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# THE EARTH'S ROTATION

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## INTRODUCTION

The Earth's rotation is not constant. Instead, both the rate of rotation and the position of the rotation axis vary with time. Changes in the rotation rate are directly proportional to changes in the length of a day (LOD). In addition, the time integral of the LOD variability is proportional to fluctuations in Universal Time, the measure of time as determined by the overhead transits of celestial objects.

Variations in the position of the rotation axis are usually classified either as "polar motion" or as "nutation," where "polar motion" describes motion of the axis with respect to the Earth's surface, and "nutation" denotes motion of the axis with respect to inertial space. The distinction between polar motion and nutation is somewhat artificial, since, in general, nutation cannot occur without some accompanying polar motion, and vice versa. In practice, though, axis motion caused by an individual excitation process is mostly either one or the other, depending on the time scale. Excitation at periods much longer than one day as seen by an observer on the Earth causes mostly polar motion: The rotation axis does not move much with respect to inertial space compared with its motion with respect to the Earth. Thus, since processes originating within the Earth capable of affecting rotation generally have long time scales, they cause polar motion. Conversely, excitation with a nearly diurnal (retrograde) period as seen from the Earth causes axis motion that is mostly nutation. For example, the gravitational attraction of the Sun and Moon causes nutational motion, since the Sun and Moon have nearly diurnal periods as seen from the Earth.

This article is a survey of rotation observations and, especially, of the geophysical implications of those observations for all three types of



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variations in rotation: LOD, polar motion, and nutation. The subject of rotation touches on diverse fields in the geophysical sciences, including solid Earth geophysics, meteorology, and oceanography, and only a brief summary is presented here. More detailed descriptions can be found in Munk & MacDonald (1975), Lambeck (1980), and Rochester (1984).

## OBSERVATIONAL TECHNIQUES AND RESULTS

Variations in rotation are detected by observing the apparent motion of objects in space from fixed points on the Earth. Until recently, all such observations involved using optical telescopes to monitor the apparent angular positions of stars. Detailed observations of this sort were made over the last century or more. Furthermore, long-period terms in the LOD over the last few hundred years and a linear trend over the last two to three thousand years have been resolved from historical records of eclipses and planetary occultations. For example, recorded solar times of Babylonian eclipses differ by up to several hours from the solar times predicted using the present positions of the Sun and Moon and the assumption that the mean rotation rate has remained constant over the last few thousand years (see, for example, Brosche & Sundermann 1978).

Within the last decade or two, however, several new techniques have been implemented that have significantly greater accuracies. These include lunar laser ranging, satellite laser ranging, and very-long-baseline interferometry (VLBI). Lunar laser ranging (LLR) involves the measurement of the distance between powerful Earth-based lasers and the Moon by recording the round-trip travel time of laser pulses reflected from mirrors on the lunar surface. In satellite laser ranging a satellite is tracked by measuring the round-trip travel time of laser pulses originating from the Earth and reflected from mirrors attached to the outside of the satellite. Currently, the most accurate satellite results come from tracking the satellite LAGEOS.The VLBI technique uses widely separated radio antennas to detect signals from distant astronomical radio sources. By comparing the recordings of the same signals detected at two antennas, the length and orientation of the baseline vector between the antennas can be determined.

Although all three of these techniques provide excellent rotation results, VLBI is probably the most versatile. Satellite laser ranging, for example, is not presently as accurate at long periods due to uncertainties in the satellite orbit, although short-period variability is very well determined. LLR has not yet had enough ground-based lasers in simultaneous operation to give routine, reliable polar motion data, although the LOD results have been excellent. Furthermore, both these techniques are clearly inferior to VLBI for determining nutational motion because VLBI is tied directly to an inertial coordinate system defined by the distant radio sources.

The accuracies of the results from these new techniques are improving rapidly. Currently, the LOD can probably be determined from all three techniques to better than 0.1 ms, and polar motion from VLBI and satellite laser ranging to better than 0.002 arcsec, where both these numbers refer to values averaged over 3–5 days or less. The results improve substantially when longer averaging times are used (for an assessment of the accuracies, see Robertson et al 1985). VLBI nutation results at specific nutation frequencies are accurate to better than 0.2 milliarcseconds (mas).

Each of the new techniques involves observations of the propagation times of electromagnetic waves passing through the Earth's atmosphere. The results are, thus, sensitive to uncertainties in the atmosphere's index of refraction. This, in fact, is currently the limiting source of error for the VLBI results. (The error is mostly related to the uncertainty in the atmospheric water vapor content.) However, these errors are nowhere near as large as the refraction errors in the traditional stellar optical results. Uncertainties in the index of refraction have a much greater relevant effect on apparent angular positions than on propagation times. Still, the old results from the stellar optical technique are invaluable when investigating variability at decade and longer time scales.

The observational results for the LOD fall roughly into three categories. First, there is a linear increase in the LOD of about 2 ms per century, as determined from the ancient astronomical record. Second, there are irregular decade fluctuations of about 4 to 5 ms over 20 to 30 yr. Figure 1 shows the sum of the linear increase and the irregular fluctuations, as determined from about 150 yr of astronomical data. Note that even over this long time period it is impossible to cleanly separate the linear increase from the decade fluctuations.

