THE ASTRONOMICAL JOURNAL VOLUME 100, NUMBER 6 DECEMBER 1990

ASTEROID FAMILIES. I. IDENTIFICATION BY HIERARCHICAL CLUSTERING AND RELIABILITY ASSESSMENT

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Received 11 April 1990; revised 21 June 1990

ABSTRACT

Substantial discrepancies between the existing classifications and inconsistencies with the results of physical studies have motivated a research program aimed at deriving an improved classification of asteroids in dynamical families. We analyzed a set of 4100 numbered asteroids, whose proper elements had been computed by a new second-order, fourth-degree secular perturbation theory [Milani and Knežević 1990, Celestial Mech. (submitted)], and checked with numerical integrations to assess their long-term stability. A multivariate data analysis technique (*hierarchical clustering*) was applied to build for each zone of the belt a *dendrogram* in the space of proper elements, with a distance function related to the incremental velocity needed for orbital change after ejection from a fragmented parent body. Families were then identified by comparing this dendrogram with a similar one, derived for a quasirandom distribution of elements matching the large-scale structure of the real distribution. A significance parameter was associated with each family, measuring its departure from random concentrations, and two robustness parameters were obtained by repeating the classification procedure after varying the elements by small amounts (consistent with the results of numerical tests of their long-term stability) and changing the coefficients of the distance function. The most significant and robust families are those associated with Themis, Eos, and Koronis, that collectively include about 14% of the known main-belt population; but 12 more reliable and robust families were found throughout the belt, most of which partially match those found in previous classifications. In the Flora region of the inner belt, a reliable identification of families is difficult, since the background has a high density and the accuracy of proper eccentricities and inclinations is poor, mainly because of the proximity to the strong ν_6 secular resonance. Other results include: a relatively populous Eunomia family, lacking large C-type members; a small family having Vesta as its largest object; the disappearance of the unlikely association in one family (Nysa-Hertha) of M, F, and E types; the existence of two small, but robust families with sizeable largest members in the Themis region, at moderate inclinations.

I. INTRODUCTION

After a long standstill in the last 15 yr, the subject of asteroid families has been raising increasing interest and motivating new investigations. There are several reasons for this development: (i) recent work [for reviews, see Froeschlé *et al.* (1988); Valsecchi *et al.* (1989)] has clarified subtle issues concerning the long-term dynamical evolution of asteroid orbits, whose modeling is an essential prerequisite for the derivation of proper elements (which, in turn, are the basic dataset for family classification purposes); (ii) the availability of physical data on sizes, shapes, taxonomic types, and rotation rates for many hundreds of asteroids has prompted new analyses of families, searching for correlations and/or peculiarities that may throw light on the properties (in particular, the internal structure) of the parent bodies and on the mechanism of their fragmentation (Gradie *et al.* 1979; Zappalà *et al.* 1984; Binzel 1988; Chapman *et al.* 1989; Paolicchi *et al.* 1989; Bell 1989; Housen and Holsapple 1990); (iii) as families are widely believed to represent the outcomes of very energetic asteroidal impacts, their abundance and properties provide an obvious observational counterpart for theoretical models of the asteroid collisional history, and may put important constraints on both the values of the main parameters (impact strength, energy partitioning, etc.) controlling the collisional outcomes, and the initial abundance of solid material in the asteroid belt (Farinella *et al.* 1982; Davis *et al.* 1985, 1989).

However, there is an outstanding obstacle to the exploitation of family data for the purposes outlined above. As pointed out by Carusi and Valsecchi (1982), on such basic issues as the number of existing families, their memberships, and their distribution in the asteroid belt there has always been poor agreement among different investigators. This is related to the use of different starting sets of osculating elements, different secular perturbation theories used to derive proper elements, and different procedures and criteria aimed at "extracting" families (i.e., assumed nonrandom clusters in the multidimensional space of proper elements) from the quasirandom background of "field asteroids." For instance, the two family classifications presented in *Asteroids* (Kozai 1979; Williams 1979) were strikingly different; and some authors (e.g., Carusi and Massaro 1978) have claimed that apart from the outstanding "populous" families (Eos, Themis, Koronis, and a few others), all the remaining ("small") families have a low level of significance. On the other hand, the number of existing families and the sizes of the parent bodies are critical parameters for assessing the intensity of asteroid collisional evolution and, consequently, the likely ages of the families, too (Farinella *et al.* 1989).

Other problems have arisen from physical studies of families. First, while the populous families appear both fairly homogeneous in composition and clearly distinct from the background (Gradie *et al.* 1979; Zellner *et al.* 1985; Chapman *et al.* 1989), it has been difficult to make physical and geochemical sense from most other families identified by celestial mechanicians. For instance, some analyses of Williams' families (Gradie *et al.* 1979; Chapman 1986; Bell 1989) have concluded that many small families provide assemblages of taxonomic types that correspond to materials which, from a cosmochemical point of view, are not likely to have ever resided together in a single parent object. Chapman's *et al.* (1989) distinctness criterion, on the other hand, showed that while 9/10 of Williams' families with 12 or more members are (or probably are) taxonomically distinct from the background, 37/46 of those with less than five members are definitely not distinct (not necessarily implying that they are genetically "unreal," however). An additional problem evidenced by physical studies is related to the geometrical properties of the ejection velocity fields of family members from their parent bodies, as inferred from the distributions of proper elements. As pointed out by Brouwer (1951) and Zappalà *et al.* (1984), most families derived by both Brouwer and Williams show a systematic anisotropy of the three-dimensional distribution of ejection velocities in an orbit-bound reference frame. According to these authors, the anisotropy is more likely related to a limited accuracy of proper eccentricities and inclinations than to a real feature of the breakup events—a conclusion later supported, for the proper elements used by Brouwer, by the numerical experiments of Carpino *et al.* (1986).

With these motivations, we have decided to perform a new classification of asteroids in families. The improvements with respect to previous work in this field are the following: (i) a larger and updated set of osculating elements, including 4100 numbered asteroids, was used as starting data; (ii) proper elements were derived with a very refined secular perturbation theory, whose accuracy (namely, stability in time) has been extensively checked by long-term numerical integrations; (iii) an objective, automatic, and bias-free multivariate data analysis technique was employed to find nonrandom groupings in the space of proper elements and also to quantitatively estimate the statistical significance of these groupings; (iv) an assessment was made of the robustness of the statistically significant families with respect to small, random variations of proper elements, due to the finite accuracy of the theory used to derive them.

The remainder of this paper is organized as follows. In Sec. 2 we are going to describe our database and the *hierarchical clustering* technique applied to the identification of families; we shall also introduce a new method to quantitatively assess the statistical significance of families and their robustness with respect to inaccuracies of the proper elements and changes in the distance function defined in the proper element space. In Sec. III we present a list of the families identified in different parts of the belt with this procedure, give their memberships and their significance and robustness parameters, and briefly discuss their main properties. The main conclusions and perspectives for future work in this field are outlined in the final section.

II. THE HIERARCHICAL CLUSTERING METHOD

Our basic dataset was the list of asteroid proper elements computed by Milani and Knežević [Version 4.2, see Milani and Knežević (1990)] by means of a second-order (in the planetary masses), fourth-degree (in the eccentricities and inclinations) secular perturbation theory, based on the Lie series technique, and including an iterative algorithm to obtain more accurate secular frequencies and more stable elements. A computer-readable file containing these proper elements can be requested via *e*-mail at the address TWIN2 at ICNUCEVM.BITNET. For earlier versions and presentations of the theory and the elements, the reader is referred to Knežević (1986, 1989), Valsecchi *et al.* (1989), and Knežević and Milani (1989); notice that the list given in the latter reference has now been replaced by a new one, derived by an improved version of the same theory (containing the direct secular perturbations of Saturn in the fundamental frequencies and the forced terms) and expanded to a larger asteroid set. Short-periodic perturbations were eliminated analytically, as explained in detail by Knežević (1988) and Knežević *et al.* (1988). The elements are of course quasi-integrals of the motion, within an accuracy depending in a complex way on the location in the belt; this accuracy was quantitatively determined by carrying out a comprehensive set of numerical integrations of asteroid orbits for spans of time up to a few millions years. Later we shall discuss in detail how the limited accuracy of the theory used to derive proper elements can affect the identification of families.

The dataset includes 4100 asteroids, namely all the 4265 asteroids numbered up to 1989, minus the Hildas, the Trojans, and the Earth-approaching objects. For each asteroid, we used the proper semimajor axis (a'), the proper eccentricity (e'), and the sine of the proper inclination ($\sin i'$). Figures 1 and 2 show the distributions in the (a', $\sin i'$,) and (a', e') planes. To make a better comparison with an inhomogeneous background (see later), and also to speed up the classification process, the whole dataset was divided in subsets, to which the clustering method was applied separately. This was achieved by dividing the asteroid belt in eight zones, mostly delimited by the main mean motion commensurabilities with Jupiter (as it is *a priori* unlikely to find families across a low-order resonance, due either to depletion effects, or to poor accuracy of the proper elements); both for the resonance-defined boundaries and for the only other boundary, that at 2.3 AU between zone 2 and zone 3, we anyway checked *a posteriori* that no overlapping family exists, with just one exception (due to a few members of Eos' family, as we shall discuss later). Table I shows the boundaries in semimajor axis of the zones, the corresponding resonances, and, for each zone, the total population of bodies and the least-numbered asteroid residing in it.

