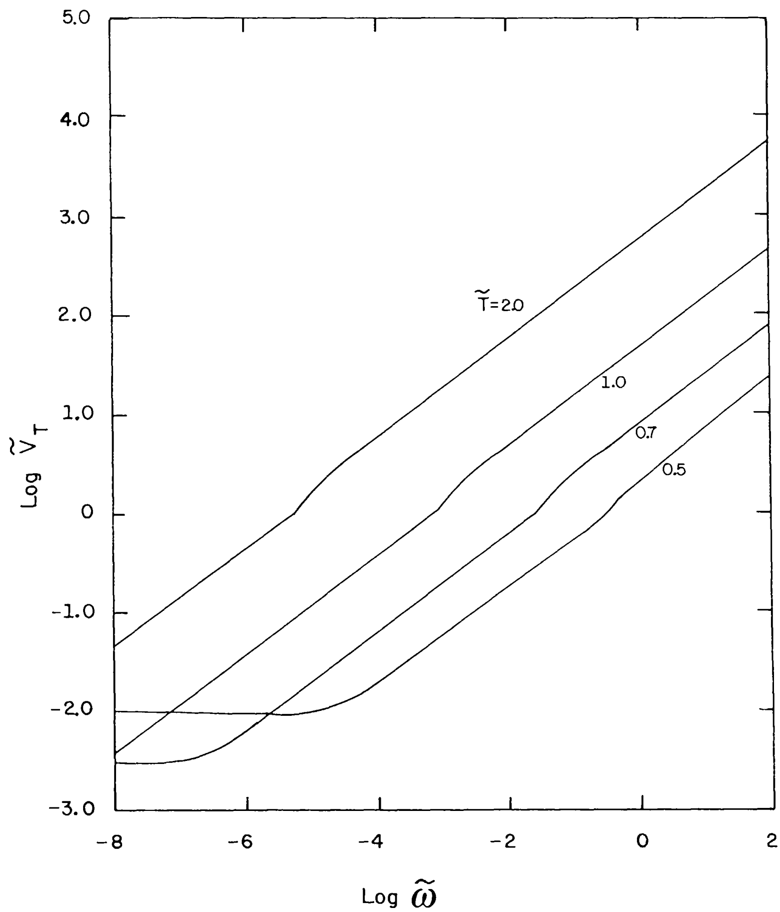

Fig. 5c. The dimensionless phase velocity of the thermal waves $\tilde{v}_T$ as a function of the dimensionless frequency $\tilde{\omega}$ for several values of $\tilde{T}$ and $T_* = 5770$ K.

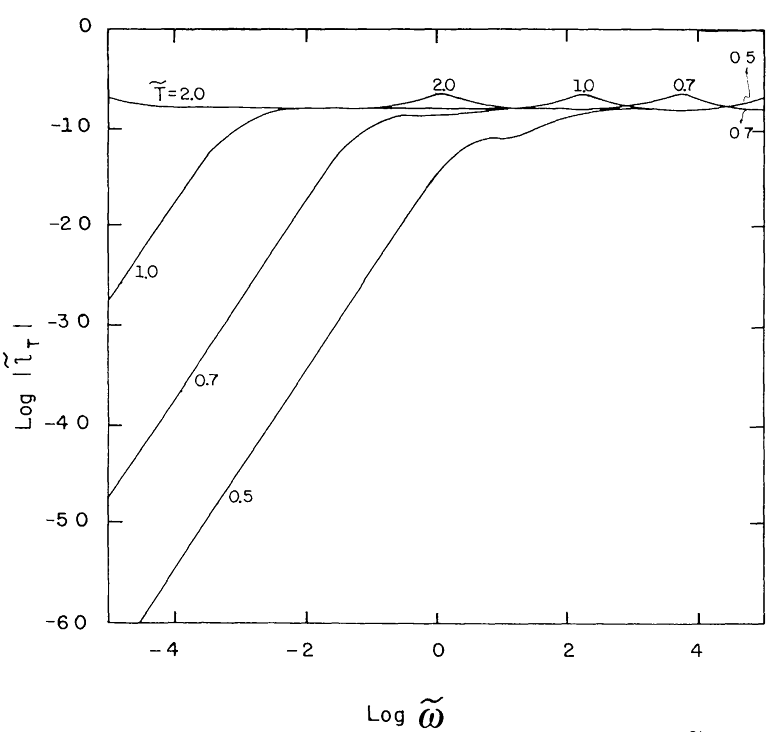

Fig. 5d. The dimensionless scale length for damping of the thermal waves $\tilde{l}_T$ as a function of the dimensionless frequency $\tilde{\omega}$ for several values of $\tilde{T}$ and $T_* = 5770$ K.