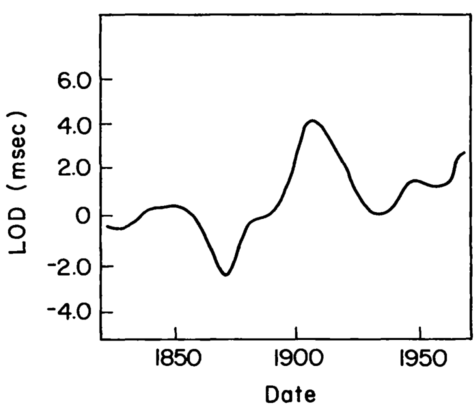


*Figure 1* The length of day (LOD) is variable over a wide range of time scales. At long periods are decade fluctuations (caused by the transfer of angular momentum between the Earth's fluid core and solid mantle) and a linear increase in the LOD (caused by a combination of tidal friction in the oceans and the effects of the last ice age). Here we show astronomical results for the long-period variability during 1820–1975, using data from Morrison (1979). Note that even 150 yr of data is inadequate to cleanly separate the linear increase in the LOD from the decade fluctuations.

The third category includes those variations in the LOD with periods shorter than about 5 yr. The solid line in Figure 2 shows a typical example of the short-period LOD variability, as determined from a combination of LLR, VLBI, and LAGEOS results (the effects of tides have been removed, as described below) during 1982–86. Although the linear increase and decade fluctuations have not been removed from the results shown in Figure 2, they are only marginally evident in the data, since the latter cover such a short time span.

The observed variability of polar motion is much less complex. As an example, VLBI results for the position of the pole from November 1983 to April 1987 are shown in Figure 3. Note that the pole follows a roughly circular, counterclockwise path about its mean position. This short-period variability can be separated into an annual oscillation (the "annual wobble") and a 14-month oscillation (called the "Chandler wobble" after the American astronomer S. C. Chandler, who first reported the motion in 1891), both with amplitudes of about 0.1 arcsec. At long periods there is also evidence from a century or so of optical data for a linear drift of the rotation axis and for perhaps a 30-yr periodic variation (Dickman 1981). No other significant variability has been observed.

Nutational motion occurs at discrete, nearly diurnal frequencies determined by the orbital periods of the Earth and Moon. The amplitudes at these frequencies are sensitive to details of the Earth's internal structure, as we discuss in more detail below.

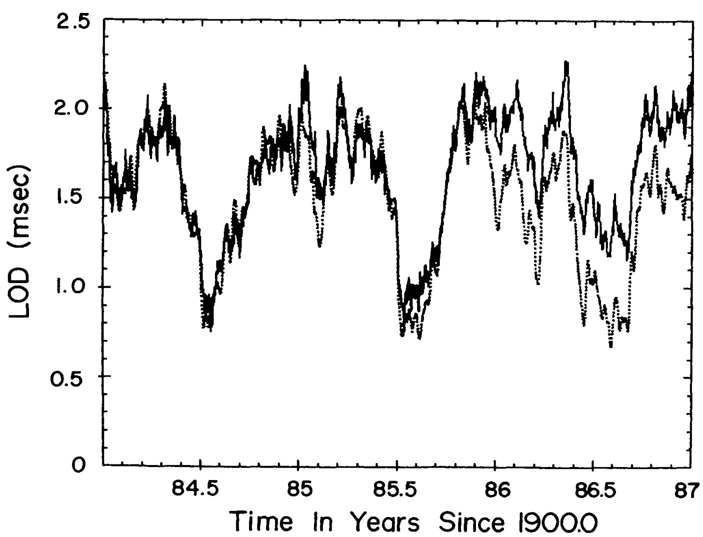


*Figure 2* The atmosphere is responsible for much of the short-period variability in the LOD. Here, results from atmospheric wind and pressure data are compared with 1982–86 LOD data obtained with a simultaneous solution using VLBI, LLR, and LAGEOS observations. (Results provided by Marshall Eubanks.) The effects of tides have been removed from the LOD results.

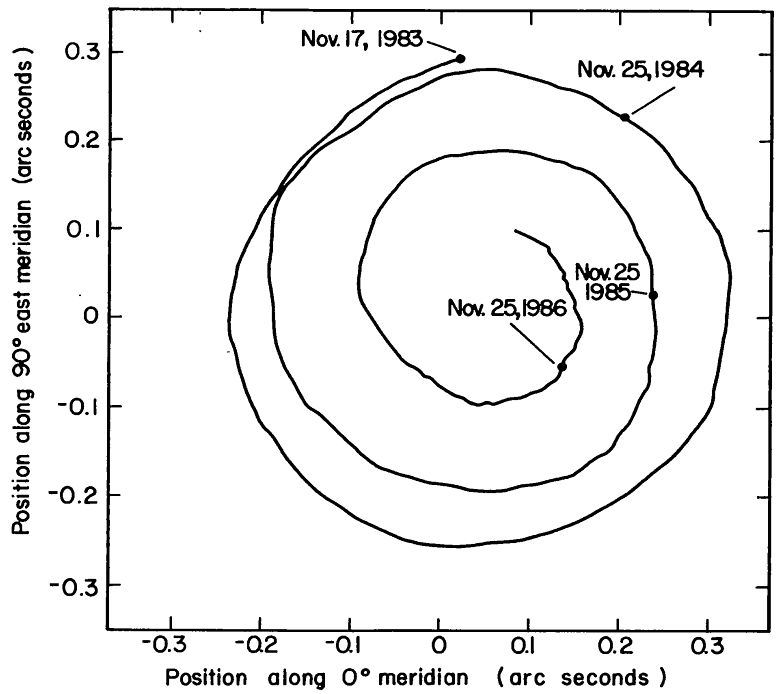


*Figure 3* The Earth's rotation axis does not remain fixed with respect to the Earth's surface. The motion of the pole from 17 November 1983 through 19 April 1987 is shown above, using VLBI results that have been reduced by the US National Geodetic Survey. The $x$- and $y$-coordinates represent the amount the pole is tipped along the Greenwich and 90°E meridians, respectively. The pole moves in a counterclockwise direction along a roughly circular path. This motion is a superimposition of annual and 14-month oscillations.

