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THE LUMINOSITY FUNCTION OF WHITE DWARFS

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ABSTRACT

A variety of new data—trigonometric parallaxes, optical colors, and spectrophotometry—are presented for a sample of mostly cool white dwarfs having large proper motions. These and existing data from the literature are used to derive an empirical luminosity function for cool white dwarfs using the $1/V_{max}$ method. To facilitate comparison with theoretical cooling curves, new relations for cool white dwarfs are estimated for T_{eff} versus M_V and for M_V versus M_{bol}. Model atmospheres with helium composition are adopted for fitting the coolest stars, although the effect of using hydrogen-rich atmospheres on the result is discussed. These relations are then used to derive the luminosity function in bolometric magnitude intervals, including that for hotter white dwarfs determined from the color-selected Palomar-Green survey sample and to update the observational data presented by Winget and coworkers.

The results show that a downturn occurs in the distribution of cool degenerate stars near log $(L/L_{\odot}) \sim -4.4$. The reality of this deficiency is supported by the results of other kinds of investigations. Our results would likely be affected only minimally by the addition to this sample of the very low luminosity white dwarf ER 8, an object discovered recently in a southern proper-motion survey. The indicated local space density of observed degenerate dwarfs is 0.003 stars pc^{-3}, which corresponds to $\sim 1\%$ of the dynamical mass density in the solar neighborhood.

Given the shape of the derived luminosity function, it is difficult to attribute the downturn at the faint end to any physical effect other than the near-equality of the cooling times of the lowest luminosity observed degenerates to the age of the Galactic disk. The latter is indicated to be in the (7–10) $\times 10^9$ yr range, but the cooling times are sensitive to several parameters in the cooling models. The observed distribution also constrains the application of a bimodal star formation theory.

Subject headings: luminosity function — stars: stellar statistics — stars: white dwarfs

I. INTRODUCTION

The luminosity function (LF) and total space density of white dwarf stars in the solar neighborhood are important measures of the star formation history of the Galactic disk, and provide an independent method for measuring its age. Using largely the early compilations of Eggen and Greenstein (1965, 1967), the first empirical estimates of the LF for degenerate stars were those of Weidemann (1967), Kovetz and Shaviv (1976) and Sion and Liebert (1977). With the completion of the Luyten proper-motion survey, however, the data base included many fainter white dwarfs. In a first analysis of a fainter sample, Liebert *et al.* (1979*b*) argued that there is a paucity of white dwarfs fainter than $M_V \sim +16$ with respect to predictions of theoretical cooling models extending in time past 10^{10} yr and under the simple assumption of a constant stellar deathrate. The comparison implied that the bulk of star formation in the disk began $< 10^{10}$ years ago, an effective disk age billions of years younger than the ages estimated at that time for globular clusters in the halo (see also Liebert 1980). These results also implied that degenerate dwarfs do not contribute substantially to the local mass density, despite the likelihood that up to half of the local mass is presently unaccounted for by known objects (Bahcall 1984*a*, *b*, *c*).

The last several years have brought considerable improvements in both the theoretical calculations and the observational data base. Iben and Tutukov (1984) presented new evolutionary models and rediscussed the published oberva-tional data, offering alternative explanations for an apparent paucity of very cool degenerates. In addition, the application of a bimodal star formation hypothesis to the Galactic disk (Larson 1986; Olive 1986) could also influence strongly the interpretation of an empirical LF for white dwarfs. Most recently Winget *et al.* (1987) used new theoretical calculations, combined with a preliminary version of the observational results presented in this paper, to estimate an age of the Galactic disk of $(9.3 \pm 2.0) \times 10^9$ yr although the derived value is sensitive to several physical parameters in the models (Winget and Van Horn 1987). The purpose of this paper is to document the improved observational data base, updated from the version utilized in that work, and to discuss more fully some of its implications.

In the following assessment, we present in § II new observations of a specifically defined sample of (predominantly) cool white dwarfs taken from the LHS Catalog (Luyten 1979). New results include trigonometric parallaxes, photometry, and spectrophotometry from which improved absolute magnitude estimates are obtained. In § III an LF for cool white dwarfs is derived, with the use of the "$1/V_{max}$" method (Schmidt 1975) to correct for the kinematic bias. In order to present these results in bolometric magnitude units, we must discuss different means of estimating the bolometric corrections for degenerate stars. Thus the LF for both the cool white dwarfs and the hotter stars derived by (Fleming, Liebert, and Green 1986, hereafter FLG) is transformed into bolometric units. This LF is compared with previous and alternative estimates, derived from different observed white dwarf samples. The reality of the apparent "cutoff" or "shortfall" of white dwarfs fainter than $M_V = +16$ is reinforced by other kinds of observational data discussed in § IV, and its possible implications for star formation in the Galaxy are briefly outlined. Finally, in § V we suggest some avenues for future research.

II. THE OBSERVATIONAL DATA BASE

For the range of luminosity in which blue color is an effective discriminant, the LF for white dwarfs is reasonably well known. The largest and probably best observed sample with such a determination is the white dwarf subset of the Palomar-Green survey (Green, Schmidt, and Liebert 1986), from which FLG have presented an LF for $M_V \leq +13$ corresponding to degenerates more luminous than log $(L/L_{\odot}) \sim -3.3$. Other recent estimates of the LF of hot white dwarfs (Ishida *et al.* 1982; Downes 1986) are in at least fair agreement. These results constrain the death rate of stars in the local Galactic disk over the last $\sim 10^9$ yr.

For the fainter absolute magnitude intervals whose numbers dominate the space density of white dwarfs, available samples contain stars found primarily through proper-motion catalogs or as binary companions. Proper-motion selection may inject a very strong kinematic bias in favor of stars with large tangential velocities (cf. Hansen 1983). Spectroscopists and photometrists initially experienced considerable difficulty (cf. Greenstein 1971) in obtaining a high yield of cool white dwarfs, since low-luminosity and high-velocity main-sequence stars dominate the catalogs. Moreover, the modest limiting magnitudes and blue colors of the Lowell and early Luyten proper-motion plate material greatly restricted the volume of space accessible to search for cool, low-luminosity white dwarfs. However, the availability of the Luyten Palomar (LP) survey catalogs, especially the Luyten Half-Second (LHS) Catalog (Luyten 1979), made possible more efficient searches for such degenerate stars. Indeed, numerous authors referenced above and in § IV*a* have added dozens of cool white dwarfs to the empirical data base in the past decade and a half.

The data base employed in the present investigation consists of the 43 spectroscopically confirmed white dwarfs contained in the LHS Catalog (Luyten 1979) which have (1) proper motions $\geq 0\farcs8$ yr^{-1}, (2) locations in the sky north of $\delta = -20°$, and (3) $M_V \geq 13.00$.

Table 1 summarizes the basic observational data for these 43 objects. The first column gives the number in the LHS Catalog and the WD number using the notation specified by McCook and Sion (1987). The second column gives the primary name of the star (which generally reflects the original discoverer) and the number assigned to the star in the updated version of the Yale Parallax Catalog which is currently in preparation (van Altena 1986). The third column gives the absolute annual proper motion adopted from the LHS Catalog and the tangential velocity calculated using the adopted parallax which is given in the fourth column along with its mean (i.e., standard) error. The adopted value of the parallax is generally the mean trigonometric value derived for the new version of the Yale Parallax Catalog (van Altena 1986) but updated with unpublished preliminary USNO photographic and/or CCD determinations. In the absence of trigonometric parallaxes, photometrically estimated values were adopted with the corresponding entries in the table followed by a "p."

The results of broad-band photometric observations made at the Flagstaff station are presented in the fifth through eighth columns where the $V-I$ is on the system of Kron and Mayall (1960). The three integers on the top line of the ninth column indicate the number of independent observations of (1) V and $B-V$, (2) $U-B$, and (3) $V-I$, respectively. The corresponding formal mean errors of the mean values computed from multiple observations are given on the second line. A J following an entry indicates that the quantity represents measures of the joint light from close components. The second line of the ninth column gives the spectral classification on the system of Sion *et al.* (1983) either extracted from McCook and Sion (1987) or observed for the present investigation. The tenth column gives M_V along with its formal uncertainty based on the adopted values of the absolute parallax and apparent visual magnitude as quoted in the fourth and fifth columns, respectively.

The notes following the table include information on other designations, membership in binary systems, possible astrometric perturbations that have been reported, and photometric results from other investigators if the disagreement with the tabulated values exceeds 0.1 mag. Asterisks are employed in the table to call attention to specific explanations given in the notes.

III. THE LUMINOSITY FUNCTION

a) The $1/V_{max}$ Method: Background

This method employs a generalization of the approach pioneered by Schmidt (1968) for the study of quasars. As applied in this study, the method specifies inclusion of stars in particular ranges of both apparent magnitude and proper motion, for which the proper-motion catalog can be assumed to be complete. Since each star's contribution to the space density is itself a function of its proper motion in the $1/V_{max}$ approach, the effect is to remove directly the kinematical bias associated with proper-motion samples (Hansen 1983). Its success in doing this can be tested in part by comparing, for example, the mean tangential velocities for selected stars as a function of absolute magnitude.

Schmidt has applied this technique to samples of Lowell proper-motion stars in order to estimate the LF of the halo (Schmidt 1975). In addition, Eggen (1983) has applied the method to a sample of brighter ($V \leq 15.0$), primarily southern ($\delta < +30°$) stars, including degenerates.

With the $1/V_{max}$ method, each object does not make an equal contribution to the space density within its absolute magnitude bin. Rather, each star is weighted by the inverse of the maximum volume (V_{max}) within which that same star hypothetically could be found, given the presumed completeness of the sample in both proper motion and apparent magnitude. A nearby star with proper motion just higher than the adopted lower limit of the sample would have a relatively small value of V_{max} in comparison with most other stars in its magnitude bin and would, therefore, make a correspondingly higher contribution to the space density for that bin. Consequently, we should not expect the formal uncertainties for the $1/V_{max}$ space density determinations to improve as the square root of the total number of stars in the bins. This is in contrast to methods employing straightforward number counts over some volume-complete sample. In the present application we assume that the uncertainty in each star's contribution to the space density is

TABLE 1

Observational Results

LHS WD	Name YPC	P.M. Vtan	Pi m.e.	V m.e.	B−V m.e.	U−B m.e.	V−I m.e.	n,n,n Sp.	M_V m.e.
7 0046+052	vMa2 160.00	2.980 61	0.2317 0.0024	12.39 0.00	0.54 0.00	0.02 0.01	0.35 0.01	4,3,7 DZ7	14.21 0.03
27 0426+589	Stein 2051B 986.04	2.383 62	0.1812 0.0011	12.44	0.32	−0.51		* DC5	13.73 0.01
32 0552−041	HL4 1363.01	2.377 72	0.1559 0.0023	14.45 0.01	1.05 0.01	0.76 0.05	0.85 0.02	10,5,7 DZ9	15.41 0.03
46 1334+039	W489 3112.00	3.870 148	0.1240 0.0035	14.64 0.02	0.95 0.03	0.29 0.05	0.88 0.02	5,2,3 DZ9	15.11 0.06
56 1756+827	LP9−231 4149.01	3.587 268	0.0635 0.0030	14.31 0.01	0.35 0.01	−0.52 0.02	0.33 0.01	6,2,4 DA7	13.32 0.10
69 2251−070	LP701−29 5548.01	2.570 98	0.1245 0.0044	15.65 0.01	1.91 0.02		1.07 0.02	11,–,8 DZ9	16.13 0.08
147 0145−174	LP768−500 376.01	1.186 358	0.0157 0.0106	17.66	0.43	−0.50		1,1,– DC7	13.64 1.47
151 0208+397	G74−7 442.02	1.144 88	0.0618 0.0037	14.51 0.01	0.31 0.02	−0.52 0.03		4,4,– DAZ7	13.46 0.13
153 0213+427	G134−22 463.01	1.009 94	0.0510 0.0042	16.19 0.01	0.72 0.02	−0.02 0.04		4,4,– DC9	14.73 0.18
179 0341+183	W219 791.00	1.204 107	0.0533 0.0027	15.18 0.01	0.31 0.01	−0.56 0.01	0.24	4,4,1 DQ8	13.81 0.11
194 0435−089	L879−14 1026.01	1.520 68	0.1054 0.0026	13.76 0.00	0.32 0.01	−0.53 0.01	0.41 0.01	7,4,2 DQ7	13.87 0.05
212 0553+054	G99−47 1366.01	1.056 40	0.1249 0.0041	14.11 0.01	0.60 0.01	−0.17 0.02	0.62 0.02	8,3,5 DAP9	14.59 0.07
230A 0727+483A	G107−70A 1766.01	1.295 68	0.0900 0.0011	15.27* 0.05	0.99J 0.01	0.37J 0.05	0.89J 0.01	8,2,6 DC9	15.04 0.06
230B 0727+483B	G107−70B 1766.01	1.295 68	0.0900 0.0011	15.57* 0.11	0.99J 0.01	0.37J 0.05	0.89J 0.01	8,2,6 DC9	15.34 0.11
235 0738−173	L745−46A 1813.00	1.252 53	0.1120 0.0027	13.06 0.02	0.24 0.00	−0.62	0.19	2,1,1 DZ6	13.31 0.06
239 0747+073B	LP543−32 1850.01	1.778 153	0.0551 0.0013	16.98 0.00	1.34 0.07		0.99 0.18	2,–,2 DC9	15.69 0.05
240 0747+073A	LP543−33 1850.01	1.778 153	0.0551 0.0013	16.69 0.02	1.12 0.05		0.86 0.26	3,–,3 DC9	15.40 0.05
262 0912+536	G195−19 2199.02	1.550 75	0.0973 0.0017	13.85 0.00	0.35 0.01	−0.52 0.02	0.18 0.02	9,4,4 DXP7	13.79 0.04
282 1022+010	LP610−10	1.101 290	0.018* 0.007	18.06 0.11	0.84 0.16	0.13 0.14		3,3,– DC8	14.3* 0.9
285 1033+714	LP37−186	1.997 186	0.051p 0.012	16.87 0.02	1.05 0.02		1.18 0.05	3,–,3 DC9	15.4p 0.5
290 1043−189	LP791−55 2512.00	1.980 120	0.0784 0.0078	15.5:	0.55			1,–,– DQ9	15.0* 0.3
291 1042+593	LP93−21 2510.01	1.770 682	0.0123 0.0054	17.94	0.13	−0.85		1,1,– DQ8	13.39 0.96
304 1121+216	R627 2642.00	1.050 66	0.0759 0.0027	14.24 0.02	0.29 0.01	−0.54 0.01	0.29 0.01	6,3,3 DA7	13.64 0.08