The single-linkage hierarchical clustering method we have used in the present work can be summarized as follows: we have first found the distances of all asteroids (in the proper elements space) to nearby neighbors, and then have looked for clusters such that each cluster member is less than some limiting distance from at least one other cluster member. More in detail, given N objects with known coordinates in the three-dimensional space of the proper elements, we have defined a metric assigning a distance function δv (with the dimension of a velocity, see later) between any pair of objects. Then we carried out four steps: (1) identify the two closest objects, labeled (say) j and k; (2) agglomerate them, i.e., replace j and k with a new object (in fact, a grouping)

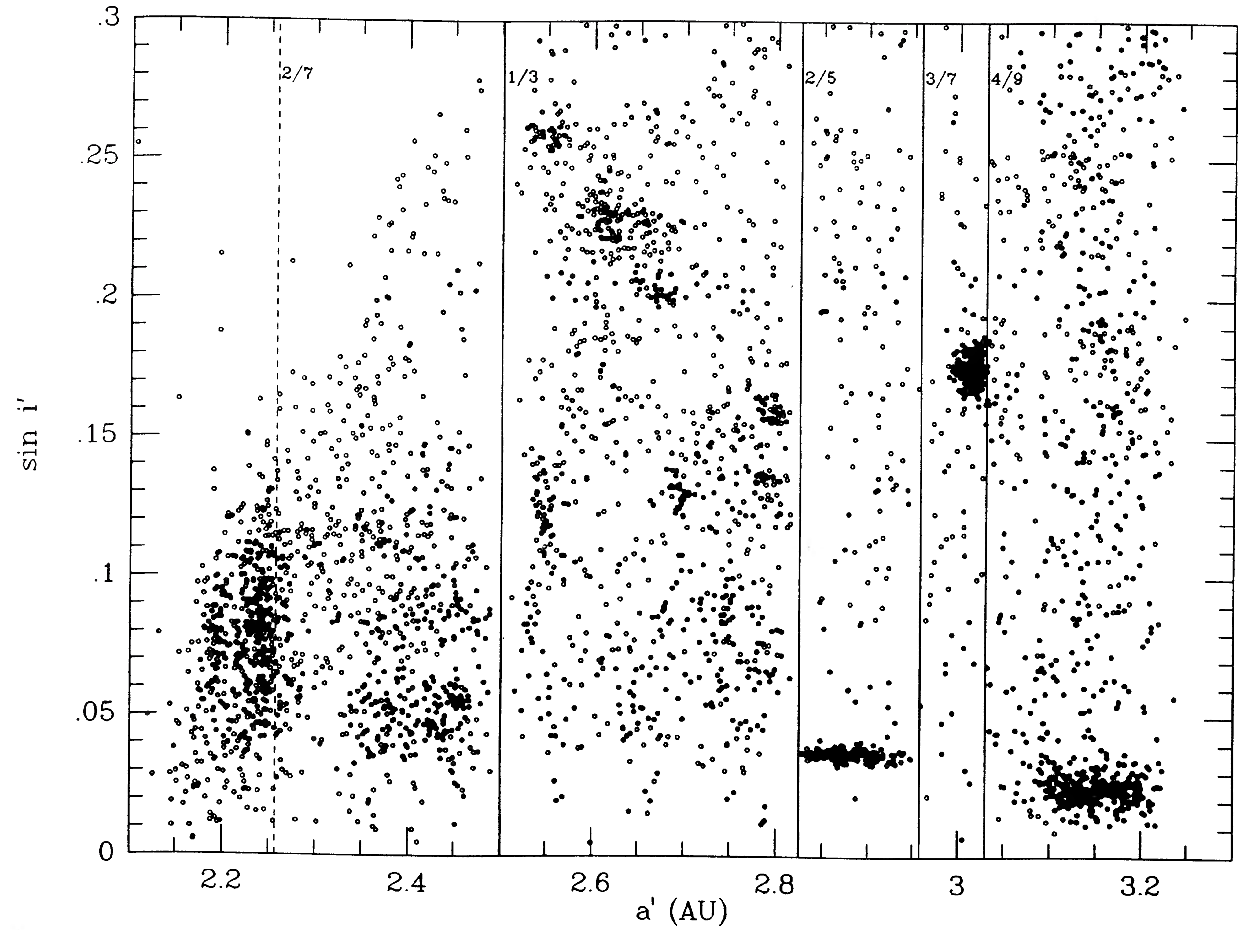

FIG. 1. Distribution in the (a', sin i') plane of the 4100 main-belt asteroids used for family classification.

$j+k$; (3) update all the distances, according to the following rule: the distance $\delta v(j+k,i)$ between $j+k$ and any other object i is defined as the minimum between $\delta v(j,i)$ and $\delta v(k,i)$; (4) as long as at least two objects survive the agglomeration process, return to step (1). By means of this algorithm, we have been able to obtain a *dendrogram* connecting all the objects, which contains the whole information needed for the identification of families: for any threshold value $\delta v'$ of the distance function, it is easy to list all the existing groupings (families) of objects with mutual distances $< \delta v'$. Notice that the rule introduced in step (3) for updating the distances includes in our definition of "grouping" (or "family") not only real three-dimensional clusters, denser than surrounding background, but also unusual "filaments" and two-dimensional "disks." For a more detailed discussion of hierarchical clustering and other multivariate data analysis techniques of current use in astronomy, we refer to Murtagh and Heck (1987).

How is it possible to define a sensible metric function in the three-dimensional space of the elements? If we start from the idea (dating back to Hirayama) that families were generated by the explosive breakup of parent asteroids, for any pair of fragments from the same parent we could use Gauss' equations [see Brouwer and Clemence (1961), p. 299] to connect the δ (elements) with the components of the post-breakup ejection velocity in Gauss' orbit-related reference frame. Following Brouwer (1951) and Zappalà *et al.* (1984), and neglecting terms proportional to the eccentricity, we obtain

$$\begin{aligned} 2\delta v_1/na &= \delta a/a, \\ \delta v_2 \sin(f)/na + 2\delta v_1 \cos(f)/na &= \delta e \qquad (1) \\ \delta v_3 \cos(\omega+f)/na &= \delta i, \end{aligned}$$

where a, e, i, ω, and f are the osculating elements of the parent body (semimajor axis, eccentricity, inclination, argument of perihelion, true anomaly; n is the mean motion, and na is the circular velocity) at the instant of breakup, while δv_1, δv_2, and δv_3 are the components of the ejection velocity in the along-track, radial, and out-of-plane directions, respectively. Were the angles f and $(\omega+f)$ known for any given family we could compute the velocity components by taking the differences in the proper elements $\delta a'$, $\delta e'$, $\delta i'$ instead of the osculating ones (for a proof that this substitution is consistent, at least in the frame of the linear secular perturbation theory, see Brouwer 1951; higher-order and -degree corrections to the proper elements are in general small

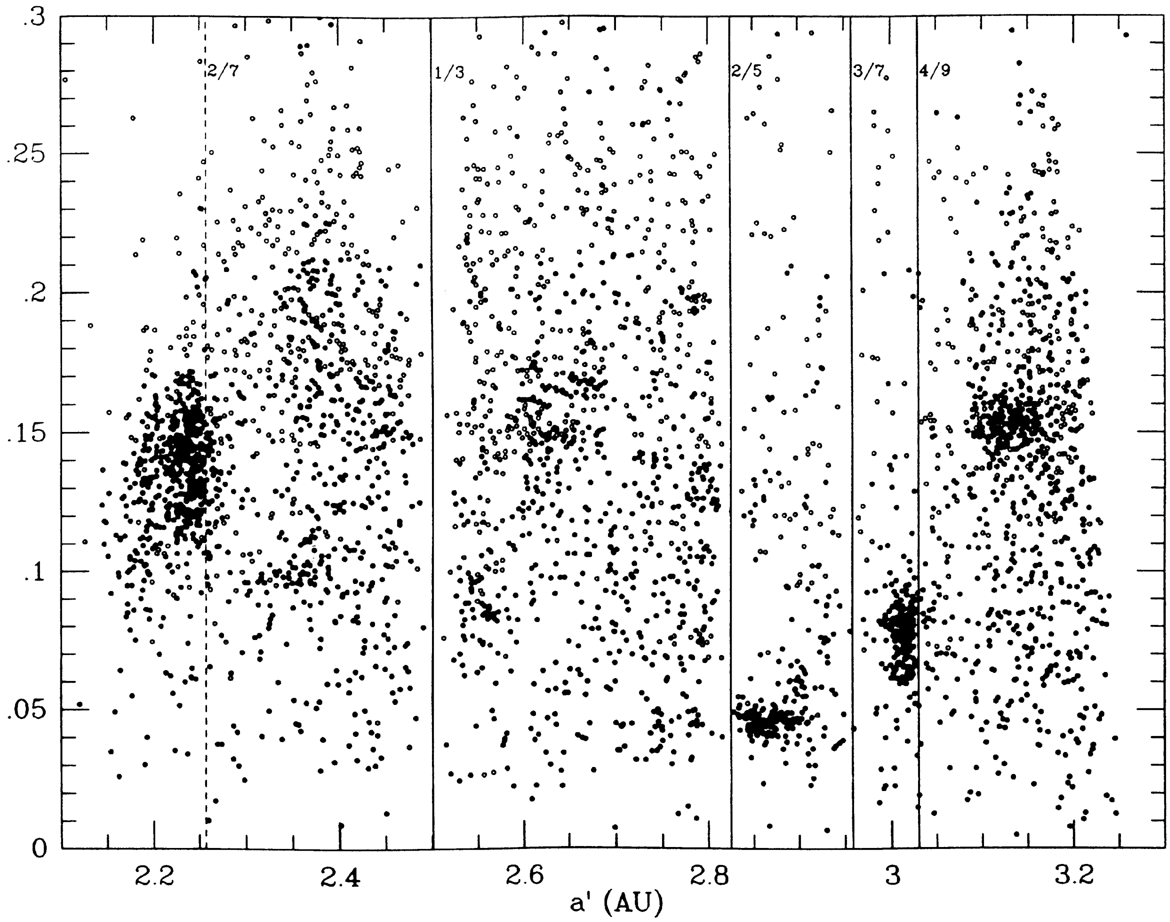

FIG. 2. Distribution in the (a', e') plane of the 4100 main-belt asteroids used for family classification.

