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DIRECT EVIDENCE FOR GRAVITATIONAL DOMINATION OF THE MOTION OF GAS WITHIN ONE LIGHT-WEEK OF THE CENTRAL OBJECT IN NGC 4151 AND THE DETERMINATION OF THE MASS OF THE PROBABLE BLACK HOLE

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ABSTRACT

A practical method is demonstrated for determining the average direction of motion of broad-line region gas in quasars. Analysis of the variability of the blue and red wings of the C IV λ1549 and Mg II λ2798 lines and the UV continuum in the low-luminosity quasar in NGC 4151 reveals that the red sides of both lines lead to blue sides in their variations. Outflow of the high-velocity gas ($v > 5000$ km s^{-1}) emitting the C IV and Mg II lines is excluded at the 99.5% confidence level. The dominant motion of the gas is most likely to be inward, although totally random or chaotic motions are excluded only at the 93% level. It is suggested that the gravitational domination of the motion of the broad-line region clouds at small radii is a consequence of their high densities ($N_e > 10^{11}$ cm^{-3}). The demonstration that the broad-line region has its motions dominated by gravity (versus domination by a wind or radiation pressure) puts black hole accretion models onto an observational footing and permits the first simple kinematic determination of the mass of the black hole in a galactic nucleus. The mass if $5 \times 10^7\ M_\odot$ if the gas is indeed inflowing.

Subject headings: black holes — galaxies: individual (NGC 4151) — galaxies: internal motions — galaxies: nuclei — galaxies: Seyfert — quasars

I. INTRODUCTION

Broad emission lines originating in dense ($N_e > 10^9$ cm^{-3}), rapidly moving ($V > 3000$ km s^{-1}) gas seem to be present in perhaps all members of the quasar family. This gas (the so-called broad-line region, hereafter BLR) provides a probe of conditions near the central energy source. The ubiquitousness of BLRs suggests a close relationship between the energy source and the BLR (for example, the ratio of line emission to continuum luminosity is roughly constant over an enormous range in luminosity; Yee 1980; Shuder 1981). The nature of the motions of the BLR gas is an important and long-standing question. Systematic asymmetries in high-ionization lines (Gaskell 1982) require that the motions of at the least the high-ionization lines be radial, and there are good theoretical reasons for doubting that clouds can be in Keplerian motion for long (Mathews 1982*b*). The sign of the velocity in the innermost regions (i.e., whether inflow or outflow) has remained unknown but most indications are that the gas further out is outflowing (see Mathews and Capriotti 1985). This must certainly be the case where blueshifted intrinsic absorption lines are seen. The BLR probably consists of at least two distinct types of cloud—large predominantly neutral optically thick clouds and smaller more fully ionized optically thin ones (Gaskell 1987). The neutral clouds are more distant from the central photoionizing source (Gaskell and Sparke 1986). There is evidence that the motions of the optically thick clouds are dominated by nongravitational forces (Gaskell 1985). The standard model for the energy source of a quasar involves accretion of matter onto a massive black hole (although other alternatives still receive serious attention—see recent reviews by Kundt 1987 and Stone and Ptak 1987). The idea of identifying the BLR with the matter being accreted onto the postulated black hole (see, for example Wandel, Milgrom, and Yahil 1985; Kazanas 1986) is an attractive one and evidence for infall of BLR material at small radii would be important.

It has long been recognized that study of the variability of the line profiles of the broad emission lines in quasars could settle the issue of the motions of the clouds (e.g., Fabrica 1980*a*, *b*; Capriotti, Foltz, and Peterson 1982; Blandford and McKee 1982). For example, if the clouds were indeed all outflowing, as the central photoionizing source varied, the blue wings of the lines should vary first and the red wings should follow with a delay of the order of the light-crossing time. In most cases seen so far both wings of the broad lines seem to have come and gone together and this has been taken as evidence for chaotic motions (e.g., Ulrich *et al.* 1984). Gaskell and Sparke (1986) have, however, recently established that the sizes of BLRs are considerably smaller than had been thought to be the case (see also Peterson *et al.* 1985; Peterson and Gaskell 1986; Gaskell and Peterson 1987). In Akn 120 and NGC 4151 much of the BLR flux is coming from a region of radius 1 light-week. The light-crossing time is therefore *not* much greater than the continuum variability time scale (contrary to what had been assumed). Gaskell and Sparke stated that kinematic mapping of BLRs was therefore going to be difficult.