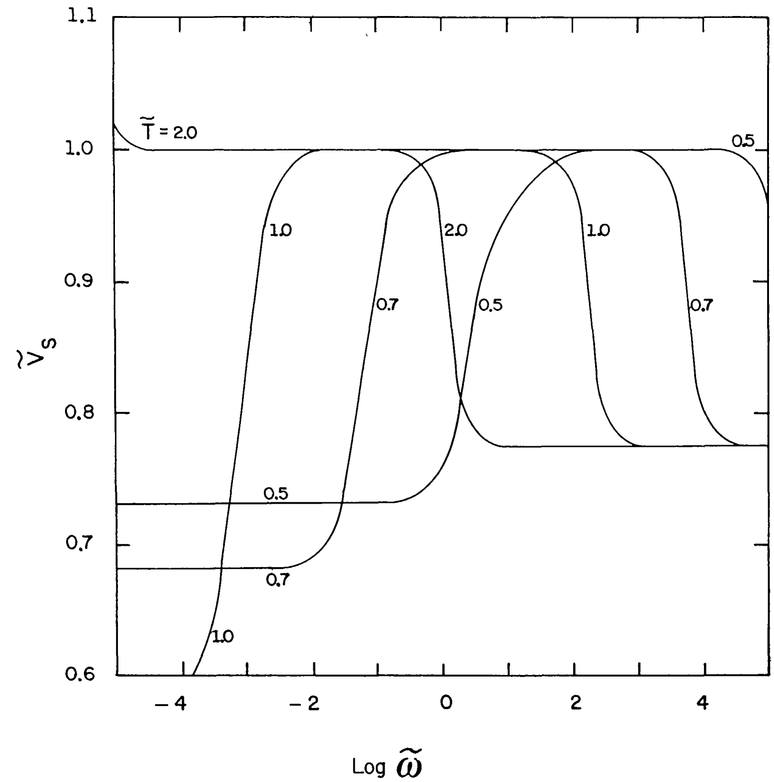

Fig. 6a. As Figure 5(a) for $T_* = 10^4$ K.

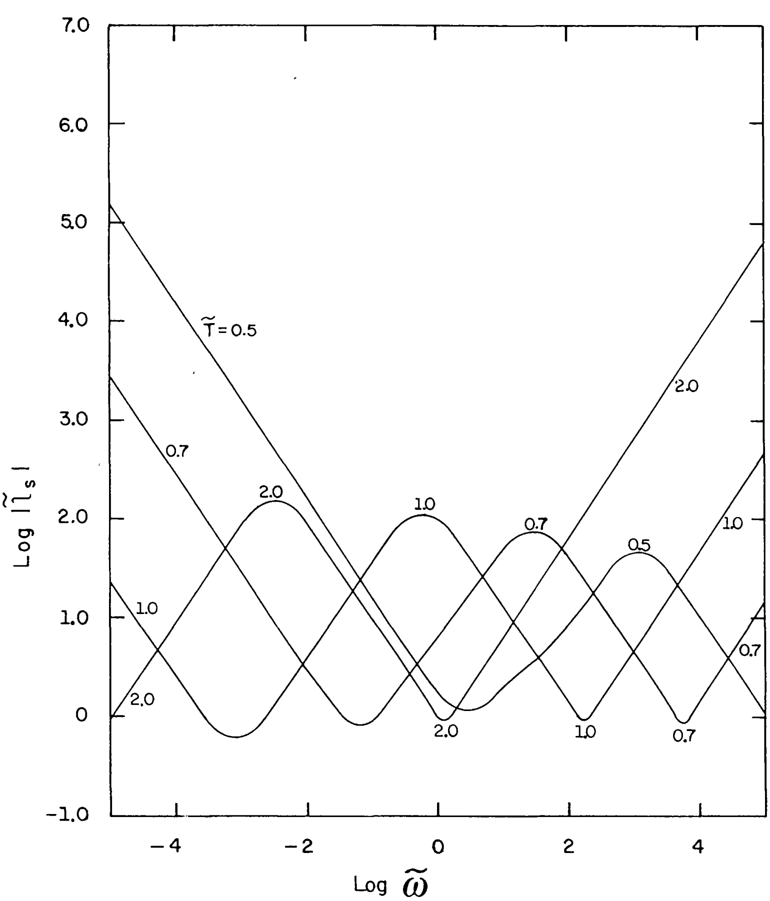

Fig. 6b. As Figure 5(b) for $T_* = 10^4$ K.

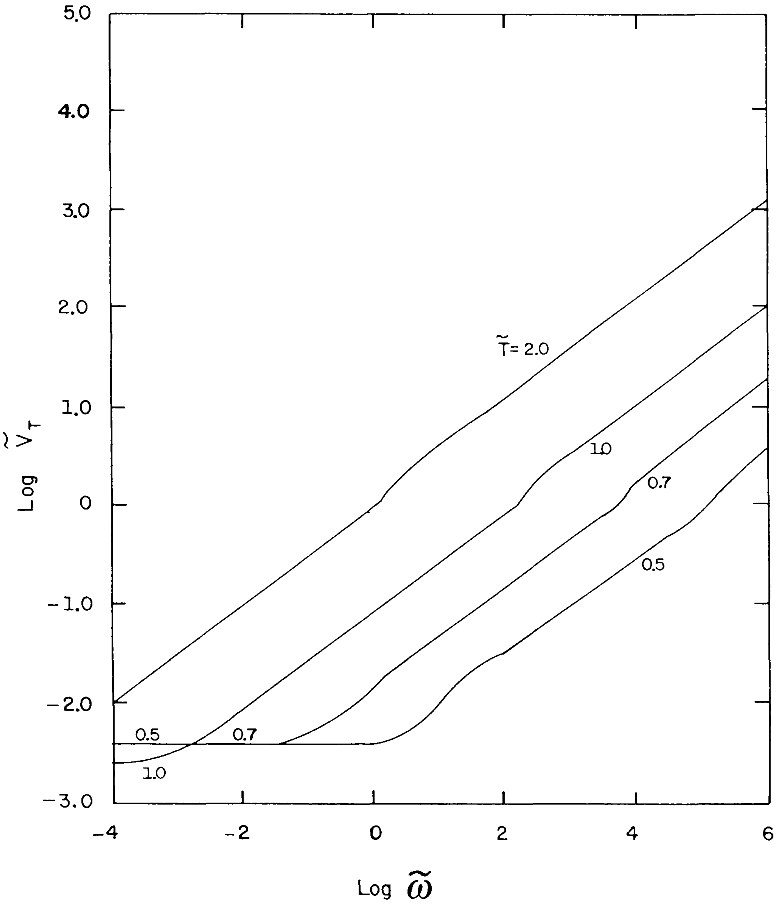

Fig. 6c. As Figure 5(c) for $T_* = 10^4$ K.