## ROTATION THEORY

In the following sections, we discuss what can be learned about the Earth from the observational results. The initial step in understanding an individual variation is to identify the excitation source. Then, the observed fluctuation in rotation can be used either to learn more about the excitation process or, if that process is already well enough understood, to learn about the Earth's deformational response to the excitation source (the amount of deformation sometimes affects the rotational response, as is described below) and thereby to help constrain our knowledge of material properties and structure within the Earth. In this section, we discuss how to compute the effects of a given excitation process on polar motion and the LOD. The theory of nutation is more complicated and is briefly described in a later section.

Suppose we are in a coordinate system rotating with respect to inertial space with angular velocity vector $\boldsymbol{\omega}(t)$. Let $\mathbf{H}(t)$ be the angular momentum vector of the Earth as seen in our coordinate system, and let $\mathbf{L}(t)$ be the external torque on the Earth. Then

$$\partial_t \mathbf{H} + \boldsymbol{\omega} \times \mathbf{H} = \mathbf{L}, \tag{1}$$

and **H** has the form

$$\mathbf{H} = \mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{h}, \tag{2}$$

where **I** is the Earth's time-dependent inertia tensor, and

$$\mathbf{h} = \int_{\text{earth}} \rho \mathbf{r} \times \mathbf{v}\, d^3\mathbf{r} \tag{3}$$

is the relative angular momentum, representing the net contribution to **H** of all motion relative to the coordinate system. [The variables $\rho$, **r**, and **v** in (3) are the internal density, particle coordinate, and particle velocity, respectively.]

Let us now remove the arbitrariness of $\boldsymbol{\omega}$ and define it as the time-dependent, mean rotation vector of the mantle, which is the quantity detected by the observations. For this choice of $\boldsymbol{\omega}$, there is no net contribution to **h** from motion in the mantle. There may, however, be contributions to **h** from the core, the atmosphere, or the oceans.

Suppose that in its equilibrium state, the entire Earth is rotating about the $\hat{\mathbf{z}}$ axis with uniform angular velocity $\boldsymbol{\Omega} = \Omega\hat{\mathbf{z}}$. In this state, the Earth is elliptical and has diagonal inertia tensor

$$\mathbf{I}_0 = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix}, \tag{4}$$

where $A$ and $C$ are the Earth's principal moments of inertia and differ by about one part in 300. The Earth's equilibrium angular momentum is then

$$\mathbf{H}_0 = \mathbf{I}_0 \cdot \boldsymbol{\Omega}. \tag{5}$$

Now we do something to perturb the Earth's angular momentum. We can accomplish this by exerting an external torque **L** on the Earth, by changing the Earth's inertia tensor to

$$\mathbf{I} = \mathbf{I}_0 + \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}, \tag{6}$$

or by inducing motion in the fluid portions of the Earth so as to give rise to a relative angular momentum **h**. The result of any of these perturbations is that $\boldsymbol{\omega}$ must change so that (1) and (2) remain satisfied.

Let the perturbed rotation vector for the mantle be

$$\omega = \mathbf{\Omega} + \Omega \mathbf{m} = \mathbf{\Omega} + \Omega \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}. \tag{7}$$

Here, $m_1$ and $m_2$ represent polar motion (two parameters are needed to describe polar motion, since it takes two parameters to define the angular position of an axis passing through the origin), and $m_3$ represents a change in the LOD. (The change in the LOD is $-m_3 2\pi/\Omega$.)

Using (2), (4), (6), and (7) in (1) and ignoring second-order terms in the perturbation gives

$$\partial_t m_3 = -\partial_t \left( \frac{h_3}{\Omega C} + \frac{c_{33}}{C} \right) + \frac{L_3}{\Omega C}, \tag{8}$$

$$\Omega A \partial_t m_+ - i\Omega^2 (C-A) m_+ + (\partial_t + i\Omega)(h_+ + \Omega c_+) = L_+, \tag{9}$$

where $m_+ = m_1 + im_2$, $h_+ = h_1 + h_2$, $c_+ = c_{13} + ic_{23}$, and $L_+ = L_1 + iL_2$. Here (8) and (9) represent equations for the variation in the LOD and for polar motion, respectively. Note that (9) is a complex equation for $m_+$ and so represents two real equations for $m_1$ and $m_2$.

These two equations allow us to find $m_3$ and $m_+$ once we know $L_i$, $h_i$, and $c_{i3}$ for a given excitation process. $L_i$ can be found directly from sufficient knowledge of the excitation. However, finding $h_i$ and $c_{i3}$ is more involved, because these quantities depend not only on the mass redistribution and relative motion associated with the excitation process, but also on $\mathbf{m}$. This is because the incremental centrifugal force caused by a change in rotation deforms the Earth.