In this paper it will be shown that the same cross-correlation techniques used by Gaskell and Sparke (1986) to estimate BLR sizes can also be used to study the kinematics and that existing undersampled data for the low-luminosity quasar in NGC 4151 can be used to show that the high-velocity gas is probably inflowing.

II. DATA

To investigate the kinematics of the BLR one needs to study the flux in the wings of the broad lines. The best-studied Seyfert 1 galaxy is NGC 4151. Ulrich *et al.* (1984) present measurements of the half-width at zero intensity (HWZI) of both the blue and red wings of the broad C IV λ1549 and Mg II λ2798 emission lines for 25 epochs over a 2 yr period from mid-1978 to mid-1980. A more detailed discussion of the C IV profile at a smaller number of more recent epochs has been given by Stoner, Ptak, and Gregory (1984) and Stoner and Ptak (1985). The line expected to be most sensitive to changes in the photoionizing continuum is C IV (see Gaskell and Sparke 1986). The changes in the width of this line are very striking (see Fig. 1 of Ulrich *et al.* 1984 or Fig. 1 of Stoner *et al.* 1984). When the continuum is faint the HWZI can be less than 5000 km s^{-1} while when the continuum is bright the blue HWZI approaches 20,000 km s^{-1}. As is expected (see Gaskell and Sparke 1986), Mg II varies less, but it too can vary in width by a factor of 2 (see for example Fig. 1 of Ulrich *et al.* 1984). In this paper the analysis will be confined to the HWZI values published by Ulrich *et al.* The HWZI can be taken as a (highly nonlinear) measure of the fraction of the total line flux at velocities greater than (say) 5000 km s^{-1}. Ideally one wants the actual fluxes in the line wings but these are not yet available and as far as the cross-correlation analysis is concerned the nonlinear relationship between HWZI and the line flux in a wing is unimportant. The continuum fluxes used were those measured at 1450 Å and reported by Perola *et al.* (1983).

Perola *et al.* (1983) claim an accuracy of 5% in the 1450 Å continuum fluxes. Ulrich *et al.* (1984) do not quote errors for their FWZI values but comparisons of measures on spectra taken on the same day suggests a 1 σ error of 1200 km s^{-1}. Inspection of the line wing autocorrelation function (ACF—see below) at small but nonzero time lags also suggests errors of the order of 1200 km s^{-1} as did remeasurement of some of the published line profiles by the author. The contribution of measurement error to the sample standard deviation is much greater for the HWZI values (45%) than for the continuum flux (20%).

III. ANALYSIS

a) The Cross-Correlation Program

The treatment is essentially that developed by Gaskell and Sparke (1986), and further details and improvements can be found in Gaskell and Peterson (1987). Ideally one wants two infinite time series $a(t)$ and $b(t)$ sampled every day (or more often). The cross-correlation function (hereafter CCF; see Jenkins and Watts 1968 for a formal definition) at a lag u is calculated by cross-correlating the $a(t)$ with the $b(t-u)$. In reality there are only a finite number of elements in the $a(t)$ and $b(t)$ series and their sampling is not uniform. Following Gaskell and Sparke (1986), $a(t)$ and $b(t)$ are linearly interpolated in time and the ends of the data are extended at a constant value equal to the end values. Each *observed* point in $a(t)$ is correlated with the corresponding point $b(t-u)$ in the other series (this point will probably not be an observed point but an interpolated one). The correlation is then performed the other way round, i.e., each *observed* point in $b(t)$ is correlated with the corresponding point $a(t+u)$. The CCF at lag u is then taken to be the average of the two. This reduces the errors due to the linear interpolation (which, as explained by Gaskell and Peterson, is the dominant error in most cases) and gives an additional check (used in next section).

b) Error Analysis

In this paper we will only be concerned with the positions of the peaks in the CCFs. Since important conclusions will be drawn on the basis of the position of these it is clearly important to assess the errors. The effect of random or sampling errors has been calculated in three different ways, as follows.

i) *Theoretical Error*

Cross correlation techniques have been widely used in radial velocity work for some 20 yr now. The theoretical errors in radial velocities calculated by cross-correlation techniques have been discussed by Griffin and Gunn (1974) for the case of analog radial velocities from the Griffin radial velocity meter (Griffin 1966) and by Tonry and Davis (1979) for the case of digitized galaxy spectra. The former techniques involve real-time cross correlation of a star spectrum with a mask consisting of clear and opaque regions at irregular intervals corresponding to prominent features in stellar spectra. The latter involves rebinning uniformly sampled digitized spectra onto a logarithmic wavelength scale and cross-correlating them in a computer. Tonry and Davis (1979) show that the error in the position of the peak in the CCF (which in their case gives the radial velocity) depends only on the ratio of the height of the peak in the CCF to the noise level in the CCF, and the ratio of the wavenumber of the half-power point in the Fourier transform to the number of bins. The latter quantity is related to the half-width at half maximum of the peak in the CCF (to of the "dip" in the analysis of Griffin and Gunn 1974).