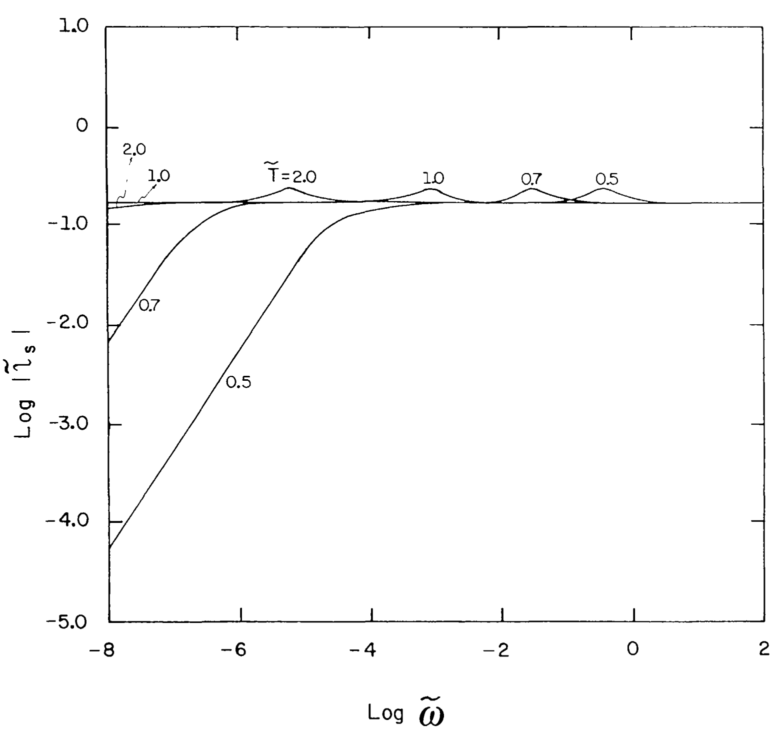

Fig. 6d. As Figure 5(d) for $T_* = 10^4$ K.

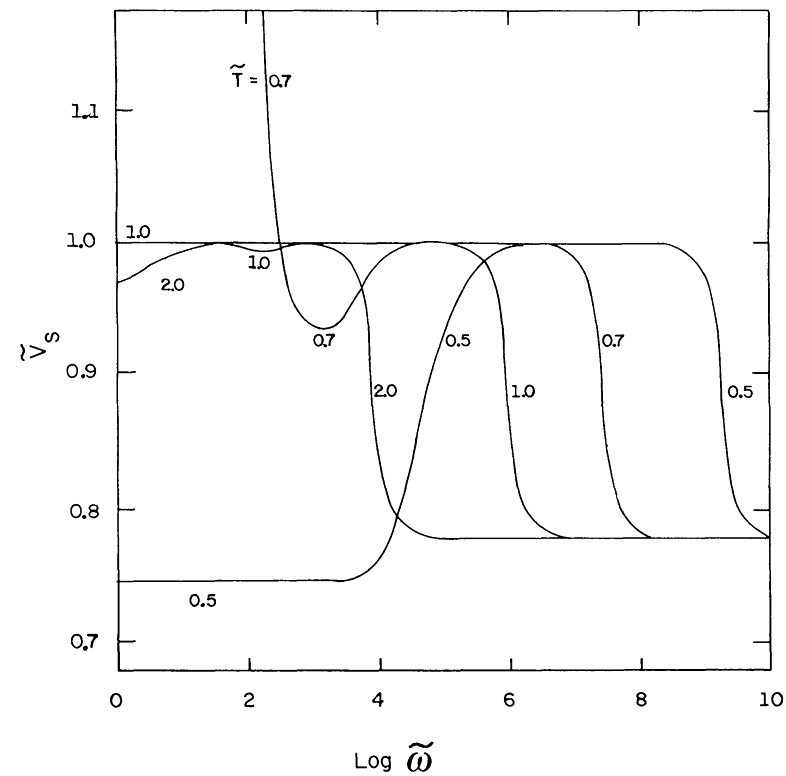

Fig. 7a. As Figure 5(a) for $T_* = 2 \times 10^4$ K.

