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THE COMMON ENVELOPE PHASE IN THE EVOLUTION OF BINARY STARS

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ABSTRACT

We examine the common envelope phase in the evolution of binary systems. We identify three parameters which characterize the efficiency of energy deposition, the importance of three-dimensional effects, and the efficiency of spin-up of the envelope. We demonstrate that the efficiency of deposition of orbital energy into envelope ejection can be quite low. We find that significant spin-up of the envelope can be expected to occur in relatively early stages, when the spiralling-in occurs inside evolved supergiant envelopes. In normal giants spin-up can occur only in the final stages of the spiraling-in process. We present the results of a simplified three-dimensional numerical calculation of the common envelope phase and discuss the implications of the results for the formation of planetary nebulae with binary nuclei, double white dwarf systems, and FK Com stars.

Subject headings: stars: binaries — stars: evolution

I. INTRODUCTION

Common envelope evolution is widely assumed to be an agent capable of reducing the separation of initially wide binaries and transforming them into close binaries. As such, the common envelope is supposed to be responsible for the formation of cataclysmic variables (Paczyński 1976, 1985; Eggleton 1986), of some low mass X-ray binaries (e.g., 4U 1830−30, Bailyn and Grindlay 1987; 2A 0620−00, Eggleton and Verbunt 1986) and of double white dwarf systems that are supposed to be the progenitors of Type I supernovae (Iben and Tutukov 1984; Iben and Webbink 1987; Webbink and Iben 1987).

In spite of its wide applicability, very few actual calculations of the common envelope (hereafter CE) phase exist. This is a consequence of the fact that the spiraling-in process of two stars (or a star and a core), embedded in a CE, involves a large number of hydrodynamic and thermodynamic processes, occurring on a very wide range in both *time scales* and *length scales*. In addition, in most cases the calculations are impeded by the absence of a spherical or cylindrical symmetry. Nevertheless, some attempts to follow the CE evolution have been made. In a pioneering work, Sparks and Stecher (1974) calculated the orbital decay resulting from a local tidal interaction between an orbiting white dwarf and a giant envelope. This calculation, however, neglected hydrodynamic effects and did not treat the response of the giant to mass loss. Some of the basic equations describing the orbital evolution of a compact object moving through the distended atmosphere of a giant, acted upon by a gravitational accretion drag, have been formulated by Alexander, Chau, and Henriksen (1976). Taam, Bodenheimer, and Ostriker (1978) have followed the evolution of a 16 $M_\odot$ giant inside which an additional energy source has been included, resulting from the drag luminosity generated by a spiralling-in neutron star. This work was the first to realize the possible importance of efficient energy transport (at least in the spherically symmetric case). It was found that when energy transport (by convection) was rapid, no significant mass motion was obtained. The energy generated by friction was efficiently transported to the surface instead of being deposited into envelope ejection.

Meyer and Meyer-Hofmeister (1979) calculated the CE phase of an evolved 5 $M_\odot$ (possessing a 1 $M_\odot$ degenerate core) giant and a 1 $M_\odot$ main-sequence star. They modeled the configuration by means of a corotating region around the core–main-sequence star binary, coupled to a differentially rotating envelope. This situation, applicable perhaps to cases in which the initial giant envelope is not far from corotation, resulted in a rather peaceful evolution with no mass ejection (since the frictional energy source did not perturb significantly the giant's energy generation and transport). It has been speculated, that envelope ejection could result in the final stages, when the main sequence star's outer layers start to be affected by tidal interaction with the giant's core. The main uncertainty concerning the applicability of the Meyer and Meyer-Hofmeister (1979) results to the real situation, originates from the assumption of spherical symmetry, since, as we shall see, non-spherical effects can be extremely important. Some of the differences in CE evolution between the corotating and noncorotating (of the giant's envelope) cases, have been demonstrated by Morris (1981), who used particle trajectories in an attempt to model the formation of bipolar nebulae. Livio and Soker (1984*a*, *b*) and Soker, Harpaz, and Livio (1984), examined the possibility of forming CVs with low-mass secondaries from star–brown dwarf binaries. The CE phase in their case involved the core of an evolved giant and a very low mass ($M_s \lesssim 0.025\ M_\odot$) secondary. Many of the assumptions made, concerning the spiraling-in process, could be justified in such a calculation, due to the smallness of the perturbation introduced by the secondary star. These calculations have again demonstrated the importance of efficient energy transport (when spherical symmetry is assumed).

The only existing two-dimensional hydrodynamic calcu-

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lation of the CE phase is that of Bodenheimer and Taam (1984). The most important result of this calculation has been the realization that nonspherical effects can be extremely important. In particular, it was found that the energy deposition into envelope ejection is quite inefficient, because a relatively small amount of mass is accelerated to velocities larger than those necessary to escape. A discussion of some aspects of CE evolution can be found in Webbink (1986) and a more complete description of existing calculations can be found in deKool (1987).

In the present work, we first examine analytically some of the physical processes involved in CE evolution; this is done in § II. In § III we present a preliminary three-dimensional calculation of a spiraling-in process. The implications of the results of §§ II and III for the formation of CVs, planetary nebulae with binary nuclei, double white dwarf systems, and FK Com stars are discussed in § IV.

II. PHYSICAL PROCESSES AND TIME SCALES IN COMMON ENVELOPE EVOLUTION

We shall discuss mainly the spiraling-in process in the case that the giant envelope is not corotating. This requires a relatively small initial mass ratio $q = M_s/M_p \lesssim 1/6$ (e.g., Sparks and Stecher 1974, where M_s, M_p are the masses of the secondary and primary, respectively). Such mass ratios represent very probably cataclysmic variable (hereafter CV) progenitors, since even in the present systems typically $q < 1$ and the primary had to be considerably more massive initially. We assume that the initial separation is a few tens to $\sim 1500\ R_\odot$, so that the primary fills its Roche lobe in the giant or AGB phase of its evolution. Because of the fact that giants with deep convective envelopes expand upon mass loss and the mass transfer takes place from the more massive to the less massive component, a dynamical time scale mass transfer process ensues (e.g., Paczyński 1965; Paczyński, Ziolkowski, and Zytkow 1969). Overwhelmed by the high mass accretion rate, the secondary star is driven out of thermal equilibrium and starts expanding and a CE configuration is obtained (e.g., Webbink 1977; Yungelson 1973; Prialnik and Livio 1985).

The secondary star and the giant's core start spiraling-in as a result of gravitational drag and tidal forces. In most cases the gravitational drag dominates and leads to an orbital decay which can be formulated as (Bondi and Hoyle 1944; Shima *et al.* 1985; Livio *et al.* 1986)

$$-\frac{GM_s M(a)}{2a^2}\frac{da}{dt} = \xi(M)\pi R_A{}^2 \rho V^3 , \tag{1}$$

where a is the separation, $M(a)$ is the mass in the giant inner to radius a, ρ is the local density in the CE, and V is the relative velocity between the secondary and the CE. The accretion radius R_A is given approximately by (Bondi 1952; Shima *et al.* 1985)

$$R_A = \frac{2GM_s}{V^2 + V_s{}^2} , \tag{2}$$

where V_s is the speed of sound. The function $\xi(M)$ which determines the dissipation rate, is a function of the Mach number and is of order 2–4 in the supersonic case (Shima *et al.* 1985; deKool 1987) but can be considerably less than one in the subsonic case. The energy lost from the orbit is deposited mostly into heating the envelope and partly into rotating it. At the same time orbital angular momentum is deposited into spin angular momentum of the envelope. In principle at least, the energy deposited into the envelope can cause its ejection if it exceeds the binding energy of the envelope. This is essentially the mechanism, originally proposed by Paczyński (1976), for the formation of CVs.

We define an efficiency parameter $\alpha_{\rm CE}$ in the following way. Suppose that $\Delta E_{\rm orb}$ represents the change in the orbital energy of the binary between the time of the formation of the CE (the beginning of the spiraling-in process) and the time that the entire envelope is ejected (if it indeed is ejected). We then define

$$\alpha_{\rm CE} \equiv \frac{\Delta E_{\rm bind}}{\Delta E_{\rm orb}} , \tag{3}$$

where $\Delta E_{\rm bind}$ is the (actual) binding energy of the envelope (obtained by integrating the density times potential distribution). Our $\alpha_{\rm CE}$ is closely related to the parameter α defined by Iben and Tutukov (1984). They, however, used very approximate expressions for $\Delta E_{\rm bind}$, $\Delta E_{\rm orb}$ in their definition. In terms of the binary separation, $\alpha_{\rm CE}$ is given by

$$\alpha_{\rm CE} \approx \frac{a_f}{a_f{}^0} , \tag{4}$$

where a_f is the *actual* final separation of the binary emerging from the CE and $a_f{}^0$ is the separation that would have been obtained, if the deposition of orbital energy into envelope ejection would have been 100% efficient. Before we examine the question of what can effect the value of $\alpha_{\rm CE}$, we would like to define a few other physical parameters that characterize the spiraling-in process. From equations (1) and (2) we can define a ratio of the decay time scale $\tau_{\rm decay} = a/\dot{a}$ to the Keplerian time scale $\tau_{\rm Kep}$. This is given by

$$\beta_{\rm CE} \equiv \frac{\tau_{\rm decay}}{\tau_{\rm Kep}} = \frac{1}{12\pi} G(M) \left[\frac{M(a) + M_s}{M_s}\right]\left(\frac{V_s}{V_{\rm Kep}}\right)\left(\frac{\bar{\rho}_a}{\rho_a}\right) , \tag{5}$$

where ρ_a is the local density at separation a, $\bar{\rho}_a$ is the average density in the giant interior to radius a, and $V_{\rm Kep}$ is the Keplerian orbital velocity. $G(M)$ is a function of the Mach number given by

$$G(M) = \frac{1}{\xi(M)} \frac{(M^2 + 1)^2}{M^3} . \tag{6}$$

The parameter $\beta_{\rm CE}$ measures the importance of local (three-dimensional) effects in the spiraling-in process. When $\beta_{\rm CE} \lesssim 1$, energy is deposited locally and no spherical (or cylindrical) symmetry can be assumed. It can be expected that under such circumstances, relatively small amounts of mass located in the orbital plane will acquire velocities larger than those necessary to escape. This has the effect of reducing $\alpha_{\rm CE}$. As can be seen from equations (4) and (5), $\beta_{\rm CE}$ depends on the relative velocity between the secondary and the envelope, on the degree of central concentration of the giant (which determines $\bar{\rho}_a/\rho_a$) and on the mass ratio. As long as the relative velocity is Keplerian, then, since the Keplerian Mach number is of the order of 1–4 for typical giant envelopes (Livio and Soker 1984*a*; deKool 1987), local effects are more important the less centrally condensed the giant and the more massive the secondary. If the relative velocity decreases due to spin-up of the envelope (to be discussed shortly), local effects become less and less important and the energy deposition can be expected to become more cylindrically symmetric (and eventually more spherically symmetric if coupled to a moderately efficient energy transport).