enough to make us confident that this procedure stays meaningful); a' is the average of the proper semimajor axes of the two objects whose δv is being computed. But even with f and $(\omega + f)$ unknown, Eqs. (1) show that if we choose a distance function in the proper elements of space of the form

$$\delta v = na\sqrt{[k_1(\delta a'/a')^2 + k_2(\delta e')^2 + k_3(\delta i')^2]}, \quad (2)$$

with coefficients k_1, k_2, and k_3 of order unity, our metric will give an order-of-magnitude estimate of the velocity increment causing separation of the two orbits. In order to choose the coefficients, we can note that by squaring Eq. (1), averaging over f and $(\omega + f)$, and then substituting the δ (elements) in Eq. (2), we get

$$\delta v = \sqrt{x\langle \delta v_1^2 \rangle + y\langle \delta v_2^2 \rangle + z\langle \delta v_3^2 \rangle}, \quad (3)$$

with

$$x = (4k_1 + 2k_2), \quad y = k_2/2, \quad z = k_3/2. \quad (4)$$

Unfortunately this shows that, using this averaging method, we cannot obtain $x = y = z = 1$ with $k_1, k_2, k_3 > 0$ (an obvious requirement for a usable distance function). Instead, we have chosen as *standard* metric coefficients $k_1 = 5/4$, $k_2 = 2$, $k_3 = 2$, which yields $x = 9$, $y = 1$, $z = 1$ and thus give a higher weight (by a factor 3) to the δv_1 component than to δv_2 and δv_3. This is reasonable, as according to Eq. (1) δv_1 does not depend on unknown angles or on $\delta e'$ and $\delta i'$, and it is well known that proper eccentricities and inclinations can be derived to a poorer accuracy than proper semimajor axes [see Milani and Knežević (1990), Sec. 4]. Notice, however, that for "isotropic" clusters ($\langle \delta v_1^2 \rangle = \langle \delta v_2^2 \rangle = \langle \delta v_3^2 \rangle$), our metric function would overestimate the real separation velocities by a factor $\sqrt{(11/3)} = 1.915$. As our choice of the coefficients is somewhat arbitrary, we have verified that as long as the coefficients keep order of

TABLE I. Boundaries and number of asteroids of each zone.

Zone	Boundaries: s.major axes	Boundaries: resonances	Number of asteroids	Least-numbered asteroid of the zone
1	— -2.065	— - 1/4	68	433
2	2.065–2.3	1/4 - —	695	8
3	2.3 -2.501	— - 1/3	689	4
4	2.501–2.825	1/3 - 2/5	1094	1
5	2.825–2.958	2/5 - 3/7	290	16
6	2.958–3.030	3/7 - 4/9	295	35
7	3.030–3.278	4/9 - 1/2	893	10
8	3.278- —	1/2 - —	76	65

unity, the main (and most "robust," see later for definition of "robust") resulting groupings undergo no or small changes. We shall describe in more detail later the results of the clustering search when an alternative choice of the metric coefficients is made.

The procedure outlined above has been implemented in a program which generates the dendrogram in every zone of the main asteroid belt. The output consists in a list of the existing groupings of objects having mutual δv distances less than any given threshold value $\delta v'$; the latter parameter is then varied in a wide range, typically from 300 down to 40 m/s. The results can be shown in a synthetic way by means of *stalactite* diagrams, with the number of objects present in the various groupings plotted versus $\delta v'$ (at discrete steps of 20 m/s), so that deeper stalactites indicate higher-density groupings that survive down to lower values of the threshold distance.

The next issue is: how can we be sure that a given stalactite corresponds to a nonrandom grouping? Or, better, at which level should we "cut" our stalactites to discriminate real families from random clusters? In order to solve this crucial problem, for every zone of the belt we have generated a quasirandom population in the (a', e', sin i') space, with the same total number of objects as in the set of real asteroids (see Table I). For each proper element, our quasirandom populations were chosen in such a way to match the large-scale distribution of the corresponding zone of the belt. This means that the abundance of bodies was exactly the same as in the real population when compared on a set of discrete bins, separately for each element; we have taken for a', e', and sin i' 10, 10, 10 bins in zones 2, 3, 4, and 5, 5, 5, bins in zones 5, 6, 7, respectively (the last three zones have smaller populations, while zones 1 and 8 are easily seen to be uninteresting for family classification purposes; e' and sin i' always have been assumed to range from 0 to 0.5, and the ranges of a' are those shown in Table I). Figure 3 shows for instance the distribution in the (a', sin i') plane for the real asteroid population in zone 4 and for the corresponding quasirandom population. Of course, in the zones where families account for a significant fraction of the total number of asteroids, our quasirandom populations cannot but keep some track of them; this kind of bias is unavoidable, since we have no mean to know *a priori* where are the real families. However, the importance of this effect is strongly diminished since no correlation between different elements was introduced in the fictitious populations, while such correlations are obviously

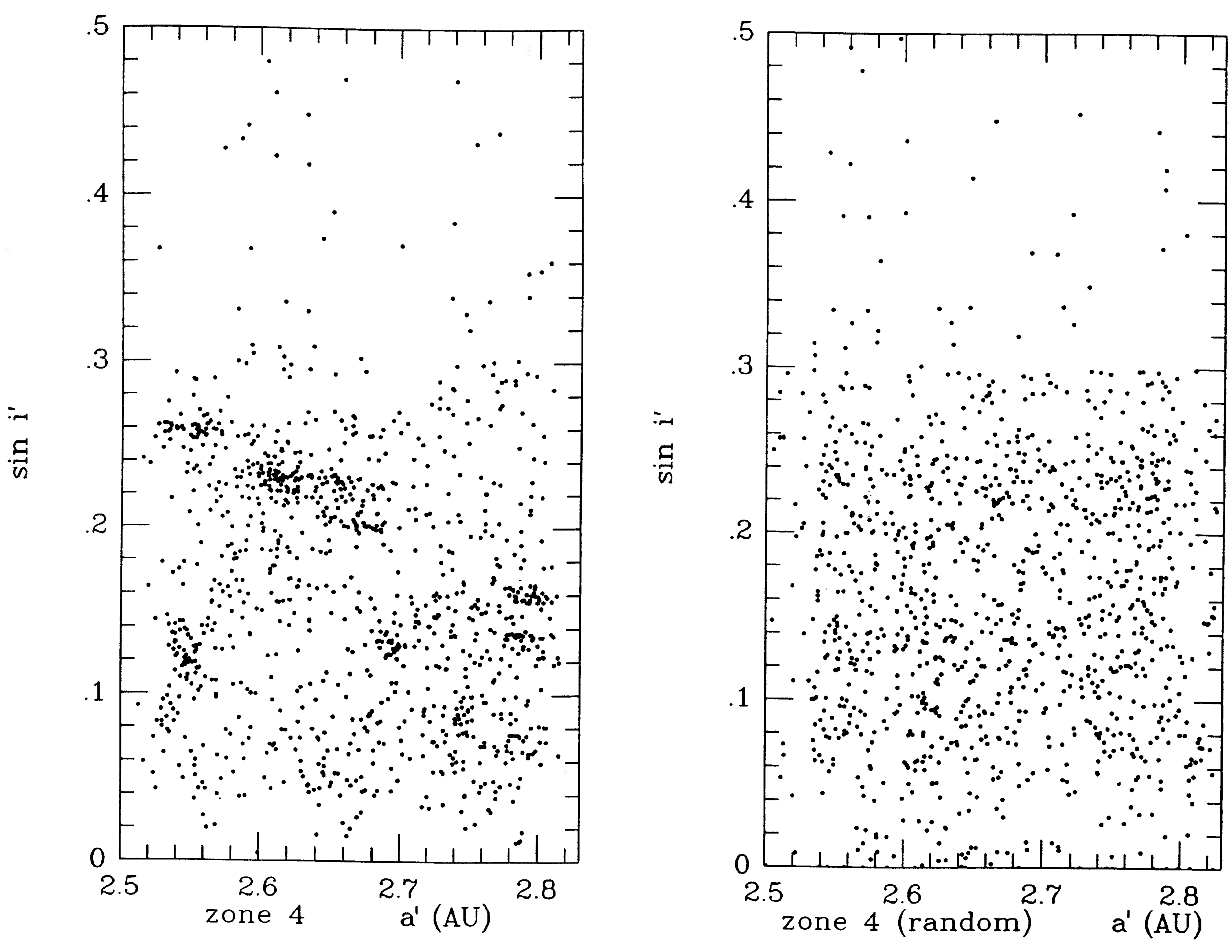

FIG. 3. Distribution in the (a', sin i') plane of the real asteroid population in zone 4 of the main belt (left), and of the corresponding quasirandom population (right). The latter is derived from the real population with the constraint that the histograms giving the abundance of bodies vs each proper element, using ten bins per element within the zone, have to coincide.

present in the real families. Then we could apply to the fictitious populations the same hierarchical clustering procedure used for the real populations, and derive the corresponding stalactite diagrams. These diagrams could then be compared with those of the real populations, and the following "rejection criterion" was chosen: a grouping corresponds to a real family only provided its stalactite either is deeper (by at least one 20 m/s step) than the deepest one found in the quasirandom population of the same zone; or it has the same depth, but more than twice the number of member objects in the deepest "layer," when compared to the deepest quasirandom stalactite. The intuitive meaning of this criterion is obvious: we look for groupings being either "denser" than the densest random clusters, or of comparable density but more populous. The members of the real families, defined as above, are just the objects belonging to them at the layer corresponding to the deepest quasirandom stalactite (*quasirandom layer*). Finally, we have excluded all the "families" having less than five members.

This criterion allowed us to introduce a quantitative parameter P_s to assess the statistical significance of each family. This *significance parameter* is derived: (1) by counting the objects belonging to the family, each of them with a weight given by the ratio between the value of $\delta v'$ corresponding to the quasirandom level defined above and the value of $\delta v'$ corresponding to the deepest level of the real-family stalactite where the object appears (this implies that family members belonging to deeper layers, that is to denser "cores," have a larger weight); (2) dividing the number obtained in the previous step by the number of objects appearing in the deepest layer of the deepest quasirandom stalactite (if two or more quasirandom stalactites have the same depth, we take the thicker one). With this definition, when the statistical significance of the family is marginal, P_s is of order unity; on the other hand, $P_s \gg 1$ (actually, of the order of 100) for the most dense and populous families.

