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GRAVITATIONAL INSTABILITY WITH HIGH RESOLUTION

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ABSTRACT

We have performed a series of very high resolution simulations of gravitational clustering in two dimensions. We show that mass resolution, achieved by having large numbers of particles, is as important as spatial resolution. We show new objects arising inside pancakes which are coherent with structure on much larger scales. The evolution of structure in hierarchical clustering models with a sufficiently flat power spectrum includes pancake-like structures, confirming the presence of filaments in cold dark matter universes.

Subject headings: cosmology — dark matter — galaxies: clustering

I. INTRODUCTION

Gravitational instability plays a very important role in the universe, resulting in the growth of primordial density perturbations and formation of the observable structure (e.g., Peebles 1980). Primeval perturbations presumably arose as vacuum fluctuations during the very early inflationary stage when the universe was expanding exponentially. At present, the scale factor characterizing the Hubble expansion of the universe $a(t) \propto t^{2/3}$, assuming that the mean density is close to the closure value. The density perturbations had a long and dramatic history before they became galaxies, clusters of galaxies, superclusters, or voids. The evolution of density perturbations and the formation of the structure in the universe is one of the most important problems in modern cosmology.

There are several rival theories to explain this process. They can be classified by the type of primordial perturbations. Some theories are based on the assumption that primeval fluctuations were random fields of the Gaussian type. Others (e.g., the model of cosmic strings) assume that initial perturbations were non-Gaussian. Here we discuss some problems arising in models based on the assumption of Gaussian fluctuations.

One of the most difficult problems arising in understanding such models is the analysis of the evolution of perturbations at the nonlinear stage, when the typical amplitudes of density inhomogeneities become larger than mean density in the universe: $\delta\rho/\rho \geq 1$. The most straightforward way to solve the problem is to do three-dimensional numerical simulations. Usually in simulations of this type the medium is assumed to consist of collisionless particles, in agreement with the popular hypothesis that most of the mass in the universe is in the form of weakly interacting particles such as massive neutrinos, photinos, or axions. However, three-dimensional N-body simulations take much storage and computer time, since the number of particles needs to be large. A practical solution to this problem is either to perform detailed simulations of an isolated object or else to make rough simulations in a rather large volume containing many objects. The former suffers from unrealistic assumptions about the environment while the latter has poor resolution of the internal structure of individual objects. There is a widespread belief that resolution is determined primarily by the dynamical range of length in the simulation (or spatial resolution), which can be quantitatively specified as the ratio of the largest to smallest scales where the gravitational force is described correctly. The number of particles is considered less important. In this short note we preliminarily report some new results of numerical simulations demonstrating the importance of both high spatial resolution and large number of particles. Full description of the results will be published elsewhere.

Our numerical simulations were performed with a two-dimensional PM (particle mesh) code (Hockney and Eastwood 1980). The code solves for the force in comoving coordinates by Fourier Transform. In the models shown, we have $512^2 = 262,144$ particles on an equivalent mesh. Our models are equivalent to a cross section of an Einstein–de Sitter universe homogeneous perpendicular to the plane and doubly periodic in the plane.

Initial conditions were of nine types. All models had random smooth Gaussian perturbations. All had pure power-law spectra $\langle \delta_k^2 \rangle \propto k^n$, where $n = 2$(Q series), $n = 0$(J series), or $n = -2$(N series). In each case, three alternative cutoffs were imposed on the initial spectrum: waves longer than $L/4$, $L/32$, or $L/256$ were kept, where L is the side of the square. All models began with $(\delta\rho/\rho) = 0.25$ on the scale of cells and were evolved and saved at expansion factors 2^{n-1} up to $n = 8$, or $(\delta\rho/\rho)$ linear theory of 32. Thus, file N4F5 corresponds to a power law $n = -2$, cut off at $L/4$, and stopped at an expansion factor of 16. Amplitudes and phases of waves are identical for models with identical cutoffs.

Our resolution is $L/512$. It is important to note that we have not only correct gravity to that *length* scale, but also we have enough particles to resolve *mass* to that scale. Other methods such as "tree codes" and P^3M can resolve such lengths but slow down greatly when a large number of particles are used. The method we use is by far the fastest when mass is resolved as well as length. When mass is not resolved as well as length, two-body scattering will mean that the code is an incorrect description of a Vlasov-type system such a dark-matter uni-

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verse. Further discussion of some numerical issues can be found in Peebles *et al.* (1989). It will be interesting to see whether multigrid methods (e.g., Villumsen 1988) can suppress unwanted two-body scattering and resolve the kinds of structures shown here, if only in one region at a time. A possible test of such multigrid methods would be to compare high-resolution two-dimensional PM simulations with a multigrid three-dimensional simulation with identical (two-dimensional) perturbations, to see if the three-dimensional simulation can reproduce the fine structure.