By inspection of the CCFs below (Figs 1–4) the half-width at half-maximum of the peaks in the CCFs is at ~10 days. The average peak "noise" level in the CCFs is expected to be $\sim(2)^{1/2}/(n-2)^{1/2} = 0.29$ (as can be seen to be the case at $|u| > 20$ days in Figs 1–4). The average peak is at ~0.67 so using equations (23) and (24) of Tonry and Davis (1979), the expected error is ±3.0 days.

Note that the analysis of Tonry and Davis does not depend

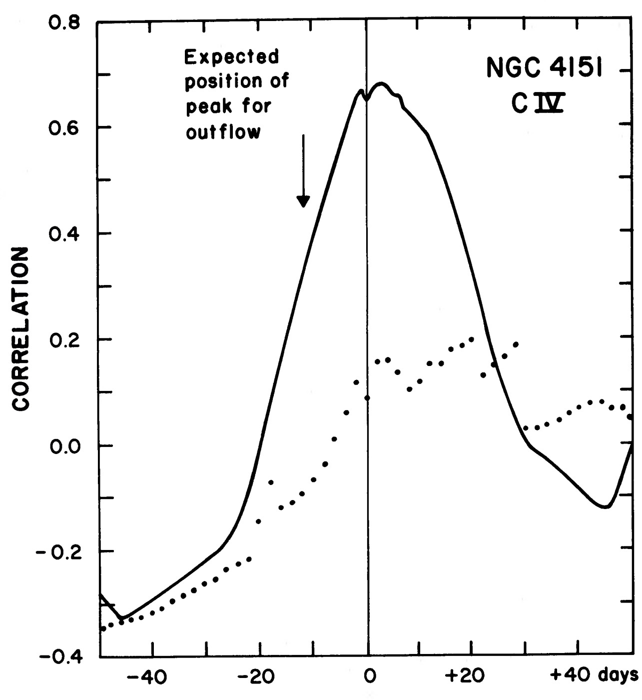

FIG. 1.—Cross correlation between HWZI of the blue and red wings of the C IV λ1549 line in NGC 4151. A positive delay corresponds to the blue wing following the red wing. Solid line is for all the available data. Dotted line shows the effect of ignoring the low state on 1980 April 21. Arrow indicates the position of the peak of the cross correlation that would be expected were the gas outflowing.

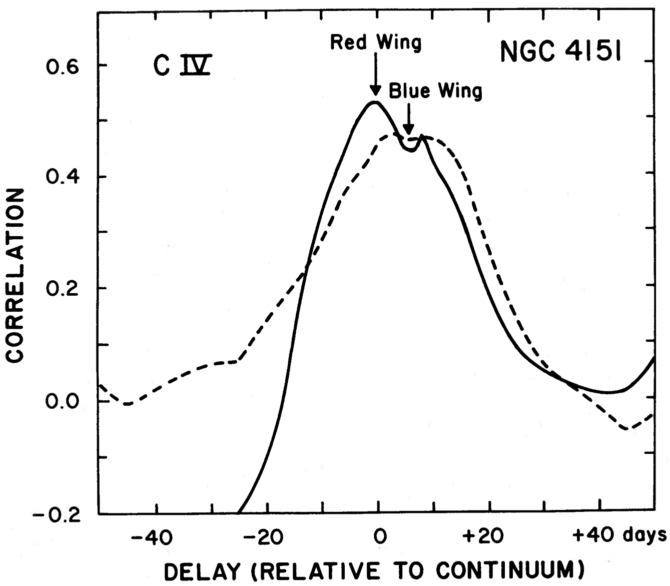

FIG. 2.—Cross correlation between the red and blue wing HWZIs of C IV and the 1450 Å continuum flux. A positive delay corresponds to the line wing variations following the continuum variations.

on what is causing the broadening of the peak in the CCF. In the case of radial velocity work the broadening is due either to the resolution (or slit width) of the spectrograph or the internal line broadening in the star or galaxy whose radial velocity is being measured (or is being used as a "template"). Here the width is determined by the largest either of the time scale of variability of $a(t)$ or $b(t)$ or by the typical sampling time. In the CCFs calculated here the width is set by the continuum variability time scale (see below). These factors influencing the accuracy are discussed in more detail in Gaskell and Peterson (1987).