Then, we have tried to assess the "robustness" of the families identified by hierarchical clustering with respect to changes in the proper elements, caused by the finite accuracy of the secular perturbation theory used to derive them. Carpino *et al.* (1986) have obtained quantitative estimates of this accuracy in the case of the linear theory, by integrating numerically a number of (real and fictitious) asteroid orbits and then deriving proper elements at different times. A similar method was used by Milani and Knežević (1990) for their higher-order and -degree theory. The results of these tests can be approximately summarized by expressing the typical size N of the changes in the (e', $\sin i'$) elements (the proper semimajor axis is also not constant, due to imperfect elimination of short-periodic effects, but its variations are in general smaller) via the following formula:

$$N = \sqrt{[K^2 + A^2/(g-g_6)^2 + fB^2/d^2]}. \quad (5)$$

Here K, A, and B are constants; the values $K = 0.002$, $A = 0.047$, and $B = 0.0005$ give a good fit to the observed variations of the elements in the numerical tests (we also checked that a somewhat different choice of these parameters, i.e., $A = 0.035$ and $B = 0.005$, would not affect in a significant way our main conclusions). The first term in Eq. (5) simply reflects the intrinsic limits of the theory, averaged over the whole asteroid belt; the second term is a function of the proximity of the asteroid's orbit to the strong ν_6 secular resonance (g and g_6 are the secular rates of the longitude of perihelion for the asteroid and Saturn, respectively, measured in arcsec/yr; they are typically of the order of 30–100); and the third term depends on the proximity to other, weaker secular resonances [$f = 1$ whenever the modulus of a secular small divisor d, that is a linear combination of the secular rates of the asteroid's, Jupiter's and Saturn's perihelia and nodes, is less than 0.5 arcsec/yr, otherwise $f = 0$; 28 such small divisors, containing up to four frequencies, are actually accounted for by the theory; see Milani and Knežević (1990), Table 3.1]. For the members of each family identified with the procedure described earlier, N was then multiplied times a random number in the interval (0,1) and added (with a random sign) to the values of e' and $\sin i'$ of the asteriod. Then, the whole family-searching procedure was repeated by using the new set of "noisy" proper elements, and new stalactites were thus obtained. Finally, for each family identified with the original elements, we defined a *robustness parameter* P_r as follows: for each layer of the family stalactite, we considered the intersection between the original members and the "new" ones (derived from the "noisy" elements); the significance parameter P_s, as defined earlier, was computed for these "intersection stalactites"; P_r is then the ratio between the original value of P_s for the family and that for the corresponding intersection stalactite. Hence, P_r is lower for families that are less robust with respect to changes in the elements; in fact, the limit cases are $P_r = 1$ when the family keeps all its members after application of the "noise," and $P_r = 0$ when the family disappears (including the case when less than five members remain together).

As a final check on the reliability of families, we have repeated the whole clustering search—including the derivation of quasirandom levels—with a different choice of the metric function (2). Our alternative coefficient set has been: $k_1 = 1/2$, $k_2 = 3/4$, $k_3 = 4$. It keeps the same sum $k_1 + k_2 + k_3 = 21/4$ as the *standard* metric; apart from a factor 2, it is very close to the metric $k_1 = 1/4$, $k_2 = 2/5$, $k_3 = 2$) which can be obtained from Gauss' equations (1) averaged over the angles when one assumes *a priori* $\langle \delta v_1^2 \rangle = \langle \delta v_2^2 \rangle$ [Williams (1990), private communication]; it corresponds to $x = 7/2$, $y = 3/8$, $z = 2$ [see Eq. (4)], so that it strongly increases the weight of δv_3 (which depends on i') with respect to both δv_1 (a') and δv_2 (a' and e'); and for "isotropic" families, it overestimates the separation velocities by a factor $\sqrt{(47/24)} = 1.399$. In our opinion, this choice is different enough from the *standard* one to provide a meaningful test for the robustness of the families with respect to variations in the metric coefficients. Actually, for each family found with the *standard* metric, we have derived a second robustness parameter P_r', also ranging from 0 to 1, defined as the ratio between the number of asteroids (provided it was $\geqslant 5$) belonging to both the original family and the same family, but derived by using the alternative metric, and the number of members of the original family. In general, the alternative metric gave family memberships only marginally different from the original ones (see Tables IV to IX, where the family members "lost" with the alternative metric are listed in brackets); actually, the corresponding discrepancies are not larger than the differences between the original families taken at two consecutive $\delta v'$ steps.

III. THE NEW FAMILIES AND THEIR PROPERTIES

Figures 4 and 5, to be compared with Figs. 1 and 2, show the locations in the proper element space of the families iden-

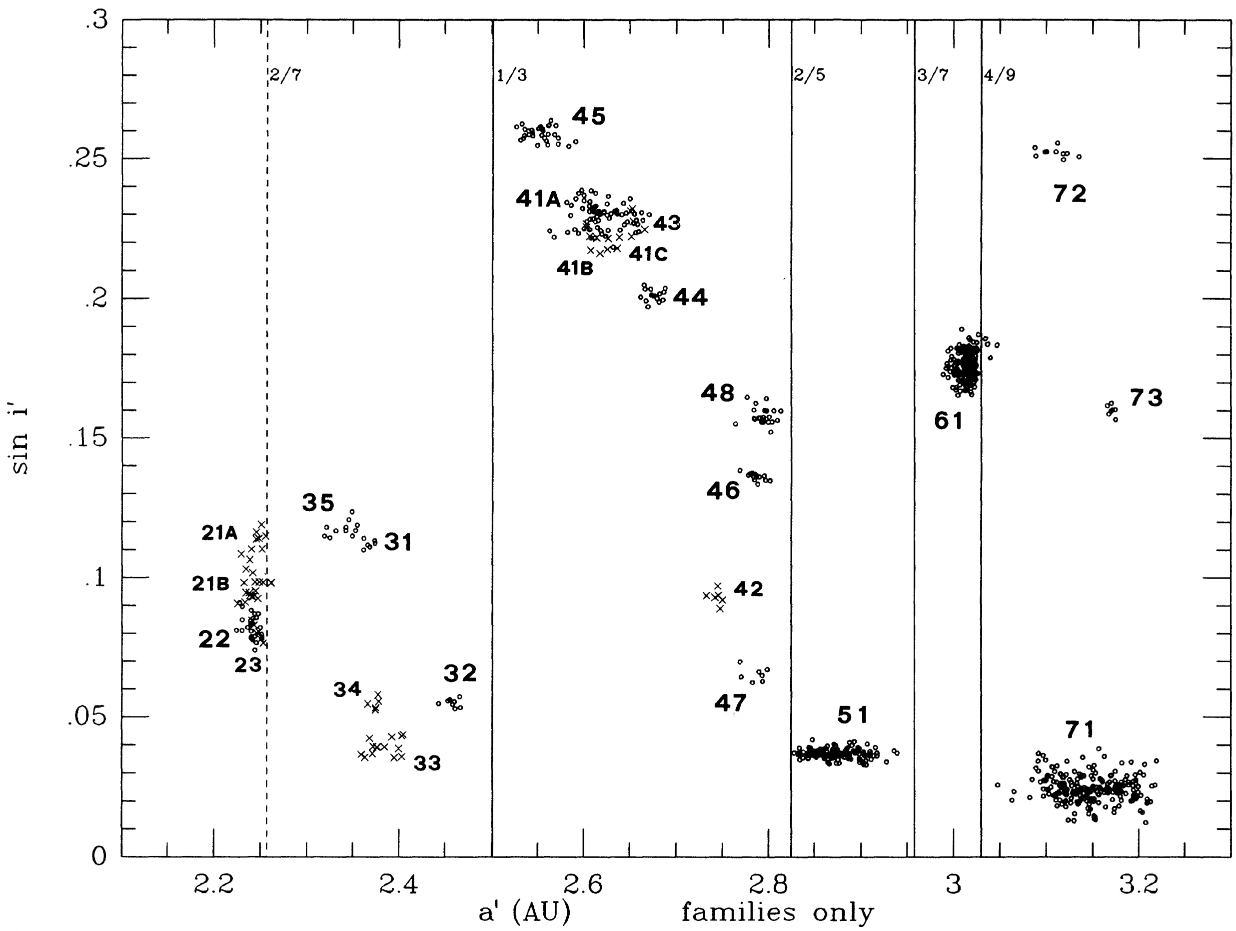

FIG. 4. Same as in Fig. 1, but plotting only the members of the 21 families identified by hierarchical clustering. For each family, the label number given in Table II is indicated.

tified as described above. These families are labeled according to the zone of the asteroid belt they belong to (first digit) and starting from families with lower-numbered member objects (second digit). Families 21 and 41 are separated in two and three subfamilies, respectively; they split only at the quasirandom level defined in Sec. II, and the addition of just one member could easily make a "bridge" and unify them again. In Table II we give, respectively, the family label number, the name of the least-numbered member asteroid (round brackets), the total number of members, the same but using the "noisy" proper elements (square brackets), the value of $\delta v'$ corresponding to the quasirandom layer, the significance parameter P_s and the robustness parameters P_r and P_r'. Table III lists, for each family, the values of the proper elements of the least-numbered object, together with the ranges of variation in the family. Figures 6–11 show the stalactite diagrams for zones 2 to 7, starting three layers above the quasirandom level of each zone; in each layer—corresponding to a 20 m/s step in $\delta v'$—the least-numbered asteroids present in the groupings are indicated. Tables IV to IX give the memberships of all the families at different $\delta v'$ levels, starting from the quasirandom layer. This information is potentially important for future physical studies, as it allows discrimination between objects located in the densest (and statistically most significant) concentrations, and those that lie at the periphery of their families, and therefore would be lost from them if a stricter rejection criterion were to be adopted. Moreover, the objects that are lost from the family when the alternative metric coefficients are used are identified by brackets.