The two-dimensional simulations suffer the disadvantage of not being fully generic, and their results cannot be directly compared with observational structure in our three-dimensional universe. On the other hand, they have definite advantages in spatial resolution and in visualizing the process. The results of our simulations with truncated initial power spectra (in particular, the $L/4$ series) most nearly describe the dynamics of collisionless particles in the hot dark matter scenario. This theory is presently unpopular, but cannot be considered totally excluded. However, the $L/32$ and especially $L/256$ series are useful for understanding gravitational processes in the cold dark matter scenario. In any case, these simulations highlight new effects in a collisionless self-gravitating medium at advanced nonlinear stages. We also have found it very useful to make different simulations (e.g., with different spectra) keeping the amplitude and phases same.

In this paper, we present a qualitative discussion of the phenomena found in our simulations. Pattern recognition is not a simple question. No statistics exist which can adequately describe all the patterns and structures one can easily see in the pictures. Since developing statistics of this type is a separate complicated problem, it seems useful to show pictures which provide a great deal of information. We hope our pictures will encourage the development of new quantitative statistics as well as intuition in the very complicated field of nonlinear dynamics in a self-gravitating medium.

The $L/4$ series simulations address the problem of the evolution of internal structure in the densest regions. This is very important for the theory of galaxy formation in the HDM model, including variants with unstable particles. In addition, it is a very interesting theoretical question related to the structures of generic singularities in a self-gravitating medium (Arnold, Shandarin, and Zel'dovich 1982). Before we begin discussing results, we will comment on some tests of numerical method.

Figures 1*a* and 1*b* (Plate 1) show the time evolution of density perturbations for one of the three different spectra discussed earlier. One can easily see the complicated internal structures arising at late nonlinear stages. However, if the figures have less particles these structures are practically invisible. This is demonstrated in Figure 2*a* which was obtained from Figure 1*a* by reducing the number of particles 64 times, to the mass resolution level one might see in a more usual cosmological simulation in three-dimensions. This shows the importance of simulations with a large number of particles. Even the plates used here cannot show all the detail which exists in the computation.

Figure 2*a* shows the importance of having a sufficiently high density of mass tracers to reveal the structures. However, there are additional effects caused by discontinuities in the mass when only a small number of particles are present in the dynamics. Such discontinuities would not exist in a dark-matter–dominated universe with a spectrum of linear density initial perturbations destined to grow into galaxies and clusters. In Figure 2*b* we show the result of a simulation with an identical initial perturbation spectrum which, in fact, only contained the lower density of particles seen in Figure 2*a* from the beginning. If Figure 2*b* were a faithful representation of a collisionless dark-matter system, Figures 2*a* and 2*b* would be identical. They are similar, but not identical. The reader can by inspection see that the internal structure of the lumps and filaments is different.

The high spatial resolution of the force calculation in Figure 2*b* is similar to that in P^3M, direct N-body, "tree" codes, and similar methods. It has not produced a correct tracing of the phase space to the limit of its resolution, because of such effects as shot noise and two-body scattering (see also Peebles *et al.* 1989). It is essentially correct in its gross features, when averaged over some length scale comparable to its mean interparticle separation, $L/64$. This leads one to ask whether a more coarse force calculation might suffice.

Figure 2*c* shows the result of such a coarse simulation with 64^2 particles on a 64^2 mesh, again with the same initial perturbations. The fine structure is somewhat incorrect in both Figures 2*b* and 2*c*; it appears that Figure 2*c* is slightly *better* than Figure 2*b* at reproducing what we can see in Figure 1*a*. However, the computational cost is dramatically different. The simulation series (through F8) that produced Figures 1*a* and 2*a* took 3.5 CPU hr on a Cyber 205; Figure 2*b* took 2.3 hr; and Figure 2*c* took 2.0 *minutes*. The great expense in doing calculations like Figure 2*b* appears not to be justified, since they do not correctly resolve any smaller structures than does Figure 2*c*. Only by using a high enough particle density, as in Figure 1*a*, can the structures be properly revealed. A subset of these particles, as in Figure 2*a*, may be used to calculate the two-point correlation, but using a simulation like Figure 2*b* will produce the wrong result.

