THE ASTRONOMICAL JOURNAL VOLUME 99, NUMBER 4 APRIL 1990

# THE SHORT-PERIOD CEPHEID EU Tau. II. PHYSICAL PROPERTIES OF THE STAR

WOLFGANG P. GIEREN[a),b)]
Observatorium Hoher List der Universität-Sternwarte Bonn, D-5568 Daun, West Germany

THOMAS J. MOFFETT
Department of Physics, Purdue University, West Lafayette, Indiana 47907

THOMAS G. BARNES III
McDonald Observatory, The University of Texas at Austin, Austin, Texas 78712

JAYMIE M. MATTHEWS[a)]
Department of Geophysics and Astronomy, University of British Columbia, 2075 Westbrook Place, Vancouver, British Columbia V6T 1W5, Canada

MARIAN L. FRUEH
McDonald Observatory, The University of Texas at Austin, Austin, Texas 78712
*Received 8 November 1989; revised 19 December 1989*

## ABSTRACT

We have performed a surface-brightness analysis on the extensive photometric and radial-velocity data of EU Tau presented in Paper I and have determined the stellar radius and distance with a $\sim 5\%$ precision. We derive and discuss new values for most of the star's physical parameters, including its luminosity, mass, effective temperature, and binary status. The physical properties of EU Tau strongly suggest that this Cepheid is pulsating in the radial first overtone mode. Amplitude ratios and phase differences derived from low-order Fourier coefficients of the light curve appear to confirm this, and so does the very small relative radius variation of the star. We present arguments for the utility of the phase difference parameter $\phi_{21}$ as a pulsation mode discriminator in classical Cepheids. We find only two pieces of evidence which do not favor first overtone over fundamental mode pulsation in EU Tau. On the Hertzsprung–Russell diagram, EU Tau is located close to the blue edge of the instability strip for fundamental mode pulsation in a region where fundamental mode and first overtone pulsation seem to be about equally likely. Also, the value of the first-order phase lag $(\Delta\phi)_1$ for EU Tau seems to be more in accord with fundamental mode than with first overtone pulsation. However, we show that the role of $(\Delta\phi)_1$ as a pulsation mode indicator for Cepheids is very doubtful at the present time. In conclusion, we find that first overtone pulsation is much more likely in EU Tau than fundamental mode pulsation. We encourage hydrodynamic model calculations for the star aiming at an improved understanding of the physical causes which determine the pulsation modes of Cepheid variables.

## I. INTRODUCTION

In Paper I (Gieren *et al.* 1989) we presented new and contemporaneous radial-velocity and photometric data of the short-period, classical Cepheid EU Tau obtained at the Kitt Peak, McDonald, and Konkoly Observatories. A frequency analysis on our new $V$ photometry and previous datasets unambiguously showed that EU Tau is pulsating with a single period near 2.1 days, rather than with two periods as suggested earlier by Gieren and Matthews (1987).

The density and quality of the photometric curves and of the radial-velocity curve presented in Paper I have made EU Tau one of the best-observed classical Cepheids. Our contemporaneous data are ideally suited to a surface-brightness analysis of the star to yield its mean radius and its distance. We perform such an analysis in Sec. II of this paper. The radius and distance results, together with other information available on the star, are then used to establish other fundamental stellar parameters (Sec. III), and to discuss in detail the pulsation mode of EU Tau (Sec. IV). We anticipate that we find strong evidence that EU Tau belongs to the apparently rare class of overtone pulsators among galactic classical Cepheids.

[a)] Visiting Astronomer, Kitt Peak National Observatory, National Optical Astronomy Observatories, operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

[b)] On leave from Observatorio Astronòmico Nacional, Apartado Aèreo 2584, Bogota, Colombia.

## II. SURFACE-BRIGHTNESS SOLUTIONS

### *a) The Visual Surface-Brightness Method*

This method has been described in considerable detail elsewhere [e.g., Gieren, Barnes, and Moffett (1989) (hereafter referred to as GBM); Moffett and Barnes (1987), and references given in these papers], and we give here only a brief account of its essentials. The visual surface-brightness parameter $F_V$ is defined as

$$F_V = 4.2207 - 0.1 V_0 - 0.5 \log \phi, \quad (1)$$

or, equivalently,

$$F_V = \log T_e + 0.1\mathrm{BC}, \quad (2)$$

where $V_0$ is the apparent visual magnitude, corrected for interstellar absorption; $\phi$ is the stellar angular diameter in milliarcsecond; and BC is the bolometric correction with respect to the $V$ bandpass. $F_V$ is linearly related to the stellar surface-brightness $S_V$.

It was shown by Barnes and Evans (1976), and later by

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Barnes, Evans, and Moffett (1978) that $F_V$ could be predicted, with a high degree of accuracy, by a simple linear relation of the form

$$F_V = b + m(V - R)_0, \tag{3}$$

where $(V-R)_0$ is the Johnson color index corrected for interstellar reddening. Equation (3) is usually referred to as the surface-brightness relation. Its correct establishment is the central problem of the surface-brightness method. The knowledge of the surface-brightness relation makes it possible to calculate the stellar angular diameter from its observed $(V-R)$ color.

In the case of a radially pulsating star like a Cepheid, this information can be combined with the independent knowledge of the displacement variation of the stellar surface obtained from an integration of the star's radial-velocity curve. The relation connecting the instantaneous linear diameter of the pulsating star with its instantaneous angular diameter is

$$\Delta D + D_m = 10^{-3} d\phi, \tag{4}$$

where $D_m$ is the mean linear diameter of the star, $\Delta D$ the instantaneous displacement from it (both in AU), $d$ the stellar distance (in pc), and $\phi$ the stellar angular diameter (in mas). Equation (4) can be solved for both the distance, and the mean diameter of the Cepheid by performing a regression analysis of $\phi$ against $\Delta D$. We note that the mean diameter derived depends only on the slope of the surface-brightness relation while the distance depends on both the slope and zero point of the relation.

### *b) Choice of the Constants*

For the surface-brightness analysis in this paper, we have adopted the set of constants which were used and discussed by GBM. From their Eq. (7), we obtain $m = -0.369$ as the appropriate slope of the surface-brightness relation for the period of EU Tau. We use $b = 3.956$ for the zero point of the relation. An independent check on this value using the effective temperature scale for short-period Cepheids of Pel (1978) (Moffett, Barnes, and Gieren 1990) has confirmed this value.

For the conversion from radial to pulsational velocity, we use $p = 1.380$ as suggested by Eq. (8) of GBM. We further use the following photometric constants:

$A_V/E(B-V) = 3.3$

$E(V-R)/E(B-V) = 0.84$

$E(B-V) = 0.15$ (Fernie 1987).

For the pulsation period of EU Tau we adopt $P = 2.102\,511\,2$ days, as derived by Fernie (1987) and essentially confirmed by our frequency analysis in Paper I.