In addition to giving formulae for the error in the position of the peak in the CCF, Tonry and Davis give a formula for the confidence that a peak is not spurious (their eq. [36]). There is a 95% probability that each peak found in this paper is real. If one does not have an *a priori* reason for believing that a peak is "real" then this additional uncertainty means that the errors in the peak position are non-Gaussian (in particular, the tails of the distribution function are much more extended).

ii) *Subsets of the Data*

The data sets correlated in Figs 1–4 were subdivided into halves (into first and last halves and also into even and odd alternate pairs of points). The correlations of $a(t)$ on $b(t-u)$ and $a(t+u)$ on $b(t)$ were also inspected separately. For Mg II the standard deviation in the position of the peaks was estimated to be ± 3.1 days. For C IV it was ± 3.3 days but it was found that the C IV cross-correlation was strongly influenced by the low continuum state on 1980 April 21 (see Fig. 1).

iii) *Monte Carlo Simulation*

Gaskell and Peterson (1987) have also investigated the accuracy of cross correlations using unevenly sampled data and compared these results with the analytical results. Their agreement is reasonably good. Similar simulations have been carried out to investigate the accuracy of the determinations of the peaks in Figures 1–4. The interested reader can find full details of the simulations in Gaskell and Peterson (1987). To simulate the real cross correlations as closely as possible (i) the spacing of the "observations" in the simulations is taken to be *exactly* the same as the real observations; (ii) the ACF (calculated in the same way as the CCF) of time series a (modeled as a simple first-order autoregressive process; see, e.g., Jenkins and Watts [1968, p. 168] or Box and Jenkins [1970, chap. 1] for details) was made to have the same width as the real ACF of series a; (iii) a "transfer function" was assumed that gave an ACF of series b (the dependent series) that matched the real ACF of time series b (in this case it was just a phase shift), and series b was derived using it; and (iv) noise was added to match the measuring errors (estimated in § IIa above).