In the following we offer some short and preliminary comments on these results. More detailed analyses will be the subject of forthcoming papers. We identified a total of 21 families, 15 and 20 of which have $P_r > 0$ and $P_r' > 0$, respectively; of these, 8 have $P_s > 5$. This result is closer to that of Carusi and Massaro (1978; 13 families), who also used an automatic family-searching procedure, than to those of Kozai (1979; 72 families) and Williams (1979; 104 families). In our opinion, the most likely reason is that the latter authors have been too liberal in giving family status to clusters

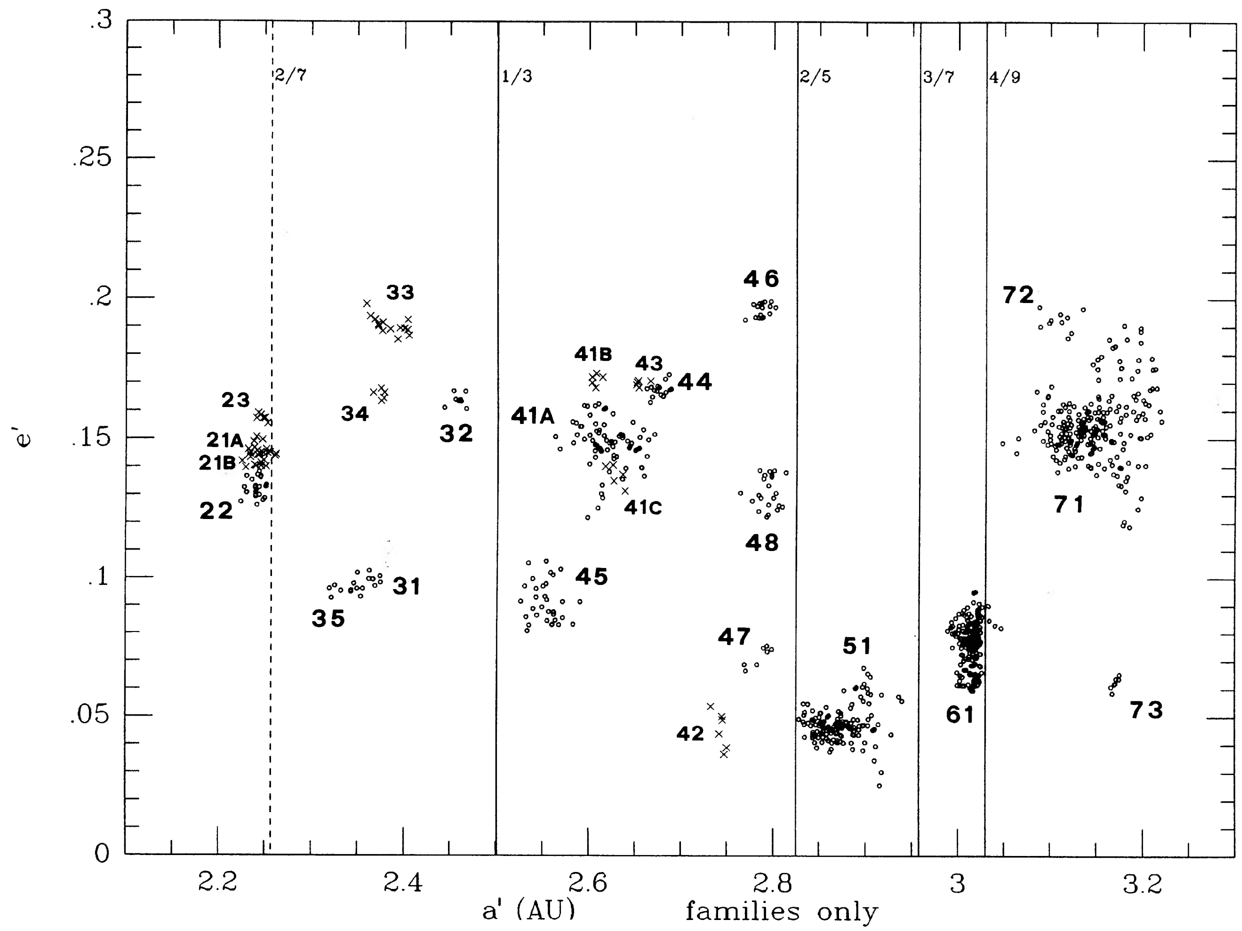

FIG. 5. Same as in Fig. 2, but plotting only the members of the 21 families identified by hierarchical clustering.

TABLE II. Asteroid families and their reliability parameters.

Family		Members	q.r. $\delta v'$ (m/s)	P_s	P_r	P_r'
21A	(525 Adelaide)	9[—]	120	1.4	0	0
21B	(883 Matterania)	17[—]		3.0	0	0.71
22	(763 Cupido)	24[10]		3.7	0.43	0.42
23	(1047 Geisha)	6[—]		1.0	0	0
31	(4 Vesta)	7[7]	140	1.8	0.72	1
32	(650 Amalasuntha)	8[7]		1.8	0.89	1
33	(878 Mildred)	14[—]		2.7	0	0.57
34	(1378 Leonce)	5[—]		1.0	0	1
35	(1933 Tinchen)	10[6]		1.9	0.63	1
41A	(15 Eunomia)	71[51]	160	19.8	0.63	0.93
41B	(1392 Pierre)	5[—]		1.1	0	0
41C	(2649 Oongaq)	5[—]		1.1	0	1
42	(110 Lydia)	6[—]		1.3	0	1
43	(141 Lumen)	5[—]		1.1	0	1
44	(145 Adeona)	15[14]		6.0	0.93	1
45	(170 Maria)	32[21]		8.4	0.65	1
46	(668 Dora)	16[16]		7.4	0.85	1
47	(847 Agnia)	7[6]		1.6	0.87	0.86
48	(1272 Gefion)	22[20]		6.4	0.87	0.95
51	(158 Koronis)	137[137]	160	78.0	0.91	1
61	(221 Eos)	202[198]	120	91.2	0.94	0.98
71	(24 Themis)	228[221]	160	73.0	0.97	0.95
72	(137 Meliboea)	10[9]		2.3	0.91	1
73	(490 Veritas)	7[7]		2.8	0.96	1

TABLE III. Proper elements of the least-numbered object and boundaries limits of each family.

Family Number	Least-Numbered Object No	a′	e′	sin i′	Family Ranges Δa′	Δe′	Δsin i′
21A	525	2.24522	0.1413	0.1137	2.22924–2.25560	0.1397–0.1461	0.1063–0.1189
21B	883	2.23816	0.1490	0.0945	2.22522–2.26172	0.1419–0.1506	0.0907–0.1030
22	763	2.24070	0.1317	0.0836	2.22401–2.25125	0.1263–0.1381	0.0739–0.0910
23	1047	2.24097	0.1572	0.0846	2.24097–2.25312	0.1555–0.1591	0.0765–0.0846
31	4	2.36155	0.0996	0.1099	2.34966–2.37405	0.0972–0.1027	0.1099–0.1148
32	650	2.45817	0.1636	0.0545	2.44336–2.46716	0.1606–0.1670	0.0529–0.0573
33	878	2.36328	0.1938	0.0356	2.35937–2.40510	0.1854–0.1981	0.0355–0.0438
34	1378	2.37484	0.1634	0.0533	2.36630–2.37830	0.1634–0.1679	0.0526–0.0580
35	1933	2.35296	0.0933	0.1169	2.31948–2.35506	0.0928–0.0981	0.1143–0.1236
41A	15	2.64375	0.1474	0.2263	2.56313–2.67062	0.1219–0.1625	0.2183–0.2385
41B	1392	2.60780	0.1733	0.2172	2.60294–2.61428	0.1682–0.1733	0.2172–0.2266
41C	2649	2.62671	0.1350	0.2214	2.61781–2.63865	0.1314–0.1407	0.2162–0.2219
42	110	2.73294	0.0537	0.0936	2.73294–2.75001	0.0366–0.0537	0.0889–0.0970
43	141	2.66590	0.1706	0.2246	2.65057–2.66590	0.1683–0.1707	0.2222–0.2321
44	145	2.67273	0.1670	0.2034	2.66196–2.68833	0.1630–0.1730	0.1971–0.2048
45	170	2.55378	0.0979	0.2608	2.52646–2.59102	0.0808–0.1061	0.2544–0.2637
46	668	2.79678	0.1991	0.1349	2.77772–2.80161	0.1924–0.1991	0.1334–0.1384
47	847	2.78282	0.0687	0.0623	2.76897–2.79857	0.0665–0.0754	0.0623–0.0699
48	1272	2.78372	0.1300	0.1571	2.76392–2.81335	0.1221–0.1388	0.1521–0.1647
51	158	2.86881	0.0457	0.0372	2.82828–2.93942	0.0255–0.0676	0.0329–0.0419
61	221	3.01251	0.0801	0.1731	2.98859–3.04638	0.0594–0.0951	0.1654–0.1891
71	24	3.13416	0.1521	0.0189	3.04761–3.21982	0.1188–0.1906	0.0123–0.0387
72	137	3.11855	0.1861	0.2496	3.08775–3.13513	0.1861–0.1972	0.2496–0.2556
73	490	3.17492	0.0653	0.1568	3.16591–3.17492	0.0586–0.0653	0.1568–0.1626

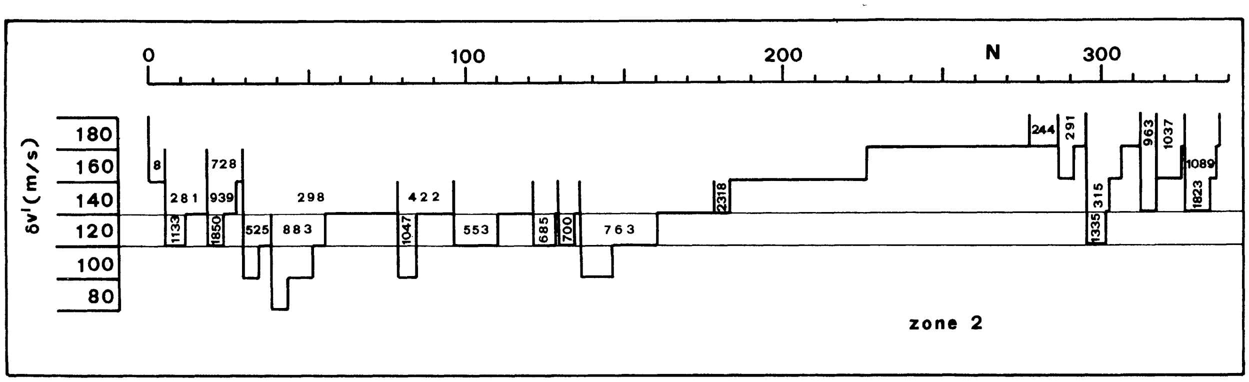

FIG. 6. *Stalactite diagram* of zone 2. The abundance of objects present in different groupings separated by hierarchical clustering is plotted versus the threshold $\delta v'$ distance used for "cutting" the dendrogram of the zone. The starting (maximum) value of $\delta v'$ has been chosen at three layers (each corresponding to a 20 m/s step) above the quasirandom level of each zone; the latter is defined as the level reached by the deepest stalactite found in the quasirandom population of the zone. $\delta v'$ labels refer to the upper edge of the corresponding layers. In each layer, the least-numbered asteroids present in the different groupings are indicated.