It is convenient to include this rotational deformation separately by writing $c_{i3}$ and $h_i$ as sums of terms dependent directly on the excitation process and of terms dependent on the components of $\mathbf{m}$. The former terms will be denoted here by $\overline{c_{i3}}$ and $\overline{h_i}$, and the latter terms are linear in $\mathbf{m}$, to first order, and vanish for a nondeformable Earth.

The effects of the $\mathbf{m}$-dependent terms turn out to be negligible on the $m_3$ equation (8). Thus (8) is approximately valid as written, with $h_3$ and $c_{33}$ replaced by $\overline{h_3}$ and $\overline{c_{33}}$. The resulting equation is easy to integrate for $m_3$, once $L_3$, $\overline{h_3}$, and $\overline{c_{33}}$ are known. The problem of modeling LOD variability, then, reduces to finding the latter quantities from knowledge of the excitation process.

The $\mathbf{m}$-dependent terms *are* important, however, in the polar motion equation (9). After modeling and separating out these terms, transforming to the frequency domain where all time dependence is assumed to be of the form $\exp(i\lambda t)$ (where $\lambda$ is the angular frequency), and assuming that the forcing period is much longer than one day so that $\lambda \ll \Omega$, Equation (9) reduces to

$$m_+ = -\frac{1}{\Omega A_m}\left[\frac{iL_+ + \Omega^2\overline{c_+} + \Omega\overline{h_+}}{\lambda - \lambda_{\mathrm{CW}}}\right], \tag{10}$$

where $\lambda_{\mathrm{CW}}$ is the Chandler wobble eigenfrequency (see the discussion in a later section) given by

$$\lambda_{\mathrm{CW}} = \Omega\left[\frac{C - A - \kappa\Omega^2 a^5/3G}{A_{\mathrm{m}}}\right]. \tag{11}$$

Here $A_{\mathrm{m}}$ is a principal moment of inertia of the mantle, $G$ is Newton's gravitational constant, and $\kappa$ is a dimensionless parameter describing the deformation induced by the incremental centrifugal force and depending on the elastic and anelastic parameters within the Earth.

Equation (10) implies that the frequency spectrum for polar motion should be resonant at $\lambda = \lambda_{\mathrm{CW}}$, about one cycle per 14 months. By using observations to estimate the resonance frequency, $\kappa$ can be determined. In fact, the resonant frequency can be estimated without detailed knowledge of $L_+$, $\overline{h_+}$, or $\overline{c_+}$. If these quantities are reasonably independent of frequency near $\lambda = \lambda_{\mathrm{CW}}$, then $\lambda_{\mathrm{CW}}$ can be determined directly from the observed frequency dependence of polar motion near $\lambda_{\mathrm{CW}}$.

# INTERPRETATION

## *Tidal Friction*

Most of the observed linear trend in the LOD is due to gravitational tides in the Earth and oceans caused by the Moon and Sun. The Moon, for example, deforms the Earth and oceans into the ellipsoidal shape shown greatly exaggerated in Figure 4. The orientation of the ellipsoidal bulge is fixed with respect to the Moon, while the Earth rotates at 1 cycle day$^{-1}$ relative to that bulge. The resulting lunar tides are time dependent, with frequencies equal to integral multiples of 1 cycle day$^{-1}$, modulated by the frequencies of the lunar orbit, such as 1 cycle per 27.7 days and 1 cycle per 13.7 days.

If there were no energy dissipation in the Earth and oceans, the ellipsoidal tidal bulge shown in Figure 4 would be oriented exactly toward the Moon. However, since there is some dissipation, the Earth and oceans take a short time to fully respond to the Moon's gravitational force. The maximum tidal uplift occurs shortly after the Moon is overhead, and the bulge leads the Earth-Moon vector by a small angle $\delta$, shown greatly

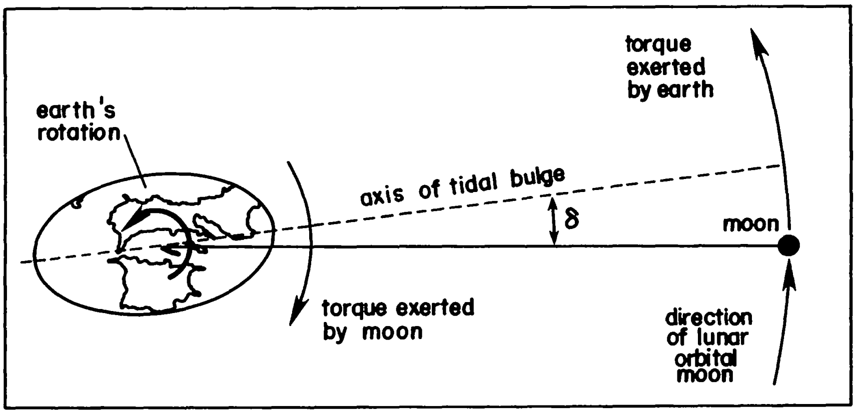


*Figure 4* The gravitational force from the Moon deforms the Earth as shown here (greatly exaggerated), producing tides in the solid Earth and oceans. Because of energy dissipation in the oceans, the tidal bulge leads the Earth-Moon vector by the small angle $\delta$. The Moon's gravitational force acts on the bulge to produce a clockwise torque on the Earth, and so to increase the LOD. The bulge causes a counterclockwise torque on the Moon, leading to an increase in the Moon's orbital period.

exaggerated in Figure 4. The Moon's gravitational force acts on the tidal bulge to produce a clockwise torque on the Earth [a time-independent $L_3$ in Equation (8)], opposite to its rotation. The result is a steady decrease in the rotation rate, and thus an increase in the LOD. There is a similar, although somewhat smaller, effect from the Sun.