These comparisons suggest there is little to be gained in adding spatial resolution below the mean interparticle separation. More accurate simulations mean more particles. This motivated the choice of *PM* early in our research programs on dark-matter (Doroshkevich *et al.* 1980; Melott 1983*b*). Continual improvement is possible with growing computer power.

Careful inspection of previous stages (not shown) gives evidence that the first objects to form at the beginning of the nonlinear stage were pancakes but located close to the places where clumps will later form. It is well known (Zel'dovich) 1970; Arnold *et al.* 1980; Shandarin and Zel'dovich 1989) that pancakes form from Lagrangian regions containing maxima of the largest eigenvalue of the deformation tensor. It is determined by one-dimensional contraction along the direction determined by the eigenvector related to this eigenvalue. The formation of clumps is connected with contraction along the other principal direction and probably begins near the Lagrangian sites of maxima of the second eigenvalue, as predicted by the Zel'dovich (1970) approximation. In generic fields, maxima of the largest eigenvalue never coincide with maxima of the smallest one, so formation of the clumps does not always happen precisely at the sites of the pancake formation. However, the Lagrangian positions of the maxima of the second eigenvalue probably correlate with the positions of the maxima of the largest eigenvalue. Thus, soon after formation the oldest part of the pancake changes its shape and becomes a clump of mass embedded in a highly asymmetric halo. Usually these parts of pancakes become nodes of the cellular structure.

But even pictures with high resolution cannot show the real structures that arise in the denses regions. Some of these structures are shown in Figures 3 and 4. These pictures show small