As we have already mentioned, spin-up of the envelope takes place as a result of orbital angular momentum being deposited into the envelope. We can define a global spin-up scale $\tau_{\text{spin-up}}$ by (see also deKool 1987)

$$\tau_{\text{spin-up}} = \frac{I(a)V}{a^2 F_{\text{drag}}} , \tag{7}$$

where $I(a)$ is the moment of inertia of the envelope interior to radius a and F_{drag} is the gravitational drag force (eq. [1]). Using equations (5) and (7) we can define a parameter γ_{CE},

$$\gamma_{\text{CE}} \equiv \frac{\tau_{\text{spin-up}}}{\tau_{\text{decay}}} = 1.2M^2\left[\frac{M(a) + M_s}{M_s}\right]\left(\frac{\tilde{\rho}_a}{\bar{\rho}_a}\right)\left(\frac{V_s}{V_{\text{Kep}}}\right)^2 , \tag{8}$$

where

$$\tilde{\rho} = \frac{5}{a^5}\int_{R_{\text{in}}}^{a} r^4 \rho(r) dr , \tag{9}$$

R_{in} being the radius at the giant's core-envelope interface. For $\gamma_{\text{CE}} \lesssim 1$ it can be expected that considerable spin-up of the envelope will occur. This has the effect of reducing the relative velocity between the secondary and the envelope and thereby significantly prolonging the spiraling-in process, since the drag force decreases. From equation (8) we see that, starting with a Keplerian relative velocity, spin-up will occur faster for more massive secondaries and for more centrally condensed giants. The last conclusion is a consequence of the fact that for very evolved AGB supergiants (very centrally condensed), the density is nearly constant over a large fraction of the envelope, so that $\tilde{\rho}_a \approx \rho_a \ll \bar{\rho}_a$. In order to demonstrate the importance of local effects and spin-up in different configurations we have plotted, in Figures 1–3, β_{CE} and γ_{CE} for an evolved AGB 0.88 $M_\odot$ (radius 400 $R_\odot$) star (Fig. 1), a 1 $M_\odot$ (radius 38 $R_\odot$) giant (Fig. 2), and a 5 $M_\odot$ (radius 65 $R_\odot$) giant (Fig. 3). The relative velocity was taken to be Keplerian in all cases. The stellar models used in Figures 1–3 were kindly supplied to us by Amos Harpaz, Peter Eggleton, and Icko Iben, respectively. The following things should be noted. In the evolved supergiant model (Fig. 1), $\beta_{\text{CE}} > 1$ everywhere and it becomes very large in the innermost 100 $R_\odot$ ($\sim$200 at 10 $R_\odot$). At the same time $\gamma_{\text{CE}} \lesssim 1$ over the entire envelope. It thus can be expected, that in very evolved supergiants, spin-up of the envelope will occur at relatively early stages, slowing down the orbital decay. Local effects are not likely to be important under such circumstances. Energy will be deposited essentially in tori and can be expected to have sufficient time to be transported, so that the deviation from spherical symmetry will not be very significant. We have also calculated the ratio $\tau_{\text{decay}}/\tau_{\text{KH}}$, where τ_{KH} is the local Kelvin-Helmholtz time scale. It was found that this ratio is also larger than one in the entire envelope, again indicating the capability to adjust thermally to the energy deposition. In contrast to the behavior in the supergiant model, an examination of Figures 2 and 3 reveals that $\beta_{\text{CE}} \lesssim 1$ over a large fraction of the envelope in the giant models. At the same time, spin-up can be expected to occur ($\gamma_{\text{CE}} < 1$) only in the innermost few solar radii. This suggests that the orbital decay in the case that the CE is encountered in the giant phase is extremely rapid and local effects are very important. Envelope ejection can be expected to be quite concentrated toward the orbital plane and a relatively small fraction of the envelope will probably escape (with velocities exceeding the escape velocity), thus reducing the value of α_{CE}.

We have also calculated a somewhat less certain spin-up time scale, based on the tidal interaction. This time-scale is given roughly by (Livio and Soker 1984*a*)

$$\tau_{\text{tidal}}^{-1} \approx C\int_{a_{\text{in}}}^{a_{\text{out}}} \frac{(r/a)^6 \eta_{\text{conv}}[M_s/M(a)]r^2\Omega_{\text{orb}}}{j(a)M(a)} dr , \tag{10}$$

where $j(a)$ is the angular momentum per unit mass, C is a numerical constant ($\sim$500), and η_{conv} is the viscosity in the convective region. The integration is performed from the inner boundary of the convective region to the secondary's Roche lobe (where accretion starts to dominate). Typical values found for $\tau_{\text{tidal}}/\tau_{\text{decay}}$ were of the order of 300 in the 1 $M_\odot$ giant model

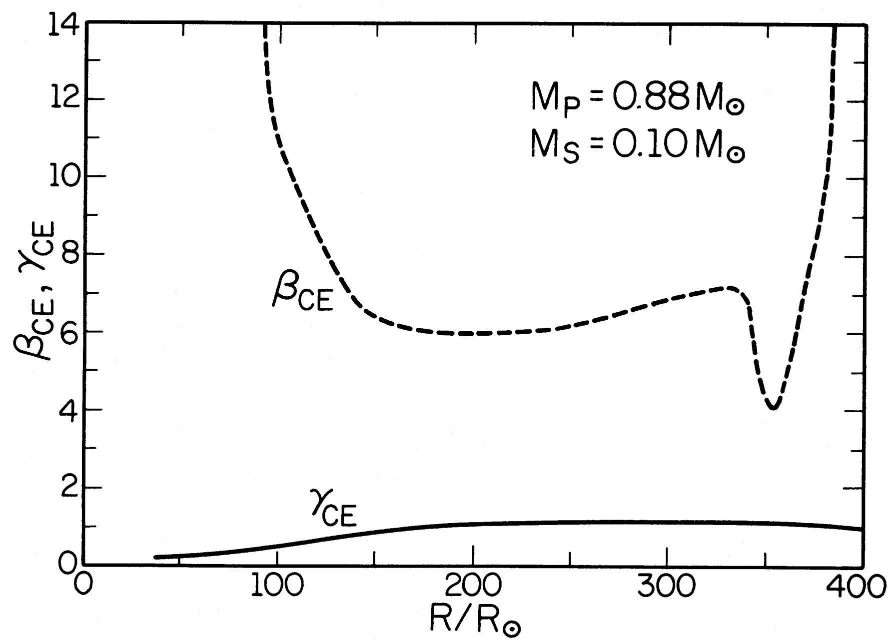

FIG. 1.—The parameters β_{CE}, γ_{CE} as a function of radius for a 0.88 $M_\odot$ supergiant model (see text). M_s is the mass of the secondary star.

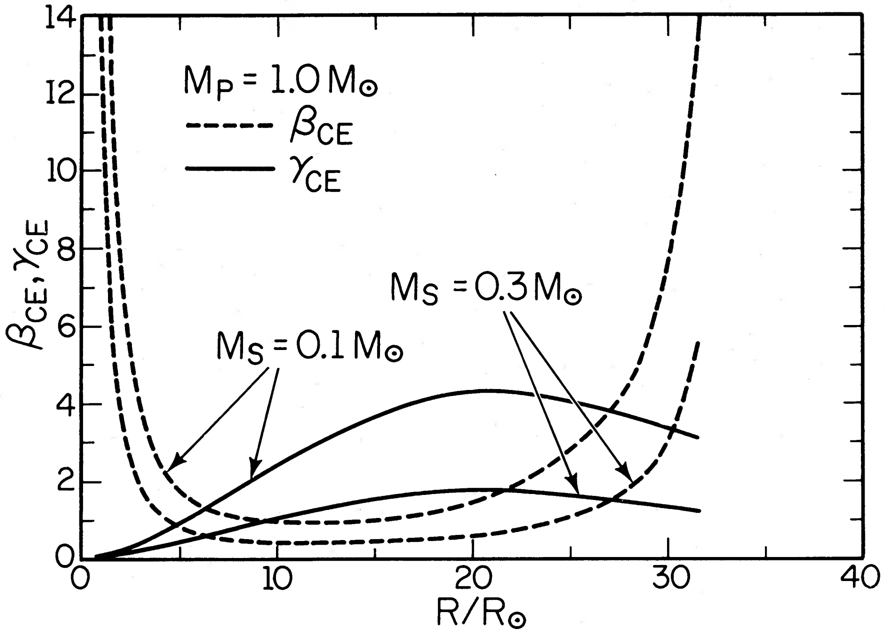

FIG. 2.—The parameters β_{CE}, γ_{CE} as a function of radius for a 1.0 $M_\odot$ giant star (see text). M_s is the mass of the secondary star.

and of the order of 20 in the evolved AGB (0.88 $M_\odot$) star. Thus, we find the same qualitative behavior as the one indicated by the values of γ_{CE}, namely, spin-up can occur much faster in evolved supergiants.

III. A THREE-DIMENSIONAL NUMERICAL CALCULATION

Realizing the possible importance of three-dimensional effects, we have attempted a preliminary three-dimensional numerical calculation of the CE phase. The results of these calculations should definitely be regarded as qualitative only. We have used a pseudoparticle method to describe the hydrodynamics, essentially identical to that used by Livio *et al.* (1986). We shall, therefore, not repeat the technical details here. The model of a 5 $M_\odot$ giant, with a radius of 65 $R_\odot$ was constructed of 30,000 particles (in half the space, using the symmetry about the orbital plane). The density and pressure distribution of the (quasi-) stable model were chosen to match as closely as possible those of the model constructed by Iben (1966). For numerical reasons, particles up to a distance of 12.5 $R_\odot$ from the center were assumed to move with the giant's core (of mass $M_c = 0.7\ M_\odot$). Consequently, we stopped all the calculations when the particles adjacent to the sphere of radius 12.5 $R_\odot$ started to move, since the results become very unreliable after that point. For the strength of the interparticle inter-

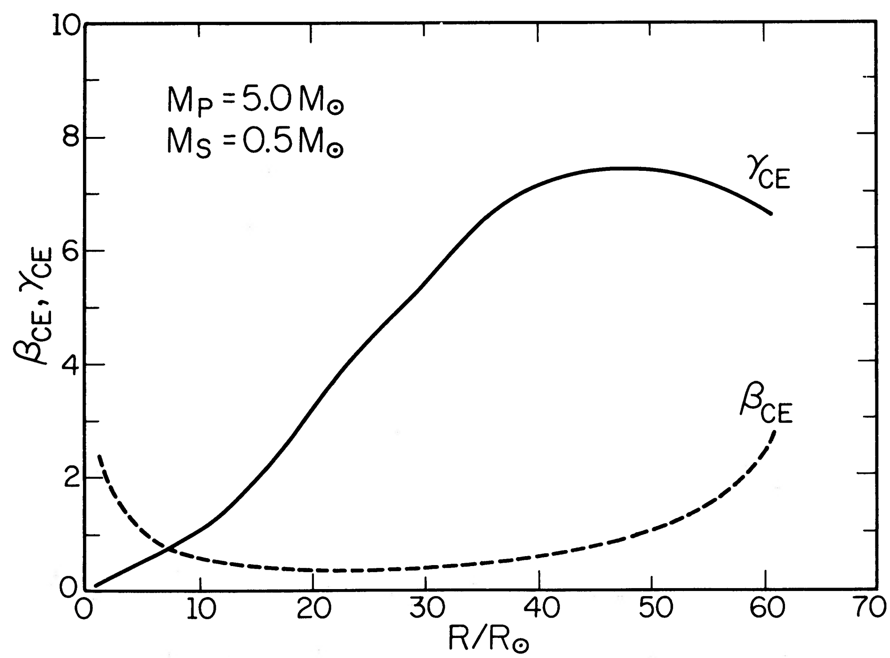

FIG. 3.—The same as Fig. 1, for a 5.0 $M_\odot$ giant star

action we have used $\alpha = 1$ in most cases (e.g., Livio *et al.* 1986; Hensler 1982), test runs with $\alpha = 0.01$ were also performed. The mass of the secondary star was chosen to be 0.3 $M_\odot$ in most cases, in one calculation $M_s = 1.4\ M_\odot$ has been used. The secondary star was represented by a point mass in the calculations. All the calculations were performed on the CRAY X-MP/48 supercomputer at the Pittsburgh Supercomputing Center; test runs were carried out on the CRAY X-MP/48 at the National Center for Supercomputing Applications at the University of Illinois.