Most of the tidal energy dissipation is believed to occur in the oceans. Frictional effects are much more important there than in the solid Earth. It is still not entirely clear, though, whether most of the dissipation occurs in shallow seas or in the deep ocean, or what the dominant frictional mechanisms are.

The lag angle $\delta$ can be determined independently of observations of the LOD by ranging to satellites such as LAGEOS. The tidal bulge perturbs the orbit of a satellite, and so $\delta$ can be found by solving for the orbit. When the satellite results for $\delta$ are used to predict the lunar torque on the Earth, the expected increase in the LOD is about 25% larger than that implied by the historical eclipse record (see, for example, Goad & Douglas 1978, Cazenave & Daillet 1981).

There is other evidence tending to confirm the satellite results. The Earth's tidal bulge acts gravitationally on the Moon, causing a counterclockwise torque on the Moon in its orbit about the Earth (see Figure 4). The torque is in the direction of the Moon's motion, and so it tends to increase the angular momentum of the Moon. The rate of increase of lunar orbital angular momentum must equal the rate of decrease of the Earth's rotational angular momentum.

The increase in lunar angular momentum causes the Moon to move farther away from the Earth and to increase its orbital period. This increase in period has been determined accurately from LLR data (Williams et al 1978), and the results predict a decrease in the Earth's rotational angular momentum consistent with the satellite estimates of $\delta$ as described above but inconsistent with the astronomical results. The likely explanation for the discrepancy is discussed in the next section.

The observed effects on the lunar orbit are surprisingly large. When the dissipation rate inferred from the LLR and satellite ranging results are used in models to extrapolate the present lunar orbit backward in time, the Moon is predicted to have been so close to the Earth 1.5 Gyr ago that it would have been torn apart by gravitational forces from the Earth. The Moon, though, is known to be over 4 Gyr old.

The implication is that tidal friction in the oceans is larger now than it has been over most of the Earth's history. The dissipation is sensitive to the shape of the ocean basins and to the rotation rate itself. Ocean basins, for example, have changed drastically over geological time as a result of continental drift. Whether these effects are large enough to sufficiently affect the oceanic dissipation is currently receiving attention (see, for example, Brosche & Sundermann 1982).

### *Postglacial Rebound*

LOD The 25% discrepancy between the historical astronomical evidence for the increase in the LOD and the LLR and satellite ranging results for the effects of tidal dissipation implies that some other mechanism is tending to decrease the LOD, thus partially offsetting the effects of tidal friction. This acceleration of the Earth is probably caused by the effects of the last ice age. When the ice over northern Canada and Scandinavia melted several thousand years ago, it left deep depressions now filled by Hudson's Bay and the Baltic Sea. The Earth behaves as a viscous fluid over long time periods, and the depressed areas are slowly uplifting as material deep within the Earth flows horizontally. There is thus a net transfer of material within the Earth toward higher northern latitudes. This decreases the Earth's polar moment of inertia [$c_{33}$ in (8)] and so increases the rotation rate.

This interpretation has recently been independently confirmed using LAGEOS ranging data. The changing internal mass distribution leads to a change in the Earth's gravitational field, which affects the LAGEOS orbit. By solving for the orbit, the change in the moment of inertia can be determined (Yoder et al 1983, Rubincam 1984). The results are consistent with the additional linear decrease in the LOD inferred from the ancient historical record.

The linear decrease in the Earth's moment of inertia depends on the rate at which material is flowing inside the Earth, which in turn depends on the viscosity of the Earth. In fact, the observed linear change in the moment of inertia has been used to place tight bounds on the viscosity of the Earth's lower mantle (see, for example, Peltier 1983).

POLAR MOTION Postglacial rebound may also be responsible for the linear drift of the pole suggested by the astronomical polar motion data taken over the last century. The horizontal readjustment of material within the Earth causes a steady drift of the Earth's figure axis [represented by $\overline{c_+}$ in (10)] relative to the Earth's surface. To conserve angular momentum, the mean position of the Earth's rotation axis (represented by $m_+$) remains coincident with the figure axis, and so the pole also drifts. The rate of drift implied by the astronomical data has been used as an additional constraint on mantle viscosity (Yuen et al 1983).

## *Decade Fluctuations*

LOD The decade fluctuations in the LOD are believed to be due to the transfer of angular momentum between the fluid core and the solid mantle. When the mantle gains angular momentum its rotation rate increases, and so the observed LOD decreases. This variability of $m_3$ can be computed from (8) either by estimating $h_3$ from assumptions about core flow or by estimating the torques $L_3$ responsible for the exchange of angular momentum.

At least two viable mechanisms have been proposed to explain the required torques. One is electromagnetic forcing. The Earth's magnetic field is caused by electric currents in the core. If these currents change with time, there will be changes in the magnetic field and so, by Faraday's Law, an electric field will be produced everywhere, including in the lower mantle. Since the lower mantle is an electrical conductor, the induced electric field gives rise to electric currents in the mantle that interact with the large, time-independent components of the magnetic field through the Lorentz force. The result is, in general, a net torque on the mantle and a resulting change in rotation. Stix & Roberts (1984) found that the electromagnetic torque is probably the right order to explain the observed LOD variability.