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TABLE 1—*Continued*

LHS WD	Name YPC	P.M. Vtan	Pi m.e.	V m.e.	B−V m.e.	U−B m.e.	V−I m.e.	n,n,n Sp.	M_V m.e.
342	LP131–66	1.286	0.0458	17.77	1.47		1.29	*	16.07
1247+551		133	0.0014	0.05	0.10		0.09	DC9	0.08
361	LP380–5	1.484	0.0845	15.64	1.10	0.47	1.01	10,3,6	15.27
1345+238	3146.11	83	0.0022	0.00	0.01	0.02	0.03	DC9	0.06
378	LP801–9	1.177	0.055p	16.48	0.94		0.95	2,–,2	15.2p
1444−175		101	0.013	0.01	0.03		0.02	DC9	0.5
422	LP101–16	1.620	0.0683	14.99	0.48		0.47	5,–,5	14.16
1633+573	3776.01	112	0.0026	0.01	0.02		0.02	DQ8	0.08
455	LP44–113	1.651	0.1638	14.15	0.41	−0.30	0.95	9,4,5	15.22
1748+709	4085.01	48	0.0020	0.01	0.01	0.02	0.01	DXP9	0.03
483	LP754–16	1.081	0.0576	16.87	1.04			3,–,–	15.67
2002−111	4763.01	89	0.0055	0.05	0.07			DC9	0.21
529	G128–7	1.269	0.0486	15.51	0.63	−0.10	0.68	10,3,7	13.94
2248+294	5534.01	124	0.0043	0.01	0.01	0.07	0.03	DA9	0.19
542	LP702–7	1.728	0.0262	18.25	1.24			1,–,–	15.40*
2316−065		312	0.0039					DC9	0.33
1044	LP704–1	0.911	0.057p	15.91	0.56		0.63	1,–,1	14.7p
0011−135		76	0.008					DC8	0.3
1547	G77–50	0.903	0.060p	16.12	0.80	0.20		3,3,–	15.0p
0322−020		71	0.014	0.01	0.01	0.02		DZ9	0.5
1670	LP535–11	0.856	0.043p	17.11	1.10		0.88*	3,–,1	15.3p
0423+044		94	0.008	0.02	0.04			DC9	0.4
1892	L886–6	0.898	0.052p	15.42	0.44		0.59	3,–,3	14.0p
0659−064	1646.11	82	0.010	0.01	0.00		0.02	DA8	0.4
1980	LP257–28	0.830	0.047p	16.85	0.90		0.95	2,–,2	15.2p
0802+387		84	0.009	0.04	0.02		0.01	DC9	0.4
2333	L898–25	0.816	0.0832	14.33	0.31	−0.53	0.20	6,3,3	13.93
1055−073	2555.02	46	0.0034	0.01	0.01	0.01	0.03	DA7	0.09
2364	LP374–4	0.816	0.039*	17.71	0.93	0.40:		2,1,–	15.67
1108+207		99	0.002	0.09	0.08			DC9:	0.14
2596	LP171–40	0.886	0.033p	16.44	0.44		0.58	3,–,2	14.0p
1236+457		129	0.006	0.01	0.00		0.03	DC8:	0.4
2661	W457	0.963	0.0647	15.77	0.64	−0.11	0.57	6,5,1	14.82
1257+038	2976.00	71	0.0035	0.02	0.02	0.05		DC9	0.12
2673	LP322–800	0.849	0.035*	18.81	1.38		1.33*	1,–,1	16.5:*
1300+263		115	0.010					DC9	0.7
2800	G63–54	0.884	0.0514	15.08	0.41	−0.49		3,3,–	13.63
1344+106	3144.01	81	0.0036	0.05	0.02	0.05		DA7	0.16
3601	vB11	0.816	0.0641	16.56	1.18		1.26	5,–,4	15.59
2054−050	5016.00	60	0.0044	0.03	0.03		0.03	DC9	0.15

NOTES TO TABLE 1

Abbreviations.—P.M. = total proper motion in arcsec yr^{-1} from Luyten 1979; Vtan = tangential velocity in km s^{-1}; Pi/m.e. = adopted absolute parallax and mean error, both in arcseconds.

LHS 7.—W28 = G1-27 = EG5 = GL 35.0. Possible small-amplitude perturbation with a period of the order of 20 yr suggested by van de Kamp 1971 not confirmed by Gatewood and Russell 1974. USNO plate material covers only 1980.6–1986.8 but shows no evidence for a perturbation.

LHS 27.–G175-34B = EG 180 = GL 169.1B. Orbital binary (separation ~7″) with LHS 26 = Stein 2051A = G175-34A. Strand 1977 estimates a period in excess of 300 yr and, in addition, suggests that Stein 2051A is an astrometric binary with $P \approx 23$ yr. Photometry is difficult owing to separation of pair; adopted values from Hardie and Heiser 1966 and Eggen and Greenstein 1967. Stein 2051B shows an essentially featureless spectrum at optical wavelengths (Wegner and Yackovich 1982), but Greenstein 1986*b* classified it as DQ?7.

LHS 32.—LP658-2 = G99-44 = EG 45 = GL 223.3. Classified DK (i.e., DZ) in Eggen and Greenstein 1965, this star is listed as DC11 + in Greenstein 1986*b*. However, the presence of Ca II is unmistakable in the spectrum displayed in Kapranidis and Liebert 1986.

LHS 46.—G62-53 = EG 100 = GL 518.0.

LHS 56.—G259-21 = EG 199 = GL 699.1.

LHS 69.—GR 453 = GL 1276.0. Spectrum heavily blanketed in blue (Dahn *et al.* 1978).

LHS 147.—GR 467. Eggen and Sandage 1967 report $V = 17.57$, $B-V = 0.37$, and $U-B = -0.48$ ($n = 2$; 200 inch [5 m] reflector).

LHS 151.—EG 168 = GL 1042.0.

LHS 153.—EG 16 = GL 91.3.

LHS 179.—G6-30 = EG 24 = GL 151.0.

LHS 194.—LP 655-42 = EG 41.

LHS 212.—GR 290 = GL 1087.0. Circular polarization discovered by Angel and Landstreet 1972.

LHS 230AB.—G107-70AB = EG 52AB = GL 275.2B. Partially resolved (Separation ~0″.7) degenerate binary (Strand, Dahn, and Liebert 1976) which is part of a wide (Separation = 103″) binary with LHS 229 = G107-69 = GL 275.2A. LHS 229 itself may be an astrometric binary (Harrington, Christy, and Strand 1981). Adopted $\Delta V = 0.30 \pm 0.05$ is based on the analyses of Harrington, Christy, and Strand 1981 and Borgman and Lippincott 1983.

LHS 235.—LP 783-3 = EG 54 = GL 283.0A. Wide (Separation = 20″.5) binary with the faint red dwarf ($V = 16.55$, $n = 1$; $M_V = 16.80$) LHS 234 = L745-46B = LP 783-2 = GL 283.0B.

LHS 239/240.—GR 426/427. Adopted parallax is the weighted mean of (1) 0″.0586 ± 0″.0020, based on van Altena's KPNO 4 m photographic determination (see Greenstein *et al.* 1977), (2) 0″.0521 ± 0″.0028 from a preliminary USNO photographic determination based on 34 plates covering 1976.9–1985.0, and (3) 0″.0530 ± 0″.0020 from a preliminary USNO CCD determination.

LHS 262.—EG 250 = GL 339.1. Circular polarization discovered by Angel and Landstreet 1971*a* and found to vary periodically with $P = 1.331$ days (Angel and Landstreet 1971*b*; Angel, Illing, and Landstreet 1972).

LHS 282.—Preliminary USNO photographic parallax determination based on only 15 plates covering 1981.0–1986.0 yields 0″.0075 ± 0″.0097 and $M_V = 12.4 \pm 2.8$. The uncertain $B-V$ value quoted above implies $M_V \sim 15.1$ corresponding to a photometric parallax of 0″.026, but Reticon spectroscopy (Liebert *et al.* 1979) does not support an absolute magnitude as faint as 15.0. Luyten's 1979 assigned color class of f–g also suggests a more luminous value. The adopted value of $M_V = 14.3$ was derived photometrically from $B-V = 0.68$, the blue 1 σ uncertainty limit for the tabulated $B-V$ photometry.

LHS 285.—Adopted parallax based on tabulated M_V, which was in turn estimated from the tabulated value of $B-V$.

LHS 290.—GL 401.0B. Common proper motion binary with Separation = 7″.5 (Luyten 1979). Adopted parallax from brighter component (LHS 289 = BD −18°3019 = L753-52 = GL 401.0A), for which we measure $V = 11.11 \pm 0.05$, $B-V = 1.46 \pm 0.02$, and $V-I = 1.84 \pm 0.04$ ($n = 2$). Single tabulated photometric observation was made through a 6″.6 aperture and is very uncertain (rough estimate: ±0.2 in V) because of the close proximity and orientation with respect to the brighter component.

LHS 291.—GR 434. Sandage 1968 reports $V = 17.73$, $B-V = 0.09$, and $U-B = -0.90$ ($n = 2$; 200 inch [5 m] reflector).

LHS 304.—G120-45 = EG 79 = GL 427.0.

LHS 342.—GR 536. Adopted parallax based on a preliminary USNO photographic determination of 0″.0557 ± 0″.0102 from 25 plates covering 1977.4–1985.4 and a preliminary USNO CCD determination of 0″.0456 ± 0″.0014 employing 21 exposures covering 1984.4–1986.4. Adopted photometry from Liebert *et al.* 1979*a*; see Liebert, Lebofsky, and Rieke 1981 for near-infrared photometry and further discussion.

LHS 361.—GL 1179.OB. Wide (separation = 188″) binary companion to LHS 362 = LP 380-6 = GL 1179.0A. Adopted parallax based on the weighted mean values for the two components.

LHS 378.—Adopted parallax based on $M_V \sim 15.0 \pm 0.5$ estimated from the spectroscopic data reported in Liebert *et al.* 1979*b*.

LHS 422.—G225-68 = EG 258 = GL 630.1B. Wide (separation = 25″.8) binary companion to LHS 421 = LP 101-15 = G225-67 = GL 630.1A, which is itself the unresolved, eclipsing binary CM Dra (Eggen and Sandage 1967; Lacy 1977). Classified DQ8 + in Greenstein 1986*b*.

LHS 455.—G240-72 = GR 372 = GL 1221.0. Elliptical polarization reported by Angel *et al.* 1974. Spectrophotometry by Greenstein 1974 revealed a 2400 Å wide absorption feature throughout the V bandpass which could affect the derived M_V value by up to 0.2. mag

LHS 483.—GR 498. Note that the M_V value derived from the trigonometric parallax implies a somewhat higher temperature (i.e., earlier DC subtype) than that estimated by Greenstein 1986*b* from photometry.

LHS 529.—GR 283.

LHS 542.—GR 555. Adopted parallax from a preliminary USNO CCD determination based on 14 exposures covering 1984.7–1986.9. This is consistent with the value of 0″.028 ± 0″.008 suggested by the tabulated $B-V$ color and the value of 0″.027 implied by the value of $M_V = 15.52$: estimated by Greenstein 1984 from MCSP observations (i.e., $V = 18.37$ and $G-R = 1.12$:). We have adopted $V = 18.31 \pm 0.06$, which is the average of the single USNO observation tabulated above and Greenstein's $V = 18.37$.

