

# Numerical simulations of accretion discs – I. Superhumps: a tidal phenomenon of accretion discs

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**Summary.** Numerical simulations of the SU UMa star Z Cha in outburst are presented which demonstrate the role of the secondary's tidal influence upon the accretion flow and provide explanations for the superhumps seen in SU UMa stars. They show that for a system with an extreme mass-ratio  $q=0.15$ , the disc is tidally unstable and can become asymmetric and slowly rotate in the inertial frame of reference. The tidal stresses raised in this disc by the secondary produce a peak in the light curve of order 30 per cent which recurs with a period 3.5 per cent longer than that of the orbit. This is interpreted as the superhump. Analytic considerations show that only for systems with mass-ratios more extreme than approximately 4::1 will such behaviour occur, which is consistent with the known mass-ratio of SU UMa stars and other cataclysmic variables. Arguments based on the observed occurrence of superhumps in SU UMa stars are advanced which suggest that outbursts are periods of high disc viscosity whereas in quiescence the disc is in a low-viscosity state. This supports the general predictions of disc-instability models over mass-transfer driven bursts. It is, however, found to be likely that mass-transfer is high during the superoutburst of SU UMa stars.

## 1 Introduction

This paper describes the results of numerical gas-dynamical simulations, made using a method developed by Whitehurst (in press, henceforth Paper II), of the accretion flow in a member of the SU Ursae Majoris subclass of dwarf novae. Of interest is how the superhumps of these stars may be produced by the dynamics of the accretion flow.

In Section 2 the problem of superhumps and superoutbursts in SU UMa stars is outlined, along with a description of various models which have been proposed to describe this behaviour. Section 3 presents the results of the simulations, compares them to the observations and provides explanations for their behaviour; these results are then used to develop a new, predictive model of superhumps. In Section 4 the implications of this model on the nature of the dwarf nova outburst mechanism are discussed.

## 2 Observations and models of superhumps

The SU Ursae Majoris stars form a subclass of the dwarf novae which are distinguished by their outburst properties. In particular they exhibit two distinct forms of outburst; short *normal outbursts* of length typically 2–3 day and long *superoutbursts* which have durations from 10 to 14 day. Intermediate length outbursts do not occur.

The SU UMa stars have been of great theoretical interest since the middle of the 1970s when Warner (1975) and Vogt (1974) reported the presence of the so-called *superhumps*. These periodic enhancements to the stars' light occur only in superoutburst, have amplitudes of 20–30 per cent of the total brightness and recur with a period a few per cent longer than the orbital period. The universality of superhumps within SU UMa stars is such that their presence has almost become the defining property of the subclass. Furthermore note that, despite attempts to observe them, superhumps have not been seen in the other subclasses of dwarf novae. The more important of the superhump and superoutburst properties are listed below; a detailed review can be found in Warner (1985).

### 2.1 OBSERVATIONS

*Period.* The superhump period  $P_s$  is remembered from one superoutburst to another for all SU UMa stars. It always exceeds the orbital period  $P_o$  by a few per cent. An empirical relationship exists between the superhump and orbital periods (Stolz & Schömb's 1984). During the course of a superoutburst the superhump period appears to decrease slightly.

*Occurrence.* Superhumps *only* occur in superoutbursts. Furthermore they do not appear immediately with the rise to maximum light but after an interval of a few days.

*Persistence.* After the decline to quiescence 'late-superhumps' have been observed in the SU UMa star VW Hydri, e.g. Häfner, Schömb's & Vogt (1979). These are a modulation of the normal orbital hump seen in VW Hyi in quiescence with the beat period  $P_b$  between the orbital and superhump periods.

*Beat period properties.* In addition to the late-superhump two other properties have been observed to occur with the beat period. These are variations in the total system brightness (Schömb's 1986) and the central or  $\gamma$  velocity of the Balmer lines in absorption (Vogt 1982 and Honey *et al.* 1988).

It is these properties, along with others such as the superhump shape and size, which provide the constraints which superhump models must satisfy.

### 2.2 MODELS

Many models have been proposed to explain the superhump properties of the SU UMa stars in superoutburst. These have included the starspot model, the eccentric orbit model and the intermediate-polar model. The first of these, proposed initially by Vogt (1974) and expanded upon by Whitehurst, Bath & Charles (1984), places a bright region upon the surface of the secondary. This star is supposed to be rotating non-synchronously which allows the visibility of this bright region to vary with a period which is slightly longer than that of the orbit, thus providing the observed superhump period. Although attractively simple this model in fact fails to provide a satisfactory explanation for many of the observed features of superhumps.

Another attractive model is that of Papaloizou & Pringle (1979) who proposed that the binary's orbit is slightly eccentric in SU UMa stars. This would provide a natural way of modulating the mass-transfer through the  $L_1$  point as the separation between the stars varies around each cycle. This modulation gives rise to the superhump through the change in the amount of energy being deposited in the disc by the stream. The tidal forces between the stars causes the precession of the

orbit's line of apsides so the period of the radial excursion becomes a few per cent longer than the orbital period. The model improves upon the starspot model in that it can explain more of the superhump's features and provide a predicted superhump period. Unfortunately it cannot explain why superhumps do not appear in quiescence and the range of predicted periods is also too small to satisfy the observations. Recent calculations by Edwards & Pringle (1987) indicate the mass transfer modulation and consequential brightening of the disc should be greater in quiescence than in outburst. In addition some features of the SU UMa stars, in particular the beat period variations, remain unexplained by this model.

A phenomenological model proposed by Warner (1985) to account for the superhump's properties is the 'intermediate polar model'. In this the white dwarf has sufficient magnetic field strength to direct the accretion flow on to one of its magnetic poles. This pole then acts as a 'searchlight' to illuminate material in the disc. The interaction between the spin period of the white dwarf (assumed to be the beat period), and the orbital period of the system gives rise to the observed superhump. The problems associated with this model are mainly theoretical (the model being constructed to fit the data), but it does seem unlikely that the white dwarf will spin so slowly despite the accumulated angular momentum from the disc. (In the intermediate-polar stars the white dwarf spin period is always less than an hour or two, often much less, whilst the beat periods required here range from 2 to 3 day.)

Two further models, also empirically based, appeal to eccentric orbits in the accreting material to generate the superhump. The model of Vogt (1982) places an elliptical ring of accreting material outside the disc. The superhump is generated by the varying amount of kinetic energy released by the bright-spot formed by the shock front between the stream and the ring. (This is at a maximum when the shock is deepest in the potential well.) However, serious theoretical and observational objections exist to this model, e.g. Whitehurst *et al.* (1984). The model of Osaki (1985) can be seen as an attempt to obviate many of these difficulties and provide some theoretical basis for the existence of eccentric orbits in the accretion flow. Here the disc itself is eccentric, and this in turn drives a variable resonant mass-transfer from the secondary. This modulated mass-transfer is now the principal source of the superhump. Whilst this model explains many of the superhump's properties it still is not clear why the eccentric disc forms in the first place. Furthermore, the existence of the phase relationships required to drive both the mass-transfer modulation and the eccentric disc is not established.

A common failing of all the above models is their lack of predictive power. They mostly fail to give even a superhump period and none explains why the SU UMa stars behave differently from the other dwarf novae. Furthermore, no model addresses the question of why SU UMa stars are all ultra-short-period binaries, indeed all but one lie below the period gap.