In model A, $M_s = 0.3\ M_\odot$, and the calculation is started at an initial separation $a_0 = 65\ R_\odot$. The velocity field in the orbital plane (in the rest frame of the giant's core) is shown in Figures 4–10 as a function of the spiraling-in time. The secondary is denoted by the heavier arrow. The most important things to note are (1) local, three-dimensional effects are very important (as could be expected for a giant model); (2) the spiraling-in process is extremely rapid, and no significant spin-up occurs (until the calculation is stopped); (3) mass ejection is quite concentrated toward the orbital plane. This can be seen in Figures 18 and 19 (taken from model D, see below) which shows the flow in the *Y-Z* and *X-Z* planes (the *z*-axis is perpendicular to the orbital plane); and (4) the efficiency or orbital energy deposition into envelope ejection is quite low (until the calculation is stopped, we are unable to determine a value of α_{CE}, because α_{CE} is defined when the entire envelope has been lost).

In order to test the effect of the secondary mass, we used in model B an extreme mass of $M_s = 1.4\ M_\odot$ (still with $a_0 = 65\ R_\odot$). The initial conditions in this case are not entirely self consistent, in that the giant can be expected to be corotating for such a high mass secondary. The interaction in this case is extremely violent and local, three-dimensional effects dominate the flow (Figs. 11–13). Material in this case is accelerated locally to velocities of order 2–3 times the escape velocity (an effect which reduces α_{CE}). The flow pattern is extremely complicated and one can observe the generation of a vortex around the secondary (Fig. 12) and of shock waves, as accelerated material near the secondary's location collides with other envelope material (Fig. 13).

In model D, we used again $M_s = 0.3\ M_\odot$, but the initial separation was taken as $a_0 = 40\ R_\odot$. Clearly this is not a self-consistent model in that the effects of the spiraling-in leading to this configuration have been neglected. However, we wanted to explore the consequences of a CE process which starts in a higher density environment. Because of the higher energy dissipation rate, material is accelerated to higher velocities than in model A (Figs. 14–17). The material is ejected in a pattern resembling a spiral arm (Fig. 17), which is quite typical for tidally interacting systems. Again the transfer of angular momentum (and mass ejection) is quite concentrated to the orbital plane (Figs. 18 and 19). This can also be seen by comparing Figure 17 with Figure 20, in which the plane at $z = 35\ R_\odot$ is shown (still in the rest frame of the giant's core).

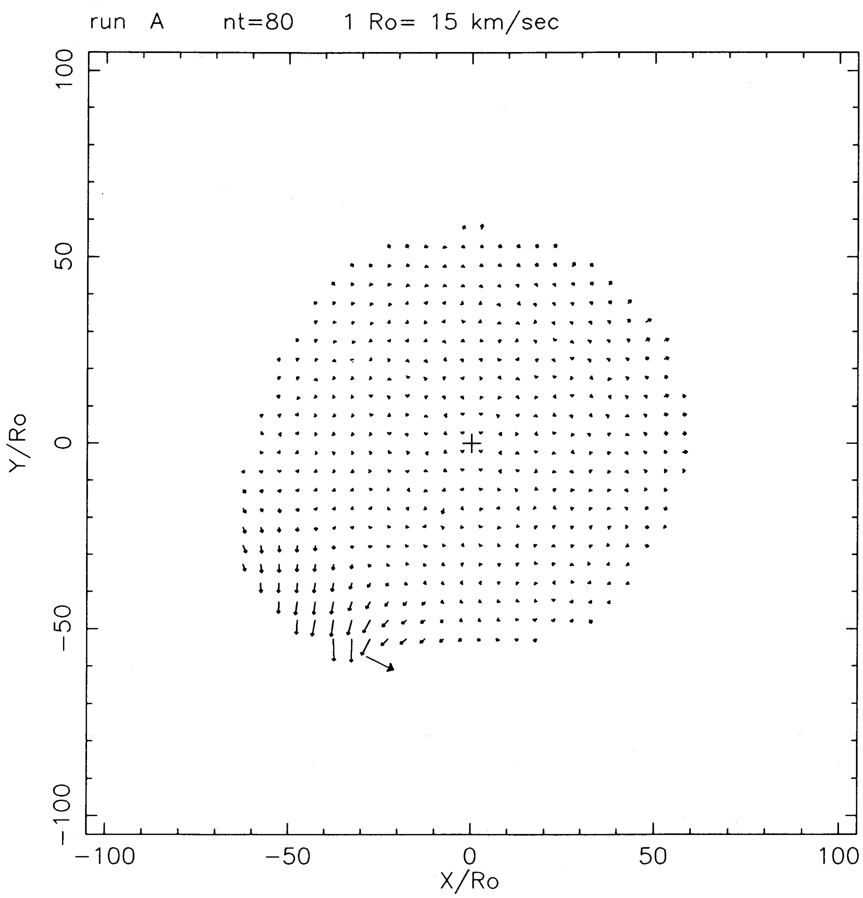

FIG. 4.—The velocity field in the orbital plane, in the rest frame of the giant's core in model A (see text). The secondary is denoted by the heavier arrow. Each time step represents 5×10^3 s (nt denotes the number of time steps).