LHS 1044.—G158-45. Adopted parallax based on tabulated M_V, which was in turn estimated from the tabulated value of $B-V$. This agrees with a very preliminary USNO photographic determination of 0″.0571 ± 0″.0078 based on only six plates covering 1984.6–1985.8.

LHS 1547.—Adopted parallax based on tabulated M_V, which was in turn estimated from the tabulated value of $B-V$.

LHS 1670.—GR 482. Adopted parallax based on $M_V = 15.29$ estimated by Greenstein 1984 from MCSP observations (i.e., $V = 17.21$ and $G-R = 0.96$), which compares with $M_V = 15.2$ implied using the tabulated value of $B-V$. Quoted $V-I$ derived from single KPNO 4 m cryocam observation.

LHS 1892.—LP 661-3 = GL 261.0. The YPC adopts a parallax of 0″.0786 ± 0″.0246 based on the preliminary determination reported by Luyten and Carpenter 1952 using a small number of Steward Observatory 36 inch (91 cm) reflector plates taken in 1951 November and 1952 March. The value quoted there was 0″.145 ± 0″.045 (p.e.), with a note that, as a preliminary value, it was probably too large and that the proper motion suggested a value more like 0″.070. Later Luyten 1970 (see footnote to Table 2) quoted 0″.070 ± 0″.026 but gave no additional information regarding the basis for this revised value. The YPC value leads to $M_V = 14.9$, which is too faint based on the present photometry and spectroscopy. The adopted photometric parallax agrees with the trigonometric value minus 1 σ. Luyten and Carpenter 1952 also mention a very faint companion with separation = 7″ and P.A. = 240° with $m_{pg} = 18.5$ and $m_{pv} = 17.6$, which they acknowledge as being "probably optical." A spectrum obtained with the Steward Observatory 2.3 m reflector, Cassegrain spectrograph, and photon-counting Reticon system (3400–6000 Å, ~4 Å resolution) shows this to be a cool DA star with Hβ through Hϵ well detected. Comparison of the measured Hγ equivalent width of 2.6 Å with the predictions from model atmospheres of Koester, Schulz, and Weidemann 1979 indicates $T_{eff} = 6600$ K. This temperature is consistent with the M_V estimated from the photometry (see Fig. 1*a*) and provides the basis for the tabulated spectral type.

LHS 2333.—LP 671-11 = G163-27 = EG 74 = GL 1140.0.

LHS 2364.—Preliminary USNO CCD parallax from 59 exposures obtained on 21 nights covering 1983.2–1987.1.

LHS 2661.—G60-54 = EG 95 = GL 492.0.

LHS 2673.—Adopted distance estimated from the single KPNO 4 m cryocam observation of $V-I$. A very preliminary CCD trigonometric parallax determination based on only eight frames covering 1987.3–1988.1 yields a relative parallax of 0″.0174 ± 0″.0049 and virtually ensures that this star does not have a luminosity *fainter* than that adopted above.

LHS 2800.—GR 360 = GL 1178.0.

LHS 3601.—R193B = EG 202 = GL 812.0B. Binary companion (separation = 14″) to LHS 3602 = R193 = GL 812.0A. Adopted parallax based on the weighted mean values for the two components. Eggen and Greenstein 1967 report $V = 16.68$, $B-V = 1.13$, and $U-B = 0.64$ ($n = 3$; 200 inch [5 m] reflector) for R193B.

equal to the value of that contribution itself, and then propagate these errors through the summation over all stars in each bin The final error bar for a bin may reflect a value for relatively few stars (i.e., those with highest weight)—as if there were actually few stars in the bin.

b) *Adopted Effective Temperatures*

For comparison with the theoretical luminosity function (Winget *et al.* 1987; see their Fig. 1), it is necessary to convert the absolute M_V scale to bolometric magnitude (M_{bol}) intervals. The M_V values for the well-studied cool white dwarfs from our sample are well determined, principally because of the availability of accurate trigonometric parallaxes. However, the assignment of corresponding M_{bol} values requires the derivation of atmospheric parameters—in particular, values of T_{eff}—from the fitting of synthetic spectrum predictions of model atmospheres to observed colors or spectrophotometric energy distributions of cool white dwarfs.

The relations between T_{eff} and various colors are different for hydrogen-rich and helium-rich atmospheres. Models applicable to the hydrogen-rich case have been calculated by Bessell, Wickramasinghe, and Cottrell (1979), Koester, Schulz, and Weidmann (1979), Mould and Liebert (1978), Shipman (1977, 1979), Wehrse and Liebert (1980), Liebert and Wehrse (1983), and Kapranidis (1985). Bolometric corrections (BCs) are published explicitly for the models of Mould and Liebert (1978). In addition, Greenstein (1976) has listed BCs for Shipman's hydrogen-rich models.

While some of the above references also give sequences for helium-rich or pure helium atmospheric compositions, the higher densities and pressures reached in cool, hydrogen-poor white dwarf atmospheres render the derived parameters less certain. For the coolest helium-rich stars at $M_V \geq +15$, we prefer to ultilize the models by Kapranidis (1985), who employed an improved equation of state, conductive energy transport, and other refinements applicable to these physical conditions.

In Table 2 we list a number of cool DA and non-DA white dwarfs with well-determined trigonometric parallaxes and M_V values, for which effective temperatures are estimated from comparison of broad-band (*BVI*) colors with models, or which are taken from the literature as referenced. The DA stars included here are regarded as a representative, though incomplete, list of well-analyzed stars with T_{eff} below 10,000 K, selected to be comparable in number to the non-DA stars. The spectral types presented are on the system defined by Sion *et al.* (1983).

At and below $T_{eff} \leq 5500$ K, the hydrogen Balmer lines are too weak to be detectable, and it is uncertain whether the dominant atmospheric constituent for a given star is H or He. Thus, it is not obvious which of the model sets should be used to derive the effective temperatures. We have long argued that, in contrast to the situation at $T_{eff} > 10,000$ K, the majority of cool DC and DZ stars are likely to have helium-rich atmospheres (Liebert 1979; Kapranidis and Liebert 1986). There are two general reasons for this view: (1) At $T_{eff} < 10,000$ K—but still high enough that the Balmer lines can be detected—there is now ample evidence that the degenerates with helium-rich atmospheres become more numerous than those with hydrogen-rich atmospheres (Sion 1984; Greenstein 1986*b*); and (2) infrared photometric observations have generally failed to reveal much evidence for expected absorption due to molecular hydrogen (Mould and Liebert 1978; Liebert, Lebofsky, and Rieke 1981; Wickramasinghe, Allen, and Bessell 1982; Lebofsky and Liebert 1984). The cause of the increased fractional percentage of helium-rich atmospheres at low temperatures is believed to be convective mixing of the thin outer hydrogen layer into a deep helium envelope. Thus, it appears likely that the helium-rich atmosphere white dwarfs are the dominant group at $T_{eff} < 6000$ K; we therefore adopt T_{eff} fits to broad-band colors from the Kapranidis (1985) models for the coolest DC and DZ stars in Table 2.

On the other hand, if, for example, accretion from the interstellar medium becomes more efficient at contaminating white dwarfs atmospheres at temperatures below 6000 K, the majority of the coolest DC and DZ stars could have atmospheres whose opacity is dominated by hydrogen. It would then be appropriate to consider using T_{eff} values derived from model atmospheres with a hydrogen-rich composition. It is difficult for us to do this explicitly, since we do not have access to what we regard as accurate hydrogen atmosphere models appropriate to these low temperatures. The models of Mould and Liebert (1978) do not extend below 4000 K (and are of dubious accuracy at this temperature). It is instructive, however, to compare temperatures obtained for the best-observed cool white dwarfs using both H-rich and He-rich models.

Shipman (1979) has fitted six cool objects (spectral types DC9 or DZ9) in common with our Table 2. A comparison of his results with our values from helium atmosphere fits is made in Table 3, with the stars listed in order of decreasing luminosity. For the hottest three stars, the derived temperatures are substantially higher with Shipman's hydrogen fits, a well-known effect due to the relative "blueness" of the He^- opacity (Shipman 1972). Fainter than about $M_V = +15$, however, the trend reverses dramatically, and the Shipman temperatures are lower for the bottom three entries. From this it is fair to infer that—were we able to apply Shipman fits to the observed *BVI* colors for the several reddest stars (i.e, LHS 239, 240, 342, 542, and 2673) in Table 1—the derived temperatures would be appreciably lower than those adopted from the Kapranidis helium models.

There is a serious problem with adopting this "hydrogen temperature scale" for the faintest stars. This dilemma has been discussed previously in Mould and Liebert (1978) and in Liebert (1979). It is well illustrated by the example of vB 11 (LHS 3601), the faintest of the well-studied cases compared in Table 3. If we use the accurately determined $M_V = +15.59$ with a modest adopted BC for 3400 K of 0.25 mag (see below), we derive $M_{bol} = 15.19$. It is fair to say that M_{bol} is more accurately determined than T_{eff}. However, a given T_{eff} corresponds to a particular radius. For Shipman's value of 3400 K, the radius is 0.021 $R_\odot$, nearly twice the mean value found for hotter DA white dwarfs (cf. Koester, Schulz, and Weidemann 1979). Using the theoretical mass-radius relations for cold degenerate configurations (Hamada and Salpeter 1961), the mass inferred from this radius (regardless of interior composition) cannot exceed 0.2 $M_\odot$. Since this mass seems improbably low for vB 11 (and for the group of cool white dwarfs), and the radius depends on the inverse square root of the temperature, the most likely explanation is that the actual temperature of the star is higher. (In fact, even the value derived from the helium models may be too low.) The derived radius also depends modestly on the adopted BC. For the above example we used the value from the model with the closest T_{eff} available to us, that for a 4000 K hydrogen-rich atmosphere by Mould and Liebert (1978). The actual correc-

TABLE 2

COOL DEGENERATES WITH REASONABLY DETERMINED STELLAR PARAMETERS

Name	WD	Spectral Type	T_{eff} (K)	References	M_V	M_{bol} (M_V+BC)	log ($L/L_\odot$) (T_{eff}, R)	M_{bol} ($L/L_\odot$)
Stars Likely Possessing H-rich Atmospheres								
W1	0011+000	DA5	9400	1	13.02	12.58	−2.994	12.18
G49-33	0955+247	DA6	8700	1	13.19	12.76	−3.128	12.52
G1-45	0101+048	DA6	8600	1	12.36	11.80	−3.148	12.57
L997-21	1953−011	DA6	8200	1	13.43	13.04	−3.231	12.78
G51-16	0827+328	DA6	8100	1	14.03	13.69	−3.252	12.83
L898-25	1055−073	DA6	8100	1	13.93	13.58	−3.252	12.83
G63-54	1344+106	DA7	7600	1	13.63	13.25	−3.363	13.11
LP 9-231	1756+827	DA6	7500	1	13.32	12.91	−3.296	12.94
R627	1121+216	DA7	7400	1	13.64	13.26	−3.409	13.22
G74-7	0208+397	DAZ7	7400	1	13.46	13.06	−3.409	13.33
G76-48	0257+080	DA7	7300	1	13.74	13.37	−3.433	13.28
G128-7	2248+294	DA9	5700	1, 2	13.94	13.60	−3.863	14.36
L97-12	0752−676	DA9	5500	3	15.18	14.97	−3.925	14.51
Stars Likely Possessing He-rich Atmospheres								
L923-21	1917−007	DB5	10600	1	12.33	11.95	−2.802	11.70
L145-141	1142−645	DQ5	9500	1	13.14	12.94	−2.975	12.14
R640	1626+368	DZA6	8800	1, 4	12.89	12.75	−3.108	12.47
L1363-3	2140+207	DQ6	8600	1	12.75	12.63	−3.148	12.57
L745-46A	0738−173	DZ6	7800	1	13.31	13.26	−3.318	12.99
Stein 2051B	0426+589	DC7	6800	5	13.73	13.72	−3.556	13.59
vMa 2	0046+052	DZ8	6000	4, 6	14.21	14.19	−3.774	14.13
LP 101-16	1633+573	DC9	5200	4, 6	14.16	24.03	−4.022	14.76
G125-3	1917+386	DC9	5200	4, 6	14.25	14.12	−4.022	14.76
G39-27	0433+270	DC9	4800	4, 6	14.72	14.47	−4.161	15.10
LP 380-5	1345+238	DC9	4700	4, 6	15.27	14.99	−4.200	15.19
HL 4	0552−041	DZ9	4600	4, 6	15.41	15.09	−4.235	15.29
LP 701-29	2251−070	DZ9	4500	4, 6, 7	16.13	15.77	−4.273	15.38
W489	1334+039	DZ9	4400	4, 6	15.11	14.69	−4.313	15.48
LP 543-33	0747+073A	DC9	4400	7, 8	15.40	14.98	−4.313	15.48
LP 543-32	0747+073B	DC9	4100	7, 8	15.69	15.11	−4.435	15.79
vB 11	2054−050	DC9	3750	4, 6	15.59	14.75	−4.590	16.18
LP 131-66	1247+551	DC9	3750	4, 6	16.07	15.23	−4.590	16.18

L923-21.—Eggen and Greenstein 1965 reported a photographic spectrum with weak hydrogen lines, although this was disputed by Wickramasinghe, Bessell, and Whelan 1978, who found an essentially featureless spectrum with an image dissector scanner. In another digital spectrum of even better quality, Wegner and Yackovich 1982 found very weak He I lines, while carbon features appeared in ultraviolet (*IUE*) spectra. Greenstein 1986*b* classifies the star as DBQA5 and notes that the spectrum is complex, containing ultraviolet C I lines, possible weak C_2 bands, Hα with $W \sim 0.8$ Å over ±30 Å, and He I λ5876 with $W = 1.0$ Å.