### 3 The accretion disc in Z Chamæleontis: simulations and stability

#### 3.1 THE INITIAL SIMULATION

In an attempt to test the claims of Vogt and Osaki the numerical method to be described in Paper II was applied to a system representing the SU UMa star Z Chamæleontis. This star was selected because as an eclipsing system its parameters are known accurately, e.g. Wood *et al.* (1986). Note that no attempt was made to 'prime' the simulation with information about superhumps, the only difference of note between this simulation and those described in Paper II for the U Gem star SS Cygni is the different mass-ratio. (The primary mass  $0.54 M_{\odot}$  and secondary mass  $0.081 M_{\odot}$  imply a mass-ratio  $q$  of 0.15, against 0.6 used for SS Cyg.)

The numerical method employed is to be described in detail in Paper II, and I shall only give a brief outline here. The computations are carried out in a two-dimensional zone extending 10

binary separations from the primary. Therefore the results presented below cover less than 1 per cent of the total computational area. The accretion flow is assumed to commence as a mass-transfer stream emerging from the secondary at the  $L_1$  point. This matter is represented by a large number of interacting test particles which carry the mass and kinetic energy of the gas.

The numerical method is essentially similar to that employed by Lin & Pringle (1976) to model steady-state accretion discs. The same method was used recently by Lin, Papaloizou & Ruden (1987) to calculate material distribution within planetary rings. As in these papers the particles are advanced through selected time-steps according to the influence of the long-range gravitational forces. Upon completing these time-steps the particles interact to simulate the action of the short-range forces within the disc, i.e. the pressure and viscosity forces.

These short-range forces of pressure and viscosity are calculated through the collisional interactions between pairs of neighbouring particles in a manner first suggested by Larson (1978). The pairs are selected so that each particle finds the particle that has the shortest *interaction time* with it, i.e. separation divided by line-of-centres approach velocity is minimized. Only particles that lie within a certain *viscous scale-length* (an imposed horizon) are considered. The artificial viscosity imposed is in fact identical to that used in smoothed particle hydrodynamics (SPH) codes in the limit of highly supersonic flows (Monaghan 1985), and is superior to that employed in particle-in-cell techniques where numerical viscosity confounds the results, e.g. Sawada *et al.* (1987). The flow is compressible and dissipation can be measured through the loss of the particles' kinetic energy during the collisions. Finally, the strength and form of the viscosity can be varied by changing the elasticity of the collisions and (more importantly) the rules determining any given particle's viscous scale-length.

The limitations of the method are linked to the strong assumption that the disc is thin, effected by the two-dimensional nature of the calculations. This has the consequence that no flows in the vertical direction can be calculated. Another, less obvious, limitation is that the radiation-transfer processes within the disc are effectively neglected by this assumption. (To do anything else is incompatible with the assumption of disc thinness.) The simulations made are therefore concerned primarily with the dynamics of the accretion disc rather than its radiation transport. Global variations in the radiation field can still be recorded, as well as other principal features such as the bright-spot.

The computations use various parameter sets in simulating the disc in these systems, some of the more important ones being given below. The primary radius is taken as  $0.02a$ , this quantity only being of importance in determining the accretion surface of the primary. Here  $a$  is the binary separation, taken as  $a = 4.42 \times 10^8$  m. Along with the particle mass  $m_p$  this is one of the parameters which determine the absolute values of the calculated quantities (such as mass-transfer rate and luminosity), but do not affect the form of the results. The particle mass used is  $m_p = 5 \times 10^{15}$  kg. For the number of particles added in this simulation this gives a quiescent mass-transfer rate of  $2 \times 10^{12}$  kg s $^{-1}$ , or  $3 \times 10^{-11} M_\odot$  yr $^{-1}$ .

For the first simulation shown below the viscous scale-length is taken as  $0.02a$ , the interactions' coefficient of restitution as  $Q=0.7$  and the pressure constant as  $C=0.019$ . (Paper II discusses the significance of these quantities.) The mass-transfer burst is simulated with the addition of 6000 particles in a Gaussian pulse with a 6 standard deviation width of  $2.5 \times 10^4$  s.

All these parameters give the evolution shown in Fig. 1. Initially, in frames (a) and (b) a small, annular disc forms with a radius determined by the initial angular momentum of the stream. This ring spreads outwards [frame (c)] under the influence of the viscosity to fill its tidal radius by frame (d). The resultant, tidally elongated disc is not gravitationally stable though, and in frames (e) and (f) the disc has become a *precessing, eccentric disc*. The transformation of the disc from one form to the other is shown in greater detail in Fig. 2, where each frame is exactly three periods apart. The switch is effected by the steady collection of the outer disc on to a 1 to 1 resonance with