### *c) Radius and Distance Solutions*

As photometric input data in our surface-brightness analysis, we used the $V$ and $(V-R)_J$ values given in Paper I, excluding for this purpose the Konkoly $V$ data because of their significantly larger scatter as compared to the McDonald and Kitt Peak data. To these data, we added the $V$ and $(V-R)_C$ observations of Gieren and Matthews (1987) which were also obtained at KPNO. These $(V-R)_C$ data were transformed to the Johnson system in the same way as described in Paper I, and an appropriate zero-point correction was applied. The final $V$ and $(V-R)_J$ curves used in our analysis are shown in Figs. 1 and 2, respectively. Each consists of more than 400 individ-

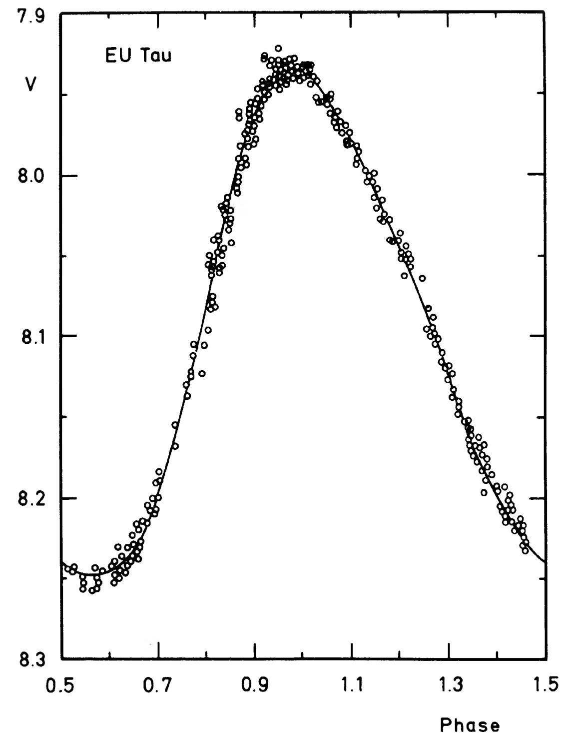


FIG. 1. The $V$ light curve of EU Tau showing the observations used in the surface-brightness analysis of this paper. The solid line is a third-order Fourier series fit to the data.

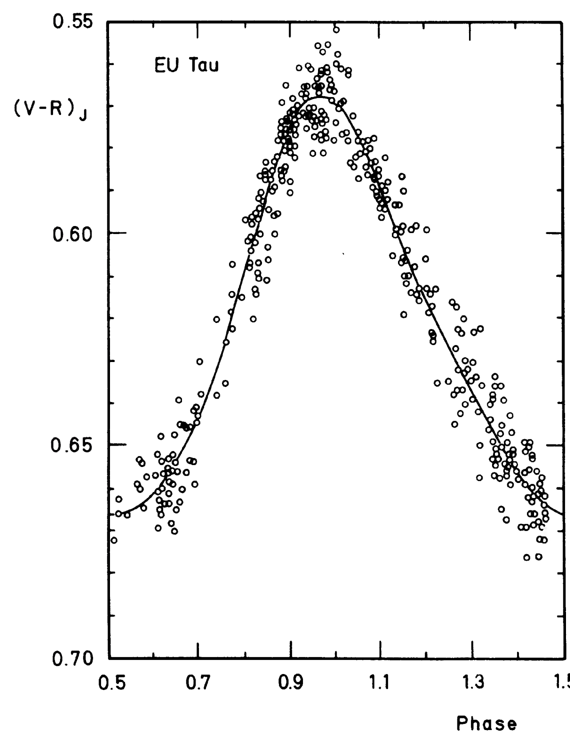


FIG. 2. The $(V-R)_J$ color curve of EU Tau showing the observations used in the surface-brightness analysis of this paper. The solid line is a third-order Fourier series fit to the data.

ual observations. The solid lines in these figures are the fitted third-order Fourier series.

As radial-velocity input data we used our own measurements, as described in Paper I, and the CORAVEL measurements of Burki (1985). The zero-point and phase adjustments made to bring both sets of observations into agreement (see Paper I) place all of the radial velocities onto the phase scale of the contemporaneous photometric observations, and onto Burki's velocity scale. The final adopted radial velocities are shown in Fig. 3. Again, the solid line is the third-order Fourier series fit to all corrected velocities.

We performed the least-squares solution of Eq. (4) in two different ways. First, we used the Fourier series representation for *all* of the observed curves (i.e., radial velocities, $V$, and $V-R$), using numerical values spaced by 0.01 in phase. In this case, the best fit was found with a phase shift of $-0.06$ (radial velocities relative to photometry), and the resulting radius and distance are

$$R/R_{\odot} = 29.52 \pm 0.63$$

$$d = (1163 \pm 25)\ \text{pc}.$$

In Fig. 4, the fit of the angular diameters to the linear displacements is shown. It is not perfect but appears to be the best that can be done with the linear surface-brightness relation. The distortions seen in the photometrically determined angular diameter curve in the phase interval 0.8 to 0.3 are similar to, but less severe than, the distortions seen in similar plots for RR Lyrae variables (see, for example, Cacciari *et al.* 1989). For the RR Lyrae stars, it is generally accepted that the distortions arise from acceleration effects upon the surface brightness during the expansion phase. Perhaps we are here for the first time seeing evidence of acceleration effects upon the surface brightness of a classical Cepheid.

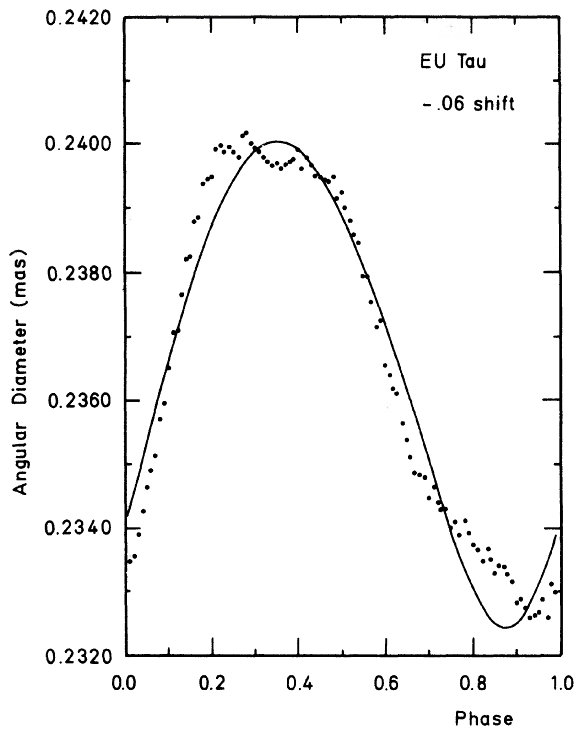


FIG. 4. The angular diameter variation of EU Tau (dots) as a function of phase. Superimposed is the linear displacement curve. A best fit between these curves is achieved with a phase shift of $-0.06$ of the velocities relative to the photometric curves. Without applying this phase shift the radius and distance results change by about one standard deviation.