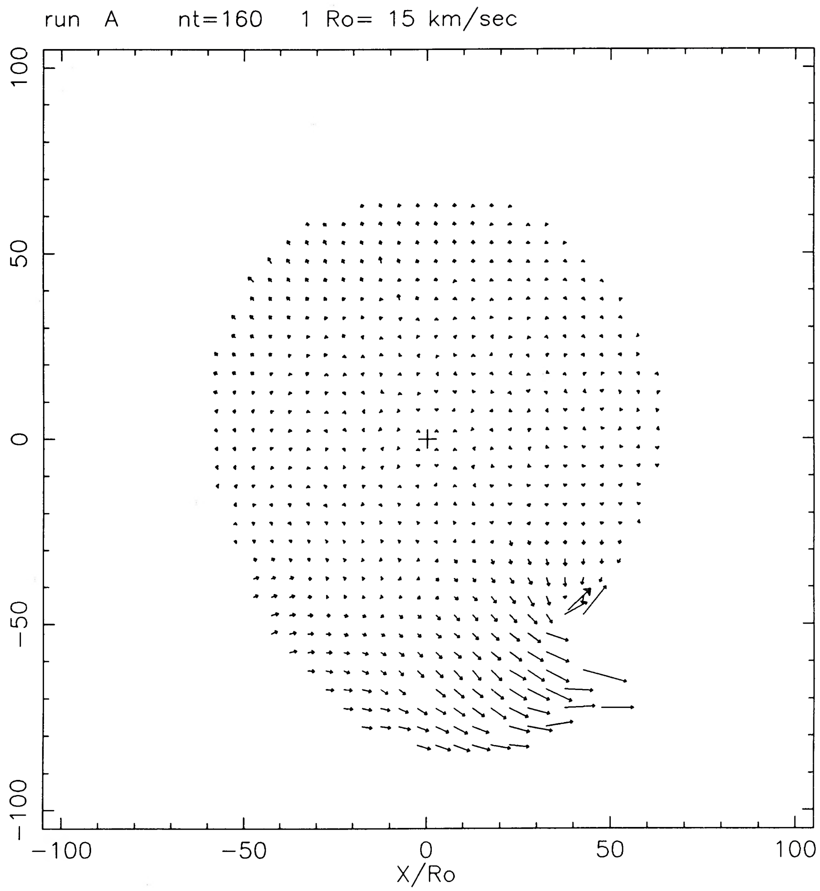

FIG. 5.—The same as Fig. 4, for a later time

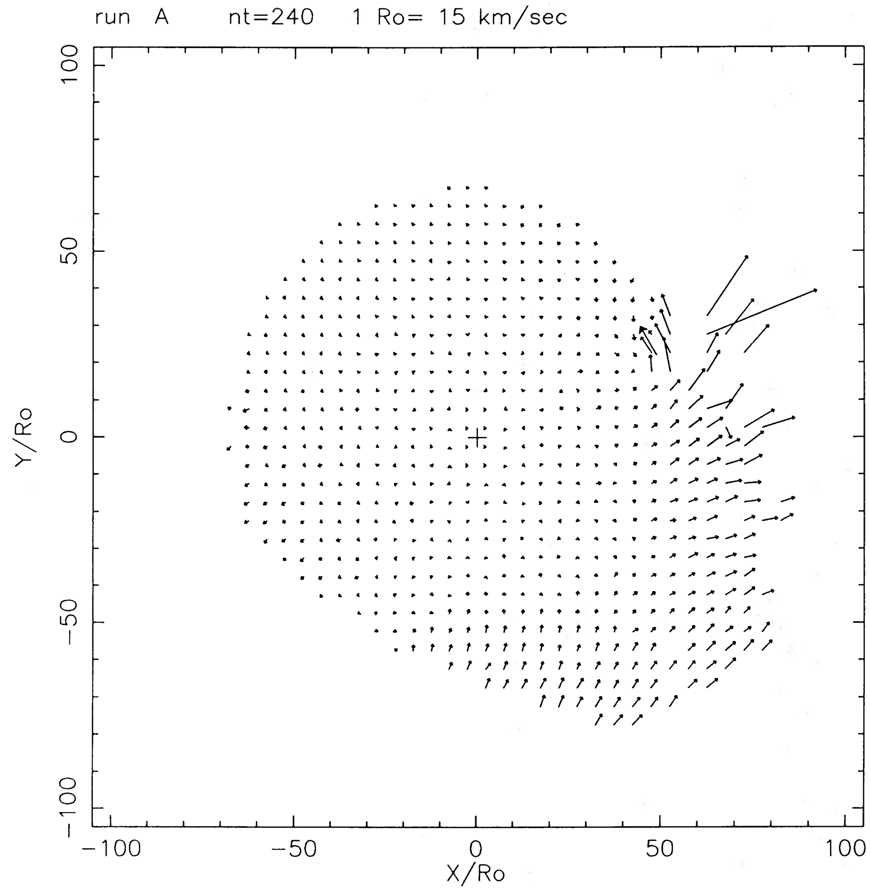

FIG. 6.—The same as Fig. 5, for a later time

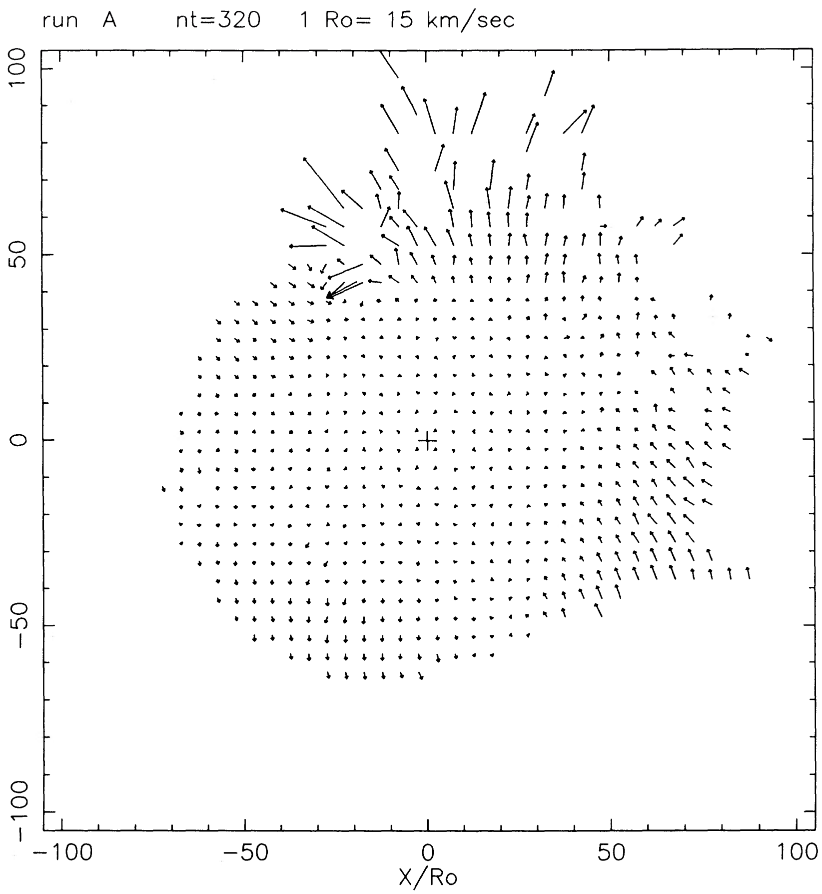

FIG. 7.—The same as Fig. 6, for a later time

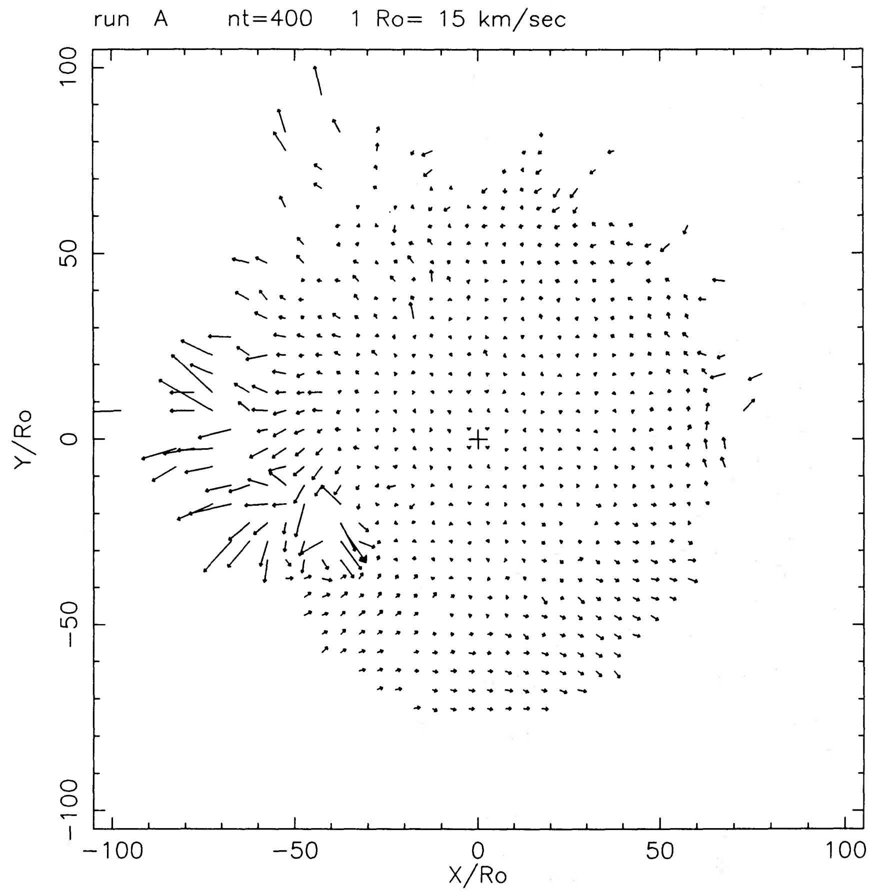

FIG. 8.—The same as Fig. 7, for a later time

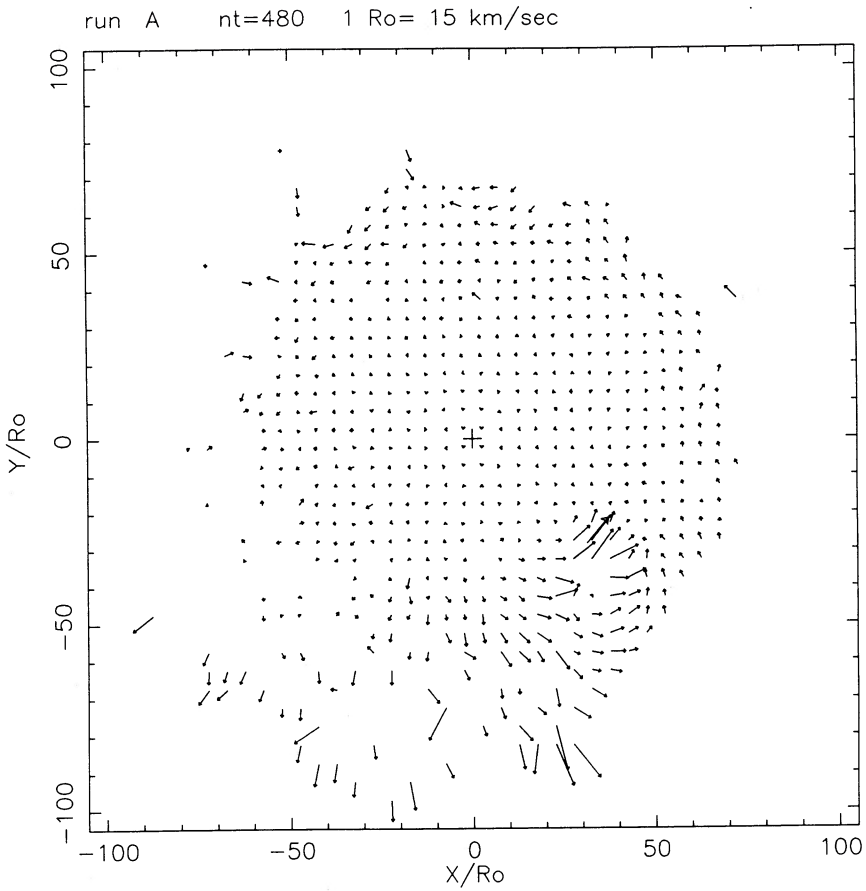

FIG. 9.—The same as Fig. 8, for a later time