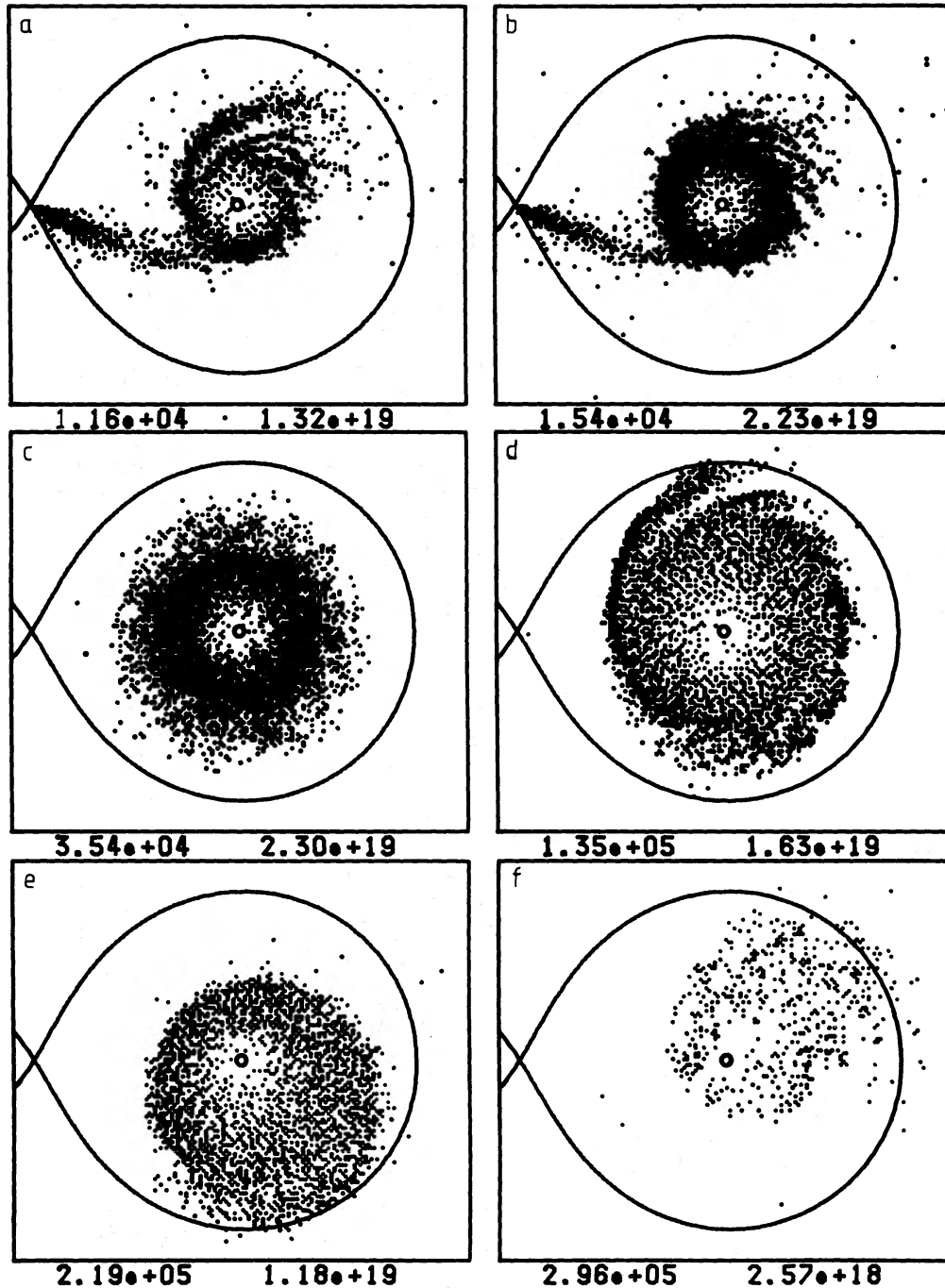


**Figure 1.** Evolution of a simulated outburst in Z Cha. As in all similar plots each point represents at least one test particle (for convenience of plotting the particle positions have been regularized on to a Cartesian grid). The sequence (a)–(f) shows a series of snapshots of the disc, the first of the two numbers under each snapshot gives time in seconds from the burst commencement. The second value is the disc mass in kilograms (each particle being assumed to have mass  $5 \times 10^{15}$  kg). The outer solid line is the Roche potential, the inner one the assumed primary radius (a free accretion surface).

the secondary. The whole process of disc growth and transformation is complete  $2 \times 10^5$  s after the initial mass transfer burst (rather more than 2 day). The average precession rate of the disc is quite slow, the disc moving by about 3.5 per cent of a cycle in each orbital period [e.g. compare Fig. 2(e) and (f)].

The outburst decays away rapidly, as illustrated by Fig. 3 which shows the total bolometric flux

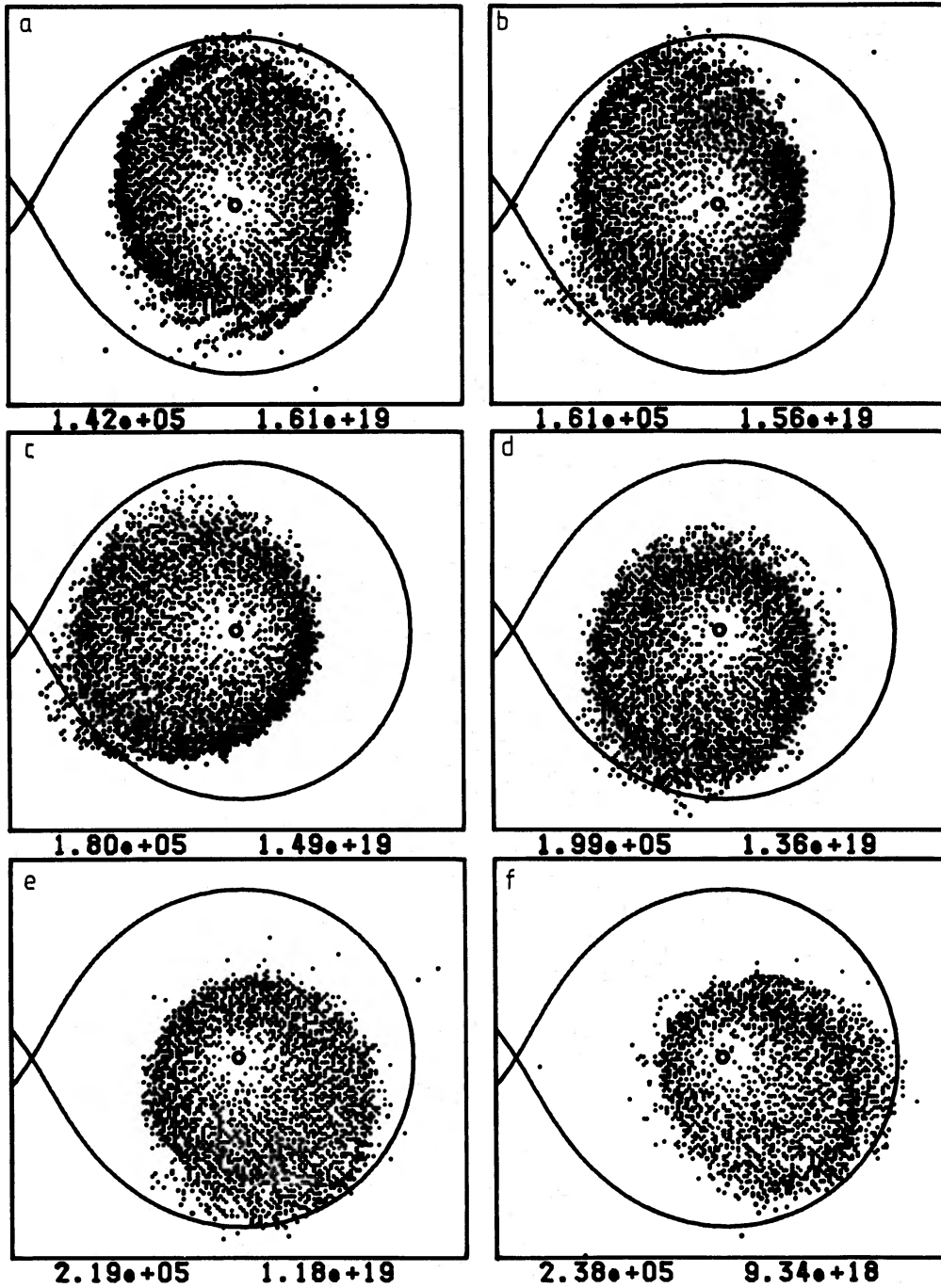


**Figure 2.** Transformation of the disc from the initial corotating mode to the eccentric 1:1 resonant mode. Each snapshot is precisely three orbital periods apart. Note that the rotation of the disc is clear in snapshots (d), (e) and (f).

dissipated in the disc by the viscosity. The rapid decay is due to the high mass-loss rate on to the secondary from the elongated disc as it sweeps by this star once every cycle. The disc retains its eccentric shape throughout the decline, as can be seen in Fig. 1(f).

The correspondence between the precessing disc and the eccentric disc models of superhumps is clear, but first we need to understand why the disc behaves in this way.

### 3.2 TIDALSTABILITY

The truncation of the disc by the tides raised by the secondary is well known, e.g. Paczyński (1977). This process forces the disc's orbital streamlines to remain inside the *tidal radius*. Note,

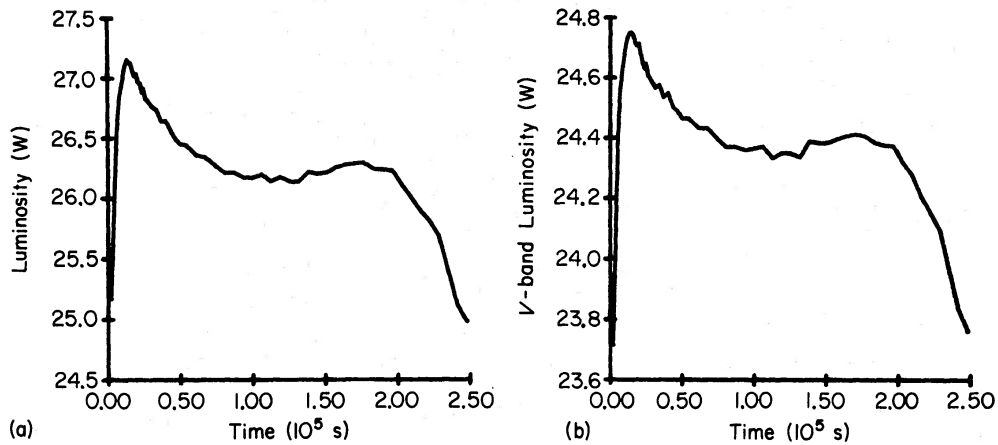


Figure 3. Integrated luminosity from the simulated Z Cha outburst against time.

however, that viscosity may allow the disc to grow a little beyond the value calculated by Paczyński.