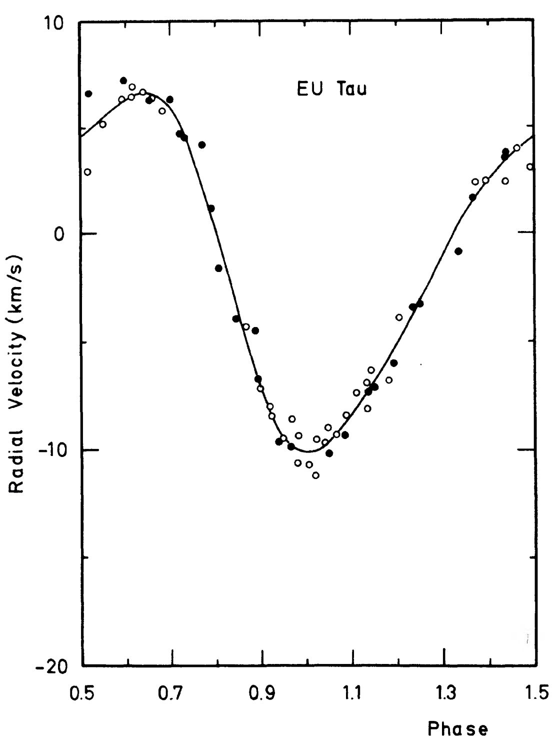


FIG. 3. The radial-velocity curve of EU Tau showing the observations used in the surface-brightness analysis of this paper. The observations are on the Burki (1985; filled circles) velocity system and on the KPNO (Paper I; open circles) phase system. The solid line is a third-order series fit to the data.

As a second approach, we used the Fourier series representation for the radial-velocity curve but used the actual observed photometric points in the analysis. In this case, the best fit was obtained with a phase shift of $-0.04$, and the results are

$$R/R_{\odot} = 30.93 \pm 1.46$$

$$d = (1218 \pm 58)\ \text{pc}.$$

They agree, within the errors, with the first solution, and so does the phase shift (we note that without applying any phase shift, the radius and distance change by approximately one standard deviation; hence, the solutions are only slightly sensitive to the adopted value for the phase shift). We believe that the $\sim 5\%$ accuracy for $R$ and $d$ obtained in the second solution reflect more correctly the true uncertainties of the radius and distance results. We note that this is the best radius and distance accuracy so far obtained for a short-period, low-amplitude Cepheid from the surface-brightness technique (GBM; Gieren 1988). This is not surprising in view of the excellent observations at our disposal. We believe that the remaining imperfection of the present solution, as reflected in Fig. 4, is almost entirely due to the star's actual surface-brightness relation being somewhat more complicated than the simple linear Eq. (3), rather than to limitations imposed by the quality of the data. Unfortunately, it is not yet possible to quantify the terms which may have a (small) effect on the surface-brightness relation of a classical Cepheid. Future work on this will be important and lead to radius and distance solutions of still greater precision.

As our final radius and distance result for EU Tau, we adopt the averages of the two determinations. This yields

$$R/R_{\odot} = 30.23 \pm 1.43,$$

$$d = (1191 \pm 57)\ \text{pc},$$

where the uncertainties correspond to those of the second solution discussed above.

## III. PHYSICAL PARAMETERS OF EU TAU

### *a) Absolute Magnitude and Luminosity*

Using the photometric constants in Sec. II*b*, a mean $V$ magnitude of $\langle V\rangle = 8.101$ determined from our light curve (Fig. 1), and the distance found in the preceding section, we obtain a mean absolute visual magnitude for EU Tau of

$$M_{\langle V\rangle} = -2.77\ \text{mag}.$$

The uncertainty of the distance alone translates into a $\pm 0.10$ mag uncertainty of this result. With an additional realistic $\pm 0.02$ uncertainty in the color excess, the total uncertainty of $M_{\langle V\rangle}$ is $\pm 0.15$ mag.

We may compare the absolute magnitude of EU Tau to the value expected from the Cepheid period–luminosity (P–L) relation for a star of its pulsation period. The expected values for $M_{\langle V\rangle}$ according to different recent calibrations of the P–L relation are given in Table I. Their mean value is $-2.19$ mag for fundamental mode pulsation. Thus, EU Tau is $\sim 0.6$ mag brighter than expected for its period. In Fig. 5, the absolute magnitudes of 52 Cepheids based on surface-brightness distances (Gieren 1988) are plotted against the period, along with the position of EU Tau. There are only two stars (Y Oph, which is very peculiar in many respects, and KN Cen which has a bright blue companion) which deviate as much from the mean P–L relation as EU Tau. A 0.6 mag deviation is perhaps on the extreme boundary, but more probably beyond both the intrinsic and observational dispersion of the P–L relation.

One possibility to resolve this discrepancy is to assume that EU Tau is pulsating in the first overtone radial mode. (More evidence supporting this assumption will be discussed in Sec. IV.) Using a period ratio $P_1/P_0 = 0.705$ [mean value obtained from the 11 double-mode Cepheids analyzed by Balona (1985)], the fundamental mode period becomes $P_0 = 2.98$ days under this assumption. Absolute magnitudes for EU Tau based on the first overtone assumption are also given in Table I. The mean of these is $-2.63$ mag in very good agreement with our surface-brightness result. The position of EU Tau on the P–L diagram is now very close to the mean line, as shown in Fig. 5. Thus, the observed distance of EU Tau lends support to the hypothesis that the star is pulsating in the first overtone mode.

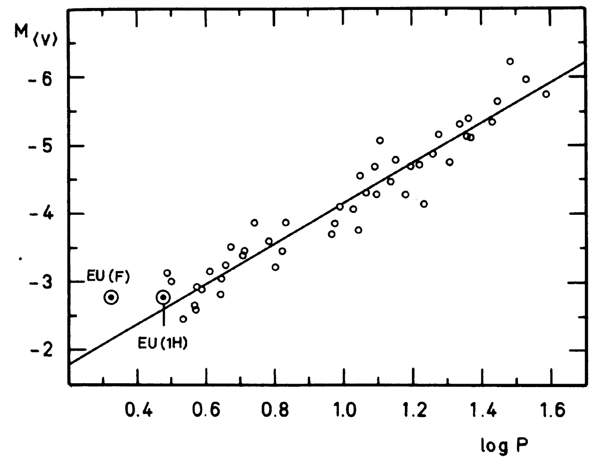


FIG. 5. The position of EU Tau relative to the Cepheid period-luminosity relation found from surface-brightness distances (Gieren 1988), assuming fundamental mode ($F$), and first overtone mode ($1H$) pulsation of the star. The open circles are the calibrating data of Gieren (1988). The uncertainty of the EU Tau absolute magnitude is $\pm 0.15$ mag.

TABLE I. Absolute magnitude predictions for EU Tau from different period–luminosity calibrations, assuming fundamental mode (F), and first overtone mode ($1H$) pulsation, respectively.

| Source | $M_{\langle V\rangle}(F)$ | $M_{\langle V\rangle}(1H)$ |
|---|---|---|
| Gieren (1988) | −2.17 | −2.61 |
| Caldwell and Coulson (1987) | −1.97 | −2.44 |
| Feast and Walker (1987) | −2.25 | −2.66 |
| Balona and Shobbrook (1984) | −2.04 | −2.45 |
| Fernie and McGonegal (1983) | −2.54 | −2.97 |
| Mean value | −2.19 | −2.63 |

Using the mean intrinsic $B-V$ color of 0.510 derived from our measurements, and the bolometric correction $-(B-V)_0$ scale of Parsons (1971), we obtain BC $= -0.02$ mag for EU Tau. With this value, the luminosity of EU Tau becomes

$$L/L_{\odot} = 1038 \pm 140,$$

where our value for $M_{\langle V\rangle}$ and $M_{\text{bol}\odot} = 4.75$ (Allen 1973) have been adopted. The quoted uncertainty corresponds to the $\pm 0.15$ mag uncertainty of the absolute magnitude.