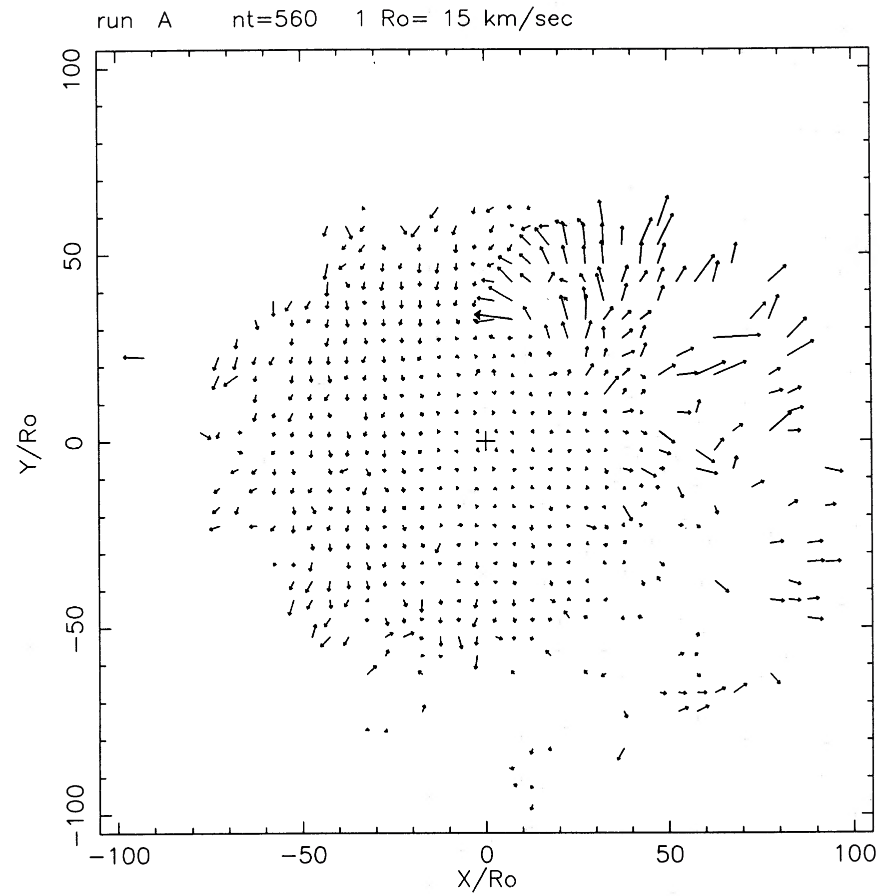

FIG. 10.—The same as Fig. 9, for a later time

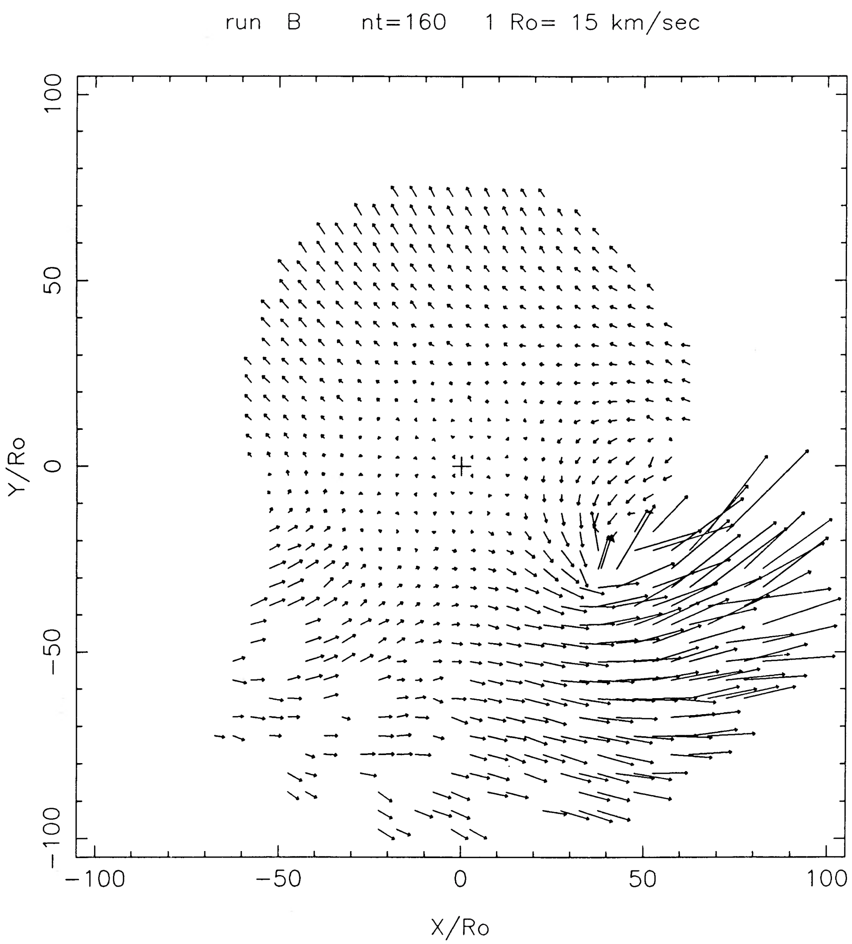

FIG. 11.—The same as Fig. 4, for model B (see text)

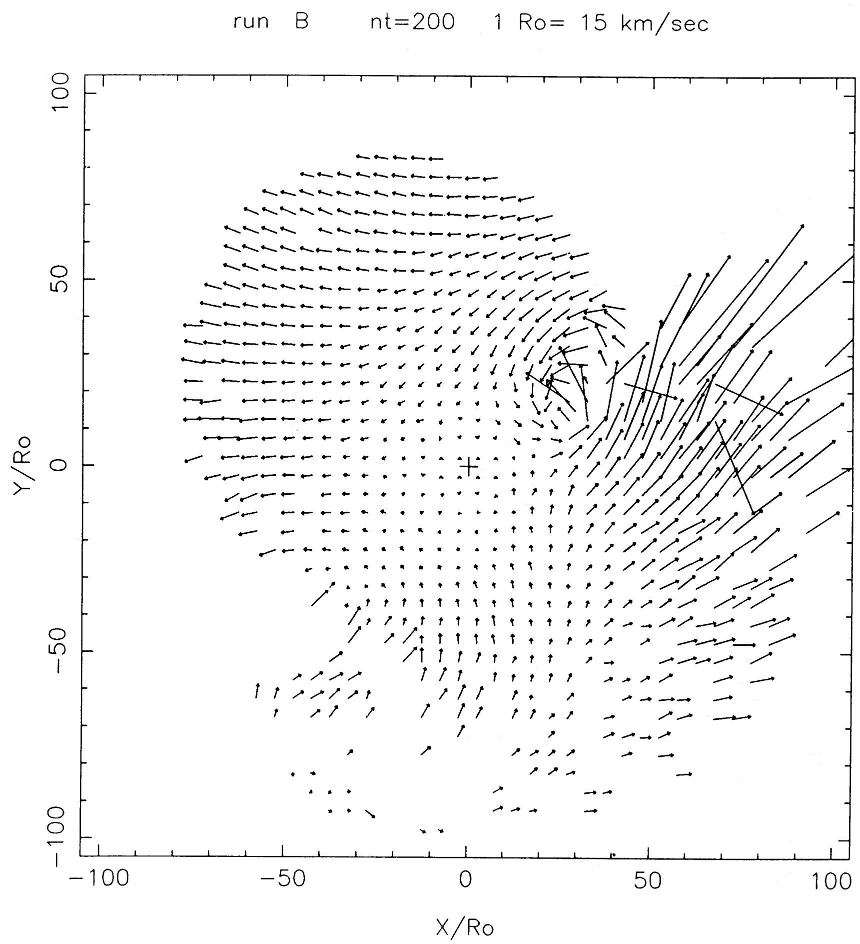

FIG. 12.—The same as Fig. 11, for a later time

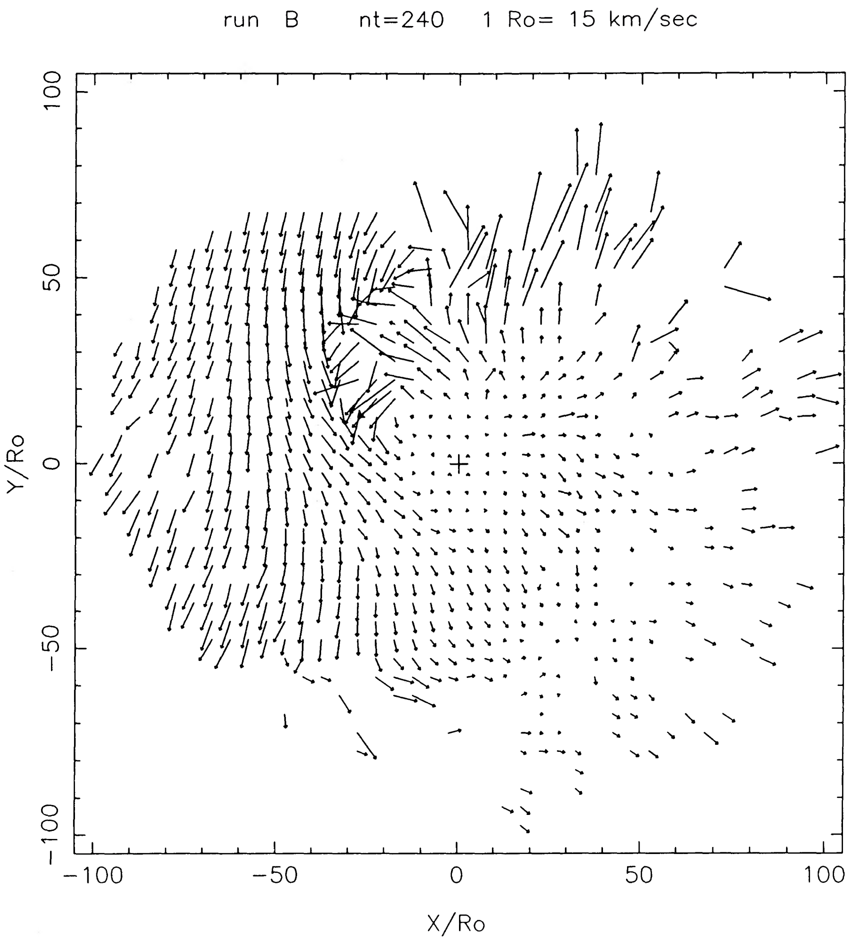

FIG. 13.—The same as Fig. 12, for a later time

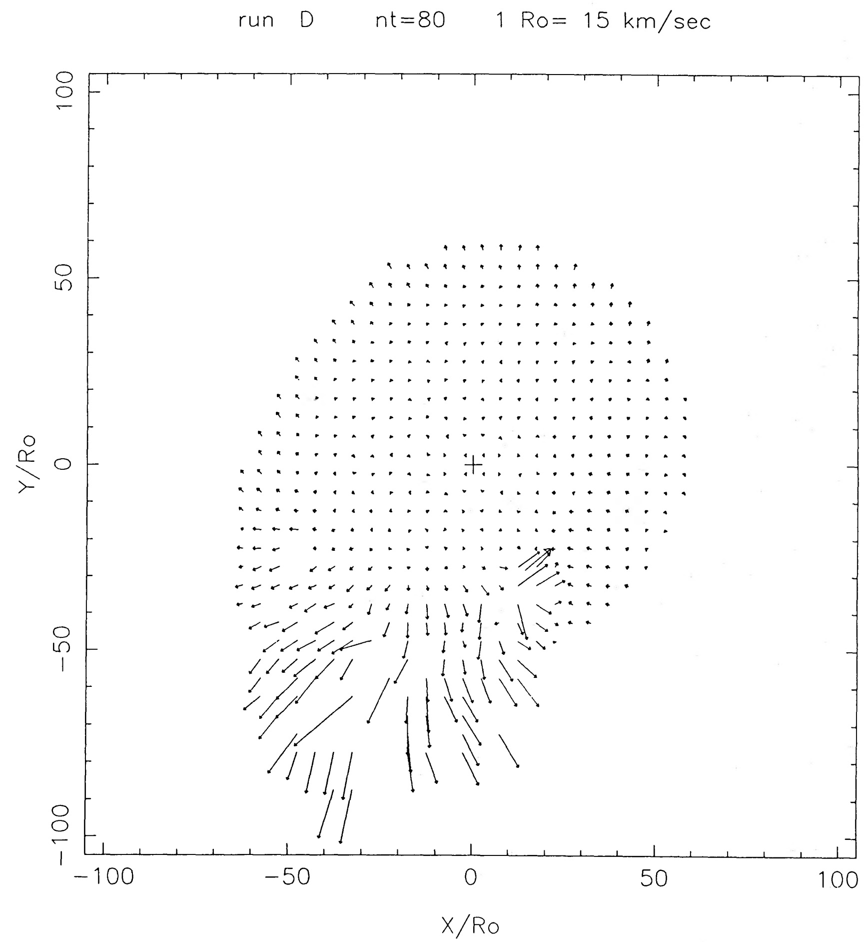

FIG. 14.—The same as Fig. 4, for model D (see text).