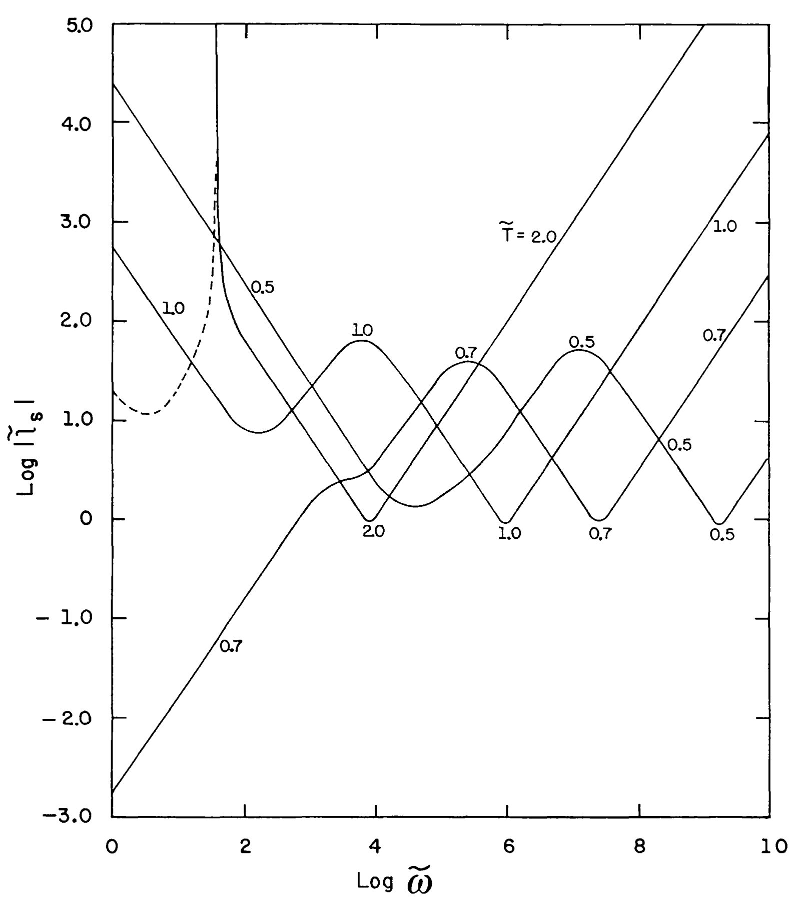

Fig. 7b. As Figure 5(b) for $T_* = 2 \times 10^4$ K.

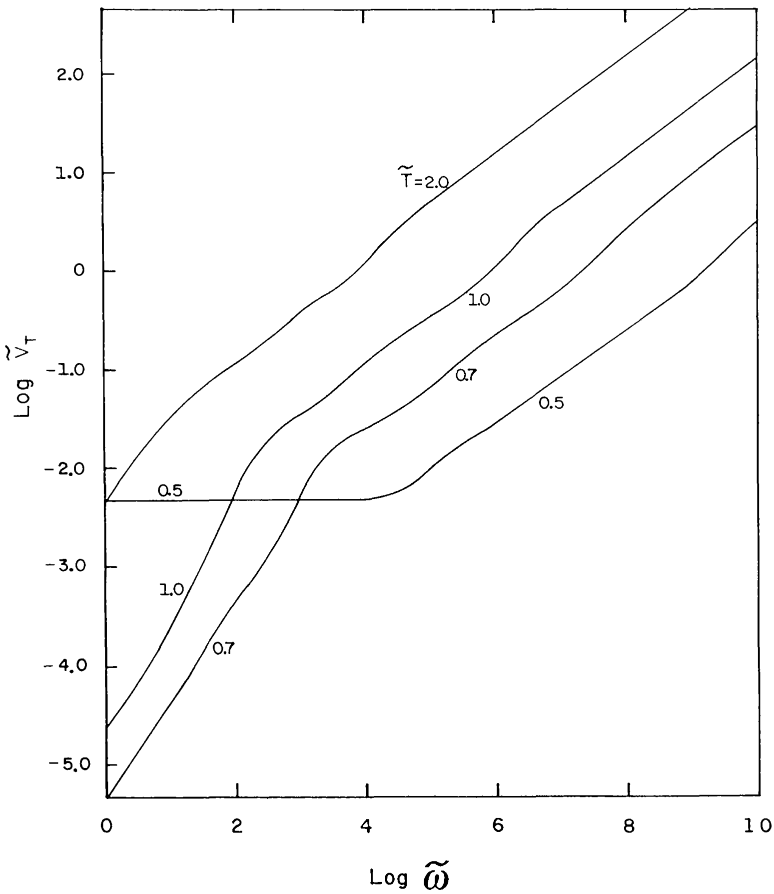

Fig. 7c. As Figure 5(c) for $T_* = 2 \times 10^4$ K.

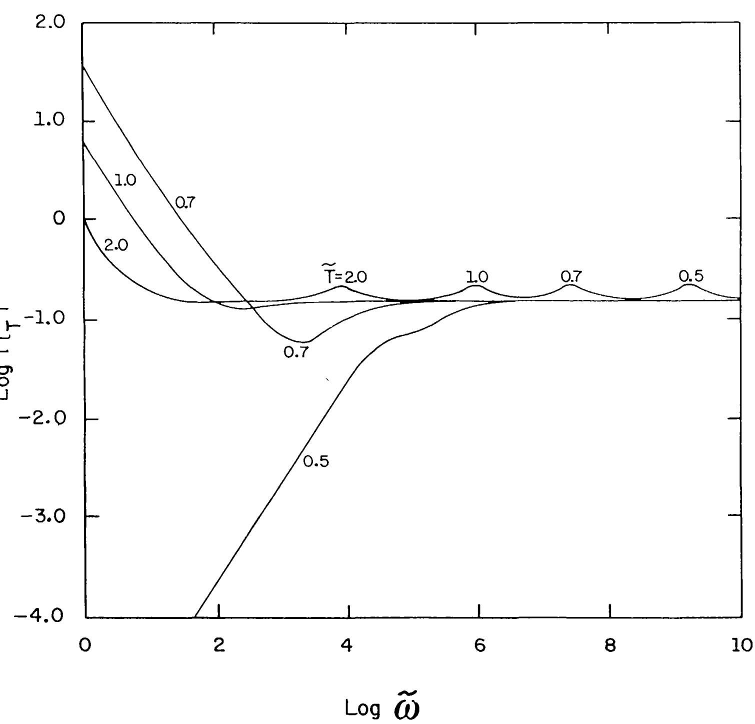

Fig. 7d. As Figure 5(d) for $T_* = 2 \times 10^4$ K.