L1363-3.—Classified as a weak DQ by Liebert 1977, Wegner and Yackovich 1982, and Greenstein 1986*b*.

G125-3.—Classified DC?7 by Greenstein 1986*b*.

G39-27.—Greenstein 1986*b* reports Hα with $W = 3.6$ Å over ±50 Å and classifies the star as DA9+. However, the atmosphere may still be He-rich.

REFERENCES.—(1) Shipman 1979; (2) Wehrse and Liebert 1980; (3) Wickramasinghe and Bessell 1979; (4) photometric data from Liebert *et al.* 1979*a*; (5) Wegner and Yackovich 1982; (6) Kapranidis 1985; (7) Kapranidis and Liebert 1986; (8) Dahn, unpublished *BVI* photometry.

tion for a significantly cooler object is likely to be larger than our adopted value, thus aggravating the radius and mass problems.

The conclusions about the adopted T_{eff} values for the lowest luminosity stars may be summarized as follows: Had we attempted to apply in a wholesale manner a set of H-rich models, the derived temperatures might be lower for some of the coolest objects. However, the generally well-determined trigonometric parallaxes provide lower limits to the luminosities, which seriously constrain how much lower the effective temperatures can be. Thus, neither the derived LFs from § III*e* nor the implications discussed in § IV would be significantly different had we adopted instead the available fits from hydrogen atmospheres for the coolest stars.

c) Bolometric Corrections and Luminosities

For the DA stars in Table 1 we obtain values of M_{bol} by graphical interpolation from the BCs listed by Greenstein (1976, his Table VI) for cool, hydrogen atmosphere models provided by H. L. Shipman. However, BCs are generally not explicitly published for cool helium atmosphere models. The helium-rich models of Mould and Liebert (1978) cover only a limited range of T_{eff} for which the input physics is too simplified. A different procedure is therefore adopted for the set of non-DA stars. In Figure 1 the T_{eff} values for stars listed in Table 2 are plotted against M_V values. We calculate M_{bol} values for the non-DA stars by applying BCs appropriate for blackbodies of the same T_{eff} to each of the non-DA stars entered in Table 2 and shown in Figure 1. Blackbody BCs as a

TABLE 3

ALTERNATIVE T_{eff} ESTIMATES USING H-RICH ATMOSPHERES FOR DC9 AND DZ9 STARS IN TABLE 2

Name	WD	Spectral Type	T_{eff} (H)	T_{eff} (He)	M_V
LP 101-16	1633+573	DC9	6300	5200	14.16
G125-3	1917+386	DC9	6800	5200	14.25
G39-27	0433+270	DC9	5700	4800	14.72
W489	1334+039	DZ9	4100	4400	15.11
HL 4	0552−041	DZ9	4400	4600	15.41
vB 11	2054−050	DC9	3400	3750	15.59

function of T_{eff} are taken from Kuiper (1938). The resulting plot of M_V versus M_{bol} values for the DA and non-DA stars is shown in Figure 2*a*. The dashed line depicts a quadratic least-squares fit to the non-DA data points. The fitted relation is

$$M_{bol} = -19.55 + 3.847M_V - 0.1042M_V^2 . \quad (1)$$

The solid diagonal line is simply the relation for no BC. Note that the blackbody BCs are essentially zero at temperatures near solar, while the corresponding model BCs for the DA stars remain a few tenths of a magnitude (due to the extensive hydrogen line blanketing).

Alternatively, one may assume that each white dwarf has a radius appropriate to the mean derived for hotter hydrogen-rich (DA) and helium-rich (DB) degenerates, that is, $R \sim 0.012$ $R_\odot$, corresponding to a mean mass near 0.6 $M_\odot$ (Koester, Schulz, and Weidemann 1979; Weidemann and Koester 1984; Oke, Weidemann, and Koester 1984). One may then determine M_{bol} directly via the relations

$$L = 4\pi R^2 \sigma T_{eff}^4 \quad (2)$$

and

$$M_{bol} = -2.5 \log (L/L_\odot) + 4.70 . \quad (3)$$

M_{bol} values determined from the derived temperatures and the assumption of a fixed radius are listed in the last column of Table 2.

Application of this second method leads to smaller implied BCs for the cooler stars than those obtained from applying the blackbody or model BCs. In Figure 2*b*, the derived M_{bol} values for each star obtained from equations (2) and (3) are plotted explicitly against M_V. While the scatter is considerably larger, the data points at the faint end cluster around the solid diagonal line for $M_{bol} = M_V$. At the bright end, the DA stars tend to fall to the right of the line, as expected because of the hydrogen line blanketing and in agreement with the models. The mean blackbody relation taken from Figure 2*a* is also shown as a dashed line. The asterisks connected by a dashdot line show the result of applying BCs for main-sequence stars at the cool end of the T_{eff} sequence. The larger BC values for cool main-sequence stars are attributable to the greater line blanketing at optical wavelengths for these stars, in comparison with blackbodies and with most cool white dwarfs.

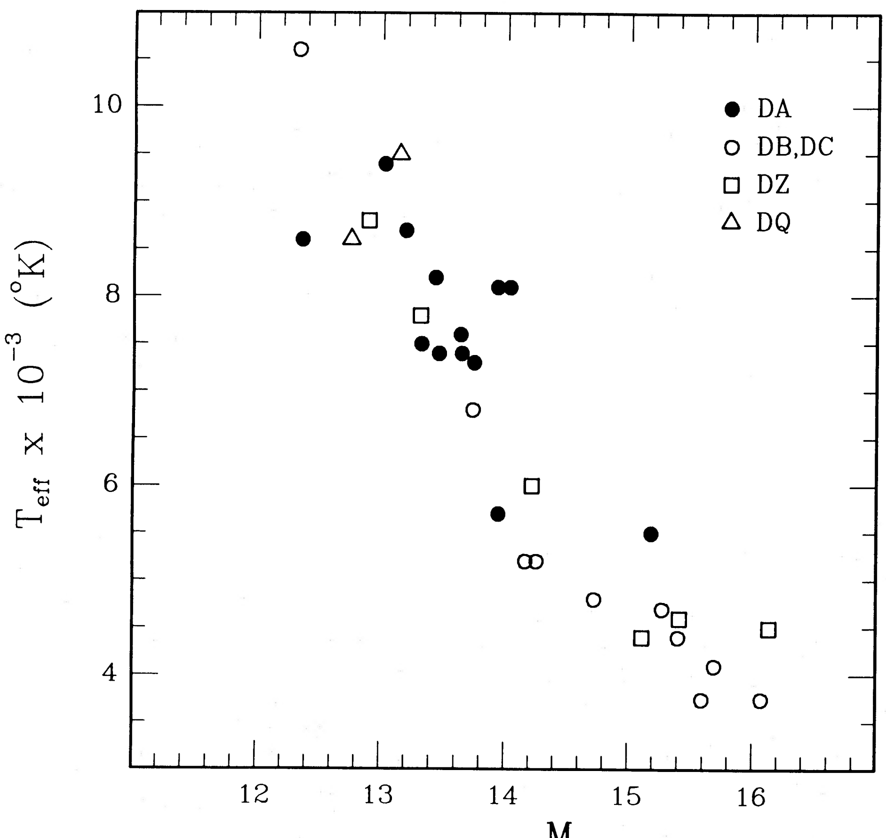


FIG. 1.—Effective temperatures plotted against absolute visual magnitudes for the stars in Table 2. The different spectral types are indicated by symbols, as shown in the key.

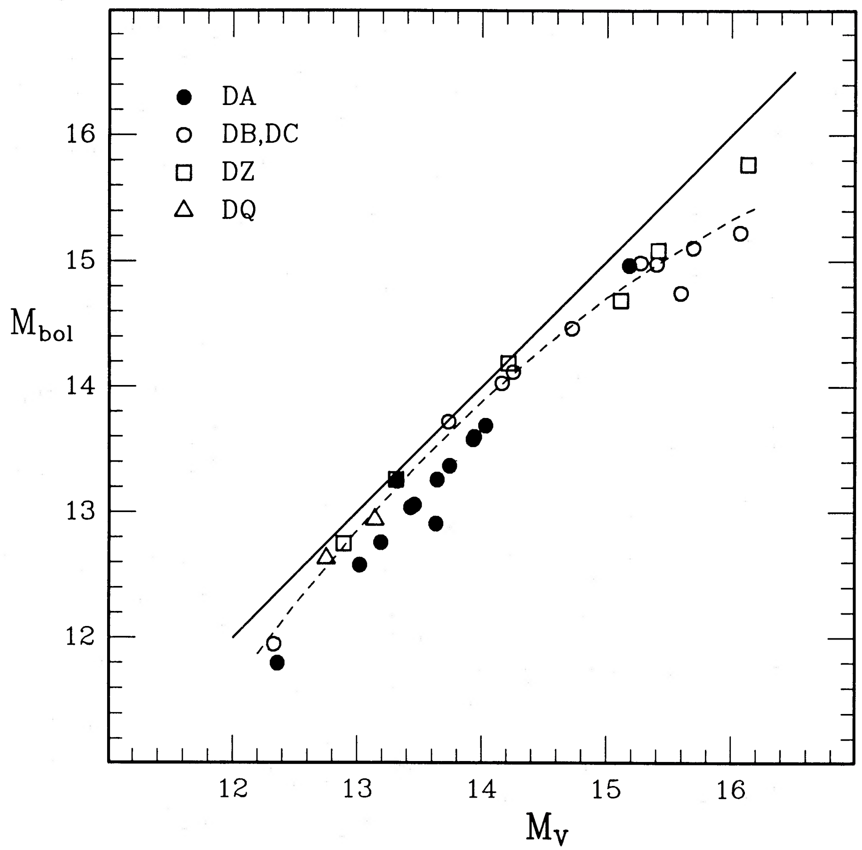


FIG. 2a

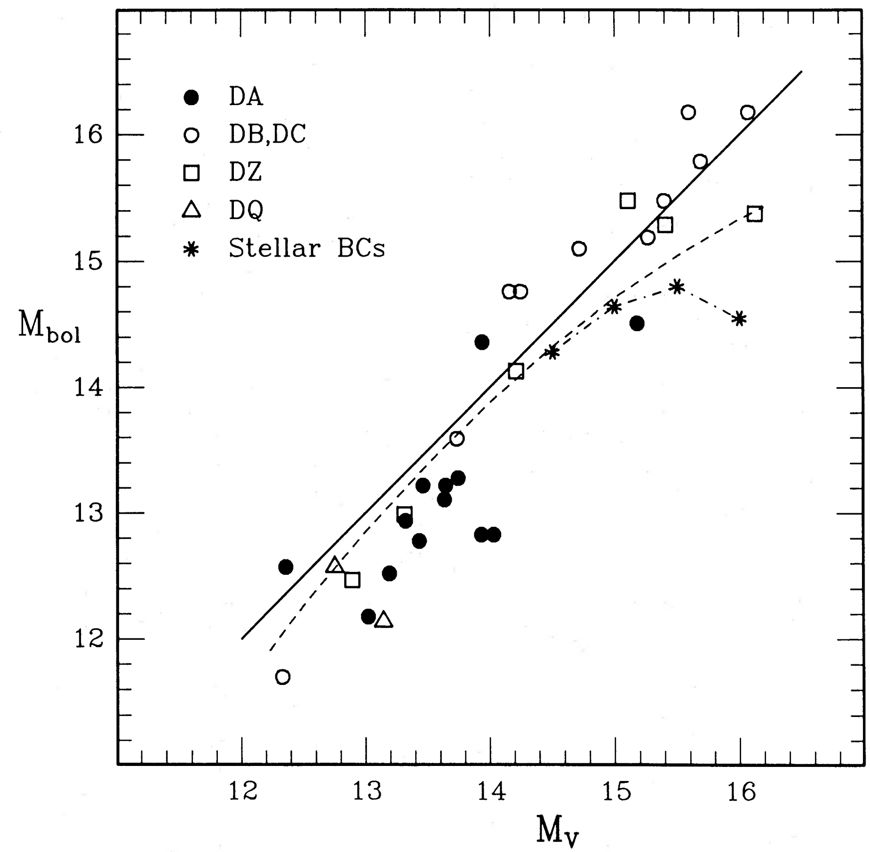


FIG. 2b

FIG. 2.—M_{bol} as a function of M_V for the sample of cool white dwarfs from Table 2, with symbols for each spectral type as shown in the key, for each of the two methods described in § IIIc. (*a*) The relation after application of the blackbody BCs to each stellar T_{eff} value for non-DA stars and model BCs for DA stars. The dashed line is a least-squares fit to the data for only the non-DA stars of Table 2, given as eq. (1) in the text. The solid line is that for zero BC. (*b*) The relation derived from direct use of the model T_{eff} vaues listed in Table 2, and assuming a radius of $0.012 R_{\odot}$. In addition to the same zero BC and blackbody curves as for (*a*), the asterisks connected by dash-dot lines show the result of applying BCs for line-blanketed main-sequence stars (taken from Schmidt-Kaler 1982) at the cool end of the T_{eff} sequence.