In addition to the tidal radius, which affects not individual streamlines but the disc as a whole, the tidal stability of each streamline needs to be considered. Imagine the disc to consist of a concentric set of orbital streamlines, increasingly distorted from the circular as the tidal radius is approached. The tidal stability of each streamline can be found by perturbing it slightly from its reference locus. If the tidal forces act to return this streamline to its original course then it can be regarded as stable; if not, as unstable.

The general problem of the stability of a coplanar orbit in a given potential has been treated by Chandrasekhar (1942). In addition Darwin (1911) and Henón (1965) have solved the problem for special cases of periodic orbits in Roche potentials. For a simple periodic orbit of the sort followed by an accretion disc's streamlines the determination of the orbital stability is quite simple. In fact

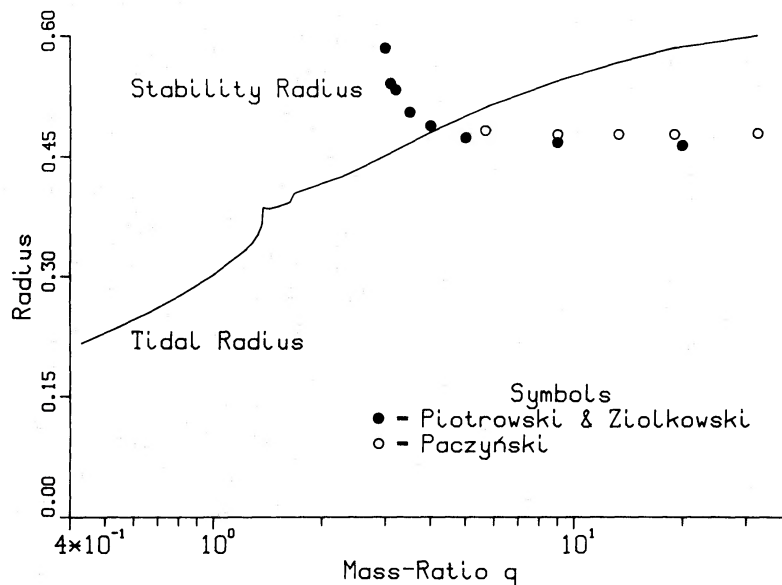


Figure 4. The solid line gives the calculated maximum size of the tidal radius of an inviscid disc as calculated by Paczyński (1977) as a function of mass-ratio. The filled and open circles show the maximum size of a stable streamline in these potentials as calculated by Piotrowski & Ziolkowski (1970) and Paczyński (1977) respectively. The intersection for inviscid flows is at a mass-ratio of 4:1.

the size of the perturbations from the initial orbit need be considered only once every cycle, as the orbit crosses the system's axis. If the perturbation grows then the orbit is unstable, e.g. Henón (1965).

The numerical calculations required to determine the stability of a given orbit have been performed by Piotrowski & Ziolkowski (1970) and Paczyński (1977). Their results are shown in Fig. 4, which gives the size of the maximum stable orbit as a function of the reciprocal mass-ratio  $1/q$ . The solid line indicates the tidal radius of an (inviscid) accretion disc within such a binary potential, and it is clear that for mass-ratios more extreme than 4::1 the disc is potentially unstable in its outer region. This was appreciated by the above authors but they had no way of knowing what the effect of this upon the disc would be.

With the help of the numerical simulation described above the implications of this tidal instability in the disc are now made clear. When the disc is unstable (as in the case of  $q=0.15$ ) it becomes dominated by a 1::1 resonance with the secondary. The disc is not *exactly* fixed in the inertial (i.e. non-rotating) frame of reference, but rotates slowly at a rate determined by the tidal interactions between the disc and the secondary.

By a simple set of approximations, outlined by Osaki (1985), an analytic expression can be obtained for the precession period of the disc as a function of the disc radius and the binary potential. This uses a symmetric potential plus a tidal perturbation from the secondary, assumed to be acting in the radial direction alone, to give the relationship:

$$\frac{\Delta P}{P_0} = \frac{\omega}{\Omega_s} = \frac{3qr^{3/2}}{4\sqrt{(1+q)}a^3}.$$

Here  $\Delta P = P_s - P_0$  the superhump period excess,  $\omega$  is the average precession rate of the disc and  $\Omega_s$  is the secondary's orbital velocity (equal to unity in the simulations). The size of the disc is  $r$  and  $a$  is the binary separation. For  $q=0.15$  and  $r$  taken to be the tidal radius the precession rate  $\omega$  is  $0.04 \Omega_s$ .

Note that this formalism applies to individual, isolated particle orbits with a given (fixed) radius  $r$ . In fact the disc is not circular and cannot even be considered as a collection of non-interacting streamlines since viscosity communicates additional information radially through the disc. The close correspondence between the analytic result and the calculated result does suggest this approximation is indeed adequate for a rough measure of the disc's rotation rate.

The greatest uncertainty in the determination of the disc's rotation rate is the selection of  $r$ . The variations in the disc's density and the asymmetry of the orbits of the material in the outer disc can be regarded as being hidden in this parameter. A reasonable upper limit for it can be taken as the maximum possible radius of the disc found by Paczyński (1977). No lower limit can be specified because it is unclear how much the viscous influence of the stable inner disc can dilute the tidal response of the outer disc. The maximum possible value of the disc's rotation rate calculated using the above relationship is  $0.10 \Omega_s$ , but this is for a mass-ratio  $q$  of 0.6, where the disc is stable. For mass-ratios more extreme than 4::1, i.e.  $q < 0.25$ , the predicted superhump period excess diminishes from a maximum of 5.7 per cent with increasingly extreme mass-ratios. This is adequate to explain all the observed superhump periods with the exception of TU Men where the superhump period is over 7 per cent longer than the orbital period. This may be due to the action of viscosity, which allows a relatively low mass-ratio system's disc to become unstable.

### 3.3 SUPERHUMP SIMULATIONS

The precessing, elliptical disc produced by the above simulation is the basis of the superhump model proposed by Osaki (1985). Furthermore, its period is in agreement with that required to

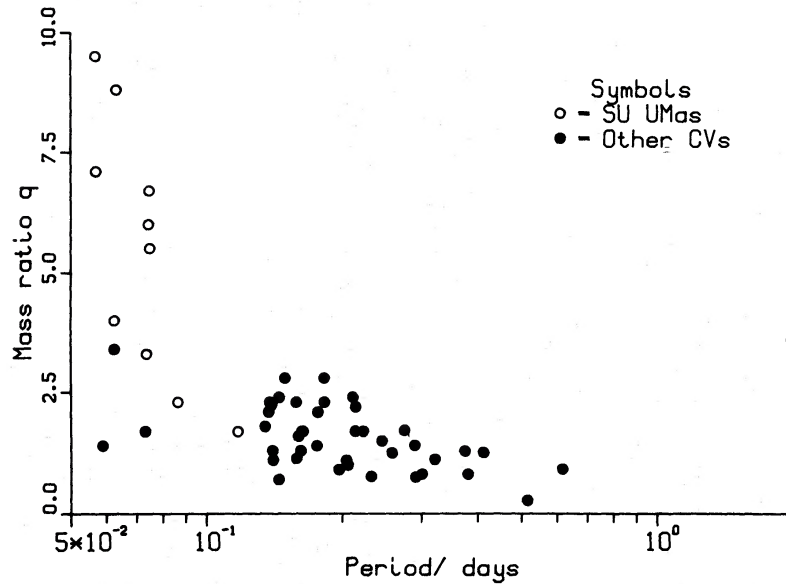


**Figure 5.** The observed mass-ratio versus period relationship for SU UMa stars (filled circles) and other non-magnetic cataclysmic variables (open circles). The data are taken from Ritter (1987) and demonstrate that SU UMa stars have as a group the required extreme mass-ratios to generate the tidal instability required to produce superhumps, whilst other CVs do not.

provide the superhump clock and with the analytically derived result of the period. This is the first prediction of a new superhump model, that the superhump is a property of the disc and that its period is a consequence of tidal actions upon the disc.