### *b) Effective Temperature*

There are various ways to estimate the effective temperature of EU Tau. First, we can use the radius and luminosity results of this paper, and the $L/L_{\odot} = (R/R_{\odot})^2(T_e/T_{e\odot})^4$ relationship. Adopting $T_{e\odot} = 5770$ K (Allen 1973), this yields

$$T_e = (5960 \pm 330)\ \text{K},$$

where the uncertainty given corresponds to the uncertainties of the radius and luminosity values of EU Tau derived above.

We can also use the visual surface-brightness Eq. (3). For EU Tau, the appropriate relation is

$$F_V = 3.956 - 0.369(V-R)_0. \quad (5)$$

From our photometry, we derive a mean $(V-R)$ color of 0.622 for EU Tau. Using the relations given in Sec. II, this yields $(V-R)_0 = 0.489$ as the mean intrinsic $(V-R)_J$ color of the star. From Eq. (5), we obtain $F_V = 3.776$, and from Eq. (2) we then obtain

$$T_e = 5998\ \text{K},$$

where again BC $= -0.02$ mag has been used.

This estimate of $T_e$ agrees well with the one derived above.

The two estimates are not independent, of course, because the same surface-brightness relation (5) has been used in the radius and distance calculation of Sec. II. However, we note that the results are consistent.

We can further use the intrinsic color of EU Tau to estimate its temperature. At the star's mean intrinsic $(B-V)$ color of 0.510, the effective temperature calibration of Pel (1985) for short-period Cepheids yields $T_e = 6300$ K. According to the calibration of Flower (1977), the appropriate value of $T_e$ is 6173 K. We note that the Pel and Flower calibrations are about midway between the extremes in the calibration of the $T_e - (B-V)_0$ relation for classical Cepheids (Pel 1985).

As an average from these different determinations, we adopt for EU Tau a mean effective temperature of

$$T_e = (6110 \pm 200) \text{ K}.$$

The quoted uncertainty of $\pm 200$ K is somewhat arbitrary, but seems to correspond to the various numerical results derived in this section.

### c) Mass

The mass of a Cepheid can be calculated in at least three different ways: (1) from an evolutionary mass–luminosity ($\mathscr{M}$–$L$) relation appropriate to Cepheids; (2) from a pulsational period–radius–mass (P–R–$\mathscr{M}$) relation, using a radius value derived from a Baade–Wesselink technique; (3) from the same P–R–$\mathscr{M}$ relation but using a radius value resulting from the Cepheid's observed $L$ and $T_e$.

These three mass estimates are called evolution mass, $\mathscr{M}_{\rm ev}$, Wesselink mass, $\mathscr{M}_{\rm Wes}$, and pulsation mass, $\mathscr{M}_{\rm pul}$, respectively. An exhaustive review of these masses for Cepheid variable stars has recently been given by Gieren (1989).

We calculate the $\mathscr{M}_{\rm ev}$ for EU Tau from the $\mathscr{M}$–$L$ relationship given by Becker, Iben, and Tuggle (1977). Assuming Pop. I composition ($Y = 0.28$, $Z = 0.02$), this relation is

$$\log (L/L_\odot) = 0.46 + 3.68 \log (\mathscr{M}/\mathscr{M}_\odot). \quad (6)$$

In this relation it is further assumed that the Cepheid is on its second crossing through the instability strip and has not suffered appreciable mass loss in the pre-Cepheid stage of evolution. Our value of $\log (L/L_\odot) = 3.016 \pm 0.06$ yields an evolution mass for EU Tau of

$$\mathscr{M}_{\rm ev}/\mathscr{M}_\odot = 4.95 \pm 0.20.$$

We derive the Wesselink and pulsation masses of EU Tau from the P–R–$\mathscr{M}$ relation of Fricke, Stobie, and Strittmatter (1972)

$$P_0 = 0.025(\mathscr{M}/\mathscr{M}_\odot)^{-0.67}(R/R_\odot)^{1.70} \text{ days}. \quad (7)$$

This relation is valid for standard Pop. I composition and is essentially independent of $T_e$ (or $L$) over the temperature (luminosity) range in which classical Cepheids are found. Our radius value of $(30.23 \pm 1.43)R_\odot$ yields

$$\mathscr{M}_{\rm Wes}/\mathscr{M}_\odot = 7.65 \pm 0.91,$$

assuming that the observed period of 2.1025 days corresponds to the fundamental mode. From the values of $L$ and $T_e$ derived in this study, we find a pulsation radius of $28.76R_\odot$ for EU Tau which yields a pulsation mass of

$$\mathscr{M}_{\rm pul}/\mathscr{M}_\odot = 6.74 \pm 2.53.$$

Here it is also assumed that $P_0 = 2.1025$ days.

It is evident, from these results, that the masses based on the pulsational P–R–$\mathscr{M}$ relation are substantially larger than the evolution mass of EU Tau. This result will be discussed in Sec. IV in connection with the pulsation mode of the star.

### d) Binary Status

GBM have discussed the influence of a binary companion to a Cepheid on its radius value obtained with the surface-brightness technique. Since $(V-R)$ is used as a surface-brightness indicator, the radius solution is not sensitive to a blue companion, but we expect some sensitivity to a bright yellow or red companion star. It is, therefore, important to check the observational data for evidence regarding the possible existence of such a companion star.

The classical method to prove the existence of a physical companion is the detection of a variable $\gamma$ velocity of the Cepheid. For EU Tau, three sets of radial-velocity observations are available in the literature: our new measurements reported in Paper I, the CORAVEL observations of Burki (1985), and the observations reported by Gieren and Matthews (1987). The results for the $\gamma$ velocity of EU Tau from these sources are $-4.29$, $-1.24$, and $-2.0$ km/s, respectively. The differences between these determinations are certainly small enough to be explained by observational errors and/or small systematic zero-point shifts between the velocity scales of the different instruments used. We conclude that there is no compelling evidence from the existing radial-velocity data for a physical companion to EU Tau.

Photometric tests for a binary companion to a Cepheid have been discussed, among others, by Coulson and Caldwell (1989) and Gieren (1982a) and do not produce reliable results except in the rare cases of very bright companions. We have looked at the intrinsic colors, color–color loops, amplitude ratios of different color curves, and at the phase difference between the $V$ and $(U-B)$ curves which all contain information about a possible photometric companion. We find that all these properties are normal as compared to nonbinary Cepheids of similar period. The only piece of evidence possibly suggesting a companion star is the rather red intrinsic $(B-V)$ color of EU Tau which is $\sim 0.10$ mag redder than expected from the period–color relation given by Dean, Warren, and Cousins (1978). However, it is well known that Cepheid period–color relations do have a considerable intrinsic dispersion, and that different color excess scales show systematic differences up to $\sim 0.04$ mag (Fernie 1990), making a 0.1 mag deviation from the mean line in a period–color diagram a very weak case for binarity. The apparent red excess of EU Tau may also be a consequence of the pulsation mode of the star (see Sec. IV).

We conclude that there is also no evidence in the photometry for a companion to EU Tau. All available observational data on EU Tau strongly suggest it to be a single star. This result strengthens our confidence in the radius and distance results of Sec. II.

## IV. THE PULSATION MODE OF EU TAU

One of our original motivations to study EU Tau was the suggestion by Simon and Lee (1981) that EU Tau might be an overtone pulsator. This conclusion was based on their Fourier decomposition analysis of the $V$ light curve of the star which yielded parameters remarkably similar to those found for the 1.95 d Cepheid SU Cas, the currently strongest case for overtone pulsation among galactic Cepheids (Aikawa, Antonello, and Simon 1987; Gieren 1982b).