TABLE 4

TRANSFORMED LUMINOSITY FUNCTION FOR HOT WHITE DWAFS

M_V (1)	N(DA) ($pc^{-3}\ M_V^{-1}$) (2)	N(DA + non-DA) ($pc^{-3}\ M_V^{-1}$) (3)	Mean M_{bol} (4)	$\Delta M_{bol}/\Delta M_V$ (5)	N(DA + non-DA) ($pc^{-3}\ M_{bol}^{-1}$) (6)	log N ($+\sigma$, $-\sigma$) (7)
9.5	3.08×10^{-6}	3.70×10^{-6}	5.50	3.03	1.22×10^{-6}	−5.914 (+0.178, −0.305)
10.0	2.36×10^{-5}	2.36×10^{-5}	6.88	2.34	1.01×10^{-5}	−4.996 (+0.140, −0.208)
10.5	3.68×10^{-5}	4.41×10^{-5}	7.84	2.04	2.16×10^{-5}	−4.666 (+0.126, −0.181)
11.0	1.79×10^{-4}	2.18×10^{-4}	8.92	2.28	9.56×10^{-5}	−4.020 (+0.117, −0.162)
11.5	2.30×10^{-4}	2.81×10^{-4}	10.12	2.32	1.21×10^{-4}	−3.917 (+0.111, −0.149)
12.0	2.26×10^{-4}	2.81×10^{-4}	11.24	1.86	1.51×10^{-4}	−3.821 (+0.114, −0.155)
12.5	2.88×10^{-4}	3.83×10^{-4}	11.98	1.31	2.92×10^{-4}	−3.535 (+0.113, −0.156)
13.0	5.08×10^{-4}	6.86×10^{-4}	12.55	1.13	6.07×10^{-4}	−3.217 (+0.202, −0.387)

Thus, even for the lowest temperature cases, the implied BCs derived from using explicitly the T_{eff} values determined from models are close to zero or are markedly less than the blackbody, white dwarf model or main-sequence stellar BC values. In fact, the data points for $M_V > +14$ are consistent with the solid line for zero BC. The minuscule BCs implied by this second method may not be unreasonable for T_{eff} down to ~4000 K, since the effect of a dominant He^- continuous opacity is to make the energy distribution for a given T_{eff} bluer than that of a blackbody of the same T_{eff} (cf. Shipman 1972). It is worth noting that the derived T_{eff} for the well-observed star LP 701-29 from Kapranidis models yields values of the radius and mass consistent with the respective means for hotter white dwarfs (Kapranidis and Liebert 1986), although this strongly line-blanketed (DZ) star has a relatively larger BC implied from the second method. Thus, the first method may overestimate the BC values. On the other hand, the second approach is very sensitive to errors in the T_{eff} determinations. There is also the possibility that the coolest white dwarfs, which are billions of years older in cooling times than the hot DA and DB stars used to derive the mean parameters, have a significantly different mean radius (and mass).

Indeed, the blackbody method may also be subjected to a self-consistency check by calculating explicitly the radius of each star. In particular, the high luminosities derived by combining the blackbody BCs obtained from the first method with the individual T_{eff} values for these stars obtained from the models imply that (1) the mean radii for these cool white dwarf are significantly larger (and therefore the mean masses are significantly smaller) than those for hotter white dwarfs; or (2) the T_{eff} values from the models may be modestly too small; and/or (3) the adopted BCs are too large. The first possibility seems least likely, since, if anything, the oldest white dwarfs might be expected to have come from more massive progenitors (Larson 1986; see § IV*c* of this paper) and, hence, may have larger mean masses. On the other hand, the second and third alternatives are viable possibilities. Regardless of which M_{bol} scale is the more appropriate, note that the well-determined trigonometric parallaxes and, hence, accurate M_V values do provide important lower limits to the bolometric luminosities (and upper limits to the cooling ages) of the cool white dwarfs.

d) Bolometric Magnitudes for the Hot White Dwarf Sample

The volume densities for hot DA white dwarfs derived from 353 Palomar-Green survey stars in FLG are listed in bins of M_V. In order to present here a more complete LF for white dwarfs, we must first augment the data for DA stars by including explicitly the non-DA (primarily DB and DO) white dwarfs as a function of luminosity. Second, the hot star space densities must be converted to bolometric magnitude units. In the FLG paper, the model curves were transformed into the observational units for comparison. However, the transformation derived below should be an improvement over that adopted in Winget *et al.* (1987).

In Table 4 we list first the space densities given by FLG (their Table 2). Columns (1) and (2) give the values directly from FLG but expressed per unit M_V interval (for consistency) rather than per 0.5 mag M_V interval. In column (3) the estimated numbers of non-DA stars are included with the DA numbers, according to the approximate ratios of DA to non-DA stars given in Table 3 of FLG. Note, for example, that no augmentation is made to the $M_V = +10$ bin, because of the absence of known DB stars in the corresponding range of T_{eff} (Liebert *et al.* 1986). Also note that the fraction of non-DA stars is believed to be higher at the lower temperatures than at temperatures above the region of the DB "gap".

Column (4) of Table 4 gives the mean M_{bol} corresponding to the M_V bin center. In order to express the space density in bolometric magnitude units, it is necessary to divide the space density per unit M_V by a correction factor ≥ 1 which accounts for the fact that, for hot stars, a bolometric magnitude interval occupies only a fraction of a unit visual magnitude. This correction factor is presented in column (5) of Table 4. The derived space densities per unit bolometric magnitude are shown in column (6), and their logarithmic values are given in column (7). Details on the derivation of the quantities presented in columns (5) and (6) now follow.

In transforming the summed space densities of DA and non-DA white dwarfs per unit M_V, we adopt a relationship between M_V and M_{bol} as calculated from models for hydrogen-rich atmospheres, under the assumption that the physics for such stars is relatively well understood. (Note also that too few hot white dwarfs have accurate enough trigonometric parallaxes for an accurate empirical calibration to be extended to the higher temperatures.) Accordingly, we list in Table 5 the BCs and derived parameters for the two sets of models—the Wesemael *et al.* (1980) pure hydrogen, log $g = 8.0$ models are used for $T_{eff} \geq 20{,}000$ K, while Shipman's results given by Greenstein (1976) are used for $T_{eff} \leq 20{,}000$ K. At 20,000 K the two models differ in the predicted BC by 0.14 mag. For the Wesemael *et al.* (1980) models, we expect that the BC values might be estimated more accurately from the set including hydrogen line blanketing, but only a sparse grid of these was calculated. Accordingly, we have used the BC values from the blanketed models and have applied small corrections (≤ 0.1 mag) to the unblanketed models at T_{eff} values in between those

TABLE 5

THEORETICAL RELATIONS FROM MODELS FOR DA WHITE DWARFS

T_{eff} (K)	Estimated BC[a]	Adopted $R/R_\odot$[b]	log ($L/L_\odot$)	M_{bol}	M_V
Values Derived from Shipman Models					
5000......	−0.39	0.0120	−4.080	14.90	15.29
6000......	−0.29	0.0120	−3.764	14.11	14.40
7000......	−0.35	0.0120	−3.496	13.44	13.79
8000......	−0.41	0.0120	−3.264	12.86	13.27
9000......	−0.47	0.0120	−3.060	12.35	12.82
10000......	−0.53	0.0120	−2.876	11.89	12.42
12000......	−0.82	0.0120	−2.560	11.10	11.92
14000......	−1.21	0.0120	−2.292	10.43	11.64
20000......	−2.14	0.0120	−1.672	8.88	11.02
Values Derived from Wesemael *et al.* Models					
20000......	−2.012	0.0120	−1.682	8.96	10.97
22500......	−2.32	0.0120	−1.478	8.44	10.76
27500......	−2.82	0.0121	−1.122	7.50	10.32
30000......	−3.013	0.0122	−0.963	7.16	10.17
32500......	−3.17	0.0123	−0.817	6.79	9.96
35000......	−3.34	0.0124	−0.682	6.40	9.74
40000......	−3.787	0.0126	−0.436	5.79	9.58
45000......	−4.17	0.0127	−0.224	5.26	9.43
50000......	−4.516	0.0130	−0.020	4.75	9.27
60000......	−5.15	0.0136	+0.335	3.86	9.01
70000......	−5.691	0.0143	+0.646	3.08	8.78
90000......	−6.56	0.0162	+1.191	1.72	8.28

[a] Values listed to millimagnitudes are taken directly from the blanketed models of Wesemael *et al.* 1980; other values are interpolated, using both unblanketed and blanketed models, as described in the text.

[b] Estimated from the evolutionary models of Koester and Schönberner 1986 as described in the text.

with blanketed models. These interpolated BC values are also listed in Table 5. For the Greenstein/Shipman set, we have adjusted the BC and $M_{1.85}$ values from those given in Greenstein (1976, his Table VI) using the quoted transformation $M_V = M_{1.85} + 0.05$. The ratio ($\Delta M_{bol}/\Delta M_V$) was estimated by graphical interpolation at each half-integer M_V for which space densities from FLG are listed and used to compute the numbers in column (6) of Table 4.

Since the mean parameters and their (small) dispersions are better determined for hot DA white dwarfs, the assumption of a mean radius (0.012 $R_\odot$) to convert from derived model T_{eff} values to a stellar luminosity (M_{bol}) has greater justification than in § III*c*, and this radius is adopted for the Shipman and Wesemael *et al.* models with $T_{eff} \leq 22{,}500$ K. For the hotter stars, however, the radii are expected to be significantly larger owing to finite-temperature effects. Accordingly, we have adjusted the assumed radii upward according to the predictions of the evolutionary models of Koester and Schönberner (1986) as a function of T_{eff}. The plot for the 0.6 $M_\odot$ model with an assumed "thin" outer hydrogen layer with $M_H \ll 10^{-4}$ $M_\odot$ (the "He" model; see their Fig. 8) was used. Note that the radii will be larger still if the hot DA stars have "thick" hydrogen layers, but we are persuaded by arguments concerning the pulsations and convective mixing behavior of cool DA stars (cf. Fontaine and Wesemael 1987) that the outer layers are not likely to be this thick. Adopting the radii appropriate to "thin" H layers, luminosities for the grid of models are calculated from equation (1), and the luminosities and BC values are used to estimate M_V values for conversion of the hot degenerate star LF of FLG. These derived values are listed in Table 5. We have omitted the three brightest luminosity bins from FLG, since (1) these M_{bol}, T_{eff}, and space density values are less certain; (2) it appears that DO stars dominate over DA white dwarfs at $T_{eff} > 70{,}000$ K so that a different set of helium-rich models should be employed; and (3) this region makes a negligible contribution to the space density of degenerate stars.

In addition to the errors quoted by FLG for the LF in units of M_V, additional sources of error for each volume density are introduced by (1) the addition of non-DA stars, for which the LF is less certain, and (2) the mapping from M_V into luminosity bins. We estimate the former to be a 10% uncertainty in the total volume densities over the range of magnitudes quoted. The latter is clearly smallest at $M_V = +13$, where the BC is smallest, but becomes quite large for the highest luminosities. In order to allow for these two effects, even if subjectively, we have somewhat arbitrarily adopted a sliding scale of errors, ranging from 40% at $M_V = +9$ to 10% at $M_V = +13$. These errors have been added quadratically to those of FLG, and the adopted total error is included in the last column of Table 4.

e) Derived LF

In application to the cooler stars listed in Table 1, we make two primary assumptions: (1) that the LHS is complete for stars with $V < 19$ and with proper motions $\geq 0\farcs8$ yr^{-1} over the approximately two-thirds of the sky north of $\delta = -20°$; and (2) that the incompleteness factor, due primarily to the lack of Galactic plane fields in the Luyten Palomar (LP) survey, is 20%. We secondarily assume that the LHS is complete for proper motions up to $10\farcs0$ yr^{-1}. Concern has been expressed about the completeness of the LP catalogs for very large motions (cf. Liebert *et al.* 1979*b*). However, our results are affected only modestly when an upper proper-motion limit of 2″ yr^{-1} is adopted. Thus, the present sample overlaps with, but is both larger and more rigorously defined than, that introduced in the earlier paper.