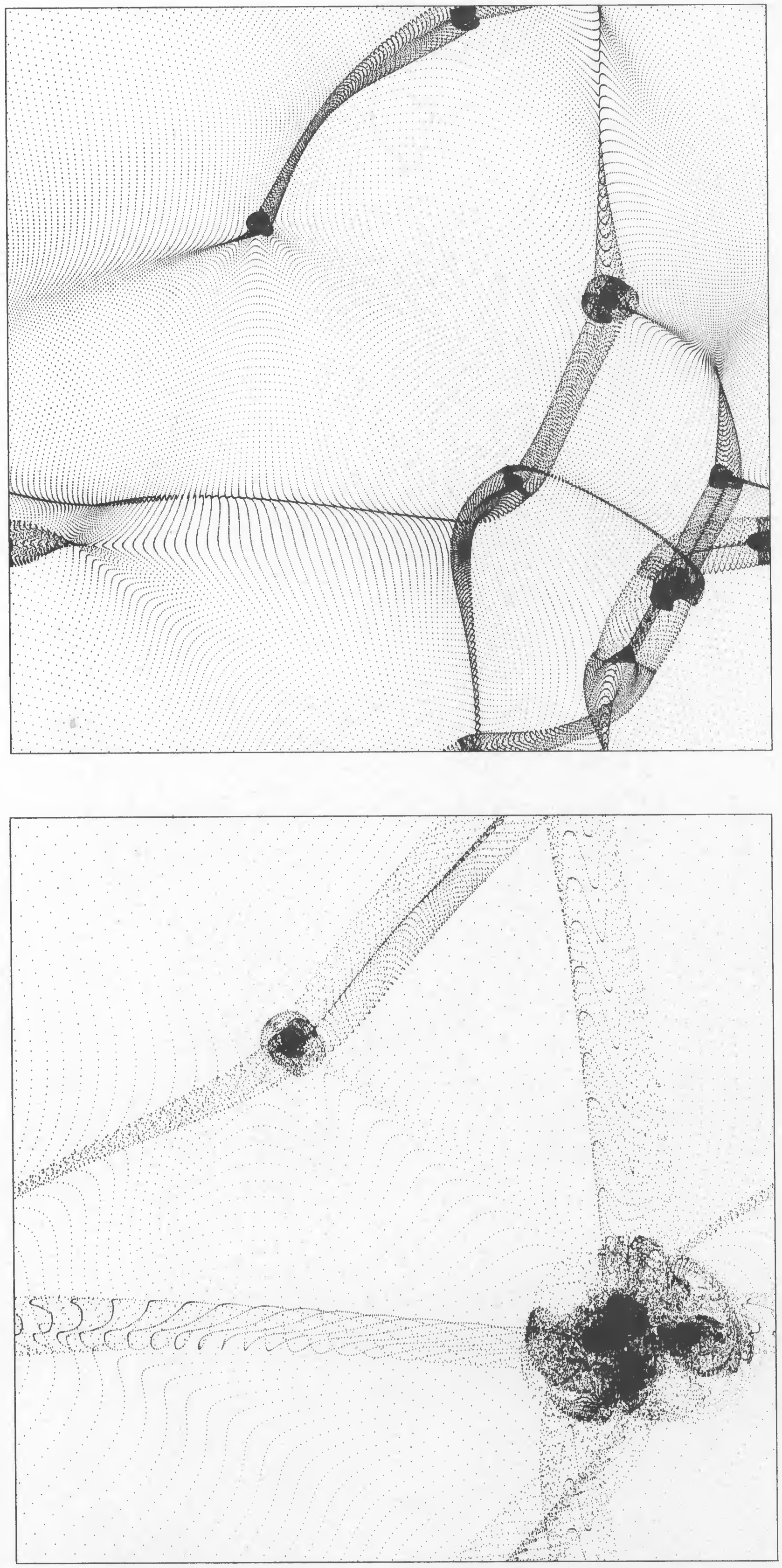

Fig. 1.—(*a*) J4F4. (*b*) J4F6.

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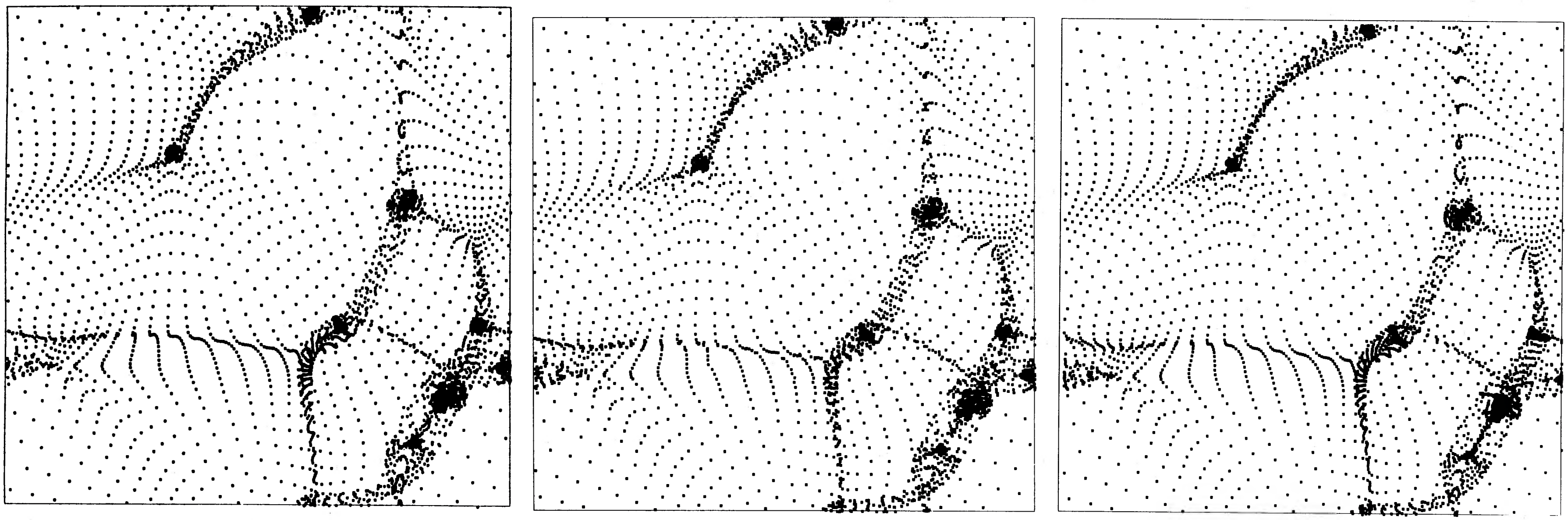

FIG. 2.—(*a*) Same as Fig. 1*a*, but only every 64th point is displayed. (*b*) a simulation with 64^2 particles on a 512^2 mesh tracing the same initial conditions as the J4 series. (*c*) A simulation with 64^2 particles on a 64^2 mesh tracing the same initial conditions as the J4 series.