Overtone pulsation seems to be a very rare phenomenon among classical Cepheids. In their recent analysis of a homogeneous set of 101 Cepheid radii, GBM have found no evidence for a substantial number of overtone pulsators among the short-period Cepheids. The same conclusion was reached on different grounds by Arellano Ferro (1984) who studied a sample of short-period, low-amplitude Cepheids. This apparent absence of overtone pulsators makes it very important to identify the few existing cases and establish their physical properties as accurately as possible. Hydrodynamic model calculations as those performed by Aikawa *et al.* (1987) might then help to understand why these stars pulsate in an overtone mode, and why overtone pulsation is so infrequent in the Cepheids of our galaxy.

In what follows, we shall use the physical parameters of EU Tau established in the preceding sections to discuss its pulsation mode.

### *a) Radius*

We have plotted the position of EU Tau on the period–radius diagram in Fig. 6, along with the surface-brightness radius data on 101 Cepheids given by GBM. If EU Tau is a fundamental mode pulsator, its radius value is ~40% larger than expected from the mean period–radius relation found by GBM which predicts a radius of $22.3R_\odot$ at a period of 2.1 days. This deviation is very large compared to the $\pm 1.4R_\odot$ uncertainty of the EU Tau radius, and compared to the scatter of the period–radius relation which is in the order of 0.04 in log $R$ around the mean relation at a fixed period. There is no other star among the ~50 Cepheids with periods less than 8 days in the sample of GBM showing such a large deviation from the mean line on the P–R diagram.

As before, if we interpret the observed period of EU Tau as the first overtone mode period, the fundamental period becomes 2.98 days, and the position of the star on the P–R diagram is now very close to the mean line (only 0.02 in log $R$ above the mean relation). Thus, the observed radius of EU Tau is consistent with pulsation in the first overtone mode and inconsistent with pulsation in the fundamental mode. We note that pulsation in the second overtone, as suggested by Burki (1985), is less likely because under this assumption the radius of EU Tau lies ~0.07 in log $R$ below the mean P–R relation, which is about twice its dispersion (although a few Cepheids which are presumably fundamental mode pulsators do lie that far from the mean curve).

We want to stress that the surface-brightness radius of EU Tau can be ideally compared to the GBM radius data because all these radii were derived with the same technique and with the same set of parameters.

### *b) Mass*

In the previous section, we have derived

$\mathcal{M}_{\rm ev}/\mathcal{M}_\odot = 4.95 \pm 0.20,$

$\mathcal{M}_{\rm Wes}/\mathcal{M}_\odot = 7.65 \pm 0.91,$

$\mathcal{M}_{\rm pul}/\mathcal{M}_\odot = 6.74 \pm 2.53,$

from the radius and luminosity of EU Tau found in this paper. The Wesselink and pulsation masses were derived under the assumption of fundamental mode pulsation. If we assume pulsation in the first overtone, the Wesselink and pulsation masses decrease to

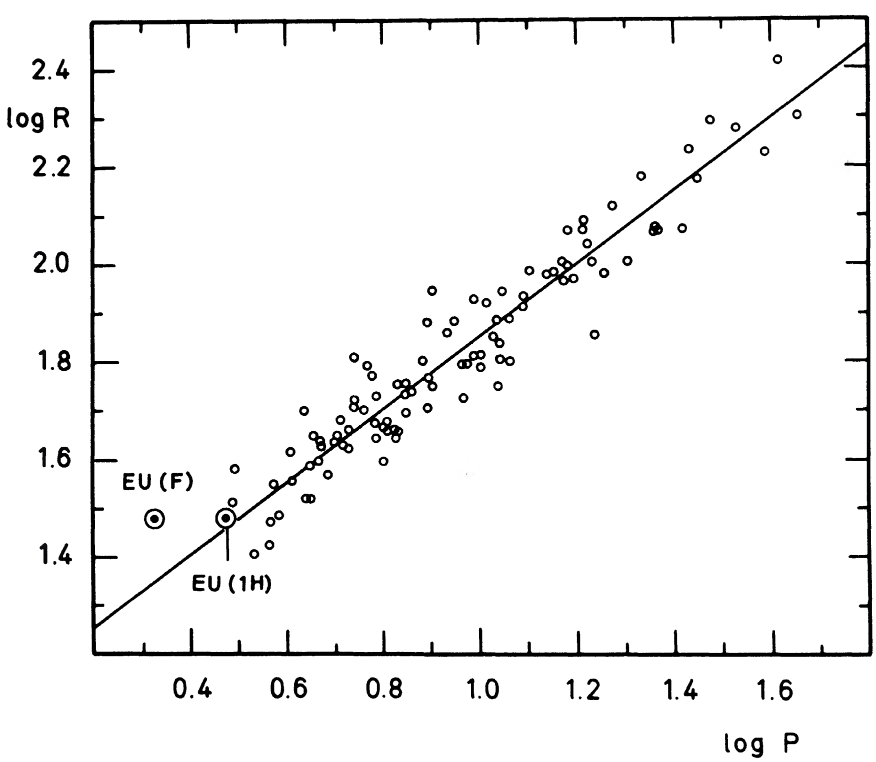


FIG. 6. The position of EU Tau relative to the Cepheid period–radius relation obtained from the surface-brightness method (Gieren, Barnes, and Moffett 1989), assuming fundamental mode ($F$), and first overtone mode ($1H$) pulsation of the star. The open circles are the calibrating data of GBM. The radius uncertainty for EU Tau is $\pm 0.02$ in log $R$.

$$\mathcal{M}_{\rm Wes}/\mathcal{M}_\odot = 4.55 \pm 0.54,$$
$$\mathcal{M}_{\rm pul}/\mathcal{M}_\odot = 4.01 \pm 1.51.$$

We find that agreement among the different mass calculations is only possible if first overtone pulsation is assumed. This is illustrated in Fig. 7 where the ratio $\mathcal{M}_{\rm Wes}/\mathcal{M}_{\rm ev}$ is plotted against period for EU Tau and for a sample of 52 Cepheids for which the necessary data exist from the study of Gieren (1989). Under the assumption of fundamental mode pulsation the observed mass ratio for EU Tau is discrepant by a factor of $\sim 2$, as compared to the expected value. This discrepancy is *the largest for any of the Cepheids in Fig. 7.* Assuming first overtone pulsation in EU Tau, the mass ratio becomes ($0.92 \pm 0.11$) which is very close to the value expected from the data in Fig. 7. The mass data illustrate even more clearly, because of the $\mathcal{M} \sim R^{2.5}$ dependence, what we already concluded from the radius value of EU Tau, namely that the star is very likely to pulsate in the first overtone mode. Its true mass is very likely between 4.5 and 5.0 $\mathcal{M}_\odot$.

### *c) Absolute Magnitude*

We have already shown in Sec. III*a* that the absolute magnitude of EU Tau is only in accord with the predictions of Cepheid P–L relations if first overtone pulsation is assumed. If we use a P–L–C relation instead of a P–L relation to predict the absolute magnitude of EU Tau, we basically obtain the same result. Adopting the P–L–C Eqs. (1) and (2) given by Feast and Walker (1987; their Table III), we obtain predictions of $M_{\langle V\rangle} = -2.12$ and $-2.18$ mag, respectively, if $P_0 = 2.10$ d is assumed, and $M_{\langle V\rangle} = -2.70$ and $-2.72$ mag, respectively, if $P_0 = 2.98$ d is assumed. The latter values are in excellent agreement with our derived value of $-2.77$ mag. This demonstrates that an application of a Cepheid P–L–C relation, rather than a P–L relation, does not change the conclusions regarding the pulsation mode of EU Tau already drawn in Sec. III*a*. The bright absolute magnitude of EU Tau clearly indicates that the star is more likely to pulsate in the first overtone than in the fundamental mode.