Given the uncertainty in the M_V versus M_{bol} transformation, particularly for the stars of lowest luminosity, it is appropriate to list first the derived LF in the observational M_V bins. Three different binning choices are given in Table 6. First, the volume densities are listed for each half of an M_V interval, with bins centered on the whole and half-magnitudes. The second listing has the same spacing, but with the centers shifted by 0.25 mag. Finally, densities per unit magnitude are listed.

These results correspond, of course, to the second of the limiting BC relations for cool white dwarfs presented in § III*c* (i.e., to identically zero BC values), which is a sufficient approximation to the data points of Figure 2*b*. We also employ the relation using blackbody BCs for the non-DA stars and model BCs for the DA stars, as presented in Figure 2*a*, to derive M_{bol} values for each of the stars in Table 1. The M_{bol} values were then used to determine space densities for the individual (0.5) bolometric magnitude intervals from the sample, employing the $1/V_{max}$ method. The results for blackbody/model BC scale are listed in Table 7 with the same three binning selections. Some of the LFs derived from these two BC scales are presented in Figure 3. The space densities per bolometric magnitude interval shown here for the hotter stars are as derived in Table 4.

In Figures 3*a* and 3*b* two representations of the LF are displayed with M_{bol} assumed equal to M_V for the cool white dwarfs. The first shows the cool stars binned coarsely into 1 mag intervals, as used in Winget *et al.* (1987). In this panel, a

TABLE 6

$1/V_{max}$ SPACE DENSITIES DERIVED FROM M_V DATA

M_V	Number	$\langle V_{tan} \rangle$ (km s^{-1})	Space Density × 10^4 (pc^{-3} M_V^{-1})	log Space Density [log $N(+\sigma, -\sigma)$]
Bins 0.50 mag Wide Centered on M_V				
13.5.........	8	207:	8.7 ± 4.4	−3.06 (+0.18, −0.30)
14.0.........	9	89	11.8 ± 5.7	−2.93 (+0.17, −0.29)
14.5.........	4	125	9.3 ± 7.7	−3.03 (+0.26, −0.76)
15.0.........	8	89	10.4 ± 5.2	−2.98 (+0.18, −0.30)
15.5.........	11	125	8.0 ± 3.3	−3.09 (+0.15, −0.23)
16.0.........	2	115	0.7 ± 0.6	−4.14 (+0.25, −0.64)
16.5.........	1	115	0.3 ± 0.3	−4.50 (+0.30, −∞)
Bins 0.50 mag Wide Centered on M_V				
13.25.........	4	273:	4.0 ± 3.3	−3.40 (+0.26, −0.78)
13.75.........	9	110	12.8 ± 5.9	−2.89 (+0.17, −0.27)
14.25.........	5	134	3.6 ± 2.3	−3.44 (+0.22, −0.46)
14.75.........	4	70	10.7 ± 7.8	−2.97 (+0.24, −0.57)
15.25.........	14	115	13.6 ± 5.5	−2.87 (+0.15, −0.23)
15.75.........	4	100	3.5 ± 2.4	−3.45 (+0.22, −0.48)
16.25.........	2	115	0.7 ± 0.6	−4.14 (+0.25, −0.64)
16.75.........	1	115	0.3 ± 0.3	−4.50 (+0.30, −∞)
Bins 1.00 mag Wide Centered on M_V				
13.5	13	160	8.4 ± 3.4	−3.08 (+0.15, −0.23)
14.5	9	106	7.1 ± 4.1	−3.15 (+0.20, −0.37)
15.5	18	112	8.5 ± 3.0	−3.07 (+0.13, −0.16)
16.5	3	115	0.5 ± 0.3	−4.28 (+0.21, −0.42)

drop in the LF of more than an order of magnitude occurs at log $(L/L_{\odot}) \sim -4.5 \pm 0.15$, as discussed in that paper. The theoretical fit from the Winget *et al.* models, which assumes a cutoff in white dwarf formation some 9.0×10^9 years ago, is also depicted. However, the shape of the cool end of the empirical LF, and the position of the cutoff, are somewhat sensitive to the choices of bin width and bin centers. Thus, we show in Figure 3*b* the same data set binned in 0.5 mag intervals. This version suggests a fit with a cutoff at a luminosity as much as 0.2 higher in the logarithm. In the last two panels of Figure 3, LFs based on the blackbody/model BCs are shown. Both use bins with 0.5 bolometric magnitude widths. However, in Figure 3*c* these are centered on the even and 0.5 mag points, while Figure 3*d* shows the effect of shifting the data set by 0.25 mag. With these assumed BCs, the cool stars are more luminous. Thus the drop from the peak is moved back as far as log $(L/L_{\odot}) \sim -4.2$. (The crosses in Fig. 3*d* are discussed in § IV*a*.)

We have argued previously that the two BC assumptions presented above are limiting cases. With respect to the considerable influence of this parameter on the final result, the

TABLE 7

$1/V_{max}$ SPACE DENSITIES DERIVED FROM M_{bol} DATA

M_{bol}	Number	$\langle V_{tan} \rangle$ (km s^{-1})	Space Density × 10^4 (pc^{-3} M_{bol}^{-1})	log Space Density [log $N(+\sigma, -\sigma)$]
Bins 0.50 mag Wide Centered on M_{bol}				
13.5..........	9	178	11.3 ± 5.7	−2.95 (+0.18, −0.31)
14.0..........	5	132	4.3 ± 2.7	−3.37 (+0.21, −0.42)
14.5..........	7	77	13.8 ± 8.1	−2.86 (+0.20, −0.38)
15.0..........	15	117	14.1 ± 5.6	−2.85 (+0.15, −0.22)
15.5..........	3	115	1.1 ± 0.7	−3.96 (+0.21, −0.41)
Bins 0.50 mag Wide Centered on M_{bol}				
13.25.........	5	194:	6.6 ± 3.9	−3.18 (+0.20, −0.38)
13.75.........	9	117	11.3 ± 5.7	−2.95 (+0.18, −0.30)
14.25.........	4	126	10.1 ± 7.9	−3.00 (+0.25, −0.66)
14.75.........	16	104	16.7 ± 5.8	−2.78 (+0.13, −0.19)
15.25.........	7	117	4.5 ± 2.4	−3.35 (+0.19, −0.35)
15.75.........	1	115	0.3 ± 0.3	−4.47 (+0.30, −∞)
Bins 1.00 mag Wide Centered on M_{bol}				
13.5..........	14	144	8.9 ± 3.4	−3.05 (+0.14, −0.21)
14.5..........	20	108	13.4 ± 4.9	−2.87 (+0.14, −0.20)
15.5..........	8	117	2.4 ± 1.2	−3.62 (+0.18, −0.31)

TABLE 8

PUBLISHED ESTIMATES OF THE TOTAL MASS DENSITY OF WHITE DWARFS

Reference	Method of Analysis	Mass Density[a] ($M_\odot$ pc^{-3})
Eggen 1983	$1/V_{max}$; $V \le 15.0$, $\delta < +30°$, $\mu \ge 0''.7$ yr^{-1} sample	0.004
Liebert, Dahn, and Sion	Relative completeness	0.004[b]
Liebert, Dahn, and Sion 1983	Known WDs within 10 pc of Sun	0.002
This paper	$1/V_{max}$; *LHS* WDS with $\delta \ge -20°$ and $\mu \ge 0''.8$ yr^{-1}	0.002

[a] Number densities and quoted mass densities converted to a common scale assuming a fixed mass of 0.6 $M_\odot$.

[b] Renormalized to the hot star values of FLG.

low-luminosity ends of the alternative LFs shown here may also bracket the correct answer. In particular, because of the generally accurate trigonometric parallaxes available for most of this sample, the "zero BC" solutions—Figures 3*a* and 3*b*—should provide a lower limit to the luminosity of the cutoff in the true LF. In summary, taking into account also the sensitivity of the cutoff to the bin centering and width, we can conclude only that the drop occurs within the approximate range $-4.2 \le \log(L/L_\odot) \le -4.6$.

These LFs differ from the preliminary version presented in Winget *et al.* (1987) in the following ways: (1) the data for the cool stars are binned in 0.5 bolometric magnitude bins, except for Figure 3*a*, rather than in whole visual magnitude bins which were transformed; (2) new trigonometric parallax, photometric, and spectroscopic data have been used to refine the M_V estimates for a significant number of stars; and (3) the Palomar-Green (FLG) hot star data should be on a bolometric scale of somewhat improved accuracy. Some implications of the derived LF for the ages of the oldest observed degenerate stars are discussed in § IV*c*.

The cumulative space density of local degenerate stars resulting from the sums of the individual volume densities from Table 2 of FLG for the hot stars and from Table 6 for the cool stars (where the individual values per unit M_V for every 0.5 M_V unit have been summed and divided by 2) is 0.0032 white dwarfs pc^{-3}. If the mean mass of 0.6 $M_\odot$ determined for hot white dwarfs is applicable to the bulk of the stars, this implies a contribution to the local mass density in the solar neighborhood of only 0.002 $M_\odot$ pc^{-3}. This result is in good agreement with other estimates (see § III*g* and Table 8). We discuss in § IV why it is unlikely that there are large numbers of undetected, extremely faint degenerate dwarfs.

f) Justification and Tests for the Derived LF

The recent analysis by Dawson (1986) indicates that the LHS Catalog is mostly complete over the region of the sky north of $\delta = -30°$ and above Galactic latitude $b = 10°$. Further justification for adopting the $1/V_{max}$ procedure in analyzing the LHS sample of white dwarfs is provided by study of similarly defined samples of M dwarfs and Population II main-sequence subdwarfs (Liebert and Dahn 1985). The former gives reasonably consistent results in comparison with M dwarf LFs derived from the red color survey of Gilmore and Reid (1983)—see the review of M dwarf LF estimates by Reid (1987). It is reasonable to assume that color surveys are free of kinematic bias. Yet the LHS sample of M dwarfs yields an LF with proportionately more stars past the peak near $M_V \sim +13$ than do the Gilmore-Reid samples (see also the review by Liebert and Probst 1987). Thus, if anything, the method of allowing for proper-motion bias may produce a luminosity function which is *too large* at the faint end. Indeed, follow-up work on the LHS Catalog sample has led to the identification of the least luminous known main-sequence stars (at least with accurate trigonometric or photometric parallaxes). Such stars include LHS 2924 (Probst and Liebert 1983), for which a preliminary USNO CCD trigonometric parallax (Monet and Dahn 1984) indicates $M_V \sim +19.5$—some 3 mag fainter than the lowest luminosity white dwarf yet found from the LHS. The LHS Catalog has also resulted in the identification of the least luminous known subdwarf M or Population II main-sequence stars (Hartwick, Cowley, and Mould 1984; Liebert and Dahn 1985; Liebert and Probst 1987).

Moreover, we note that the mean tangential velocities for the stars in Table 6 and 7 do not vary appreciably with absolute magnitude. This suggests that the $1/V_{max}$ method treats the kinematic bias somewhat uniformly over the observed LF. The mean values are higher than generally found for unbiased samples of K–M dwarfs. This may be readily understood as a combination of two effects: (1) the LHS Catalog, which includes only stars with proper motions larger than $0''.5$ yr^{-1} obviously will be biased toward higher tangential velocities, and (2) the white dwarfs, with cooling ages in excess of 10^9 yr, may well constitute an older sample than those typical for K–M dwarfs.

g) Comparison with White Dwarf Space Densities (LFs) Derived from Other Methods and Samples

While the sample defined in § II includes the three lowest luminosity degenerate stars currently known with reasonable trigonometric or photometric parallaxes, most of the known white dwarfs with $M_V > 14$ are not in this sample. Previous estimates of the white dwarf LF have attempted to include a greater fraction of the total empirical sample using a method of "relative completeness" (Weidemann 1967; Sion and Liebert 1977; Liebert, Dahn, and Sion 1983). This approach recognizes that an empirical catalog of all white dwarfs is incomplete because of kinematical bias and other factors. The crucial assumption of the method is that the fractional or relative completeness in the empirical sample of known cool white dwarfs must be independent of absolute magnitude or color down to a given apparent magnitude, say $V = 15.5$ or 16.5. The apparent-magnitude cutoff is chosen at the magnitude at which the cumulative numbers in each absolute magnitude bin stop increasing by the factor of 4 expected from a spatially