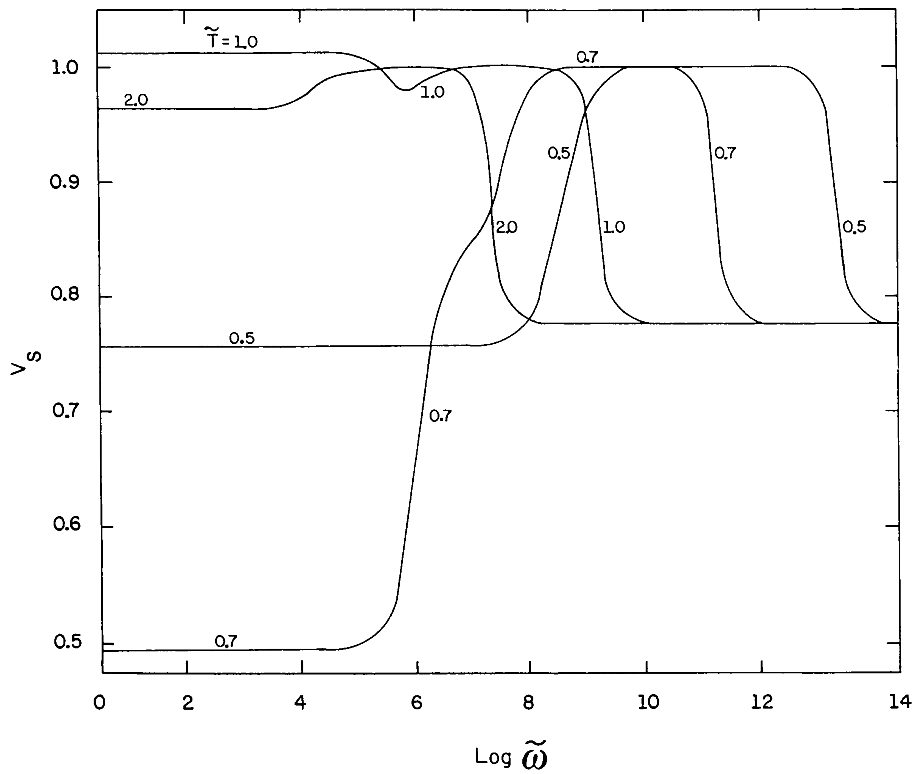

Fig. 8a. As Figure 5(a) for $T_* = 8 \times 10^4$ K.

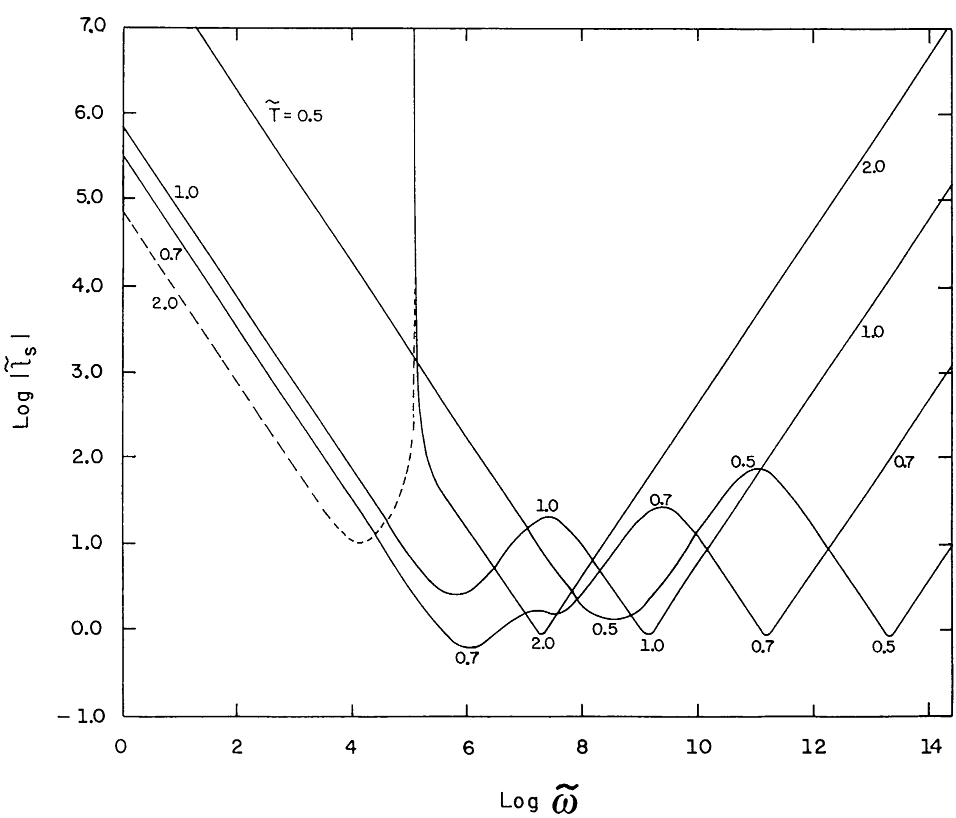

Fig. 8b. As Figure 5(b) for $T_* = 8 \times 10^4$ K.