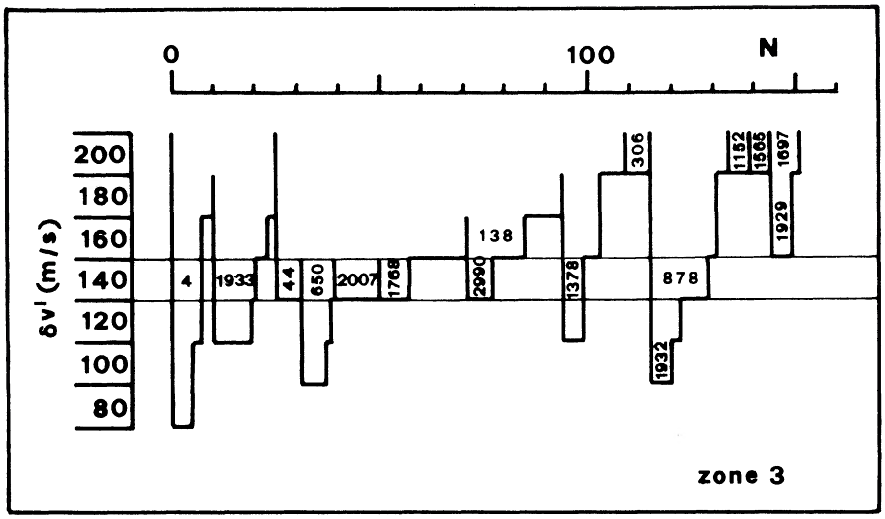

FIG. 7. Same as in Fig. 6 except for zone 3.

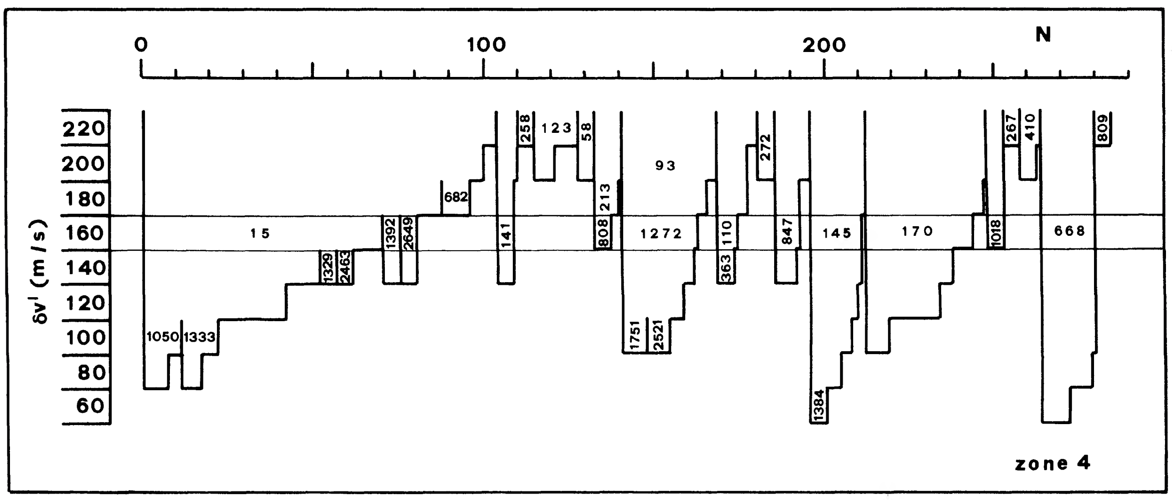

FIG. 8. Same as in Fig. 6 except for zone 4.

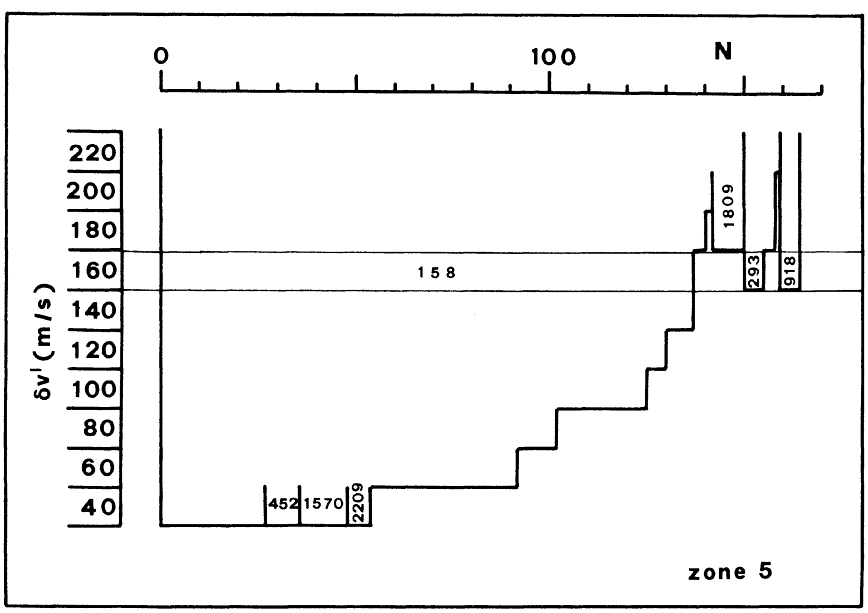

FIG. 9. Same as in Fig. 6 except for zone 5.

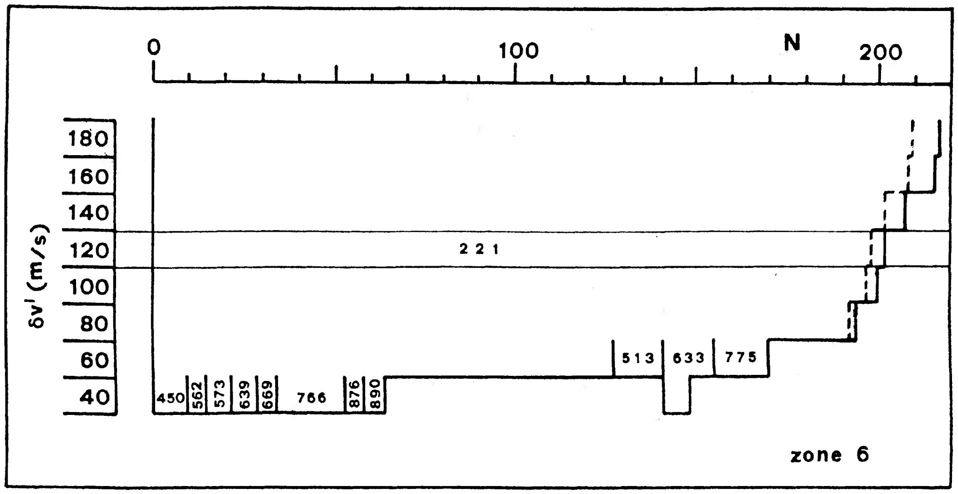

FIG. 10. Same as in Fig. 6 except for zone 6. The figure shows that Eos' family is somewhat enlarged when the clustering procedure is applied to zones 6 and 7 together.

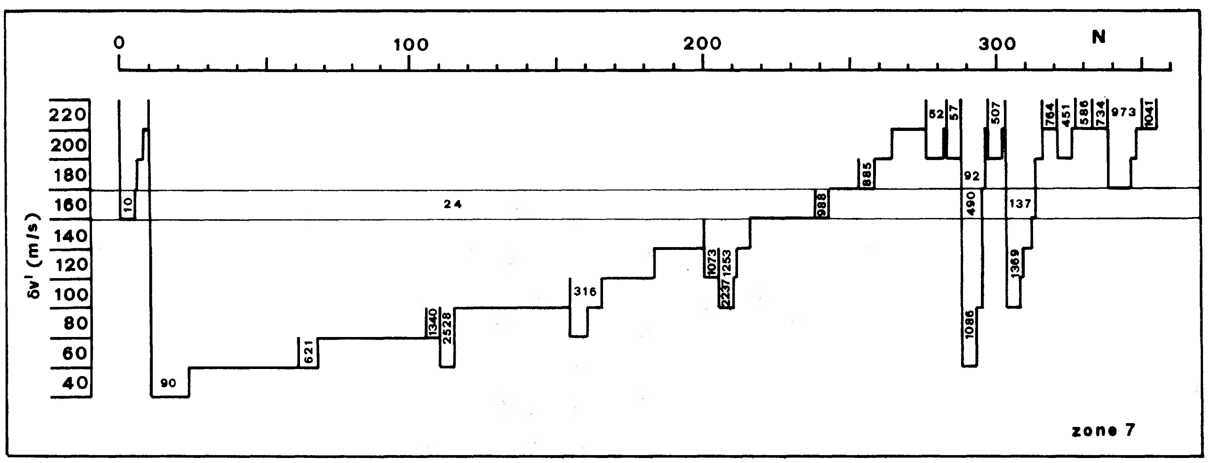

FIG. 11. Same as in Fig. 6 except for zone 7.