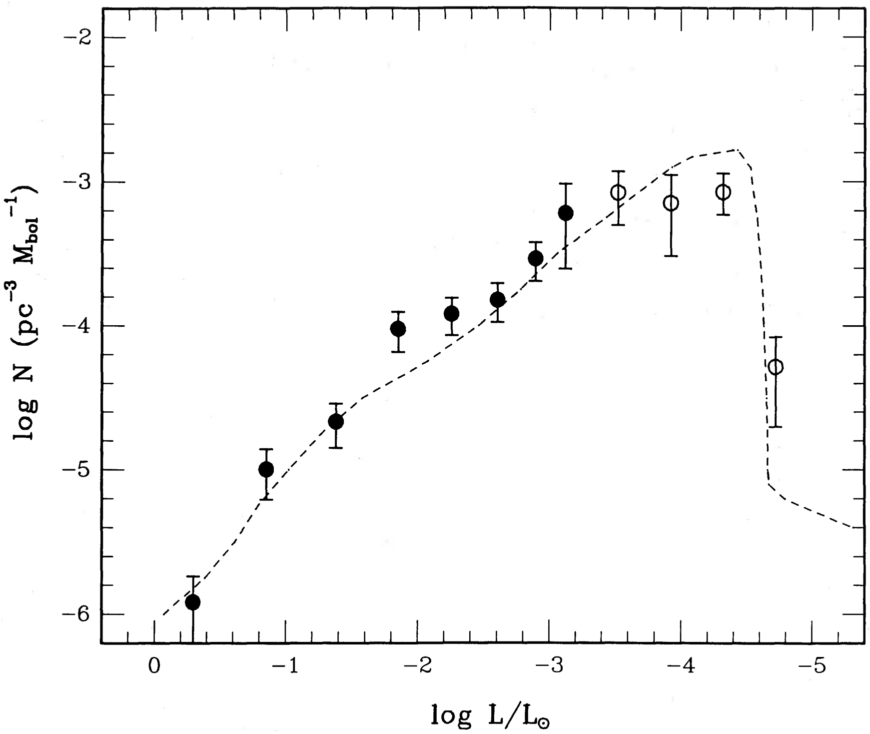


FIG. 3*a*

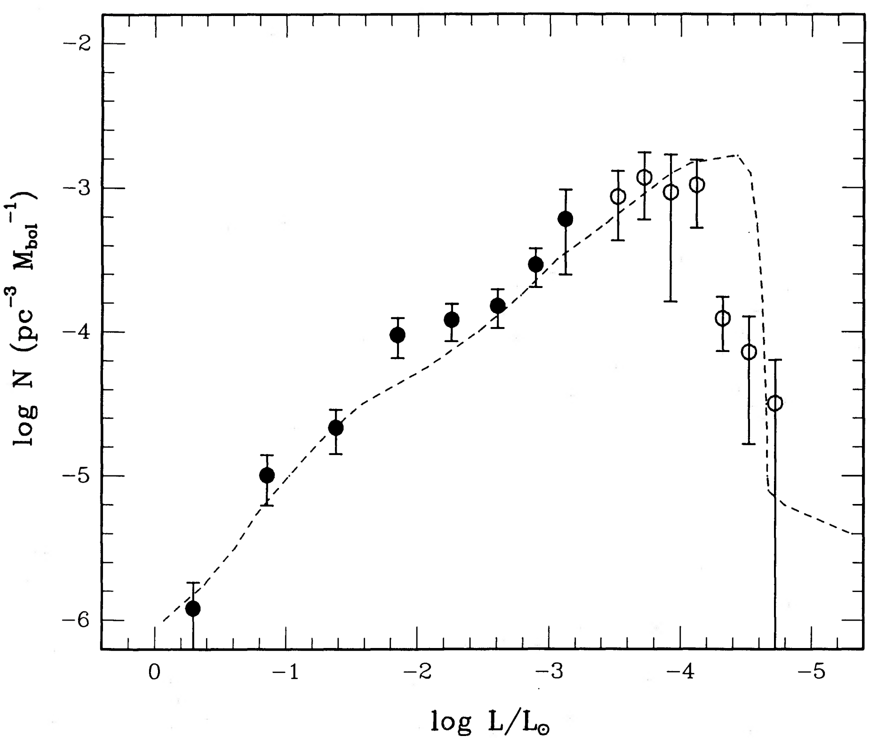
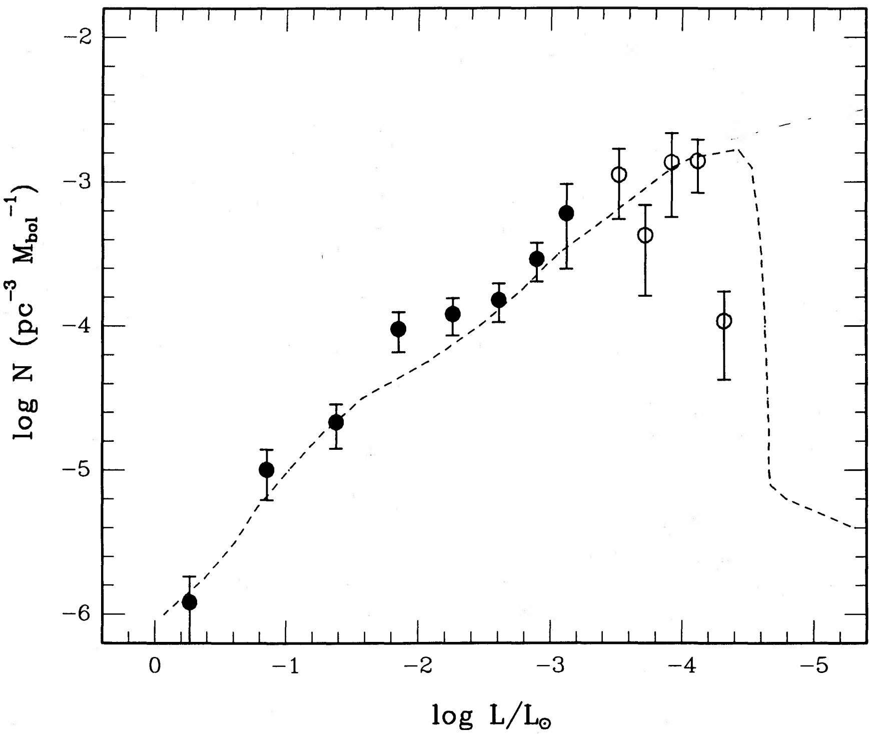


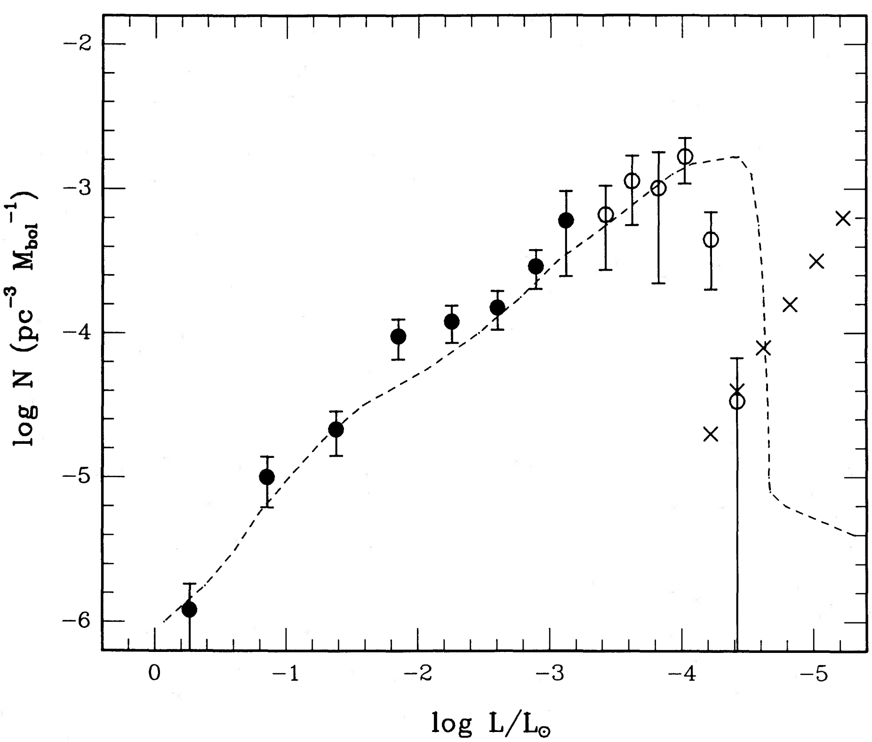
FIG. 3*b*

FIG. 3.—Derived luminosity function of white dwarfs, specifically the volume density per unit absolute bolometric magnitude M_{bol}. The filled circles are data taken from Fleming, Liebert, and Green (1986), in 0.5 unit M_V bins, transformed into luminosity as described in § III*d*. The open circles are derived from the data in Table 1, as described in § III. Also shown, as the dashed curve, is the theoretical function of Winget *et al.* (1987; their Fig. 1), assuming a constant white dwarf formation rate with time. (*a*) The cool white dwarf data with M_{bol} assumed equal to M_V, binned in 1 mag bins centered on the half-magnitudes, as listed in the bottom set of Table 6. This choice is similar to that presented in Winget *et al.* (1987). (*b*) The cool white dwarf data with no BC, but binned in 0.5 mag intervals, as given in the middle set of Table 6. (*c*) The cool white dwarf data with blackbody and model BCs, binned in 0.5 bolometric magnitude bins, as listed in the top set of Table 7. (*d*) The cool white dwarf data with blackbody/model BCs binned in 0.5 bolometric magnitude bins, but shifted in their centering by 0.25 mag, as listed in the middle set of Table 7. Crosses represent the contributions of a single "ER 8–like" star on the LF for each of several faint M_{bol} bins, as discussed in § IV*a*.



log N (pc^{-3} M$_{bol}$$^{-1}$)
−2
−3
−4
−5
−6
0
−1
−2
−3
−4
−5
log L/L$_\odot$

FIG. 3*c*



log N (pc^{-3} M$_{bol}$$^{-1}$)
−2
−3
−4
−5
−6
0
−1
−2
−3
−4
−5
log L/L$_\odot$

FIG. 3*d*

uniform distribution. This presumably marks the onset of incompleteness due to apparent-magnitude limitation (see further explanation in the above references).

The crucial assumption of the "relative completeness" method is difficult to defend. While spectroscopic investigations of faint proper-motion stars have proceeded vigorously for objects of all catalog color classes (photographic estimates), including the reddest colors, there is every reason to suspect that the kinematic bias in the available sample may depend on absolute magnitude or stellar color, In particular, stars selected at $M_V = +13$ will have higher space motions than stars of the same proper motion and apparent magnitude at $M_V = +15$ or fainter. Moreover, since an apparent-magnitude cutoff of $V = 15.5$ to $V = 16.5$ severely limits the volume of space searched for the lowest luminosity degenerates, little useful information on the number densities of stars with $M_V > 16$ is obtainable from this method. Hence, we do not discuss explicitly an LF derived from this method.

The application of the method previously by Liebert *et al.* (1983) probably results in a serious overestimate of the total number density of white dwarfs. Since the sample employed was assumed only to be "relatively" complete even at brighter apparent magnitudes, because of kinematic and other selection effects, the number density of cool white dwarfs was normalized to that derived for hot white dwarfs from the earlier determination of Green (1980). The revised space density of the hot white dwarfs from the Palomar Green sample (FLG) is reduced about a factor of 2 from the earlier determination, which was based on a smaller, poorly observed sample. Thus, the corresponding normalization for the cool white dwarfs with $M_V \geq +13$ should also be reduced by about a factor of 2 in Liebert, Dahn, and Sion (1983).

Despite our strong misgivings about the method, a new normalization of the Liebert, Dahn, and Sion (1983) numbers to the revised LF for hot white dwarfs *does result in good agreement* between the space density derived from the "relative completeness" method and that presented here from the $1/V_{max}$ method, as well as with space densities of white dwarfs derived from other sources which are presented in Table 8.

IV. THE FAINT CUTOFF IN THE LUMINOSITY FUNCTION AND ITS IMPLICATIONS

It is apparent from Tables 6 and 7 and from Figure 3 that the luminosity function declines sharply for M_V fainter than about $+16.0$ or $M_{bol} \sim +15.5$–16, which corresponds to log $(L/L_\odot) \sim -4.2$ to -4.6. The faintest star in this sample has a lower limit luminosity of log $(L/L_\odot) \sim -4.6$. For fainter magnitude bins in which no stars were found, no useful upper limits can be estimated from the $1/V_{max}$ method. However, several additional types of evidence, including results from other samples of proper-motion stars and searches for faint binary companions, support the conclusion from our data that the sharp decine past log $(L/L_\odot) \sim -4.4$ is a real effect.

a) Other Proper-Motion Samples: A Numerical Test of the Possible Significance of ER 8

The extensive follow-up observations of faint Luyten and Lowell proper-motion stars represented in such papers as Greenstein (1984), Eggen (1983), Hintzen (1986), Probst (1985), and Bessell (1986) have added to the numbers of cool white dwarfs known in the 1980s. Many of these have proper motions too small, or lie too far south, to be included in the sample analyzed here. With the exception of Eggen's (1983) sample limited to $V \leq 15.0$, these data have not been analyzed to determine an LF for the white dwarfs. However, the complete list of spectroscopically confirmed white dwarfs (McCook and Sion 1987) now includes over 80 stars with $M_V > +14$, having reasonable trigonometric or photometric parallaxes, ranging in distance out to 50 pc. A V-magnitude limit of around 18–19 implies the ability to find white dwarfs fainter than $M_V = +17$ to distances in excess of 10 pc. We have also noted the discovery of dozens of main-sequence stars fainter than $M_V = +16$ from these samples.