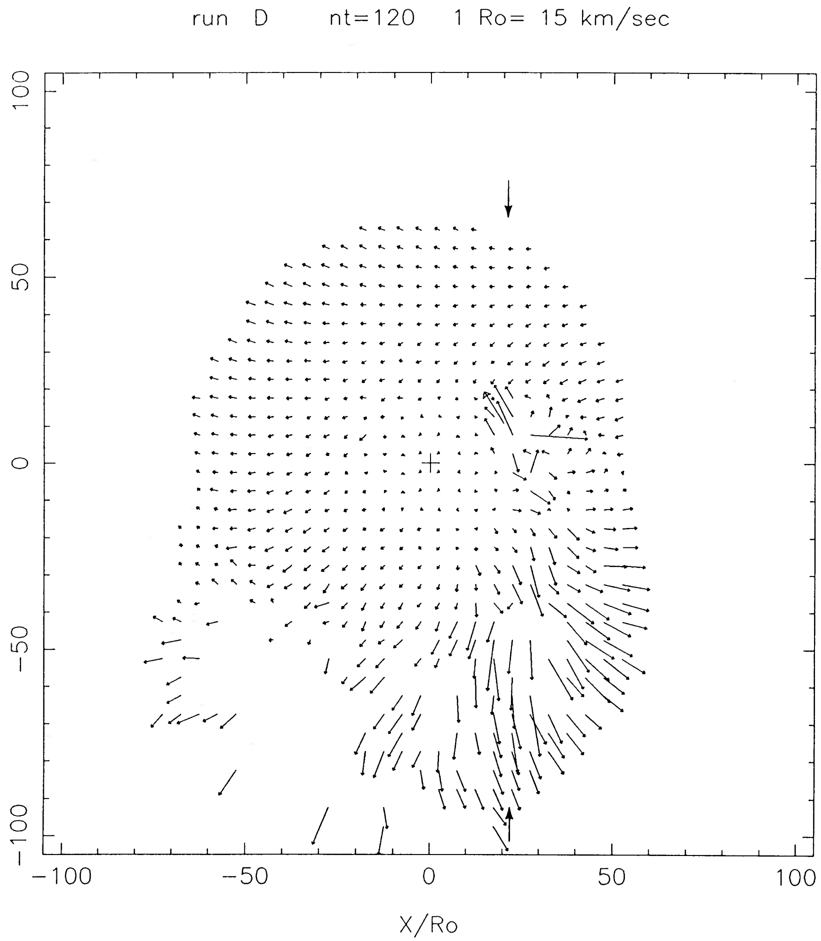

FIG. 15

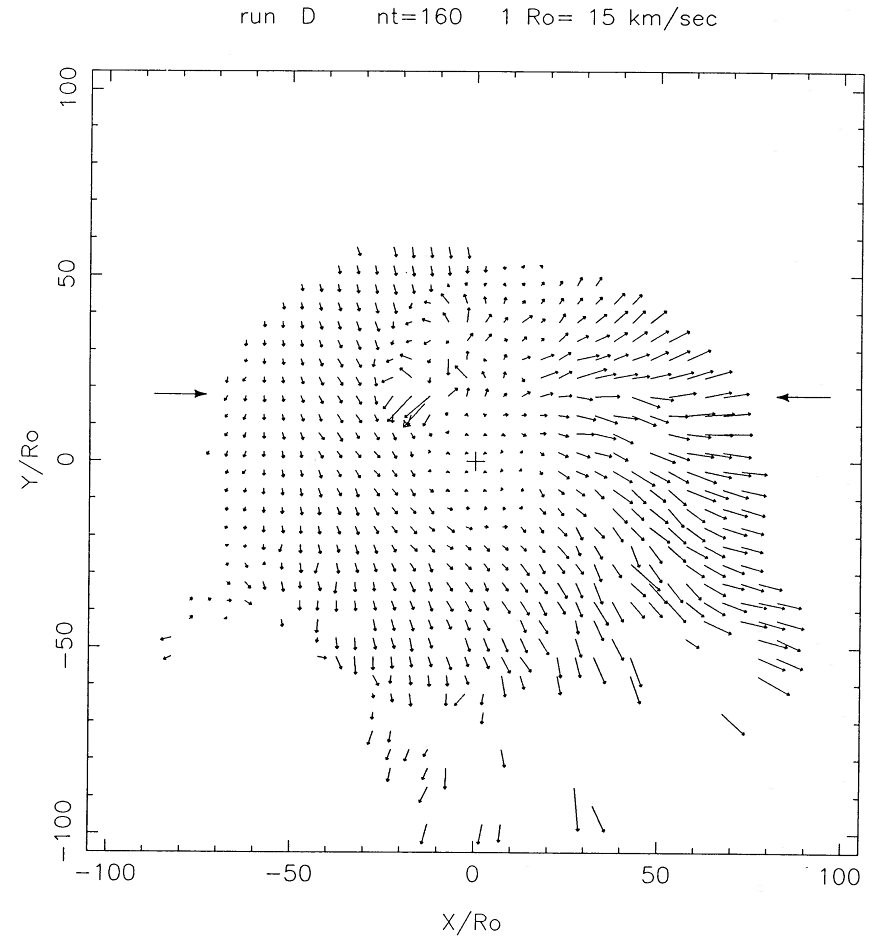

FIG. 16

FIG. 15.—The same as Fig. 14, for a later time. The two arrows on the sides indicate the value of X for which the Y-Z plane is shown in Fig. 18 (see text).
FIG. 16.—The same as Fig. 14, for a later time. The two arrows on the sides indicate the value of Y for which the X-Z plane is shown in Fig. 19 (see text).

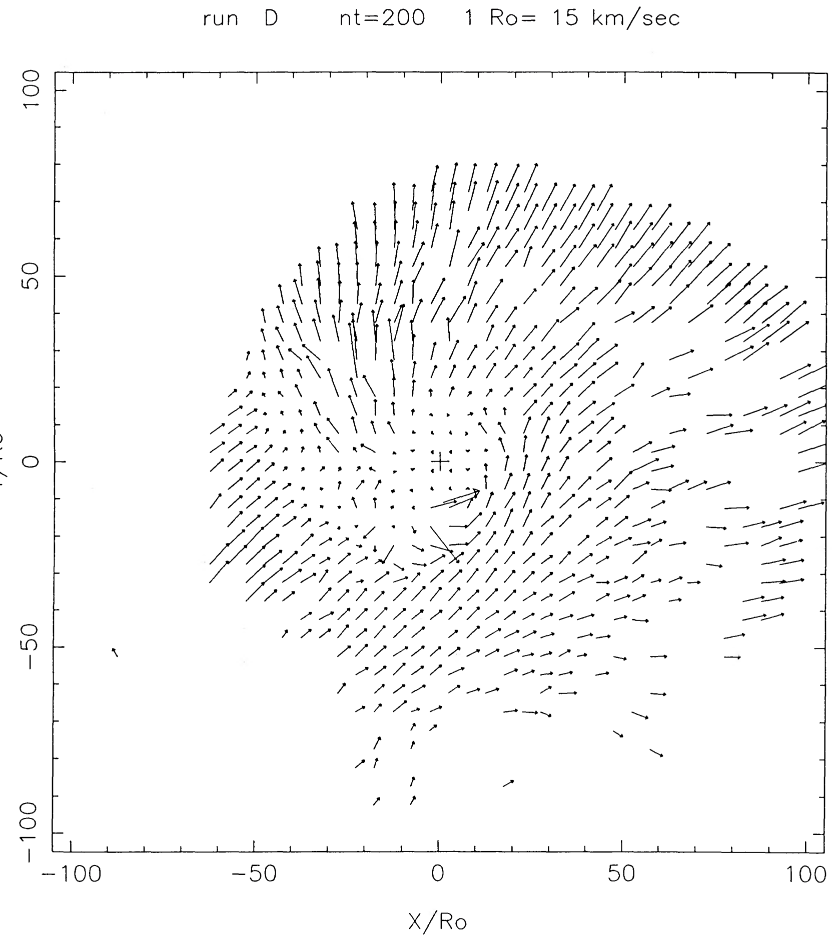

FIG. 17

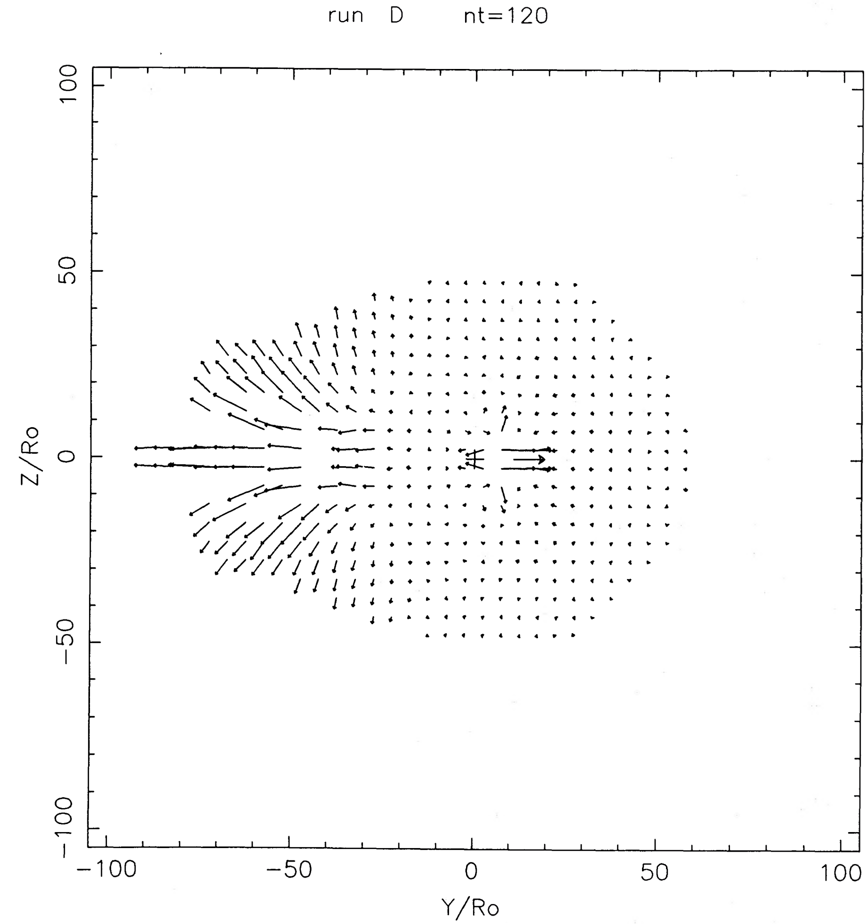

FIG. 18

FIG. 17.—The same as Fig. 14, for a later time
FIG. 18.—The velocity field in the Y-Z plane, for the X value indicated in Fig. 15 (see text)