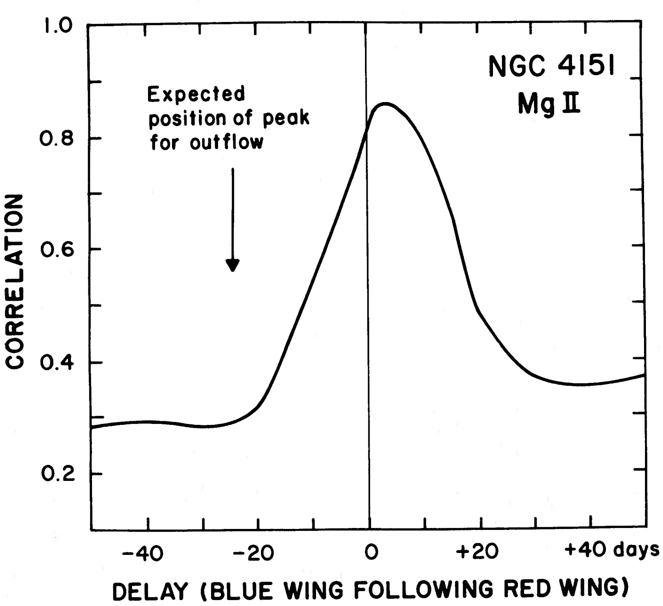

FIG. 3.—Same as Fig. 1, but showing Mg II λ2798

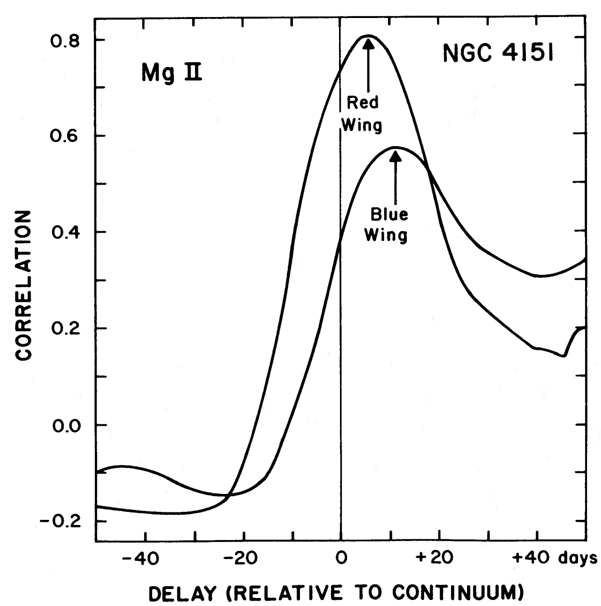

FIG. 4.—Same as Fig. 2, but showing Mg II λ2798

The resulting distribution was, as expected, non-Gaussian, and to facilitate comparison with the analytical error calculations of Tonry and Davis (1979) $\pm 1\ \sigma$ was taken to include 68% of the simulation results. About 10% of the simulations gave results outside the $\pm 2.5\ \sigma$ range, in rough agreement with the results of equation (36) of Tonry and Davis (1979). Matching Figures 1–4 as closely as possible, as just described, gave a 1 σ error of ± 3.4 days for each CCF. In the discussion of gas motions in NGC 4151 the empirical error distribution found from these Monte Carlo simultations will be used rather than that of a normal distribution.[1]

v) *Systematic Errors*

One can easily think of systematic effects that might be influencing each time series (such as line blending, errors in continuum placing, or wavelength calibration, etc.). However, all possible such effects are only a function of the continuum brightness or line strength, and *they will not systematically affect one side of the line before the other*. Their only effect can be to introduce a spurious correlation at *zero* time delay but the correlations in Figs 1–4 do *not* (with one exception) peak at zero time delay. I therefore conclude that although the shapes of the CCFs might be *influenced* by systematic effects the results below are not *caused* by systematic effects.

IV. RESULTS

a) C IV λ1549

Figure 1 shows that the C IV blue HWZI increases *after* the red HWZI (thus implying inflow). The delay is 3.4 days. This is estimated to be only 1 σ away from zero. Subdividing the data revealed that the correlation was strongly influenced by the values of just one night (1980 April 21) when NGC 4151 was at its faintest. The effect of ignoring this night is shown in Figure 1. Notice that although the correlation is weakened there is no evidence for the blue side leading to red. Note also that there is no *a priori* reason for rejecting 1980 April 21 (i.e., there is no reason to doubt that the quasar *was* faint on that night and that the lines *were* narrow).

Figure 2 shows the cross-correlation functions for the wings of the C IV line and the continuum measured at λ1450. As would be expected (for inflow) the red side of the line shows essentially zero time delay (formally +1.4 days) but the blue side shows a delay of 6.4 days. These differ at the 1 σ level.

b) Mg II λ2798

Figure 3 shows that the Mg II HWZI on the blue side of the line also increases after the red HWZI. The delay is $+4.5 \pm 3.1$ days. This differs from zero by 1.5 σ. Unlike C IV this peak is not influenced strongly by any extreme points, and (to within the errors) it has the same position when various subsets of the data are analyzed (as described in § III).

Figure 4 shows that *both* the red and the blue wings of Mg II are lagging behind the 1450 Å continuum. The red wing follows by 6 ± 3 days and the blue wing by 12 ± 3 days. The difference between these is in good agreement with Figure 3. The peak of the cross-correlation function for the red wing differs from zero at the 2 σ level. As with the Mg II red/blue wing cross correlations the positions of the peaks are insensitive to any particular nights' data or any particular event.

V. DISCUSSION

a) *Motions of the BLR Clouds*

The strongest result is that outflow at small radii is excluded. Gaskell and Sparke (1986) have estimated that the C IV emission in NGC 4151 come predominantly from a radius of ~5 light-days and that the Mg II emission comes from ~12 light days. One would therefore confidently predict that the red wings would lag the blue wings by twice the respective light travel times if the dominant motion is radial outflow. This is by ~12 or ~24 days for the C IV and Mg II lines, respectively (these positions are indicated in Figs 1 and 3). With the error of ± 3.4 days in the position of the peak in the CCF estimated in § II the predictions of the outflow model are many σ away from the observed CCF peaks. The Monte Carlo simulations exclude the outflow model at the ~95% level for each line alone or at the greater than 99.5% level for both lines taken together.