An alternative mechanism is topographic coupling, caused by fluid pressure acting against topography at the core-mantle boundary. This idea was first proposed by Hide (1969), but at that time it was not possible to meaningfully estimate the strength of the coupling. Recent developments, however, have now made such estimates feasible. Speith et al (1986) used models for fluid velocities at the top of the core from Voorhies (1986), together with the assumption of geostrophy, to estimate lateral variations in pressure at the core-mantle boundary. They combined their estimates with seismic-related results for the shape of the boundary to estimate the zonal topographic torque on the mantle from the core. Their results are the right order to explain the observed decade fluctuations. (In fact, they are several times too large.)

Whatever the nature of the torque, the assumption that the decade fluctuations are due to core flow suggests that there ought to be some correlation between the observed LOD variability and the observed time dependence in the Earth's magnetic field. It has proven difficult to find such a correlation because the magnetic field variations are attenuated as they travel upward through the conducting mantle. In fact, when correlations have been identified, the time lag between the changes in the LOD and in the observed magnetic field has been used to help constrain the mantle's electrical conductivity (see, for example, Backus 1983).

POLAR MOTION The one conceivable decade-scale variation in polar motion is the 30-yr oscillation suggested by the astronomical polar motion record. It is not clear what could cause such a variation. It is even possible that this motion might simply reflect poorly modeled local deformation at the telescopes, rather than an actual variation in the position of the rotation pole. The problem, at the moment, is far from resolved.

### *Short-Period Fluctuations*

LOD The observed short-period variability in the LOD includes fluctuations at monthly and fortnightly periods caused by the lunar tides. Figure 5 is another view of the tidal bulge in the Earth, again greatly exaggerated. As the Moon orbits the Earth, the bulge remains continually

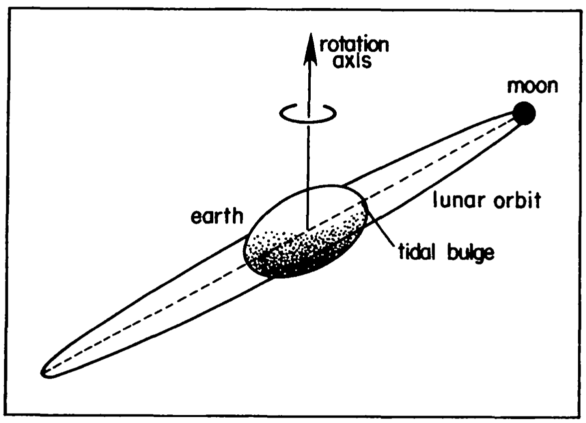


*Figure 5* The tidal bulge as seen from the equator. The bulge follows the Moon in its orbit about the Earth, causing fluctuations in the Earth's polar moment of inertia at the lunar orbital periods of 27.7 and 13.7 days. Because of conservation of angular momentum, there are corresponding oscillations in the LOD.

oriented along the Earth-Moon vector. When the Moon is at high declinations, as it is in Figure 5, the Earth's polar moment of inertia [$c_{33}$ in (8)] is decreased slightly as a result of the shift of the bulge away from the equator, and the Earth rotates more quickly. When the Moon lies in the Earth's equatorial plane, the tidal bulge is also in that plane, the polar moment of inertia increases slightly, and the rotation rate decreases. There are also variations in the size of the bulge, and so in the rotation rate, as the distance to the Moon changes during the lunar orbit.

The result is variability in the LOD at the various periods that characterize the Moon's orbit. The largest of these LOD fluctuations occur at 27.7 and 13.7 days, since these are the principal orbital periods. The amplitudes of the fluctuations depend on the Earth's material properties. For example, if there was large energy dissipation within the solid mantle at these periods, the mantle would behave more like a fluid than it would for small dissipation rates, and so the amplitudes of $c_{33}$ and of the LOD variability would be larger: A fluid is more easily deformed than a solid. Dissipation would also introduce a phase lag between the LOD fluctuations and the motion of the Moon. These effects, particularly the increase in amplitude, have been used to help learn about the anelastic behavior of the mantle at these periods (Merriam 1984).

The remaining short-period fluctuations in the LOD consist of large 6- and 12-month periodic terms and smaller, more irregular variations at other periods. Tidal deformation from the Sun is responsible for about 10% of the annual and about 50% of the semiannual variability through the same mechanisms as for the fortnightly and monthly oscillations. There are also small seasonal contributions (less than 5%) from the effects of ocean currents, mostly from the circumpolar current around Antarctica, and probably near-negligible contributions from seasonal changes in ground-water storage.

Instead, most of the annual and semiannual variability is caused by seasonal forcing from the atmosphere, particularly by the seasonal exchange of angular momentum between the solid Earth and atmospheric winds (Lambeck & Hopgood 1981). For exmaple, when the winds increase in strength from west to east, the Earth slows down. This exchange of angular momentum is accomplished by a combination of surface friction torques (due to viscous drag as the winds blow over the surface) and mountain torques (caused by higher pressure on one side of a topographic feature than on the other). These two torques contribute about equally to the coupling (Wahr & Oort 1984).