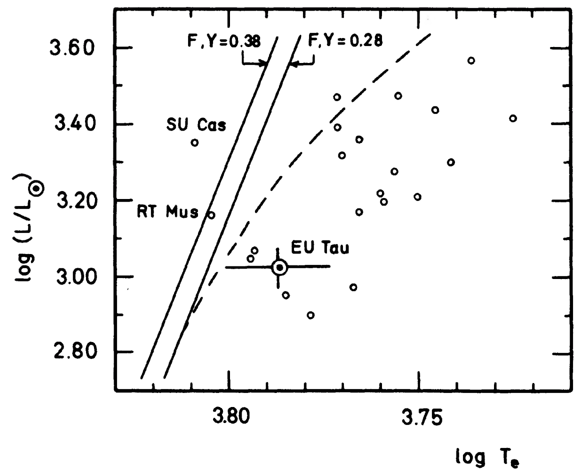


FIG. 8. The location of EU Tau in the Cepheid instability strip on the HRD, relative to the blue edges for fundamental mode pulsation and different chemical compositions. The blue edges are adopted from Iben and Tuggle (1975). Also shown are the locations of Cepheids with known surface-brightness luminosities and Pel (1978) effective temperature determinations. The broken line is a tentative observational blue edge representing these data.

### *d) Position in the Instability Strip*

In Fig. 8, we have plotted the position of EU Tau on the Hertzsprung–Russell diagram using the $L$ and $T_e$ values derived in Sec. III. For comparison, we have also plotted the positions of those short-period Cepheids which have luminosity determinations from their surface-brightness distances (Gieren 1989), and individual $T_e$ determinations from Walraven photometry by Pel (1978). The blue edges for fundamental mode pulsation and different chemical compositions shown on this diagram are adopted from Iben and Tuggle (1975).

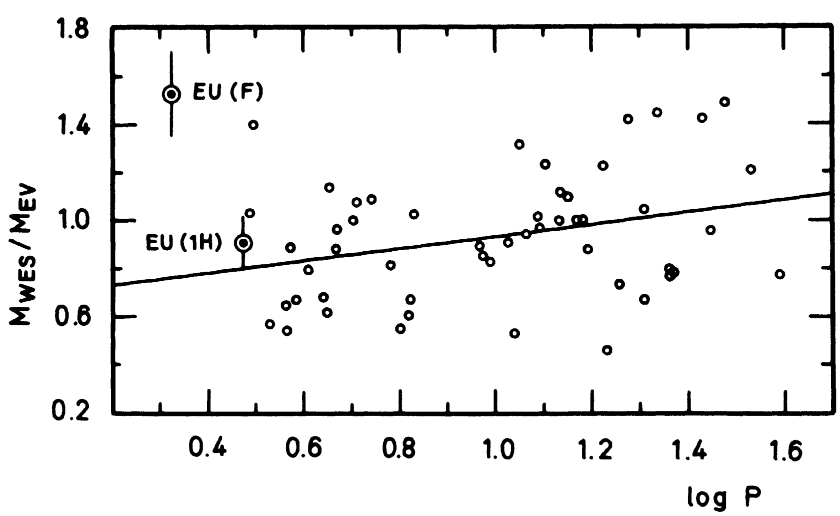


FIG. 7. The ratio Wesselink to evolution mass plotted against period, for Cepheids with surface-brightness luminosity determinations (Gieren 1989). The position of EU Tau is shown assuming fundamental mode ($F$), and first overtone mode ($1H$) pulsation, respectively.

Figure 8 suggests that EU Tau lies close to the fundamental mode blue edge for $Y = 0.28$, but still to the red side of the boundary. There are three stars on Fig. 8 having luminosities comparable to that of EU Tau which are slightly hotter, lending support to this conclusion (assuming that these stars pulsate in the fundamental mode). The location of EU Tau in the instability strip thus suggests that there is a good chance that the star is a fundamental mode pulsator. However, the probability that it is pulsating in the first overtone mode is about equal because the star is still on the blue side of the red edge for pulsation in the first overtone mode if similar widths for the fundamental and first overtone instability strips are assumed. Thus, no strong point for either of the two modes can be made from the observed position of EU Tau on the Hertzsprung–Russell diagram.

We would like to make a few additional remarks concerning Fig. 8. First, we note that at the observed luminosity of EU Tau, its effective temperature would have to be $\sim 6400$ K to bring the star right onto the fundamental mode blue edge for $Y = 0.28$. This is 300 K hotter than the value derived in Sec. III and does not seem to be consistent with the determinations discussed in this section. However, we believe that the effective temperature is not as well known as the other physical parameters determined in this study, and we would encourage independent temperature determinations for EU Tau. It might well turn out that EU Tau is somewhat hotter than our present determination and thus even closer to the blue edge than indicated in Fig. 8. It is also possible that the star does not have standard Pop. I composition but a lower $Y$ and/or a higher $Z$. In this case, the fundamental mode blue edge appropriate to such a composition would be displaced to cooler temperatures, and again the location of EU Tau would be closer to it. We do not have any detailed information on the chemical abundances of EU Tau at the present time, however, so we cannot pursue this idea further. As a final remark to Fig. 8, we note that most of the Cepheids plotted are relatively far to the red of the fundamental mode blue edge of Iben and Tuggle. This could mean that the theoretical blue edge is slightly too hot. A boundary such as the broken line drawn in Fig. 8 would seem to give a better observational fit to the locations of these Cepheids. We are aware of the fact, however, that the small-number statistics of Fig. 8 may be insufficient to draw such a conclusion and that most of the Cepheids in Fig. 8 may be really quite red. An obvious exception is the 3.1 d Cepheid RT Mus which, according to its location in Fig. 8, may be another promising candidate for overtone pulsation.

### *e) Fourier Decomposition Parameters*

Simon and Lee (1981) were the first to introduce different combinations of the low-order Fourier coefficients of Cepheid light curves in order to obtain a quantitative description of these curves. Their work was later extended by Simon and Moffett (1985) to a sample of well-observed northern Cepheids. In this work, it became clear that the famous Hertzsprung progression of Cepheid light curves could be successfully reproduced using the quantities

$$R_{21} = A_2/A_1,$$
$$\phi_{21} = \phi_2 - 2\phi_1,$$

where $A_1, A_2, \phi_1$, and $\phi_2$ are the amplitudes and phases of the first and second-order terms in the Fourier series

$$A_0 + \sum A_i \cos\left[i\omega(t-t_0) + \phi_i\right], \tag{8}$$

used to fit the light curves (the index $i$ runs from 1 to a maximum value $i_{\max}$).