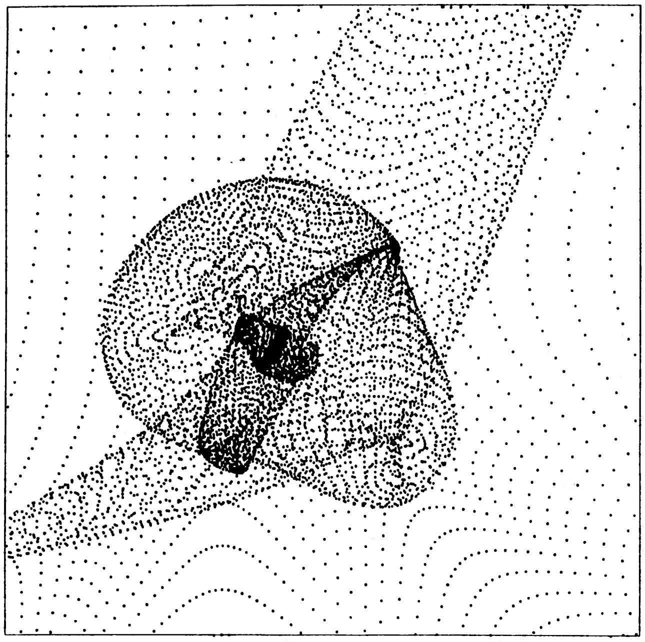

FIG. 3.—J4F4 Center 171, 363. The image is magnified by a factor of 16.

pieces cut out of Figure 1*a*. Figure 3 is a magnified view of the part that can be seen in Figure 1*a* lying close to main diagonal at about one-fourth of the distance from the upper left corner. Figure 4*a* shows the structure in the lower right corners of Figures 1*a* and 4*b* shows an even more magnified view of the structure seen near the bottom of Figure 4*a*.

The first impression of these figures is that they are complicated constructions of generic caustic patterns arising in geometrical optics or in noninteracting media (Arnold *et al.* 1982). In order to make more definite conclusions, additional analysis is needed, especially in the clusters. Some structures are so complicated that one cannot be sure that totally new structures are absent. We also cannot be sure that our resolution is enough to show all structures in the densest regions.

One common feature of these structures attracts attention: they look like a sequence of very elongated structures embedded one into another. It seems that they formed as a result of folding of the phase hypersurface in two orthogonal directions similar to what is known in one-dimensional collisionless self-gravitating systems (Doroshkevich *et al.* 1980; Melott, 1983*a*). This may explain the very strong coherence in the orientations of the structures with different scales. It is tempting to relate this phenomenon with the coherence of the Local Supercluster and much larger structures found by Tully (1987), and possibly also the "sandwich-like" structure found by de Vaucouleurs (1981).

Further evolution shows that the first clumps acquire mass by nonspherical accretion from dense bridges connecting the clumps.

Comparing the structures arising from different initial spectra shows that the larger the value of the spectra index *n*, the more "spikey" the structure (compare Figs. 5 and 6 [Plates 2–3]). The *Q* series (not shown) is even more extreme. The last stage is similar in all simulations. It is characterized by the presence of very massive clumps of mass having noticeably nonspherical shapes.

We have also studied models of types 32 and 256 (the latter with initial power right up to the Nyquist frequency). Such simulations address the question of long nonlinear evolution, which must inevitably arise in the CDM model. The large number of particles is important because at late nonlinear stages most of the mass becomes concentrated in large clumps. Thus, weak filaments as well as small clumps can be easily lost in simulations with low mass resolution. We believe we can demonstrate that pancake-like structures can be expected in