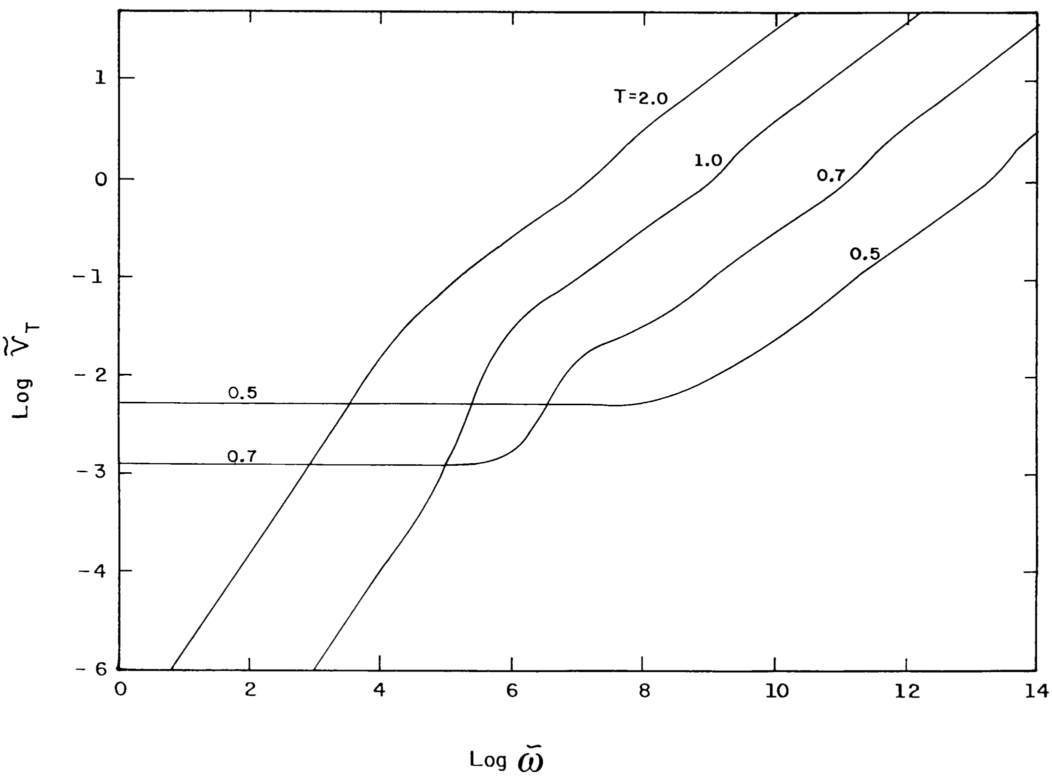

Fig. 8c. As Figure 5(c) for $T_* = 8 \times 10^4$ K.

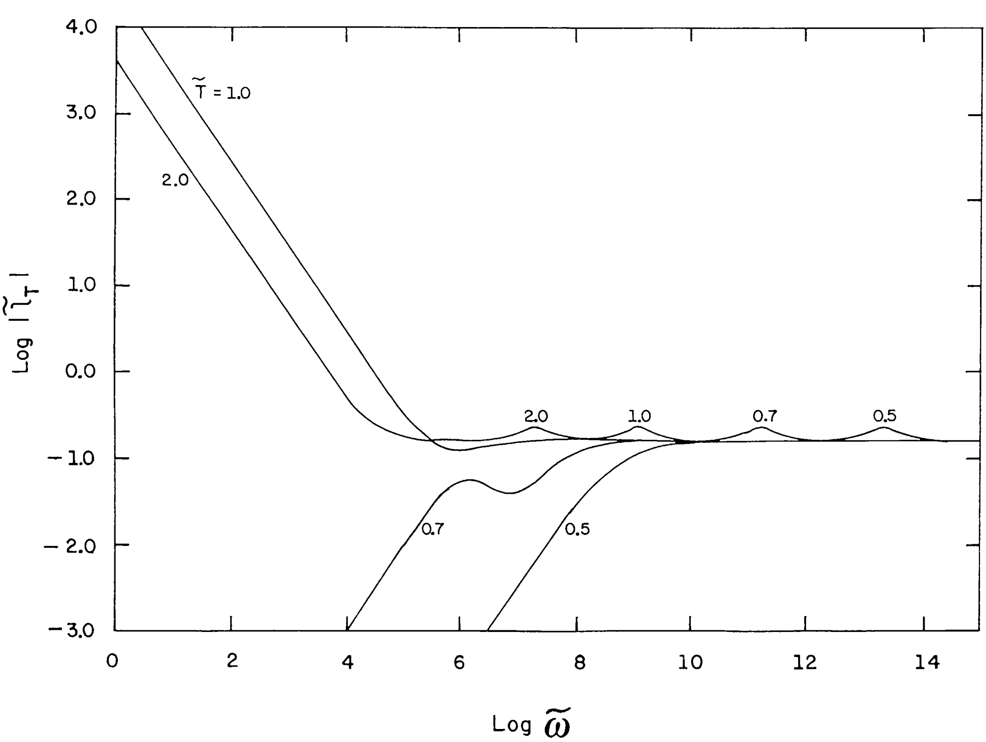

Fig. 8d. As Figure 5(d) for $T_* = 8 \times 10^4$ K.

see that for a particular value of $\tilde{T}$ the minima of $\tilde{l}_s$ are shifted towards large values of $\tilde{\omega}$ when T_* increases. Between minima values of $\tilde{l}_s$ a maximum of $\tilde{l}_s$ (minimum damping) occurs of the order of 10^2, the exact value depending on $\tilde{T}$.