Seasonal variations in global atmospheric pressure are less important than winds, but they do contribute about 10% of the observed semiannual and annual LOD variability. A change in the atmospheric pressure at a given point implies a change in the amount of atmospheric mass vertically above that point. Pressure data thus reflect the mass distribution in the atmosphere and can be used to determine the atmosphere's polar moment of inertia. Because of conservation of angular momentum, seasonal variations in the moment of inertia [$c_{33}(t)$] are accompanied by fluctuations in the LOD.

The total effects of the atmosphere can be accurately estimated using global wind and pressure data to find $h_3(t)$ and $c_{33}(t)$, respectively, and then applying these values in (8). These effects are compared in Figure 2 with LOD results for 1982–86. (The effects of tides have been removed from the LOD results.) The agreement is remarkable (see also Barnes et al 1983, Rosen & Salstein 1983, Dickey et al 1986). In fact, the atmosphere appears to be responsible for most of the nonseasonal variations as well. For example, the large maximum during the winter of 1982–83 is probably associated with the extreme El Niño event in the southern Pacific, which has often been blamed for the unusual weather patterns occurring around the globe during that period.

There is also evidence in both data sets of a 50-day oscillation (Langley et al 1981). It is not certain why such an oscillation should exist, but the good agreement between the two data sets suggests that the term is probably real. Meteorologists are presently trying to understand its origin.

POLAR MOTION The annual wobble evident in polar motion data is mostly due to the effects of annual redistribution of mass within the atmosphere (see, for example, Merriam 1982, Wahr 1983). This causes a perturbation in the inertia tensor of the atmosphere [$\overline{c_+}$ in (10)], which leads to a shift in the position of the rotation pole ($m_+$). In fact, most of the observed annual wobble is related to the large seasonal atmospheric pressure variation over central Asia: high pressure in winter and low pressure in summer.

Ground-water storage is also important in exciting the annual wobble. The Earth's inertia tensor, and so the position of the rotation pole, are affected by seasonal variations in the amount of water in snow and ice, in the water table, in rivers and lakes, and in the oceans. The effects of water storage are roughly 25% of the effects of atmospheric pressure (Van Hylckama 1970, Hinnov & Wilson 1987). The effects of wind and ocean currents are believed to be negligible (Wahr 1983).

The 14-month Chandler wobble is a free motion of the Earth. An analytical expression for its frequency is given above in Equation (11). The motion is analogous to the free nutation of a top. If the figure axis of a rapidly spinning top is initially displaced slightly from the rotation axis, the figure axis will proceed to move along a conical path about the rotation axis. The frequency of the motion depends on how nonspherical the top is, an effect evident in (11) through the $C-A$ term in the numerator.

The Earth's nonrigidity is responsible for the $\kappa$ term in (11), which lengthens the period of the Chandler wobble by about 4 months, from the 10-month period expected for a rigid Earth to the observed 14 months. In fact, observations of the Chandler wobble period and decay rate (after initial excitation, the motion damps out in several decades) have been used to solve for $\kappa$ and thus to constrain the value of mantle anelasticity pertinent to a 14-month period (Smith & Dahlen 1981).

Thus, the period and decay rate of the Chandler wobble are now well understood on theoretical grounds. The primary excitation source, however, has not yet been identified. The problem is to find a mechanism that can produce a large enough offset between the figure and rotation axes to excite the Chandler wobble to observed levels. Fluctuations in atmospheric pressure probably provide only about 25% of the necessary power (Wilson & Haubrich 1976, Wahr 1983). Other effects, including perturbations in the inertia tensor due to earthquakes, appear to be even less important (Dahlen 1973). A recent, intriguing hypothesis is that the excitation may be due to fluid pressure at the top of the core, acting against the elliptical core-mantle boundary to produce a torque on the mantle (Gire & Le Mouel 1986). Still, not much more is known now about the excitation source than was known in Chandler's time, nearly 100 years ago.

## *Nutations*

As described in the introduction, the Earth's nutational motion is caused by the gravitational attraction of the Sun and Moon. The motion can be separated into a discrete sum of periodic terms with frequencies, as seen from the diurnally rotating Earth, of 1 cycle day$^{-1}$ modulated by the lunar and solar orbital frequencies.

The Earth is believed to have a rotational normal mode, called the free core nutation (FCN), with an eigenfrequency within the diurnal band of nutation frequencies. To understand the dynamics of the FCN, suppose the fluid core and solid mantle were tipped about an equatorial axis in opposite directions and then released. If the core's internal density distribution and the core-mantle boundary were spherical, there would be no restoring torque, and the core and mantle would remain tipped relative to each other. But, because the real core/mantle boundary is elliptical and the Earth is elliptically stratified, there are, instead, restoring pressure and gravitational torques between the core and mantle. As a result, the core and mantle execute periodic twisting motion with respect to each other.

This motion is the FCN, and its frequency is 1 cycle day$^{-1}$ plus a factor dependent on the strength of the restoring torque.

Sasao et al (1980) showed that for a hydrostatically prestressed Earth, the FCN eigenfrequency has the form

$$\lambda_{\mathrm{FCN}} = \omega\left[1+\frac{A}{A_{\mathrm{m}}}(e_{\mathrm{f}}-\beta)\right], \tag{12}$$

where $e_{\mathrm{f}} \cong (C_{\mathrm{f}}-A_{\mathrm{f}})/A_{\mathrm{f}}$ is the dynamical ellipticity of the core, with $C_{\mathrm{f}}$ and $A_{\mathrm{f}}$ the principal moments of the fluid core about the polar and equatorial axes, respectively; and $\beta$ is a numerical factor that represents the effects of deformation and is effectively independent of any aspherical stratification within the Earth. For a hydrostatically prestressed Earth, $\beta$ is about 25% of $e_{\mathrm{f}}$ (Sasao et al 1980), and $\lambda_{\mathrm{FCN}} \cong (1+1/460)$ cycles day$^{-1}$ (Wahr 1981).