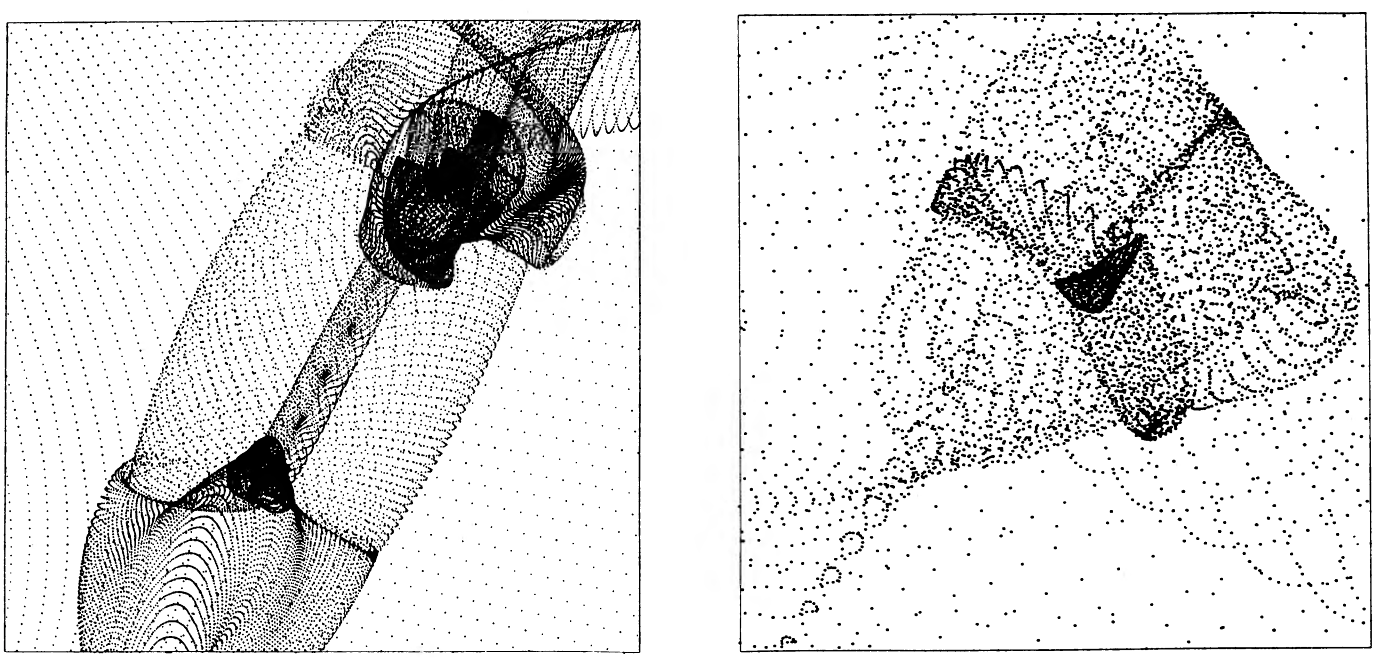

FIG. 4.—(*a*) J4F4 Center 424, 88. The image is magnified by a factor of 5. (*b*) J4F4 Center 416,64. The image is magnified by a factor of 32.