In addition the division between SU UMa stars and other dwarf novae can be explained in terms of the stars' mass-ratios. This is because in those dwarf novae where the mass-ratio is less extreme than about 4:1 the disc is always tidally stable. This prevents the formation of the precessing disc and instead the disc, although tidally distorted, remains fixed in the corotating frame of reference (see Paper II for simulations showing this). It is well known that dwarf novae with short periods have relatively extreme mass-ratios, as can be seen from Fig. 5, which plots all the available estimated mass-ratios against period for non-magnetic cataclysmic variables. With only the exception of the SU UMa star TU Mensae the mass-ratios form two non-overlapping groups with SU UMa stars having considerably more extreme mass-ratios. Thus the model's second prediction is that stars which show superhumps should have extreme mass-ratios (for inviscid discs the mass-ratio  $q$  should be less than 0.25).

Because the disc produced by the simulation is a precessing ellipse we know that it will satisfy many of the other requirements of a superhump model [by analogy with the model of Osaki (1985) for example]. But the nature of the superhump is not yet established. In order to determine the origin of the extra light which gives rise to the superhump, various further simulations were run. These varied from that described above in that an extended mass-transfer burst was included. This was meant to replace the material that is lost from the disc and so sustain the outburst for the duration required of a superoutburst. The brightness and length of the superoutburst proved easy to mimic, but the superhump itself was not reproduced. Although these simulations produced large variations in the brightness of the disc around each cycle none could reproduce the feature of a single superhump per cycle. Instead double-peaked light curves were produced with a period of  $P_s/2$ .

The difficulty was due to the relatively large amount of material present in the outer disc which was exciting the tidal shocks associated with the corotating disc's 2:1 resonances. The high density in the outer disc was a consequence of the precise form of the viscosity parameterization

used. As discussed in detail in Paper II and outlined briefly above a viscous scale-length is defined which is the horizon within which each particle searches for a neighbour. The interaction between these neighbours determines the viscous and other short range forces. The scale-length in these simulations is constant over the whole computational zone. Paper II demonstrates that in this case the shear viscosity  $\nu$  scales as  $\nu \propto r^{-3/2}$ . Now for an  $\alpha$  disc viscosity law  $\nu$  scales as  $r^{3/8}$ . It is possible to alter the chosen scale-length to give a viscosity–radius relationship which is closer to that of the  $\alpha$  disc. This is achieved by setting the scale-length to be proportional to the disc radius. In other words the particles are now allowed to know where in the disc they lie before they start to search for their neighbours. This knowledge is then used to restrict the search zone when the particle is near the centre of the disc. Because the velocity dispersion of particles increases strongly with increasing separation the particles found will now be moving much less rapidly so reducing the shear and the consequential dissipation in these areas. In fact the viscosity  $\nu$  becomes proportional to  $r^{1/2}$  when the viscous scale-length is directly proportional to the radius.

The results of a simulation run with this viscosity law are shown in Fig. 6. Immediately obvious is that the disc, as expected, is much more centrally condensed. In addition the 1:1 resonance, when it appears, is weaker. This is because the amount of material in the outer disc (where the disc is unstable), is a much smaller fraction of the total disc mass in this simulation. The density distribution of the previous simulations functioned as a ‘flywheel’, exaggerating the degree of eccentricity and the importance of the outer resonances to the structure of the disc.

The evolution of the burst is unchanged in its broader aspects, however. The initial mass burst again establishes a small disc whose radius is determined by the angular momentum of the stream. As the disc grows through the action of the viscosity it again becomes asymmetric under the tidal forces of the secondary and becomes a precessing disc.

Fig. 7 shows the form of the disc through one orbital cycle, giving views of the disc exactly  $0.2 P_0$  apart. The disc is much less purely elliptical, and in fact the shape of the disc changes considerably around each cycle. Furthermore the disc’s asymmetry (it can now hardly be called eccentricity), is much reduced. Fig. 8 shows the corresponding visible flux from the disc. Because the flux has to be integrated over each time-step those shown here are for each tenth of a period immediately prior to the displayed snapshots. The amount of the flux originating from the disc varies strongly around each orbital cycle, being a maximum in Fig. 8(c), when the disc’s long axis is oriented approximately along the stars’ line-of-centres. This variation is due to the changing tidal torques experienced by the disc as it changes its orientation with respect to the binary potential (this being also responsible for the changes in the disc’s shape around each cycle). As the tidal influence of the secondary is at a maximum along the stars’ line-of-centres it is here that the disc is stressed the most. The extra stress makes itself apparent as an increase in the kinetic ‘temperature’ of the simulations’ constituent particles which in turn increases the dissipation in the disc (by about 25 per cent in this simulation).

Again because the disc is not completely fixed, but precesses slowly in the inertial frame of reference, the times of maximum stress in the disc are separated by slightly more than one orbital period. In fact this period in this simulation is approximately  $1.035 P_0$ , which can be compared to the observed Z Cha superhump period of  $1.037 P_0$ .

It is clear that we now have the basis of a new, and predictive, superhump model. The essential feature is that the superhump is a tidal property of accretion discs. A viscously evolved (i.e. tidal radius filling), accretion disc is tidally unstable in systems with mass-ratios more extreme than 4:1. In these systems, which correspond to the SU UMa stars, the disc is driven into a precessing, roughly elliptical shape. This disc is deformed by the tidal influence of the secondary during every orbital cycle. This deformation releases a varying amount of tidal energy around the cycle which is evident as the superhump. The superhump period is a simple consequence of the precession of the disc, which means that the moments of maximum tidal stress in the disc are separated by