Anti-damping of the sound waves only occurs for high temperatures $\tilde{T} \gtrsim 2$ (completely ionized plasmas) and for high values of $T_* \gtrsim 2 \times 10^4$ K, see Figure 7(b). This result contrasts with that of Ibañez and Mendoza (1987, Figure 1(b)) for plasmas not radiated by external fields, where the amplification of sound waves also occurs for $\tilde{T} \sim 0.7$. Therefore, one may conclude that an external radiation field has a stabilizing effect on the hydrogen plasma.

The phase velocity for thermal waves $\tilde{v}_T$ and the corresponding damping length $\tilde{l}_T$, have been plotted as functions of the dimensionless frequency $\tilde{\omega}$. See Figures 5(c)–8(c), and 5(d)–8(d), respectively.

For very small values of $\tilde{\omega}$, $\tilde{v}_T$ becomes a constant depending on the values of $\tilde{T}$ and T_*; but for values of $\tilde{\omega}$ large enough, $\tilde{v}_T$ becomes an increasing function of $\tilde{\omega}$. The above behaviour of $\tilde{v}_T$ is similar to the one presented in thermal waves in a hydrogen plasma not radiated by external radiation fields (Ibañez and Mendoza, 1987, Figure 2(a)).

The length-scale $\tilde{l}_T$ for damping of thermal waves (Figures 5(d)–7(d)) shows the typical small maxima (minima of damping) at values of $\tilde{\omega}$ large enough which depend on $\tilde{T}$. They are shifted towards large values of $\tilde{\omega}$ when T_* increases, and the thermal waves do not amplify as for the case where the external radiation fields are absent (Ibañez and Mendoza, 1987; Figure 2(b)). The above results are a consequence of the stabilizing effect of the external radiation field on the thermal behaviour of the hydrogen plasma, as was pointed out previously.

In some specific applications, the dilution factor W_*, only involved in the definition of the normalizing frequency $\omega_0 = 51.4\ W_*$ (Equation (A.12)), fixes the values of the frequency measured in s^{-1}. So that, for example, assuming $W_* = 1$, for plasmas heated by an external radiation field as the solar ($T_* = 5770$ K) and highly ionized (plasmas with very low density $\sim 10^3\ cm^{-3}$) the strongest acoustic damping would occur for sound waves with characteristic periods of 6 hr for $\tilde{T} = 2$ and 2.3 min for $\tilde{T} = 1$; and, for partially-ionized plasmas ($\tilde{T} = 0.5$) it would occur for fluctuations with characteristic periods of ~ 0.3 s and ~ 1.7 hr. Small damping would occur for periods ~ 17 hr (at $\tilde{T} = 1$) and 39 s (at $\tilde{T} = 0.5$).

Due to the large shift of the above effects towards very high values of the dimensionless frequency $\tilde{\omega}$, when T_* increases, the above results have practical interest in specific applications, for plasmas radiated by very hot radiation fields, if they are sufficiently diluted: for instance, in planetary nebulae, in galactic and extragalactic jets, in H II regions and in quasar clouds.

Acknowledgements

This work was supported by CDCHT of the Universidad de Los Andes through Project C-175–81 and by CONDES of the Universidad del Zulia.

Appendix

Definitions of the coefficients appearing in the dispersion relation (5.1)

$$\beta_1 = \frac{C\hat{k}_5 - \hat{k}_6}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\left\{ - \gamma(\hat{k}_\kappa + \hat{k}_6) + \hat{k}_\kappa\left[\hat{k}_1'(\hat{k}_6 - C\hat{k}_5) + \right.\right.$$

$$\left.\left. + \hat{k}_2\left(\frac{AC}{V}\,\hat{k}_4 - \hat{k}_6\right) + \hat{k}_3\left(\frac{B}{A}\,\hat{k}_5 - \hat{k}_4\right)\right]\right\} + \frac{1}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\times$$

$$\times\left\{\gamma - \hat{k}_\kappa\left[(\hat{k}_1' - \hat{k}_2) - (1 + \gamma_1 C)\hat{k}_4 + \left(\frac{B}{A} - C\right)\hat{k}_5 + \gamma\hat{k}_6\right]\right\}, \quad \text{(A.1)}$$

$$\beta_2 = \frac{\gamma(\hat{k}_\kappa + \hat{k}_6)}{1 + (C\hat{k}_5 - \hat{k}_6)^2} - \frac{\hat{k}_\kappa}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\left[\hat{k}_1'(\hat{k}_6 - C\hat{k}_5) + \right.$$

$$\left. + \hat{k}_2\left(\frac{AC}{B}\,\hat{k}_4 - \hat{k}_5\right) + \hat{k}_3\left(\frac{B}{A}\,\hat{k}_5 - \hat{k}_4\right)\right] + \frac{C\hat{k}_5 - \hat{k}_6}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\times$$

$$\times\left\{\gamma - \hat{k}_\kappa\left[(\hat{k}_1' - \hat{k}_2) - (1 + \gamma_1 C)\hat{k}_4 + \left(\frac{B}{A} - C\right)\hat{k}_5 + \gamma\hat{k}_6\right]\right\}, \quad \text{(A.2)}$$