The plots of the quantities $\phi_{21}$ and $R_{21}$ against pulsation period reveal some interesting features. Besides the reproduction of the Hertzsprung progression, it turned out that there is a small sample of stars with periods $\lesssim 3$ days whose values of $\phi_{21}$ are clearly *larger* than expected from an extrapolation of the sequence defined by the longer-period Cepheids. Also, their $R_{21}$ values are significantly *smaller* than expected from the sequence of the longer-period stars. Simon and Lee (1981), and later Gieren (1982b) called attention to the possibility that the abnormal values for $R_{21}$, and particularly $\phi_{21}$, might be caused by overtone pulsation in these objects.

In order to study this hypothesis, Antonello and Poretti (1986) determined the Fourier coefficients of many more short-period Cepheid light curves. In their analysis, they distinguished the so-called *s* Cepheids from the classical Cepheids; *s* Cepheids are those having near-sinusoidal light curves of relatively small amplitudes. (We note that such a distinction appears to be a bit arbitrary because there is a continuous change from the Cepheids with more asymmetric light curves of larger amplitudes to the *s* Cepheids.) The essential result of their work is that the *s* Cepheids seem to define different sequences on the $\phi_{21}$ versus period and $R_{21}$ versus period diagrams than the large-amplitude classical Cepheids. While this behavior might be caused by the fact that different resonances between the fundamental and overtone modes are shaping the light curves for the two groups of Cepheids, it is also possible that the real physical difference is a different pulsation mode for the members of the two groups. We believe that a possible clue to the correct interpretation of the situation is contained in the plot of Antonello and Poretti of the $\phi_{21}$ versus period diagram which we reproduce here in Fig. 9. According to Fig. 9, the *s* Cepheids themselves are clearly split into two distinct groups with a transition occurring at a period near 3 days. The members of the group at $P < 3$ days exhibit much larger values of $\phi_{21}$ and consist of the Cepheids SU Cas, EU Tau ($\phi_{21}$ determined from our data; see Table II), DT Cyg, and AZ Cen. The interesting point is that for all of these stars except EU Tau, evidence for pulsation in the first overtone mode has been presented *independently* from their light-curve parameters (Aikawa *et al.* 1987; Gieren 1982b; Arellano Ferro 1984; Gieren 1982c; Pel and Lub 1978). A plausible picture is, therefore, that these stars are indeed overtone pulsators and that the large $\phi_{21}$ values are a consequence of the different pulsation mode, whereas the other Cepheids with $P > 3$ days (including the longer-period *s* Cepheids) are fundamental mode pulsators with considerably smaller $\phi_{21}$ values. We note that this period distribution of the pulsation modes would be consistent with the predictions of theoretical Cepheid models (e.g., Stobie 1969a,b). Our suggestion that only the *short-period s* Cepheids ($P \lesssim 3$ days) are first overtone pulsators, rather than the longer-period *s* Cepheids as well, is supported by the radius data of GBM which present clear evidence against the existence of overtone pulsators for periods in excess of about 3 days.

In spite of the arguments given above we want to emphasize that the role of $\phi_{21}$ as a pulsation mode discriminator is not yet firmly established. There may exist other mecha-

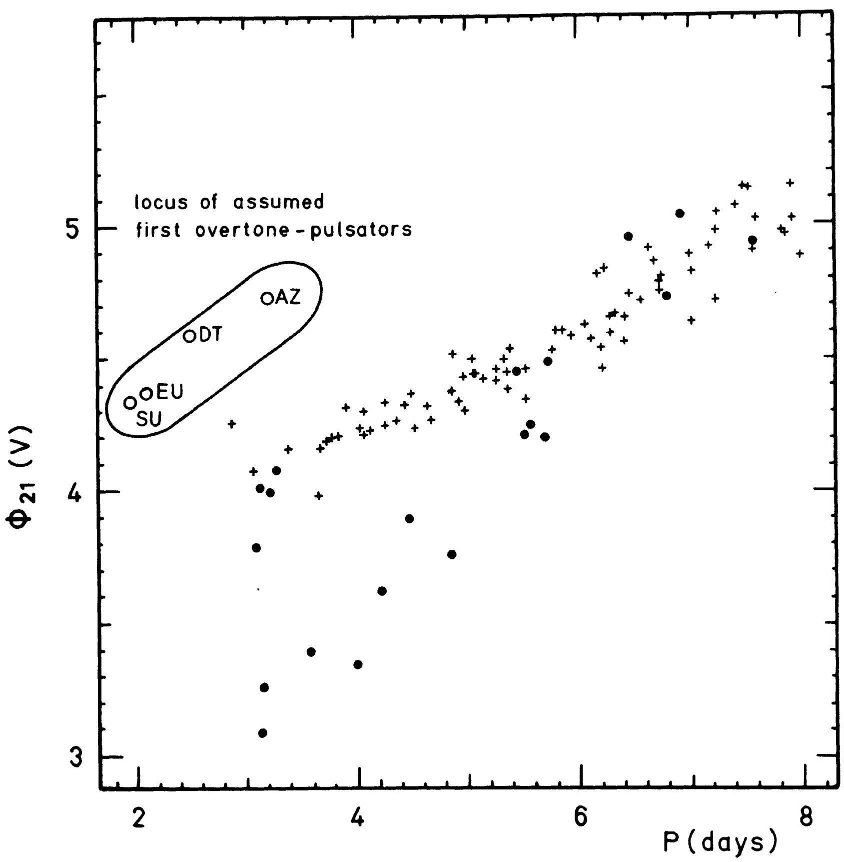


FIG. 9. The phase difference $\phi_{21}(V)$ versus period for a sample of about 100 classical Cepheids. Plus signs, "normal" classical Cepheids; filled circles, *s* Cepheids (see text). The data are adopted from Antonello and Poretti (1986). The location of EU Tau on this diagram (from the data of Table II) is close to that of the other probable overtone pulsators SU Cas, DT Cyg, and AZ Cen. The locus of first overtone pulsators on the $\phi_{21}(V)$ versus period diagram is tentatively identified.

nisms causing the split into the distinct groups of stars observed in Fig. 9. We note in this context that the hydrodynamic models of Aikawa *et al.* (1987) for SU Cas, although providing stronger evidence for first overtone than for fundamental mode pulsation in this star, produce values of $\phi_{21}$ that do *not* differ significantly with the mode. The authors attribute this to shortcomings of their models. Improved hydrodynamic models are clearly needed to resolve this question.

With respect to Cepheid radial-velocity curves, the work of Simon and Teays (1983) and more recently of Kovacs, Kisvarsanyi, and Buchler (1989) has shown that while features like the bump progression are very well described by the low-order Fourier decompositions, there appears to be no definite indication of a noncontinuous variation of any low-order Fourier components as we see it in the light curves at the shortest pulsation periods. It thus appears that overtone pulsators cannot be distinguished from fundamental mode pulsators by Fourier decomposition of their radial-velocity curves. However, the radial-velocity curve of a Cepheid may still yield information about its pulsation mode via the phase lag with respect to its light curve. Simon (1984) has investigated the dependence of the observed phase lags of 12 classical Cepheids on their periods and light amplitudes. In his study, a first-order phase lag $(\Delta\phi)_1$ is calculated from the first-order Fourier coefficients of the observed light and velocity curves according to

$$(\Delta\phi)_1 = \phi_1\ (\text{velocity}) - \phi_1\ (\text{light}) + \pi/2, \qquad (9)$$

where $\phi_1$ (light) is defined by Eq. (8) and $\phi_1$ (velocity) by the series

$$A_0 - \sum A_i \sin[i\omega(t-t_0) + \phi_i], \qquad (10)$$

fitted to the radial-velocity curves.