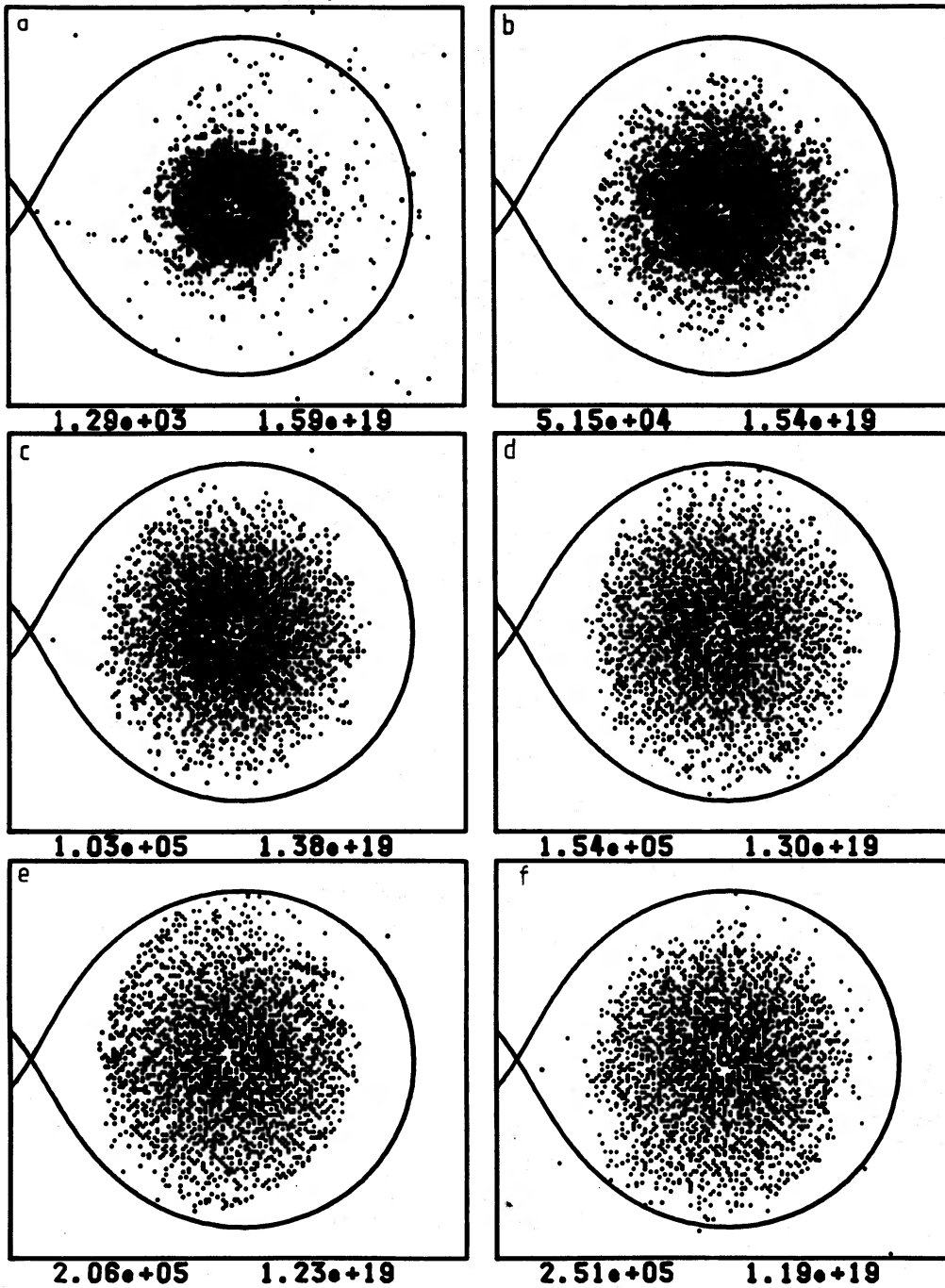


**Figure 6.** Evolution of the simulated Z Cha outburst with the new viscosity scheme. The major differences from Fig. 1 are that the disc is now much more centrally condensed and does not lose mass rapidly in the 1:1 resonant mode.

slightly more than one orbital period. The third prediction of the model is that the superhump is due to the action of the tidal forces of the secondary acting upon the disc.

### 3.4 DISCUSSION: THE OUTBURSTS OF Z CHAMÆLEONTIS

The ideas introduced so far in Section 3 are not as yet a fully developed superhump and superoutburst model. This can be seen by describing the progress of our simulated outburst and comparing it with the outburst properties observed in the star Z Cha.

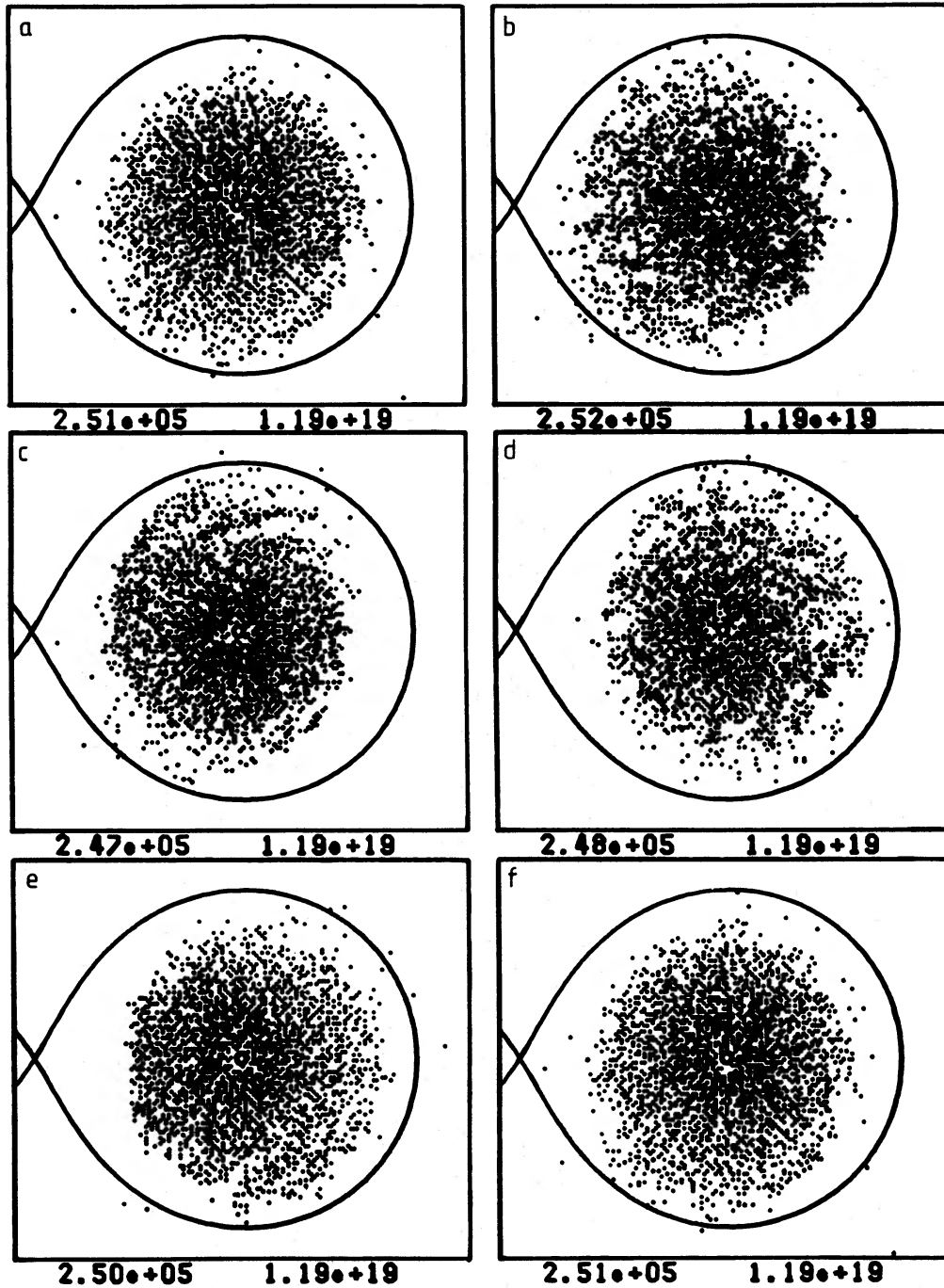


Figure 7. The form of the disc through one orbital cycle of Z Cha. The snapshots are  $0.2 P_0$  apart, (a) and (f) being identical.