TABLE IV. Memberships of the families in zone 2 at different $\delta v'$ levels.

Family = 21A (525 Adelaide)	members = 9
100	525 (1634)(1829) 2171 2243
120	(2500)(2545)(2942)(3413)

Family = 21B (883 Matterania)	members = 17
80	883 1619 2768 2897 3072
100	1365 1523 1696 2130 2512 (2784) 2845 4148
120	(2094)(3260)(3340)(3771)

Family = 22 (763 Cupido)	members = 24
100	(763) 1663 (1831) 2119 2287 (2399) 2647 3031 4033 4070
120	(915) 1056 (1446)(1590) 1810 (1857)(2350)(2438)(2575)(3180) (3306)(3459)(3764) 3825

Family = 23 (1047 Geisha)	members = 6
100	(1047)(1396)(2510)(3067)(3411)(3991)
120	——

TABLE V. Same as Table IV except for zone 3.

Family = 31 (4 Vesta)	members = 7
80	4 1906 1979 2508 4038
100	—
120	3494 4147
140	—

Family = 32 (650 Amalasuntha)	members = 8
100	650 1740 2139 2509 2923 3541
120	3130
140	750

Family = 33 (878 Mildred)	members = 14
100	1932 2607 2818 3384 3408
120	878 (3005)
140	(2210) 3530 (3857)(3891) 4027 (4227)(4251)

Family = 34 (1378 Leonce)	members = 5
120	1378 2276 3112 3247 3652
140	—

Family = 35 (1933 Tinchen)	members = 10
120	1933 2024 2590 3155 3376 3477 3498 3703 3720
140	3968

of objects observed in the proper element space, that probably did not overcome the quasirandom levels of their respective zones.

No family was found in zone 1, inside the 4/1 mean motion commensurability with Jupiter. The population in this zone is scarce and sparse, with large $\delta v'$ distances; many objects have planet-crossing orbits and/or high inclinations, and in both these circumstances the secular perturbation theory performs poorly (or does not work at all) in determining proper elements. Anyway, resonances and close encounters with planets have probably stirred up the orbits and caused a strong depletion. The remaining objects, mostly belonging to the high-inclination Hungaria group, are distributed in a broad zone and are not recognized as a family.

The situation is very different in zone 2, which contains the so-called Flora region. As it can be seen from Figs. 1 and 2, the main feature of this zone is a broad concentration of objects, displaying a complex structure and bounded in part by the ν_6 secular resonance. The high abundance of asteroids is partially (but probably not entirely) due to observational selection effects, since smaller distanced to Earth and higher average albedos allowed here the discovery of objects down to limiting sizes much smaller than in the outer belt. Some authors (e.g., Carusi and Massaro 1978; Kozai, 1979) identified a single, large Flora family; others (Brouwer 1951; Williams 1979) separated instead several families (or subfamilies) with somewhat arbitrary boundaries. We have actually found three families (one of them split in two parts), having 6 to 26 members; but their statistical significance is low (the maximum value of P_s is 3.7), they are sensitive to changes in the metric coefficients and P_r does not vanish only for the most populous family (No. 22); even this family, however, loses more than half of its members when the noise is applied to the elements. According to Eq. (5), the noise is strong in this region, since proper eccentricities and inclinations are "disturbed" by the ν_6 resonance and have typical variations of a few hundredths. It is clear that the interplay of high background density and strong noise makes the identification of families hard: the former effect results in more and denser random clusters, causing the quasirandom level to get closer to that of real families (which, in turn, get "invaded" by chance interlopers); and as the noise approaches the average distance of real families to neighboring nonfamily objects, families become mixed up in the background. Hence, we cannot exclude that a large fraction of the objects observed in this region are actually genetically related and would be recognized as one or more families, were better proper elements available for statistical analyses.

In zone 3, we have found five small families, with P_s not much larger than unity. Most of them, however, are fairly robust. The best one, family No. 32 (with eight members), separates above the quasirandom level (see Fig. 7) from a large grouping whose major body is 44 Nysa. A Nysa or Nysa–Hertha family was identified in most previous classifications; as remarked by Bell (1989) and Chapman *et al.* (1989), this family was compositionally puzzling, since it included a large E-type object (Nysa), probably made of igneous rocks, an M-type object (Hertha), the likely metal-rich core of a differentiated parent body, together several small bodies of the rare and relatively primitive F type. Bell actually suggested that both Nysa and Hertha may be interlopers in a homogeneous F-type family; our results support this view, though our family No. 32 contains only a fraction of all the F-type objects present in zone 3. Also remarkable is the finding that 4 Vesta is the major member of another small family (No. 31), with all the remaining six members having small sizes. This may have important consequences, as the assumption that Vesta's basaltic crust was not completely shattered by impacts can provide a tight constraint on the past collisional flux in the asteroid belt (Davis *et al.* 1985). On the other hand, Vesta's surface is heterogeneous, possibly due to big impact craters or basins, and both a meteorite type (the *eucrites*) and a few small Amor asteroids provide good spectral matches to Vesta (and to no other

TABLE VI. Same as Table IV except for zone 4.

Family = 41A (15 Eunomia)				members = 71
80	1050 1531 2381 3080 3707 3961 4254	1333 2302 2869 3662 3977 4164		
100	2786 2810 3305 3758	1499 2796 3252 3296 3965		
120	15 473 630 1193 1425 1431 1503 1886 2304 2660 2672 2743 2988 3387 3488 3539 3909 3934 4085 4133			
140	812 1495 1926 2181 2537 2685 2915 3017 3974 (4191)		1329 2384 3492 3729 3892	2463 2822 2993 3041 3569
160	(839) 1106 2005 (2079) 2337 (2490) 3182 3779 (4190)			

Family = 41B (1392 Pierre)	members = 5
140	(1392) 1775 3524 3767 3816
160	——

Family = 41C (2649 Oongaq)	members = 5
140	2649 2842 2970 3286 3894
160	——

Family = 42 (110 Lydia)	members = 6
140	363 2560 3124 3450 3670
160	110

Family = 43 (141 Lumen)	members = 5
140	141 390 1927 2790 4252
160	——

Family = 44 (145 Adeona)	members = 15
60	1384 1994 3205 3407 3445
80	145 1238 1936 4157
100	997 1783 3096
120	166 3725
140	3238
160	——

Family = 45 (170 Maria)	members = 32
100	170 472 575 616 695 897 3786
120	787 1158 1160 2151 2429 2638 2865 2903 2962 3055 3158 3159 3970 4104 4167
140	714 879 1677 3594
160	660 875 1996 2221 3066 4099

Family = 46 (668 Dora)	members = 16
60	668 1734 2598 2807 2940 3611 3775 4135
80	1414 1795 1836 1970 3563 3829 4220
100	3630
120	——
140	——
160	——

Family = 47 (847 Agnia)	members = 7
140	847 2401 3491 3701 4051 4261
160	(1228)

Family = 48 (1272 Gefion)		members = 22
100	1751 2373 2493 2631 2801 2977 3910	2521 2905 2911 3788 3860 4096 4182
120	1272 2053 2386 (2875)	
140	2157 2595 4020	
160	3724	

TABLE VII. Same as Table IV except for zone 5.

Family = 51 (158 Koronis)				members = 137
40	158 243 720 832 993 1079 1223 1442 1482 1762 1824 1913 2117 2144 2224 2226 2377 2555 2589 2814 2833 2931 2963 3019 3516 3778 4259	452 658 1336 1423 1955 2155 2225 2574 3409	1570 1745 1802 2092 2620 2713 2969 3117 3334 3515 3765 3781	2209 2626 3380 3623 4076 4260
60	167 208 263 321 761 975 1100 1245 1289 1389 1618 1635 1878 2051 2123 2230 2300 2338 2470 2726 2729 2785 2811 2901 2924 2953 2958 3016 3032 3191 3303 3457 3545 3726 3780 3791 3856 4123			
80	462 1741 1835 2837 3226 3337 3386 3436 4195 4206			
100	277 534 811 962 1029 1350 1363 1497 1725 1742 1774 1848 1912 2160 2188 2319 2498 2742 3207 3261 3307 3804 3975			
120	311 1894 2506 2700 3195			
140	1840 2541 2591 2683 2985 3941 4084			
160	——			

TABLE VIII. Same as Table IV except for zone 6.

Family = 61 (221 Eos)										members = 202
40	450 529 1210 1533 1767 1801 2136 3088 4041 4118	562 651 1112 3140 4207	573 653 742 1605 3062 3214 3505	639 1087 1105 1353 2216 2957 3250	669 1364 1957 3318 3896	766 1220 1286 1287 1410 1413 1552 1641 1711 1737 1812 2027 2191 2206 2345 2387 3194 3570 4058	876 1753 2263 2600 3750	890 1075 1388 2618 3736 4077	633 2111 2646 2767 2787 3232 3992		
60	221 320 1723 1732 2522 2523 3357 3469	339 579 1780 1786 2531 2622 3560 3638	590 608 1787 1852 2686 2706 3774 3820	661 807 1887 1903 2752 2804 3876 3887	1174 1186 2052 2124 2808 2889 3914 3955	1207 1291 2179 2309 2900 2928 4052 4059	1416 1464 2358 2416 3003 3126 4100 4163	1557 1649 2453 2476 3237 3310 4170	1297 1434 1654 1861 2091 2180 2459	513 798 1129 1199 1234 1339 1758 2907 2982 3366 3582 3772 4074 4115	775 1826 2315 2400 2425 2471 2677 2836 3168 3190 3312 3329 3425 3620 3950
80	520 833 1485 1532 1604 1834 1844 1882 2020 2115 2413 2443 2578 2661 2690 2711 2740 2909 3028 3328 3506 3830 4102 4210										
100	(1265) 1910 1971 (2530) 4078 4223										
120	(2562)(2632)										

* Underlined objects actually belong to zone 7

TABLE IX. Same as Table IV except for zone 7.