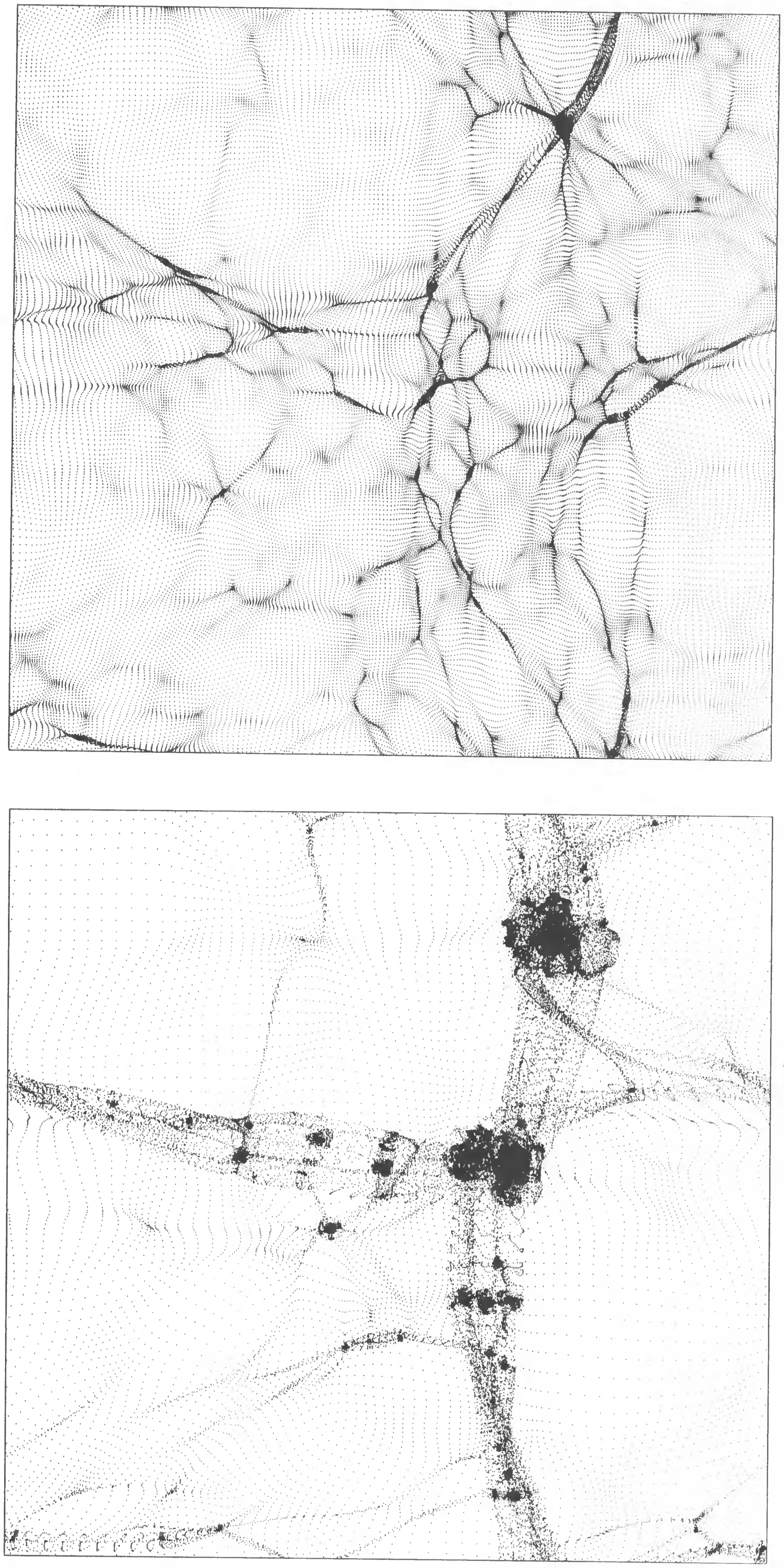

Fig. 5.—(*a*) is N32F3. (*b*) N32F5.

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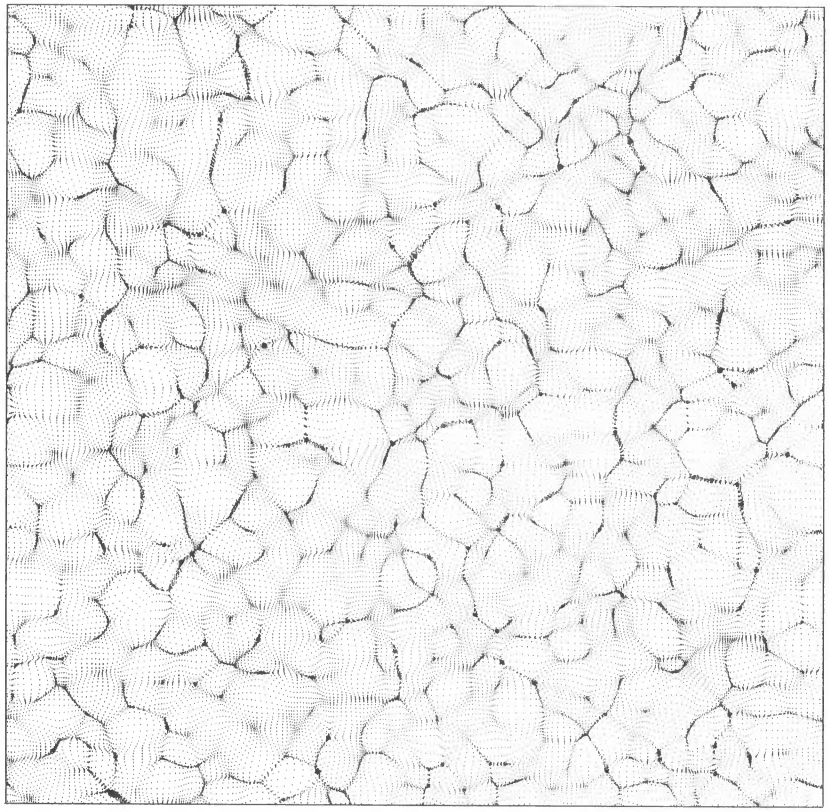