Initially a brief mass-transfer burst, with length about 12 hr, delivers some 10 times as much mass into the disc as was previously present. This has the effect of shrinking a previously steady state, and therefore asymmetric, disc down to a circular form with a radius of some  $0.17a$ . This then grows through action of viscosity to achieve, after about 2 day, a tidally limited but unstable form fixed in the system's corotating frame of reference. This soon transforms to a precessing, asymmetric disc which displays superhumps. This is now the final, stable form of the disc which persists until the next outburst, through both the weeks-long exponential decay of the outburst and the quiescent, steady-state disc. Throughout these stages the disc will continue to exhibit

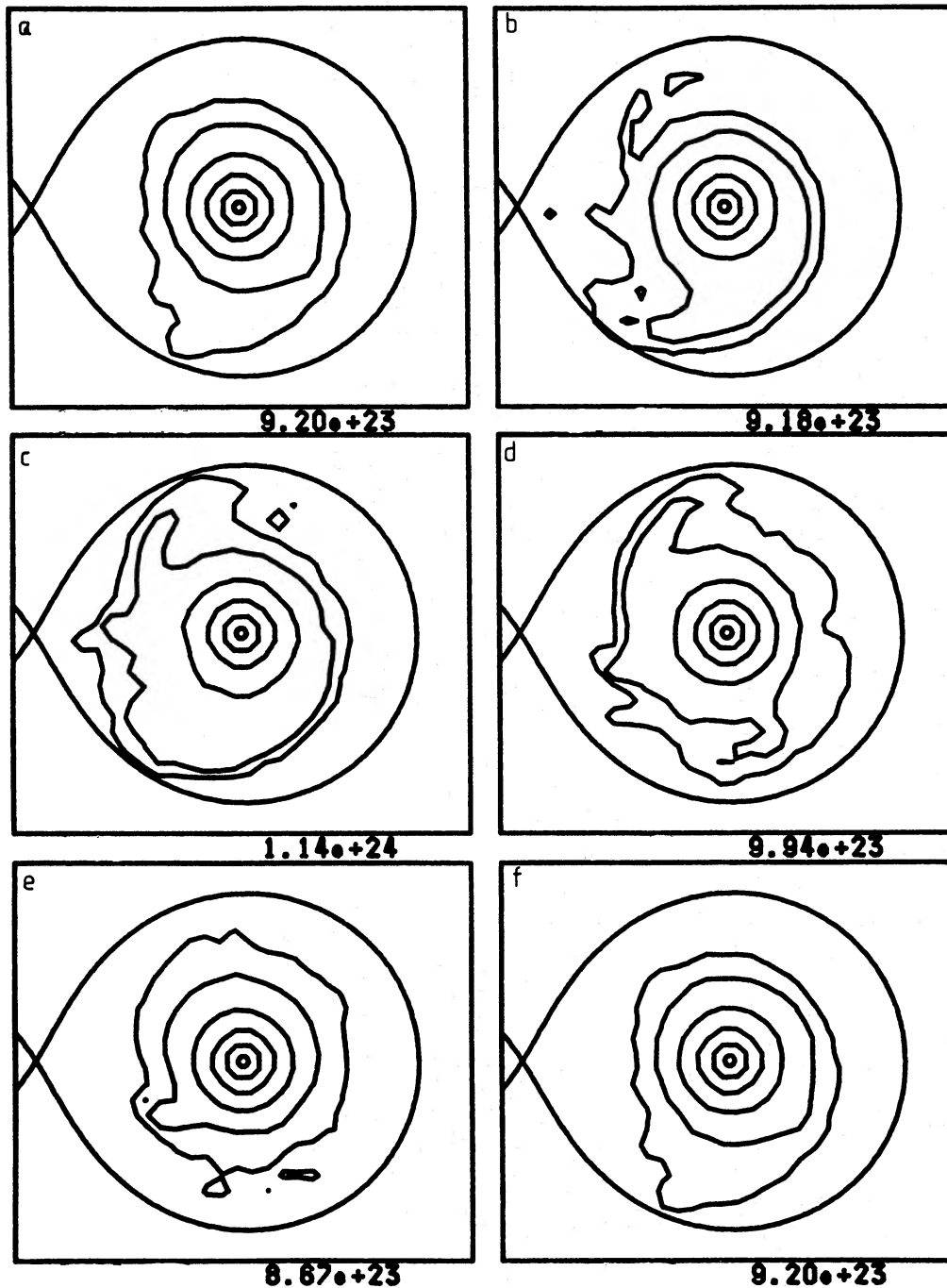


**Figure 8.** Each snapshot shows the integrated flux in the visible band (calculated assuming the disc is a collection of optically thick blackbody emitters), from the  $0.1 P_0$  preceding the moment portrayed in the corresponding frame in Fig. 7. Each contour plot is scaled so that the contour levels fall at the relative values of 1, 4, 16, 48 and 96. The highest contour always lies at 75 per cent of the maximum value. The value given under each frame is the disc's power output in the V band in W.

superhumps. This is not the behaviour observed in Z Cha or other SU UMa stars where superhumps are confined to superoutbursts. Furthermore, all outbursts end with a *rapid* decline to the star's quiescent brightness.

This problem can be resolved by arguing that the mistaken assumption is that the outburst is driven purely by a surge of mass-transfer from the secondary. Using the superhump as a probe of the state of the disc, i.e. assuming that the model described above is valid and superhumps occur

in viscously 'mature' discs, the evolution of the disc throughout the outburst cycle can be deduced. Because superhumps only occur in superoutburst it is only during superoutburst that the disc is fully viscously evolved. This is incompatible with the expectations of the mass-transfer burst model of dwarf novae outbursts where the disc is almost always viscously evolved, i.e. in steady-state. This problem can be resolved by using the predictions of disc-instability models of the outbursts. In these models the disc is *never* in steady state. Instead in quiescence the disc accumulates matter whilst it is in a cool, low-viscosity state which is then accreted rapidly in the hot, high-viscosity state during outburst. Because the dynamics of the disc's response to the outburst are independent of the details chosen for the driving mechanism it is simple to describe the expected course of such an outburst.

First, the material that in the mass-transfer burst model is deposited rapidly at the beginning of the outburst is allowed to collect slowly in quiescence. This has the same effect because the disc's viscosity is now so small that the material cannot be transported away through the disc. So a small symmetric, circular disc is formed around the primary. When the outburst begins it is marked by an increase in the disc's viscosity, so the disc brightens as the dissipation increases and material begins to move through the disc. Again the disc will grow but the fate of the disc is now determined by the subsequent behaviour of the viscosity. If this remains high for long enough the disc will reach the tidal radius and begin to precess, so producing superhumps. If, on the other hand, the viscosity soon returns to its quiescent value, the disc will stop growing and remain tidally stable. Furthermore, the disc will rapidly cool to its quiescent temperature and will not show the long protected decay. These two scenarios can be seen to correspond to superoutbursts and normal outbursts respectively.

Finally, in the cool, quiescent disc the inflowing stream material will soon come to dominate the relatively small amount of mass in the outer disc. This will be through the mixture of the stream's low specific angular momentum material with the disc. As this proceeds the disc will shrink. Even if the disc had become unstable and so started to precess this process will soon return the disc to stability and so the superhumps will cease. This evolution of the disc is precisely what is seen to occur in Z Cha, where measured values of the bright-spot radius indicate that the disc shrinks in quiescence, e.g. O'Donoghue (1986).