Are the positions of the peaks in Figures 1–4 consistent with chaotic or circular motions? The predictions for this case are that the peaks in Figures 1 and 3 should be at exactly zero days delay. For C IV the peak in Figure 1 differs from zero at only the 1 σ level and there is a 35% (two-tailed) chance it is really zero. For Mg II (Fig. 3) the difference from zero delay is slightly more significant but the Monte Carlo simulations show that there is still a 19% chance that it is really zero. Neither of these probabilities is significant enough to exclude chaotic or circular motions in itself but since the Mg II and C IV HWZI measurements are independent these probabilities can be combined to exclude totally chaotic or totally circular motions at the 93% level.

b) *Size of the Inner BLR*

If the motions of the high-velocity BLR clouds are chaotic or circular the average delay between the wings and the continuum gives the size of the region. From Figures 2 and 4 there is ~3 light-days for C IV and ~8 light days for Mg II. These radii are only slightly smaller than the radii Gaskell and Sparke (1986) estimate for all velocities. The differences are not formally significant but they are just what one expects by the virial theorem. If, as seems likely from the results just presented, the dominant motion is infall then there should be almost no delay between the variations in the red wings and the continuum while the delay in the blue wings corresponds to twice the light crossing time for the radius. Under this more likely assumption the region responsible for the high-velocity C IV is at 1.7 ± 1.7 light days while the high-velocity Mg II is at 6 ± 1.7 light days. This is just what one expects with motion dominated by gravity. The only possible problem with the infall model is that the red Mg II wing seems to be delayed relative to the continuum by 6 days, which has only a 10% (two-tailed) probability of happening by chance if it should really be zero, but since not all clouds receeding from us at high velocity will be exactly on our line of sight some small delay is expected. Detailed modeling will be needed to say whether a nonradial component is needed.

c) *General Considerations*

The evidence for infall in NGC 4151 is perhaps a slight surprise because NGC 4151 is an example of a quasar nucleus

[1] The errors quoted in an earlier account of some of this work (Gaskell 1987) assumed a normal distribution and are therefore too optimistic. In addition, one-tailed probabilities were inadvertently quoted in that paper instead of two-tailed probabilities in some cases.

definitely showing blue shifted absorption (Anderson and Kraft 1969). This absorption is variable (Cromwell and Weymann 1970; Weymann 1970) and arises in gas with conditions not too far different from those expected in the BLR (Ferland and Mushotzky 1982). The situation that might exist in NGC 4151 (i.e., inflow at small radii and outflow at large radii) has, however, been predicted by Mathews (1982*a*) who argued that there was a critical radius inside of which clouds would fall in under gravity and outside of which clouds would be radiatively accelerated. This radius occurs where the confinement of the intercloud medium makes the density of the BLR clouds exceed $\sim 10^{10}$ cm^{-3}. This critical radius occurs because radiative acceleration becomes ineffective at high densities (the precise radius depends on the ratio of the luminosity to the Eddington luminosity). Gaskell and Sparke (1986) find the density in the NGC 4151 BLR to be 10^{10}–10^{11} cm^{-3} and the high-velocity filaments studied here probably have densities one order of magnitude higher still. Conditions in the innermost of BLR of NGC 4151 would therefore seem suitable for the infall of BLR clouds even when gas further out is outflowing. Capriotti, Foltz, and Byard (1980) have shown that infall can reproduce the observed line profiles.

d) Mass of the Central Object

It has long been known that the velocities of BLR clouds can give the mass of the central object (Woltjer 1959, 1966; Dibai and Pronik 1964). To do this one must know the size of the BLR reliably and the *direction* of motion as well as the velocities. Without knowing the direction it is impossible to get an estimate of the mass—if the clouds are outflowing the velocity tells you *nothing* about the central mass.

With the evidence here against outflow of the BLR and the fairly precise estimates of the radii corresponding to given velocities one can now for the first time calculate mass of the central object in NGC 4151. If the motion is predominantly radial inflow both the Mg II and C IV wings are consistent with a mass of $5 \times 10^7\ M_{\odot}$. (The C IV wings are at a higher velocity but are also at a smaller radius.) In the (less likely) case where the motion is circular the mass will be about twice this value [since the radii are larger—see above—and because the change in direction gives a factor of $(2)^{1/2}$].

VI. CONCLUSIONS

A practical method has been presented for determining the direction of motion of gas close to the centers of quasars. Application of this method to published data for the low-luminosity quasar NGC 4151 shows that outflow of the gas within one light-week of the central object in NGC 4151 is excluded at a high confidence level. Instead, inflow is favored, although chaotic or circular motions can only be excluded at the 93% level. This gravitational domination of the cloud motion puts quasar models invoking quasi-spherical accretion into black holes on an observational footing. The mass of the central object (probably a black hole) in a galactic nucleus has been determined for the first time. Work is underway to extend the work to additional quasars.

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