The only additional white dwarf discovered in these other samples which is certainly of potential interest in further defining the limits of the empirical distribution is the star ER 8 (Ruiz *et al.* 1986), uncovered in the early stages of an important new proper-motion survey in the southern hemisphere. While the reported spectrophotometric $(B-V)$ color is marginally redder than those for the reddest stars in Table 1 (LHS 342 and LHS 2673), the object lacks a completed trigonometric parallax, and the photometric parallax estimate must be regarded as quite uncertain for this object (Liebert, Fontaine, and Wesemael 1987).

It is an interesting exercise to consider the impact of ER 8 might have had on our analysis, had it been a northern object found in the Luyten Palomar survey. This exercise illustrates the sensitivity of our derived cutoff in the LF to the discovery or failure to discover individual objects of very low luminosity where the effective search volume is minimal. Accordingly, we have run LFs with a single star added to our sample having the same apparent magnitude ($V = 17.2$) and proper motion ($2''.128$ yr^{-1}) as reported for ER 8, but for a range of assumed distances and M_V values. The adopted BC is 1.1 mag, appropriate for a blackbody of 3500 K—the color temperature reported by Ruiz *et al.* (1986). We have used the LF binning from the middle section of Table 7 and shown in Figure 3*d*, and have added single hypothetical ER 8–like stars to the sample for distances giving derived M_{bol} values of 15.75–17.75 in 0.5 mag intervals. Each of these cases is listed as a row in Table 9. The corresponding luminosities, parallaxes, and tangential velocities are also listed. The derived space density for each bin from the experiment of adding a single object with these parameters is presented in the second-to-last column, and each of these hypothetical points is added (as a cross) to Figure 3*d*. Finally, the number of similar objects required to bring the total number density back up to the peak near log $(L/L_\odot) = -4$ is shown in the final column.

The top row of Table 9 corresponds to close to the maximum luminosity ($M_{bol} = 15.75$), which we suspect is consistent with the observed optical colors. Note that the implied distance is near 15 pc and the implied tangential velocity of 118 km s^{-1} is close to the means for the cool white dwarf bins listed in Table 6. At the other extreme, the assigned $M_{bol} = 17.75$ corresponds to $M_V = 18.85$ and is close to the distance (4.6 pc) assigned by Ruiz *et al.* (1986) assuming that ER 8 belongs to the Hyades Supercluster. It is difficult to take seriously the alleged membership of a white dwarf having a cooling age of order 10^{10} yr in an association with the Hyades cluster, whose main-sequence turnoff age is around 7×10^8 yr. However, the tangential velocity would still be reasonable for an old disk star.

At the higher assumed luminosities, a single hypothetical ER 8 would have little effect on the derived LF. That is, large numbers of such objects would have to be added in order to alter drastically the derived LF reported here. If the actual

TABLE 9

POSSIBLE INFLUENCE OF "ER 8–LIKE" STARS ON SPACE DENSITIES

$\log (L/L_\odot)$	M_{bol}	Parallax	V_{tan} (km s^{-1})	$\log N$ (for 1 star)	Stars Needed for $\log N = -2.8$
−4.22......	15.25	0″.0676	149	−4.70	57.0
−4.42......	15.75	0.0851	118	−4.40	39.0
−4.62......	16.25	0.107	94	−4.10	20.0
−4.82......	16.75	0.135	75	−3.80	10.0
−5.02......	17.25	0.170	59	−3.50	5.0
−5.22......	17.75	0.214	47	−3.20	2.5

luminosity of ER 8 were less than 10^{-5} $L_\odot$, however, then the available search volume is so small that the addition of a single object would correspond to a relatively high space density. In the latter case, there would nonetheless be a substantial deficiency of stars between approximately $-4.4 < \log (L/L_\odot) < -5$. That is, the LF would have to plunge and then rise again very sharply at luminosities almost too low to be detectable in the LHS sample. Unless one invokes an extreme form of the star formation model discussed in § IV*d*, the existence of an ER 8 of very low luminosity, but missed in the sample discussed previously should have a minimal impact on the derived LF. There is in any case no justification at this time to assume such a very low luminosity of $M_{bol} > 17$ for ER 8.

b) Searches for Faint Binary Comparisons

There are three ways in which white dwarfs less luminous than the empirical limit might be found as binary companions to known, visible stars:

1. Numerous cool white dwarfs have been found as common proper motion companions, since deep plates of a given stellar field are routinely taken of identified proper-motion and parallax stars (see discussion in Liebert *et al.* 1979*b*). This method is most proficient in identifying companions at 10^2–10^3 AU separations (Greenstein 1986*a*). Yet the least luminous degenerate companions found by this method have $M_V < +16$.

2. No massive, dark, close companions have been found in perturbation analyses of the parallax solutions for nearby stars (Shipman 1983). This technique is potentially powerful, since the primary purpose of these searches is to look for companions much less massive than the visible star! This method is proficient for separations of order 1–10 AU, and a massive white dwarf companion could be completely invisible.

3. Radial velocity observations of nearby low-mass stars, again with the principal purpose of searching for lower mass companions, also have produced null results (Marcy *et al.* 1986). This particular sample of 65 M dwarf stars was chosen to exclude known binaries. With the use of multiple spectroscopic observations of very high resolution (corresponding to 500 m s^{-1} for Marcy *et al.*), this technique might uncover the presence of a massive companion to an M dwarf at a separation ranging from a small fraction of an AU to 10 AU.

The binary searches are of course difficult to quantify into a global limit on low-luminosity white dwarfs, since the frequency of occurrence of the latter in necessarily very old binaries is unknown, as is the distribution of period (see Greenstein 1986*a*). Assuming that the sharp drop near $\log (L/L_\odot) \sim -4.4$ is real, then the contribution of the white dwarfs to the total mass density locally must be small, as discussed previously. Even with a generous allowance of undiscovered components in binaries, the contribution to the local mass density from degenerate stars is unlikely to exceed 0.005 $M_\odot$ pc^{-3}.

c) Constraints on Cooling Ages and the Local Mass Density

Winget *et al.* (1987) interpret the observed break in the empirical LF near $\log (L/L_\odot) \sim -4.4$ as due to the finite age of the Galactic disk and its near-equality with the cooling ages of the oldest white dwarfs. The theoretical calculations do *not* predict that the onset of Debye cooling would be sufficiently rapid to produce such an observed break, with older white dwarfs of similar mass dropping quickly enough to much lower luminosities. In other words, the observed distribution can be reconciled with theory only by assuming a similarly sharp drop in the white dwarf formation (stellar death) rate at a time in the past equal to the cooling time required for the degenerates to reach $\log (L/L_\odot) \sim -4.2$ to -4.6. The range in cooling ages corresponding to the new log L values for a 0.6 $M_\odot$ model (Winget *et al.* 1987, Table 1) is approximately (6.4–10.8) × 10^9 yr. This interval overlaps considerably with the previous values, though the mean is $\sim 10^9$ yr younger.

A different interpretation is advanced in the paper of Mazzitelli and D'Antona (1986). The cooling time required to reach $(L/L_\odot) = -4.5$. is only (5–7) × 10^9 yr according to their models—values which these authors regarded as smaller than an acceptable age for the Galactic disk. However, Winget and Van Horn (1987) show that the age difference between the two model sets is attributable to the different physical parameters adopted in these separate calculations. Moreover, it is not clear which set of physical parameters is correct! On the other hand, nucleosynthesis chronometers also are indicating rather short time scales for the Galactic disk (Fowler and Meisl 1986; Butcher 1987).

Regardless of which is the correct time scale for the cooling models, the conclusion that even the oldest white dwarfs in the Galactic disk (except for those with luminous companions) are still detectable leaves intact the determination (Liebert *et al.* 1979*b*; Green 1980; FLG) that they contribute little to the local mass density of the disk. The value of 0.002 $M_\odot$ pc^{-3} corresponds to approximately 1% of the dynamically estimated local mass density (Bahcall 1984*b*, *c*). This result might be misleadingly low if the oldest white dwarfs have a substantially higher scale height than their hotter counterparts (Iben and Tutukov 1984), although the local mass density should be proportional (only) to the inverse of the scale height. A second possible caveat to this conclusion would be the existence of significant numbers of old remnants much more massive than 0.6 $M_\odot$, as considered below.

d) Constraints on a Bimodal Star Formation Hypothesis

The existence of at least a fairly sharp downturn in the white dwarf LF beyond $\log (L/L_\odot) \sim -4.4$ also severely constrains the bimodal star formation hypothesis advocated recently by Larson (1986). He invokes a second "high-mass" star formation mode, occurring preferentially in the early history of the Galactic disk, in order to produce a large local mass density of very faint or unobservable remnant stars. He also suggests that binaries of high mass ratio might not have been formed in this mode, so that searches for massive companions around low-mass main-sequence stars in the solar neighborhood (see § IV*a*) might not be fruitful.

The parameters of the high-mass mode would have to be adjusted in a manner different from that of Olive (1986). We require that the true LF for white dwarfs be strongly bimodal.

That is, the observed peak would be predominantly "low-mass" mode stars, while most "high-mass" remnants have cooled to much lower luminosities, or are neutron stars. The observations require a minimum between the peaks of the two modes, with the second peak not yet observed. As is illustrated by the high-mass tail of the curve in Figure 1 of Winget *et al.* (1987), the more massive white dwarfs of the same age lie at lower luminosites. Since the onset of crystallization is reached more quickly at their higher interior densities, any massive degenerates are expected to cool more quickly at the lower luminosities. Indeed, it is generally expected that a population of high-mass progenitors will produce degenerate remnants of higher mass (Weidemann and Koester 1984), so that this possibility cannot be excluded. In fact, the uncertainty in the relation between initial mass and final mass at higher masses and for various chemical compositions introduces further ambiguity in evaluating a bimodal star formation hypothesis. We also emphasize the requirement that the stars produced by a "high-mass" mode form predominantly in the early history of the disk, as specified by Larson (1986), and as favored by arguments concerning the chemical evolution of the galaxy (Wyse and Silk 1987). With these requirements, the bimodal hypothesis is very difficult to test. Assuming that the epochs of star formation begin at the same time for the two modes, the conclusions of Winget *et al.* (1987) concerning the age of the Galactic disk are unaffected by the hypothesis.

V. SOME FURTHER PURSUITS

Since the conclusions of this paper are summarized in the abstract, we finish by suggesting some ideas for future research which might further the goals of this work.

1. It is important to determine whether dark or very low luminosity degenerates exist, and in what approximate numbers. Note that even the model fit of Winget *et al.* (1987), assuming a mass distribution of remnants sharply peaked at the standard value of 0.6 $M_{\odot}$, has a low-luminosity tail due to the high-mass end of this distribution. Further searches for massive, dark companions to nearby stars deserve some priority, despite the absence of such discoveries to date.

2. It will be rewarding to clarify the shape of the visible luminosity function at the faint end. For this purpose, the rich harvest of the Luyten Palomar survey may be nearing an end. The extension of this work to the southern hemisphere (cf. Ruiz *et al.* 1986) is certainly important. Accurate distances and stellar parameters must then be determined for potentially important discoveries such as the star ER 8. A new analysis of the northern hemisphere fields using the new epoch of the Palomar Sky Survey may be useful in showing whether the LP missed any critical stars of faint apparent magnitudes, especially with proper motions larger than 2″ yr^{-1}. Better still would be the development of an effective technique for a survey based on colors alone, or some other kinematically unbiased criterion.

3. The parameter space of the theoretical cooling models needs to be explored systematically. Winget and Van Horn (1987) argue that different theoretical codes derive similar answers with similar input physics, but the cooling age for the empirical limit can range over a factor of 2 with different input physics. It is obviously crucial if the white dwarf luminosity function is to be used as a Galactic chronometer to determine which input physics the degenerate stars use.

4. The cooling ages of the models badly need an improved empirical calibration. The oldest known white dwarfs in clusters are apparently those in the Hyades. Old degenerate stars might be found as companions to demonstrably old stars, such as the likely companion to the unique, carbon-rich subdwarf star G77-61 (Dahn *et al.* 1977; Dearborn *et al.* 1986), which has halo kinematics. A search for well-separated, cool white dwarf companions to main-sequence early G stars might permit an upper limit on the cooling age of the former using the main-sequence lifetime of the latter, as discussed for the star VB 3 (Kunkel, Liebert, and Boroson 1984). Sadly, it appears that the end of the white dwarf sequences in the nearest globular clusters may be beyond the reach of the Hubble Space Telescope, though much may be learned by observing the shapes of the detected LFs. The calibration opportunities afforded by the long-term studies of pulsating white dwarfs are outlined in Winget *et al.* (1987).

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