### 3.5 THE PROPERTIES OF SUPEROUTBURSTS

Before considering what differentiates the normal and superoutbursts of SU UMa stars I shall discuss some of the important properties that any superhump model needs to address. Note that in many respects the simulation technique used produces a two-dimensional sketch of the accretion disc in these systems. Despite this, it can indicate the major properties of the disc, particularly those to do with its dynamics. However, the simulations cannot be expected to reproduce all the detailed behaviour of the real, three-dimensionally accreting gas flow in these systems. Despite this, explanations of many of the superhump's properties are possible.

#### (i) *The late superhump*

An important property of the superoutburst is the late-superhump. This is visible as a modulation of the orbital hump seen immediately after the end of a superoutburst in the high-inclination system VW Hydri. The modulation occurs with the beat period but the phasing is such that it is shifted by about half a cycle from the times of superhump maximum in superoutburst. The late-superhump decays away during a period of a few days to weeks after the end of the superoutburst. It is possible to explain these features in terms of the new model. At the end of the superoutburst the disc remains precessing and distorted, and this condition persists until the addition of low

specific angular momentum material from the secondary reduces the disc to a stable, circular form. During this stage the normal superhump is absent, due to the low viscosity of the disc which weakens the dissipative processes and the consequential superhump variation. However, the mechanism proposed by Vogt (1982) for the superhump itself can now be invoked. The disc is still asymmetric and so the bright-spot's luminosity will vary around the disc. This variation (on the superhump period) beats with the visibility of the spot (which varies with the orbital period) to give a modulation on the beat period. This property will disappear as the disc circularizes. The phase shift appears because the superhump has a maximum when the disc's long axis is pointed *towards* the secondary, whilst the maximum energy release in the bright-spot will occur when the long axis points *away* from the secondary. This provides a phase shift of roughly 0.5 cycles. This explanation predicts that the late-superhump will be eclipsed in Z Cha and OY Carinae.

#### (ii) *The superhump eclipse*

One failing of the previous eccentric disc models is their inability to explain the forms of eclipses of the superhump, i.e. when the eclipse falls at superhump maximum. These, as shown by Whitehurst *et al.* (1984) and Horne (1985), are only compatible with either a secondary star model or a *broad distribution* of light within the disc. Unfortunately both the models of Vogt (1982) and Osaki (1985) require the superhump to be localized in the bright-spot. The tidal model described above produces the superhump light distributed over a considerable fraction of the disc area, as required to explain the eclipse profiles.

#### (iii) *Brightness variations with beat phase*

The variation of the system's light with beat period can be explained as being due to the eccentricity of the disc. When the short axis is projected towards the Earth the obscuration of the bright inner disc by the (flared) outer disc is at a maximum. This means that the system's light is at a minimum at the phases required to explain the observations of Schömb's (1986).

#### (iv) *The $\gamma$ velocity variation*

Another beat period property which is an independent check on the phasing of the disc at superhump maximum are the  $\gamma$  velocity variations. These, if supposed to be due to the absorption arising in the cool outer disc as it is projected against the hot inner disc, can be modelled using the data available from the simulations. This was done by Honey *et al.* (1988) who report that the fit to the observations is remarkably good, with the exact phasing and velocity dispersion observed being reproduced by the simulations.

### 4 The nature of the outbursts

The problem as to what is the distinguishing factor between normal outbursts and superoutbursts has been neglected in the preceding section. It is not possible to state from the results of the above simulations what this factor might be, since it is beyond the scope of the calculations. Considerable evidence has been accumulated in recent years that during superoutburst the mass-transfer rate from the secondary is considerably enhanced, and that this is the consequence of an instability triggered in the secondary by a normal outburst, e.g. Osaki (1985), van der Woerd & van Paradijs (1987).

Taking the normal outburst to be controlled by changes in the disc's viscosity, as suggested by disc instability models, the question of why the disc never becomes tidally unstable needs to be

addressed. First of all, the normal outbursts are quite short, varying from 1.5 to 3 day, which is roughly the time required for the disc to become unstable from the commencement of the outburst. This could not exclude the occasional occurrence of superhumps in the later stages of a normal outburst though. One possibility is that the transformation to a lower viscosity is closely correlated to the density of the disc, as predicted by disc instability models, which means that it is in turn determined by the size of the disc. If this critical size is smaller than the stability radius then a purely viscosity driven outburst could not drive the disc tidally unstable. Since the above simulations do not deal with variable viscosity outbursts this suggestion cannot as yet be proven to be correct.

The superoutburst is to be kept active through the inflow of material at some enhanced rate from the secondary. Since at maximum light a superoutburst in Z Cha is one magnitude brighter than a normal outburst an enormously increased mass-transfer rate is not required. This means that the disc will not be appreciably shrunk during the outburst by the low specific angular momentum material leaving the secondary. It is quite possible that the changes in the outer disc's structure engendered by this inflowing material are responsible for the secular changes in superhump profile and period observed in superoutburst. During the course of the superoutburst both superhump amplitude and period decrease, which is precisely the effect expected of a slow reduction in the disc's size due to the presence of a high mass-transfer rate from the secondary.

Circumstantial support for the superoutburst being a combination of disc instability and enhanced mass-transfer can be seen in the behaviour of the atypical dwarf nova WZ Sagittae. This has remarkably infrequent, bright and long outbursts (every 33 yr with amplitudes of about 7 mag and durations around 100 day). Furthermore, during the early part of its 1978 outburst Patterson *et al.* (1981) observed a brilliant bright-spot, consistent with a very high mass-transfer rate into the disc. By interpreting the outburst as one dominated by mass-transfer rather than variable viscosity (which may well also be present), these curious anomalies can be explained. The outburst intervals are due to the properties of the secondary and not the disc, the outburst amplitude and duration are dominated by the mass-transfer process and not by the disc's viscosity. A further factor which can now be explained is the observation that the superhumps did not occur until 10 day into the outburst, after the bright-spot had faded. This can now be seen to be due to the need of the disc to grow beyond its stability radius before superhumps could occur, something it could not do while a high rate of mass-transfer kept the bright-spot prominent and the disc small.

## 5 Conclusion

The generation of 'superhumps' by a computer simulation technique that was primed with nothing more than the basic physical properties of the system is important in two regards. First, it resolves many of the uncertainties surrounding superhump models. We can now claim with some justification that the superhump is essentially understood, as indeed is the distinction between SU UMa and U Gem stars. Secondly the ability of the simulations to reproduce this property accurately gives considerable confidence in the results. This allows strong conclusions to be drawn, using the calculations as a basis for argument.

(i) In non-magnetic dwarf novae the accretion disc appears to have a varying viscosity, this being high in outburst and low in quiescence. The narrow outbursts of these stars are characterized by an increase in the viscosity of the disc, the abrupt end of the outburst not being compatible with the viscous time-scale of the disc but rather with a rapid change in disc viscosity.

(ii) In those systems with extreme mass-ratios (greater than 4:1), the disc is potentially tidally unstable when fully viscously evolved. These stars can be identified as the SU UMa stars, and the

consequence of the instability is the appearance of superhumps. These are the result of the tidal stresses working on a slowly precessing accretion disc, which periodically distort the disc around each orbital cycle.

(iii) Superhumps will therefore not be observed in U Gem and Z Cam stars where the mass-ratios are not extreme and the disc is not tidally unstable.

(iv) The superoutburst is probably due to stimulated high mass transfer from the secondary after a normal outburst has started.

Future study should concentrate on the mechanisms involved in variable viscosity outbursts and the better determination of the mass-ratio to superhump period relationship. Evidence of similar multi-periodic behaviour in other accretion discs in sufficiently extreme mass-ratio systems should be sought, as related phenomena could well occur in such stars.

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