$$\varepsilon_1 = -\frac{\gamma\hat{k}_\kappa}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\,[\hat{k}_1' - \hat{k}_4 + C\hat{k}_5 + (\hat{k}_6 - C\hat{k}_5)(\hat{k}_1'\hat{k}_6 - \hat{k}_3\hat{k}_4)]\,, \quad \text{(A.3)}$$

$$\varepsilon_2 = \frac{\gamma\hat{k}_\kappa}{1 + (C\hat{k}_5 - \hat{k}_6)^2}\,[1 + \hat{k}_3\hat{k}_4 + \hat{k}_6(\hat{k}_6 - \hat{k}_4 - C\hat{k}_5) - C\hat{k}_5(\hat{k}_1' - \hat{k}_4)]\,; \quad \text{(A.4)}$$

where the dimensionless wave numbers $\hat{k}_j$ are given by

$$\hat{k}_1' = \frac{\tilde{n}}{A\tilde{\omega}}\left[\xi^2\tilde{\beta}' + \left(\frac{\chi}{k_B}\right)\xi(1 - \xi)(\tilde{q}' + \tilde{\phi}')\right], \quad \text{(A.5)}$$

$$\hat{k}_2 = \frac{\tilde{n}}{A\tilde{\omega}\tilde{T}}\left[\xi^2\tilde{T}\tilde{\beta}_B + \left(\frac{T_1}{T_0}\right)\xi(1 - \xi)(\tilde{q} + \tilde{\phi})\right], \quad \text{(A.6)}$$

$$\hat{k}_3 = \frac{\tilde{n}}{B\tilde{\omega}\tilde{T}}\left[2\tilde{T}\tilde{\beta}_B\xi^2 + \left(\frac{T}{T_0}\right)(1 - 2\xi)(\tilde{q} + \tilde{\phi}) + \left(\frac{T_*}{T}\right)\psi\right], \quad \text{(A.7)}$$

$$\hat{k}_4 = \frac{BT_0}{A} \frac{\tilde{T}\tilde{n}}{\tilde{\omega}} \left[\tilde{\alpha}'_B \xi^2 - \tilde{q}' \xi(1 - \xi) \right], \tag{A.8}$$

$$\hat{k}_5 = \frac{\tilde{n}}{\tilde{\omega}} \left[\tilde{\alpha}_B \xi^2 - \tilde{q} \xi(1 - \xi) \right], \tag{A.9}$$

$$\hat{k}_6 = \frac{1}{\tilde{\omega}} \left[2\tilde{n}\tilde{\alpha}_B \xi - \tilde{n}\tilde{q}(1 - 2\xi) + 1 \right], \tag{A.10}$$

$$\hat{k}_\kappa = 1.04 \times 10^{-3} \frac{T_0^{-3/2}}{\beta_0} \frac{A\tilde{n}\tilde{c}^2}{\tilde{\kappa}\tilde{\omega}} ; \tag{A.11}$$

where

$$\tilde{\omega} = \frac{\omega}{k_0 c_0} , \quad k_0 c_0 = 51.4 W_* , \tag{A.12}$$

$$\tilde{c} = |(1 + \xi)\tilde{T}|^{1/2} , \tag{A.13}$$

$$\tilde{\kappa} = \frac{1.36 \times 10^8}{T_0^2} (1 - \xi)\tilde{T}^{1/2} + \frac{\tilde{T}^{5/2}\xi}{\ln \Lambda} . \tag{A.14}$$

References

Blandford, R. D.: 1985, in D. Mihalas and K. H. A. Winkler (eds.), 'Radiation Hydrodynamics in Stars and Compact Objects', *IAU Colloq.* **89**, 387.

Bodo, G., Ferrari, A., Massaglia, S., and Tsinganos, K.: 1985, *Astron. Astrophys.* **149**, 246.

Capriotti, E.: 1971, *Astrophys. J.* **166**, 563.

Cowie, L. L. and McKee, C. F.: 1977, *Astrophys. J.* **211**, 135.

Ferch, R. L. and Salpeter, E. E.: 1975, *Astrophys. J.* **202**, 195.

Hummer, D. G.: 1963, *Monthly Notices Roy. Astron. Soc.* **125**, 461.

Hummer, D. G. and Seaton, M. J.: 1963, *Monthly Notices Roy. Astron. Soc.* **125**, 437.

Ibáñez, S. M. H.: 1986, *Astrophys. J.* **308**, 382.

Ibáñez, S. M. H. and Mendoza, B. C. A.: 1987, *Astrophys. Space Sci.* **137**, 1.

Ibáñez, S. M. H. and Mendoza, B. C. A.: 1989, preprint.

Ibáñez, S. M. H. and Parravano, B. A.: 1983, *Astrophys. J.* **275**, 181.

Ibáñez, S. M. H. and Plachco, F. P.: 1989, *Astrophys. J.* **336**, 875.

Mathews, W. G.: 1986, *Astrophys. J.* **305**, 187.

McKee, C. and Cowie, L. L.: 1977, *Astrophys. J.* **215**, 213.

McKee, Ch. and Ostriker, J.: 1977, *Astrophys. J.* **218**, 148.

Parker, E. N.: 1953, *Astrophys. J.* **117**, 431.

Parravano, A.: 1987, *Astron. Astrophys.* **172**, 280.

Schatzman, E.: 1970, *Physics and Astrophysics*, CERN, Geneva.

Spitzer, L.: 1962, *Physics of Fully Ionized Gases*, Interscience, New York.

Spitzer, L., 1978, *Physical Process in the Interstellar Medium*, Wiley and Sons, New York.