Wahr (1987) showed that (12) is valid even if the mantle is not hydrostatically prestressed. In this case, the ellipticity of the core-mantle boundary need not equal the hydrostatic value, and in fact the boundary need not be an ellipse at all. Instead, its radius could depart from its mean spherical value with any arbitrary latitudinal and longitudinal dependence, so long as this departure is small.

As it happens, though, the quantity $e_{\mathrm{f}}$ in (12) depends only on the $Y_2^0$ spherical harmonic term of the core internal density field and on the $Y_2^0$ component of the shape of the core-mantle boundary. It does not depend on any other spherical harmonic component of the structure. Thus, observational results for $\lambda_{\mathrm{FCN}}$ could yield information on coefficients of this one spherical harmonic.

The free nutational motion at the period $\lambda_{\mathrm{FCN}}$ has not yet been clearly observed. Evidently, it is not excited sufficiently by any internal process. However, because $\lambda_{\mathrm{FCN}}$ is so close to the frequencies of the lunar-solar nutations, the amplitudes of these forced nutations are affected by the presence of the mode by up to 20 or 30 mas, a perturbation that can be readily detected even by conventional optical observations of nutation. These effects of the FCN are included in the standard forced nutation model adopted by the International Astronomical Union (IAU), which is based on the rigid Earth values of Kinoshita (1977) and corrections for nonrigidity from Wahr (1981), the latter assuming a hydrostatically prestressed Earth.

Herring et al (1986) found that VLBI results for the forced nutations disagree with the IAU adopted theory by almost 2 mas, particularly at the annual frequency of $\lambda = \Omega(1+1/365.25)$ cycles day$^{-1}$. This is the forcing frequency closest to $\lambda_{\mathrm{FCN}}$, and it suggests that the FCN frequency may be somewhat larger than expected, close to $\lambda_{\mathrm{FCN}} = \Omega(1+1/435)$ cycles day$^{-1}$ (Gwinn et al 1986).

Equation (12) suggests that $\lambda_{FCN}$ can be modified by changing either $\beta$ or $e_f$. Wahr & Bergen (1986) (see also Dehant 1987) considered the effects of mantle anelasticity on $\beta$ and concluded that the corresponding perturbation of $\lambda_{FCN}$ is too small and, more importantly, has the wrong sign. Although the effects of anelasticity are, in principle, observable using the current VLBI observations, the larger discrepancy between observation and theory must first be resolved before the VLBI results can be used to study diurnal anelasticity.

Probably the most likely explanation for the discrepancy is, instead, uncertainty in $e_f$, as postulated by Gwinn et al (1986). Recent results based on seismic tomography suggest that the shape of the core-mantle boundary may diverge appreciably from hydrostatic equilibrium. It is not straightforward to compute the effects on $e_f$ given only the shape of the core-mantle boundary. A perturbed boundary and an aspherical density distribution in the mantle will cause perturbations in the internl density surfaces in the core, and these will affect $e_f$. However, current models of the boundary based on seismic data suggest values for the nonhydrostatic portion of $e_f$ that are as large or larger than the result inferred from the nutation observations. In fact, the VLBI nutation observations are proving to be a valuable independent constraint on the seismic models.

## SUMMARY

There is now at least some general understanding of what causes most of the various observed fluctuations in rotation. Some aspects of the subject are understood very well. For others, there are many missing details. The hope is that in the process of filling in these details, we will learn more about the Earth and its environment.

In this review we have given some representative examples of ongoing research. One geophysical goal mentioned several times above is to better constrain the values of mantle anelasticity. There are, of course, other ways to study anelasticity. Particularly useful are observations of the attenuation of seismic waves traveling through the Earth following earthquakes. These observations, however, only tell us about dissipation at very short periods, from seconds to minutes. The frictional mechanisms responsible for dissipation are probably different at different time scales and are not clearly understood in any case. Rotational observations offer a unique opportunity to see the effects of anelasticity at much longer time periods.

There are, in fact, many opportunities, only some of which are described above, to use rotation data to learn about the solid Earth. But what about the fluid portions of the Earth? The oceans and, especially, the atmosphere have large effects on the rotation. Can the data be used to learn about them as well?

The answer is yes, but probably only in a limited sense. As an example, an unexplained variation in rotation could be a clue that there are some unknown processes in the atmosphere or oceans that should be studied further. Or, the rotation data could help confirm the reliability of certain meteorological or oceanographic results either as inferred directly from atmospheric or oceanic data or as deduced from numerical or analytical models. For instance, the agreement between the short-period LOD data and meteorological data has convinced meteorologists that the 50-day oscillation apparent in the atmospheric data is probably real and deserves attention. It is unlikely that the rotation data can help constrain many details of an atmospheric or oceanic disturbance. What they can do is to suggest or confirm that a disturbance exists.

Still, it is not clear just what the future holds. Rotation results have improved dramatically over the last few years and have already provided much new valuable information. As more high-quality data become available and as the techniques further improve, we should be able to resolve many of the long-standing problems in the field and, if we are lucky, discover new ones.

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