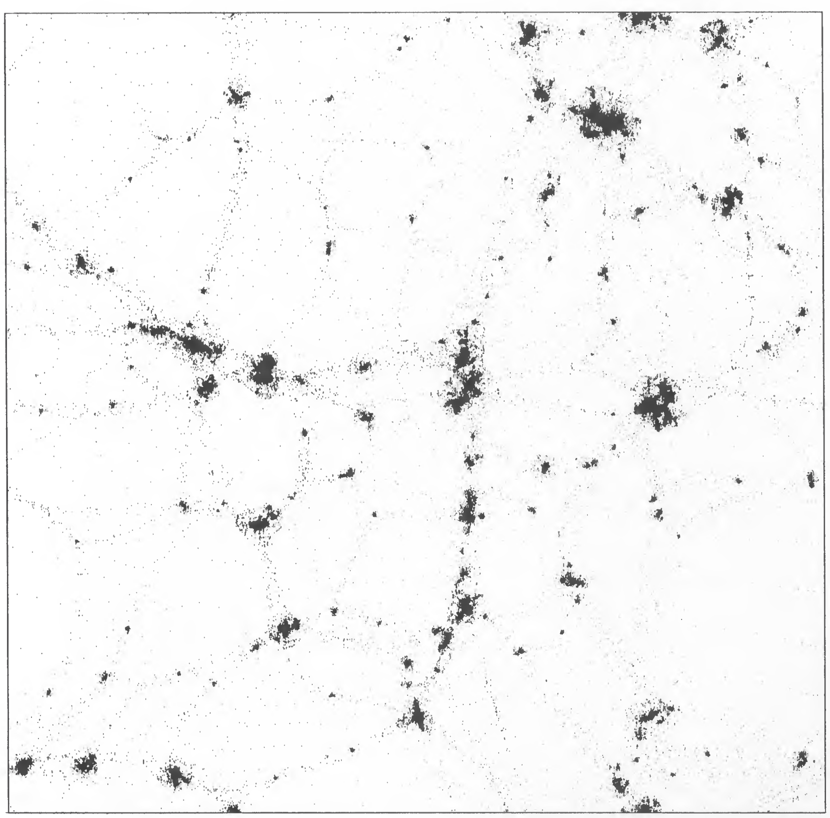

FIG. 6.—(*a*) J32F3. (*b*) J32F7.

models with spectra flatter than $n = 0 (n \leq 0$ in two dimensions and $n \leq -1$ in three dimensions) even in hierarchical clustering. Figures 5 and 6 show models with $n = -2$ and $n = 0$ power spectra, with the same phases.

One of interesting features of the N32 simulation is that at the beginning of the nonlinear stage (Fig. 5*a*), small pancakes having the size of the shortwave cutoff form a pattern that definitely reflects the long wave part of the spectrum which one can easily see at Figure 5*b* and 6*b*.

However, the most remarkable feature of the J32 simulation (which is similar to $n = -1$ in three dimensions) is that it demonstrates the existence of cellular structure for rather long time. The scale factor between Figures 6*a* and 6*b* has increased 16 times. But the cellular structure is still obvious. At this stage the typical sizes of the cells are noticeably larger that at Figure 6*a* and are not related to the initial cutoff. Obviously, structures of clumps in Figure 6*b* coincide with filaments in Figure 5*b*, and are related to long-wave amplitudes. Also, the filaments appear to be oriented parallel to nearly linear sequences of massive clumps. This can justify the presence of filaments in cold dark-matter simulations (Melott *et al.* 1983; White *et al.* 1987).

The later stages of the N32 model show quite rapid evolution resulting in the formation of a few large clumps of matter connecting by filaments. On the other hand, the frame J32F7 shows a much more irregular distribution of matter with much smaller clumps. However, the whole pattern still resembles the regular structure of N32 model.

As we have already mentioned, no direct comparison of two-dimensional simulations with a three-dimensional universe is possible. Yet two-dimensional simulations are widely used in all kinds of numerical work to achieve resolution. They can provide guidelines about what to expect in three dimensions, which must of course be checked. However, in many cases theoretical models are formulated independent of dimensionality. For instance, the assumption that high peaks of the smoothed density field in the linear regime possess similar statistical properties to galaxies and clusters, which are highly nonlinear (Kaiser 1984) must be valid in any dimensionality. Such assumptions can be checked much more easily in one-dimensional or two-dimensional numerical simulations.

We have used simple power-law spectra for initial conditions with $n = 0$ and $n = -2$ because they are similar to $n = -1$ and $n = -3$ in three dimensions, in the following sense: integrals of the spectrum determining statistical properties of random fields behave the same way. The linear spectrum of CDM after decoupling gradually changes from $\langle \delta_k^2 \rangle \propto k^{-3}$ to $\langle \delta_k^2 \rangle \propto k^{-1}$ or k^0 as the scale increases from globular clusters through galaxy clusters and superclusters. We believe that study of simple power-law spectra can give rough but useful ideas about features of dynamics with other spectra, because they clearly show the dependence of morphology on spectral index.

Summarizing our results, we wish to stress the main points. When interpreting the results of numerical particle simulations, one must keep in mind the possible influence of insufficient mass or spatial resolution, or both. Low resolution of either kind can make it impossible to see internal structure in dense regions. It can also erase the filaments or pancakes (in three dimensions) at advanced nonlinear stages. However, lower resolution simulations do correctly describe structures to the limit of their spatial or mass resolution, whichever is worse. PM methods, used here, keep these comparable.

Both the complicated internal structure of pancakes and the pancake/hierarchical hybrid character of models with negative power spectral indices need considerable future study.

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