Family = 71 (24 Themis)							members = 228
40	90 383 1623 1778 2016 2163 2264 2361 2519 2919 3832 3930 4009						
60	24 62 171 222 461 492 526 656 710 767 846 954 1003 1027 1302 1487 1576 1615 1686 1691 1782 2058 2203 2222 2270 2293 2325 2489 2551 2627 2718 2769 2894 3591 3615 3666 4013	621 1953 2009 2418 2884 3276 3884	2528 2534 3814 4139 4193				
80	379 468 936 996 1082 1445 1539 1687 1805 1851 2046 2153 2165 2217 2310 2404 2450 2461 2499 2524 2525 2781 2803 3049 3128 3174 3208 3245 3292 3399 3441 3499 3502 3790 3799 3980 4073 4176		——	1340 1895 2039 2492 3962	316 561 2164 2918 4079 4174		
100	431 515 637 938 991 1074 1247 1259 1669 1764 1788 1898 1939 2182 2220 2228 2240 2248 2297 2372 2439 2517 2549 2587 2592 2723 2749 2882 3148 3154 3186 3297 3705 3878 3898 3946 3981 4061 4198				1581 1674 1698 3008 3599	2237 2505 2981 3164 3597	
120	1229 1624 1956 2114 2142 2342 2405 2533 (2657) 2800 2978 3071 3213 3495 3797 4098 4126 4192					1253	1073 1633 2250 2336 2757
140	268 848 981 1986 2197 (2563) (2688) 2708 2722 3061 3179 (3507) 3543 (3766) 3785 4211 4234					2296 2721 2921 3010 3358	
160	555 946 (1171) 1383 (1489) (2003) 2238 2667 2673 (2707) (2848) 2863 3183 3264 3418 3598 3601 3899 (3916) 4153 4178 4187						

Family = 72 (137 Meliboea)	members = 10
100	1369 1452 1498 2829 4004
120	2040
140	137 791 2152
160	2374

Family = 73 (490 Veritas)	members = 7
60	1086 2147 2428 3090 3542
80	——
100	490 2934
120	——
140	——
160	——

main-belt asteroid). In this context, we should take note of the fact that family 35, though splitting from Vesta's family two steps above the quasirandom level (and therefore treated as a separate family), has a fairly close position with respect to it both in the (a', sin i') and in the (a', e') planes (see Figs. 4 and 5). We also point out the absence in our classification of a Phocaea family, found by most previous investigators. As pointed out by Williams (1971), however, the Phocaea region is isolated by secular resonances; it probably does not contain genetically related bodies. Our procedure does not find a Phocaea family simply because the objects of this group have too large mutual distances in the proper elements space (though this may be related to the poor accuracy of proper elements for such high-inclination orbits).

Eight families have been identified in zone 4. The most populous one, family 41, has 81 members and fairly high values of P_s, P_r, and P_r'. Its largest member is the 270 km sized asteroid 15 Eunomia, whose unusual light curve has been considered by Cellino *et al.* (1985) as a hint to a possible binary nature; as discussed by Farinella *et al.* (1982), such binary (or triaxial) asteroids could be the outcomes of angular momentum transfer by off-center shattering impacts, with reaccumulation of part of the fragments and loss of others, ending up as minor family members. Actually, according to Gaffey's and Ostro's (1987) rotationally resolved spectral reflectance study, Eunomia's surface appears to sample a range of depths within a differentiated parent body. The Eunomia family was identified also by Williams (1979). While many small asteroids (e.g., 630, 812, 1050, 1193, 1329, 1333) are members of the Eunomia family in both Williams' and our classification, our family 41 does not include the two large objects (more than 100 km in size) 85

Iσ and 141 Lumen, belonging to Williams' Eunomia family; as remarked by Gradie *et al.* (1979), this association was puzzling, since Eunomia is an S-type asteroid while Iσ and Lumen are C types, and it is not plausible to consider these three objects as huge pieces of a single parent body (see also Chapman 1986; Bell 1989). In our classification, indeed, Lumen appears as the major body of another small, albeit not robust, family (No. 43), which separates from Eunomia's family well above the quasirandom level (see Fig. 8). Another remarkable family in zone 4 is the Maria family (No. 45), also found by all previous researchers apart from Carusi and Massaro (1978); this family appears to be fairly homogeneous in composition, S types being clearly predominant, but has the peculiar feature of including several bodies of comparable size (about 50 km). Such a size distribution is an unusual outcome for shattering impacts (see, e.g., Zappalà *et al.* 1984); it is possible that in this case the ejection velocity of fragments was close to the escape velocity of the parent body and anisotropically distributed, causing reaccumulation into several separate clumps of material. Family No. 44, with 15 members, has $P_r = 0.93$ and has some members in common with Williams' family No. 138, but does not include its least-numbered members 54 Alexandra and 70 Panopaea. Another small, but compact and robust family found in this zone (No. 48) is associated with 1272 Gefion; it was previously identified by Carusi and Massaro (1978) and Williams (1979; his label is 127).

Zone 5 contains only the well-known Koronis family, whose membership (137 asteroids) is not affected by superposition of noise [according to Eq. (5), N is small in this region] or changes in the metric coefficients. An inner core of the family survives down to $\delta v' \approx 40$ m/s (a similar remark was made by Williams *et al.* 1989). Like Maria's family, Koronis' family is composed of S types (with a significant contrast to the background in this zone, where C types are also abundant), and it includes at least three major bodies of similar size, about 40 km.

Zone 6 also includes just one very populous (about 200 members) and robust ($P_r = 0.94$, $P'_r = 0.98$) family, that associated with 221 Eos. Some intriguing features of this family (defined according to Brouwer's and Williams' classifications) were recently discussed by Farinella *et al.* (1989). Although the 4/9 resonance appears to effectively border one side of the family, a small "enclave" (four objects at the quasirandom level) actually lies beyond the resonance, that is in zone 7 (see Figs. 4, 5, and 10). We also note that according to Milani and Knežević's (1990) tests, Eos' family is very close (or actually overlaps) the secular resonance whose critical argument is $(g - g_6 + s - s_6)$ (g and s are the secular rates of perihelion and node, with the label 6 referring to Saturn); as a consequence, at any time it is likely that a few members crossing the narrow resonant strip get proper elements out of the family range. We can also note that the discovery of small Eos family members has been favored by their albedo, higher than the average in this part of the belt; observational biases would therefore play against the detection of other similar families with lower albedo.

In zone 7 we have found three families. The most important one is Themis' family, that has more than 220 members (some 5% of the whole set of numbered asteroids). Proper elements here are quite accurate, in spite of the neighboring 2/1 mean motion resonance (corresponding to the Hecuba gap) and contrasting with those derived from the linear theory, used in several previous classifications (see Carpino *et al.* 1986). Themis' family shows quite homogeneous spectrophotometric properties, with all of its members belonging to low-albedo C, B, and F classes [contrasting with a statement by Gradie *et al.* (1979), the large object 171 Ophelia is also a C type]. According to Bell (1989) and Chapman *et al.* (1989), the parent body was probably made of carbonaceous material, subject to some degree of metamorphism. From Fig. 11, it is apparent that the stalactite of Themis' family is much less steep than that of Eos' family, indicating that this family is formed by a dense "core," including Themis itself and another sizeable asteroid, 90 Antiope, plus a surrounding "halo" of decreasing density. This conclusion is similar to that reached by Williams (1979), who actually distinguished two such components by labeling them 1A and 1, respectively [see also Williams *et al.* (1989)]. Also interesting in this zone is the identification of two small, robust families at moderate proper inclinations, having as largest members two objects in the 100–150 km size range, Meliboea and Veritas (the latter cannot but be a "true" family!) They appear to closely match two small Williams' families (Nos. 113 and 106, respectively).

IV. CONCLUSIONS

We have shown that an entirely automatic procedure, applied to a large set of proper elements and taking into account their finite accuracy, can lead to a reliable family classification. At the same time, our method allowed us: (i) to assess, for each family, its statistical significance and its robustness; (ii) to discriminate between members of dense family "cores" and peripheral objects; (iii) to take into account, in the comparison with the background of field asteroids, the complex large-scale structure of the belt; (iv) to give a rough estimate, via the metric function (2), of the relative velocities needed to separate "neighboring" family members (that of course is not the same as the mean ejection velocity from the parent body).

A preliminary analysis of the properties of the 21 families identified with this method shows both areas of agreement and discrepancies with respect to previous classifications. About two thirds of these families appear significant and robust enough to make us confident that their members are genetically related. The three well-known major families associated with Themis, Eos, and Koronis contribute about 14% of the total sample of numbered asteroids. Examples of significant inferences about the properties of the parent bodies and the breakup mechanism have been discussed. The way is now open to further physical studies of families.

This work is a stage of a long-standing and cooperative research program that will continue in the future. We plan: (i) to further test and improve the reliability of our classification, by studying further the sensitivity of the results to changes in the clustering procedure (choice of the metric function, definition of the quasirandom background, etc.), performing new numerical integrations and exploring the possibility of defining improved proper elements, best suited to specific zones of the phase space, in agreement with the complex dynamical structure of the asteroid belt; (ii) to carry out more systematic comparisons with other classifications, both applying the hierarchical clustering method to other sets of proper elements, and analyzing any discrepancy when other classification methods are applied to the same dataset; (iii) to carry out physical studies (e.g., analyzing spectrophotometry and light-curve data, and the mass and velocity distributions) of individual families and of family asteroids in general. Families will hopefully make us better understand the nature of asteroids and their evolution.

We are grateful to J. G. Williams for his very careful review and for many constructive remarks, and to C. R. Chapman, D. R. Davis, Ch. Froeschlè, Cl. Froeschlè, A. Milani, P. Paolicchi, and G. B. Valsecchi, for helpful discussions and comments. This research has received financial support from the Ministry of University and Scientific Research and the National Research Council (CNR) of Italy, and from the EEC collaborative research contract SC1-0011-C (GDF).

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