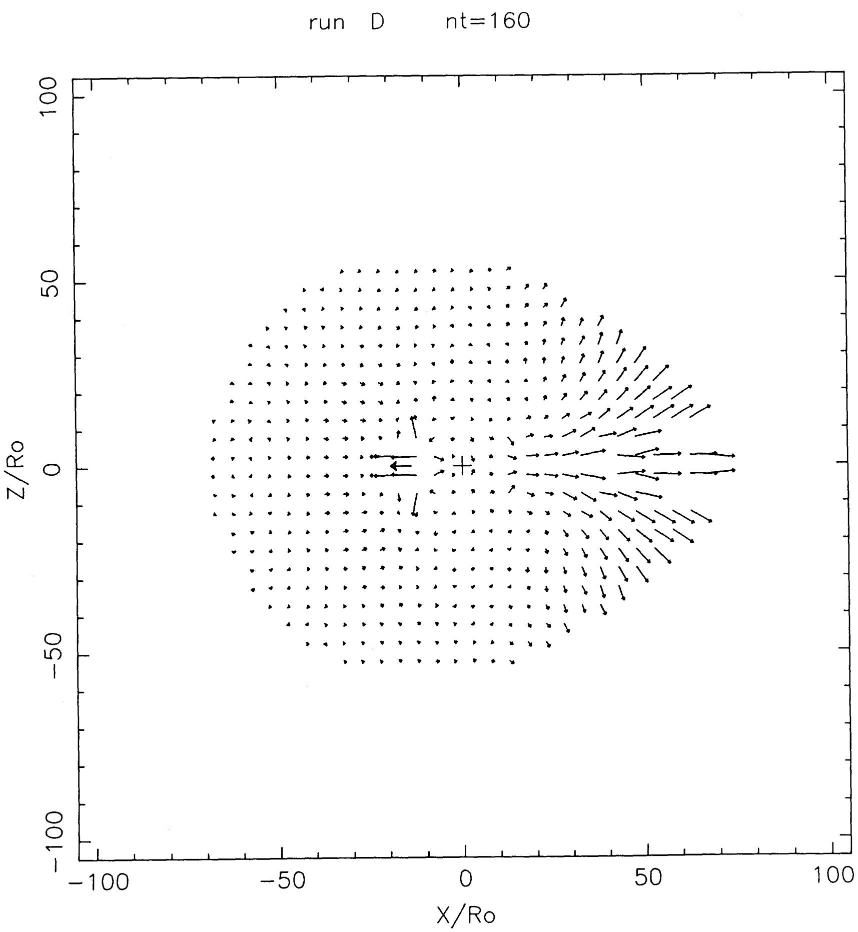

FIG. 19

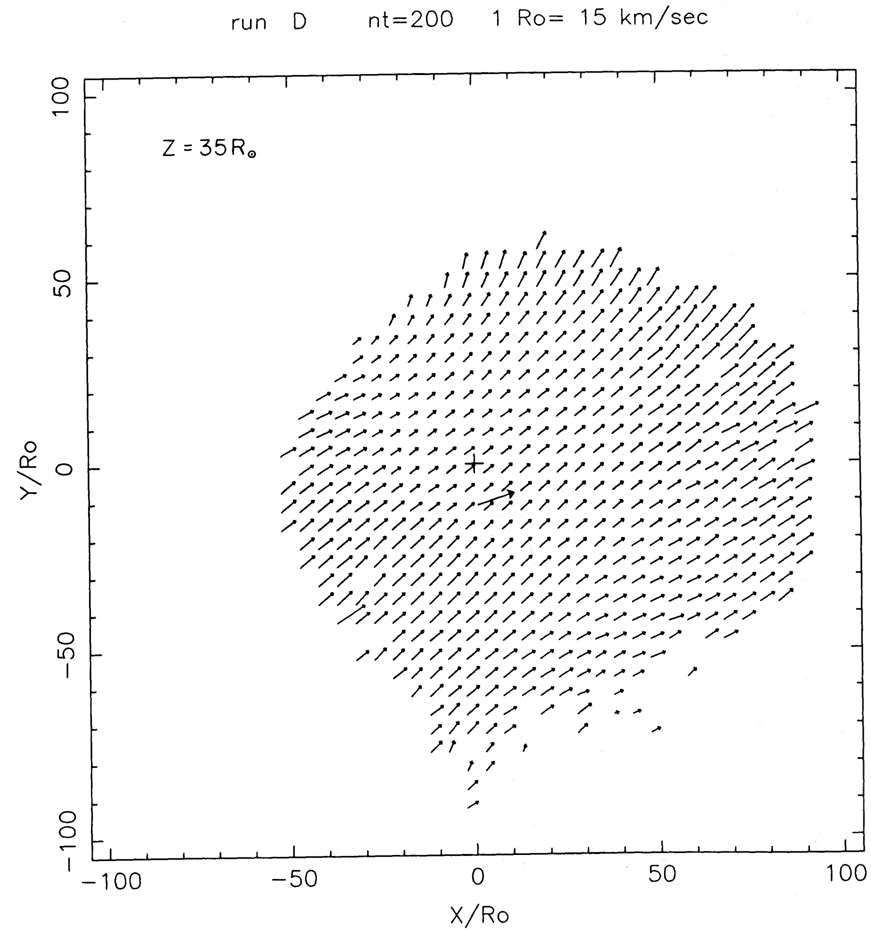

FIG. 20

FIG. 19.—The velocity field in the X-Z plane, for the Y value indicated in Fig. 16 (see text)
FIG. 20.—The same as Fig. 17, but in the $Z = 35\ R_{\odot}$ plane (see text)

Changing the value of the interparticle interaction strength did not change the qualitative picture obtained.

IV. DISCUSSION AND APPLICATIONS

The analytic considerations and numerical calculations presented in the present paper, emphasize the importance of the parameter α_{CE} in determining the outcome of CE evolution. This parameter, which describes the efficiency of the deposition of orbital energy into envelope ejection, essentially determines the separation of the binary emerging from the CE. We have shown that α_{CE} can be considerably smaller than one mainly due to two effects: (1) Efficient energy transport to the surface, which can cause a significant increase in the giant's luminosity but does not result in direct mass motion; and (2) three-dimensional effects, which result in the deposition of excessive energy into a relatively small amount of envelope material.

It should be noted, that the increase of the luminosity (point [1] above) can increase the mass loss via a stellar wind significantly (e.g., Kudritzki and Reimers 1978), but the obtained mass-loss rate is still small compared to dynamical mass ejection. It is perhaps possible that once the obtained luminosity exceeds the Eddington limit, a much more substantial mass-loss rate ensues (as suggested by Delgado 1980).

One of the criteria for the importance of point (2) above is provided by the value of the second parameter we have introduced, β_{CE}. If $\beta_{CE} \lesssim 1$, orbital decay proceeds on essentially a dynamical time scale so that energy is deposited locally, only into envelope mass located near the orbital plane. Orbital decay can be slowed down considerably if significant spin-up of the envelope occurs. A measure of this effect is given by the parameter γ_{CE}, which is the ratio of the spin-up to orbital decay time scales. We would like to emphasize that discussions of the CE phase in the past have invariably used the value $\alpha_{CE} = 1$ (e.g., Webbink 1984; Iben and Tutukov 1984; Bailyn and Grindlay 1987). Based on our preliminary calculations as well as on the calculations of Bodenheimer and Taam (1985) and of Soker, Harpaz, and Livio (1984), *we think that a value of* $\alpha_{CE} \approx 0.3$ *should be regarded as a more appropriate one* (being still quite conservative). Of course the exact value depends on the configuration, the mass ratio, the nature of the secondary star (main sequence or white dwarf, see below), etc., so the proposed value should only be regarded as a crude average value.

An important point noted in the present work (but mentioned already by Livio and Soker 1984*b*), is the difference between spiraling-in processes occurring inside giant versus supergiant (AGB) envelopes. Spin-up occurs more rapidly in the more evolved (more centrally condensed) configurations, slowing down orbital decay. In relatively less evolved giant envelopes the orbital decay is very fast, with spin-up possibly occurring only in the final stages. *The final separation in a CE phase involving a very evolved supergiant can be quite large.* This may explain the existence of systems similar to precataclysmics, but with relatively long orbital periods such as Feige 24, 39 Ceti, BE UMa, HD 128200, and others (see also Bond 1985 and Eggleton 1986). These wider systems may, on the other hand, be a consequence of a rapid, tidally enhanced mass loss ("companion reinforced attrition": see, e.g., Eggleton 1986; Tout and Eggleton 1987). A second difference is related to the degree to which the ejected envelope material is concentrated toward the orbital plane. In very evolved supergiant envelopes, the slowing down of orbital decay allows for transport of the deposited energy and the "density contrast" between the equatorial and polar directions is less pronounced than in the less evolved giant case.

In a very recent paper, Livio and Bond (1988) discussed the formation and morphology of planetary nebulae with binary nuclei. These objects can be considered the most direct evidence for the occurrence of the CE phase. Livio and Bond (1988), coupled the results of CE evolution as presented here, with the interacting stellar winds model (Volk and Kwok 1985; Kahn and West 1985) for the shaping of planetary nebulae. They have shown that planetary nebulae resulting from CE ejection are expected to be of elliptical or butterfly types, according to the morphological classification proposed by Balick (1987). Using recent photographs and CCD images of all the planetary nebulae with binary nuclei, Livio and Bond (1988) have shown that the observed morphology is generally consistent with the predicted one.

Common envelope evolution plays a most crucial role in scenarios expected to lead to Type I supernovae (Iben and Tutukov 1984; Iben 1988; Webbink 1984). A typical Iben and Tutukov scenario starts with a binary in which two stars of masses 5–9 $M_{\odot}$ are at an initial separation of 70–1500 $R_{\odot}$. After two CE phases the system evolves into a double white dwarf binary, with a period of order 12 minutes to 14 hr. The subsequent merger of the two white dwarfs (Iben 1988), brought together by gravitational radiation (the lighter white dwarf being dissipated first into a heavy disk) is supposed to produce a Type I supernova explosion. An important consequence of this scenario is the fact that there should exist a population of close, double white dwarf binaries (see also Paczyński 1985). In a recent paper, Robinson and Shafter (1987) reported the results of a search for the existence of such double white dwarf systems with orbital periods between 30 s and 3 hr. They looked for radial velocity variations in 44 DA and DB white dwarfs *without finding any binary.* They concluded that the fraction of white dwarfs that are binaries (in the given period range) is less than 1/20 with a 90% probability and less than 1/37 with a 70% probability. Making the additional assumption that *white binaries are formed with orbital periods longer than 3 hr* (the maximum period of their range), evolve across their observed period range (30 s to 3 hr), and evolve into Type I supernovae (after emerging from the lower end of the period range), Robinson and Shafter (1987) suggested that the space density of binary white dwarfs was too low to account for the rate of Type I supernovae in our Galaxy. This last conclusion is highly uncertain because (1) it has been recently suggested that Tamman's supernova rates may be too high by a factor ~ 3 (van den Bergh, McClure, and Evans 1987), and (2) white dwarf binaries are expected to form within the period range observed by Robinson and Shafter (and not just evolve across it) and thus, spend shorter times (especially if they form with short orbital periods) in that period range. Nevertheless, Robinson and Shafter's (1987) findings are intriguing, in that if one uses the period distribution for binary white dwarfs obtained by Webbink (1984) *adopting an effective* $\alpha_{CE} = 1$, then one to two systems are expected to be found in Robinson and Shafter's sample. *The observations are consistent with theoretical expectations if a lower value of* α_{CE} *is used* (say $\alpha_{CE} \approx 0.3$), as suggested by the present work. Reducing the value of α_{CE} may, however, cause a problem in a different area: the occurrence of Type I supernovae in elliptical galaxies. The problem there has been to achieve a delay of $\sim 10^{10}$ yr (after the major phase of active star formation has presumably ceased). In the Iben and Tutukov (1984) scenario, this delay is achieved by forming white dwarf binaries with periods longer than ~ 3 hr, so that the time scale to reduce their separation by gravitational radiation is of order $\tau_{GR} \approx 10^{10}$ yr. Reducing the binary separations (by reducing α_{CE}) thus eliminates this clock mechanism for generating the necessary delay. A possible solution of this difficulty may be provided by the fact that if a metallicity appropriate to Population II is used, the mass-loss rate during the red giant phase may be reduced significantly, so that even stars with an initial mass 1–2 $M_\odot$ can be expected to form massive CO cores. In such a case, the delay would be provided by the sum of the main-sequence lifetimes of the two binary components. This difficulty, of course, does not exist if active star formation is taking place in ellipticals, although infrared observations seem to indicate that this is not the case (Impey, Wynn-Williams, and Becklin 1986). In any case, the above discussion emphasizes the importance of searches for white dwarf binaries. In particular, an increase by a factor of 5–10 of the Robinson and Shafter sample (definitely not an easy task), can prove crucial for testing Type I supernova scenarios and for the understanding of the CE phase.