Simon's results indicated that $(\Delta\phi)_1$ may be correlated with the stellar pulsation mode, but there is also a definite possibility that the phase lag is determined primarily by the light amplitude. From our Fourier coefficients for the $V$ light

TABLE II. Fourier decomposition coefficients for the $V$ light curve, the $(V-R)_J$ color curve, and the radial-velocity curve of EU Tau.

| | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $\phi_1$ | $\phi_2$ | $\phi_3$ |
|---|---|---|---|---|---|---|---|
| $V$(mag) | 8.101 | 0.155 | 0.021 | 0.007 | 2.975 | 4.033 | 4.871 |
| $(V-R)_J$ (mag) | 0.622 | 0.047 | 0.007 | − 0.002 | − 59.685 | 3.948 | 0.968 |
| $RV$(km/s) | − 1.24 | − 8.068 | − 1.650 | − 0.529 | − 0.373 | 0.398 | − 73.51 |

Note to TABLE II

The fitted Fourier series have the form $A_0 + A_i \cos(i\omega t + \phi_i)$ $(i = 1-3)$.

curve and the radial-velocity curve of EU Tau, which are given in Table II, we obtain

$$(\Delta\phi)_1 = -0.206.$$

On both diagrams discussed by Simon, this value seems to correspond more closely to the bulk of stars which are thought to be fundamental mode pulsators than to the loci of SU Cas and AZ Cen. While this can be taken as an argument for fundamental mode pulsation in EU Tau, we think that such a conclusion is premature. It is clearly necessary to determine precise first-order phase lags for many more Cepheids (in particular, short-period ones) to better understand the physical information contained in the phase lags. This requires precise and *simultaneous* photometry and spectroscopy of a large sample of stars. When these data become available, we will be in a better position to judge the utility of $(\Delta\phi)_1$ as a pulsation mode indicator in classical Cepheids, and it may well turn out that the phase lag is indeed a useful parameter in this context. At the present time, however, we believe that there is stronger evidence for the role of $\phi_{21}(V)$ as a pulsation mode discriminator, than there is for $(\Delta\phi)_1$.

### *f) Relative Radius Variation*

Pel and Lub (1978) have studied the relative radius variation of a sample of classical Cepheids with well-determined effective temperatures. On a diagram plotting the relative radius variation $\Delta R/R$ as a function of the Cepheid's effective temperature difference from the fundamental mode blue edge $(\Delta \log T_e)_{FBE}$ (measured at the appropriate luminosities), they were able to identify two distinct groups of stars: one small group clustering around $(\Delta \log T_e)_{FBE} = 0$ and exhibiting very small $\Delta R/R$ values (~5%), and a much larger group of Cepheids showing much larger (~20%) $\Delta R/R$ values extending from the fundamental mode blue edge to the red side of the instability strip. This finding is somewhat reminiscent of the behavior of the type c and type ab RR Lyrae stars and, hence, suggests these two groups of Cepheids might be first overtone and fundamental mode pulsators, respectively. It is interesting to note that apparently *all* Cepheids for which overtone pulsation has been suggested on different grounds belong to this small group of relatively hot stars with low relative radius variations.

For EU Tau, we find from our data a radius variation amplitude of $0.262R_\odot$, and a relative radius variation of

$$\Delta R/R = 0.01,$$

which appears to be the smallest value reported for any classical Cepheid. This clearly puts EU Tau into the group of the low-radius-variation stars, although its value of $(\Delta \log T_e)_{FBE} = -0.02$ is slightly larger than for the other members of the group. We note, however, that the Pel (1978) $T_e$ calibration which was used for the work of Pel and Lub yields $T_e = 6300$ K for EU Tau which reduces the temperature difference from the fundamental blue edge to $-0.007$. This value and $\Delta R/R = 0.01$ make the star coincide perfectly with the small radius amplitude group.

We believe that the very small relative radius variation of EU Tau, in connection with its position close to the fundamental mode blue edge of the instability strip, provides an additional piece of evidence favoring first overtone pulsation in the star over fundamental mode pulsation.

TABLE III. Physical properties of EU Tau.

| | |
|---|---|
| Observed period: | $P = 2.1025112 \pm 0.0000031$ d |
| Radius: | $R = (30.23 \pm 1.43)\ R_\odot$ |
| Relative radius variation: | $\Delta R/R = 0.01$ |
| Distance: | $d = (1191 \pm 57)$ pc |
| Absolute visual magnitude: | $M_{(V)} = -2.77 \pm 0.15$ mag |
| Luminosity: | $L = (1038 \pm 140)\ L_\odot$ |
| Effective temperature: | $T_e = (6107 \pm 200)$ K |
| Evolution mass: | $\mathcal{M}_{ev} = (4.95 \pm 0.20)\ \mathcal{M}_\odot$ |
| Wesselink mass: | $\mathcal{M}_{Wes} = (4.55 \pm 0.54)\ \mathcal{M}_\odot$ (first overtone mode assumed) |
| Pulsation mode: | first overtone |
| Amplitude of $V$ light curve: | $A(V) = 0.313$ mag |
| Intrinsic color: | $(B-V)_0 = 0.510$ mag |
| Color excess: | $E(B-V) = 0.15$ mag |
| Amplitude of $RV$ curve: | $A(RV) = 16.6$ km/s |
| $\gamma$-velocity: | $\gamma = (-1.24 \pm 2.0)$ km/s |
| First-order phase lag: | $(\Delta\phi)_1 = -0.206$ rad |

## V. CONCLUSIONS

We have performed a surface-brightness analysis on the extensive photometric and radial-velocity data of EU Tau presented in Paper I, and have obtained the radius and distance of the star with a ~5% precision. Based on these and other observed parameters for EU Tau, we have derived and discussed the physical properties of the star, including its luminosity, mass, effective temperature, pulsation mode, and binary status. A summary of these properties is given in Table III.

The derived radius, absolute magnitude, and Wesselink mass of EU Tau strongly suggest that the star is pulsating in the first overtone mode. The amplitude ratios and phase differences obtained from the low-order Fourier coefficients of the $V$ light curve fit this picture, and so too does the very small relative radius variation of the star in connection with its fairly high effective temperature. The observed position of EU Tau in the Cepheid instability strip suggests that the star is about equally likely to pulsate in the fundamental and first overtone modes. This and the star's value of the first-order phase lag are the only pieces of evidence which do not favor first overtone over fundamental mode pulsation in EU Tau.

The weight of the evidence found in this work suggests that there is a high probability that EU Tau is pulsating in the radial first overtone mode. The star is thus a welcome addition to an apparently extremely small sample of stars. Future studies must address the question, via improved hydrodynamic model calculations, of the physical conditions that determine the stellar pulsation mode. The set of physical parameters determined in this work for the Cepheid EU Tau, and their precision should assist future research along these lines.

W. P. G. gratefully acknowledges a research fellowship from the Alexander von Humboldt-Stiftung and the hospitality of the Hoher List Observatory of Bonn University. He also acknowledges financial support received from the Universidad Nacional de Colombia, and from Colciencias. T. G. B. and M. L. F. gratefully acknowledge financial support from the McDonald Observatory. J. M. M. acknowledges financial support for this work through grants to W. H. Wehlau and G. A. H. Walker from the Natural Sciences and Engineering Research Council of Canada, and Postdoctoral Fellowships from NSERC and the Isaak Walton Killam Trust.

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