Another important consequence of the present work is the fact that coalescence of the main-sequence star with the giant's core may be a quite likely outcome of CE evolution, in the case that the CE is encountered in a relatively unevolved giant phase. Coalescence is particularly favored if the secondary is of a low mass, since in that case spin-up of the envelope is less likely to occur (see eq. [7]). The possible outcome of such a coalescence can be discussed only somewhat speculatively. In a recent calculation, Soker *et al.* (1987) simulated the collision between a low-mass (0.2 $M_\odot$) main-sequence star and a white dwarf. In their calculation, the main-sequence star was entirely smeared out to form a massive, relatively thick disk, around the white dwarf. The subsequent evolution of such a configuration depends on the stability of the disk and on the viscosity within it. In any event, it seems likely that as a result of accretion the white dwarf envelope will expand to giant dimensions (e.g., Neo, Miyaji, and Nomoto 1977; Kutter and Sparks 1980), thus producing a rapidly rotating giant, similar perhaps to FK Comae stars (see also Eggleton 1986). While other evolutionary scenarios assumed to form FK Comae stars exist, in particular via the coalescence of W UMa binaries (Webbink 1976), it may be difficult to distinguish between the two scenarios. The list of objects suggested to belong to the FK Comae class is given in Table 1. Radio and infrared observations of these objects can prove very useful for the study of circumstellar material around them (e.g., Hughes and McLean 1987 for FK Comae and Fleming *et al.* 1987 for 1E 1751 + 7046).

TABLE 1

FK Comae Stars

Star	Spectral type	$V_e \sin i$ (km/s)	Reference
FK Comae	G2 IIIa	100	1
HD 199178	G5 III/IV	90	1
HD 32918	K2 III	50	2
HD 36705 (?)	G8 III	70	2
1E 1751+7046	K5 IV–V	30–40	3
Star I-1 in NGC 188	G8 IIIb	24	4

Note.—HD 36705 is now considered to be a rapidly rotating main-sequence dwarf (P. Eggleton, private communication).

References.—(1) Bopp and Stencel 1981; (2) Collier 1982; (3) Fleming *et al.* 1987; (4) Harris and McClure 1985.

The following point should also be noted: coalescence is an *unlikely* outcome when the secondary is a white dwarf (as in the second CE phase supposed to lead to Type I supernova), rather than a main-sequence star. This is a consequence of the fact that two white dwarfs can reach a final separation that is ~ 50 times smaller than that of a white dwarf–low-mass main-sequence binary. Thus, even if the value of α_{CE} is much smaller than one, a double white dwarf system can release sufficient orbital energy to eject the common envelope. This is the case, even without relying on additional energy sources (such as nuclear burning), which may increase α_{CE}. Coalescence can probably be avoided in the same manner, when the spiraling-in object is a neutron star, as suggested for the formation of 4U 1820−30 (Verbunt 1987; Bailyn and Grindlay 1987).

In summary, we note the following conclusions:

1. The efficiency of deposition of orbital energy into envelope ejection can be quite low. A typical value may be considered $\alpha_{CE} \approx 0.3$.
2. Local deposition of energy occurs in giant (less centrally condensed) envelopes. In very evolved supergiants energy is transported to form a more spherically symmetric distribution.
3. Significant spin-up of the envelope occurs in relatively early stages in evolved supergiant envelopes. In giant envelopes spin-up can occur only in the final stages of the spiraling-in process.
4. The "density contrast" in the ejected envelope between the equatorial and polar directions is expected to be high in the case of giants and mild in the case of very evolved AGB supergiants. A high density contrast may be related to a "butterfly" morphology of the planetary nebula with a binary nucleus, a mild contrast may produce an elliptical nebula.
5. Coalescence may be a likely outcome of a spiraling-in process occurring inside a giant envelope. The results of such a coalescence may be the formation of a rapidly rotating giant, similar perhaps to FK Comae stars. Common envelope evolution involving very evolved supergiants, on the other hand, may produce relatively wide binaries. Coalescence is unlikely when the secondary star is a compact object.

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REFERENCES

Alexander, M. E., Chau, W. Y., and Henriksen, R. N. 1976, *Ap. J.*, **204**, 879.
Balick, B. 1987, *A.J.*, in press.
Bailyn, C. D., and Grindlay, J. E. 1987, *Ap. J.* (*Letters*), **316**, L25.
Bodenheimer, P., and Taam, R. E. 1984, *Ap. J.*, **280**, 771.
Bond, H. E. 1985, in *Cataclysmic Variables and Low-Mass X-Ray Binaries*, ed. D. Q. Lamb and J. Patterson (Dordrecht: Reidel), p. 15.
Bondi, H. 1952, *M.N.R.A.S.*, **112**, 195.
Bondi, H., and Hoyle, F. 1944, *M.N.R.A.S.*, **104**, 273.
Bopp, B. W., and Stencel, R. E. 1981, *Ap. J.* (*Letters*), **247**, L131.
Collier, A. C. 1982, *M.N.R.A.S.*, **200**, 489.
deKool, M. 1987, Ph.D. thesis, University of Amsterdam.
Delgado, A. J. 1980, *Astr. Ap.*, **87**, 343.
Eggleton, P. P. 1986, in *The Evolution of Galactic X-Ray Binaries*, ed. T. Truemper *et al.* (Dordrecht: Reidel), p. 87.
Eggleton, P. P., and Verbunt, F. 1986, *M.N.R.A.S.*, **220**, 12P.
Fleming, T. A., Gioia, I. M., Maccacaro, M., and Mereghetti, S. 1987, preprint.

Harris, H. C., and McClure, R. D. 1985, *Pub. A.S.P.*, **97**, 261.
Hensler, G. 1982, *Astr. Ap.*, **114**, 309.
Hughes, V. A., and McLean, B. J. 1987, *Ap. J.*, **313**, 263.
Iben, I., Jr. 1966, *Ap. J.*, **143**, 483.
———. 1988, *Ap. J.*, **324**, 355.
Iben, I., Jr., and Tutukov, A. V. 1984, *Ap. J. Suppl.*, **54**, 335.
Iben, I., Jr., and Webbink, R. F. 1987, in *IAU Colloquium 95, Second Conference on Faint Blue Stars*, ed. A. G. D. Philip, D. S. Hayes, and J. Liebert (Schenectady: L. Davis Press), in press.
Impey, C. D., Wynn-Williams, C. G., and Becklin, E. E. 1986, *Ap. J.*, **303**, 572.
Kahn, F. D., and West, K. A. 1985, *M.N.R.A.S.*, **212**, 837.
Kudritzki, R. P., and Reimers, D. 1978, *Astr. Ap.*, **70**, 227.
Kutter, G. S., and Sparks, W. M. 1980, *Ap. J.*, **239**, 988.
Livio, M., and Bond, H. E. 1988, in preparation.
Livio, M., and Soker, N. 1984*a*, *M.N.R.A.S.*, **208**, 763.
———. 1984*b*, *M.N.R.A.S.*, **208**, 783.
Livio, M., Soker, N., deKool, M., and Savonije, G. J. 1986, *M.N.R.A.S.*, **222**, 235.
Meyer, F., and Meyer-Hofmeister, E. 1979, *Astr. Ap.*, **78**, 167.
Morris, M. 1981, *Ap. J.*, **249**, 572.
Neo, S., Miyaji, S., and Nomoto, K. 1977, *Pub. Astr. Soc. Japan*, **29**, 249.
Paczyński, B. 1965, *Acta Astr.*, **15**, 89.
———. 1976, in *IAU Symposium 73, Structure and Evolution of Close Binary Systems*, ed. P. Eggleton, S. Mitton, and J. Whelan (Dordrecht: Reidel), p. 75.
Paczyński, B. 1985, in *Cataclysmic Variables and Low-Mass X-Ray Binaries*, ed. D. Q. Lamb and J. Patterson (Dordrecht: Reidel), p. 1.
Paczyński, B., Ziolkowski, J., and Zytkow, A. 1969, in *Mass Loss from Stars*, ed. M. Hack (Dordrecht: Reidel), p. 237.
Prialnik, D., and Livio, M. 1985, *M.N.R.A.S.*, **216**, 37.
Robinson, E. L., and Shafter, A. W. 1987, *Ap. J.*, **322**, 296.
Sima, E., Matsuda, T., Takeda, H., and Dawada, K. 1985, *M.N.R.A.S.*, **217**, 367.
Soker, N., Harpaz, A., and Livio, M. 1984, *M.N.R.A.S.*, **210**, 189.
Soker, N., Regev, O., Livio, M., and Shara, M. M. 1987, *Ap. J.*, **318**, 760.
Sparks, W. M., and Stecher, T. P. 1974, *Ap. J.*, **188**, 149.
Taam, R. E., Bodenheimer, P., and Ostriker, J. P. 1978, *Ap. J.*, **222**, 269.
Tout, C., and Eggleton, P. 1987, preprint.
van den Bergh, S., McClure, R. D., and Evans, R. 1987, *Ap. J.*, **323**, 44.
Verbunt, F. 1987, *Ap. J.* (*Letters*), **312**, L23.
Volk, K. M., and Kwok, S. 1985, *Astr. Ap.*, **153**, 79.
Webbink, R. F. 1977, *Ap. J.*, **211**, 486.
———. 1984, *Ap. J.*, **277**, 355.
———. 1986, in *Critical Observations vs. Physical Models for Close Binary Systems*, ed. K.-C. Leung, and D.-S. Zhai (New York: Gordon and Breach), in press.
Webbink, R. F., and Iben, I., Jr. 1987, in *IAU Colloquium 95, Second Conference on Faint Blue Stars*, ed. A. G. D. Philip, D. S. Hayes, and J. Liebert (Schenectady: L. Davis Press), in press.
Yungelson, L. R. 1973, *Nauch. Informatsii*, **27**, 93.

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