Astron. Astrophys. 196, 327–337 (1988)

ASTRONOMY AND ASTROPHYSICS

Synchrotron radiation in a thermal plasma with large-scale random magnetic fields

A. Crusius and R. Schlickeiser

Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-5300 Bonn 1, Federal Republic of Germany

Received January 9, accepted October 27, 1987

Summary. The synchrotron emission from relativistic electrons in a thermal plasma with large-scale random magnetic fields is considered. In this case, the spectral synchrotron power of a single electron can be given in closed form allowing exact analytical expressions for the synchrotron emissivity, absorption coefficient, intensity and total energy loss of particles to be derived. The influence of various physical parameters such as gas density, magnetic field strength, particle's Lorentz factor on the resulting emissivities, intensities and energy loss is discussed in detail. Below the Razin-Tsytovich frequency $\nu_R = 20$ Hz $(n_e/1\ \mathrm{cm}^{-3})$ $(\mathrm{B}/1\ \mathrm{Gauss})^{-1}$, the spectral appearance of synchrotron radiation both in the optically thin and thick case is quite different from the vacuum behaviour. Since in quasar broad line regions, ν_R is of the order of 10^{11} Hz the suppression of synchrotron radiation may explain why most quasars are radio quiet. Likewise, the necessary physical conditions for the occurrence of synchrotron masering in the optically thick case are given. We obtain optical depth $|\tau| \gg 1$ for compact non-thermal sources. The total energy loss of a single particle is shown to be exponentially reduced at Lorentz factors less than $\gamma_R = 2.1 \cdot 10^{-3}$ $(n_e/1\ \mathrm{cm}^{-3})^{1/2}$ $(\mathrm{B}/1\ \mathrm{Gauss})^{-1}$.

Key words: radiation processes – plasma – synchrotron radiation – relativistic electrons – quasar – maser

1. Introduction

The non-thermal continuum emission from radio to optical frequencies of powerful extragalactic sources is commonly interpreted as synchrotron radiation of relativistic electrons in the magnetic fields of these sources. Although present, the measured degree of linear polarization is much smaller than the value theoretically expected for a completely uniform magnetic field configuration (Angel and Stockman, 1980, Miley, 1980; Kellermann and Pauliny-Toth, 1981). This usually (e.g. Moffet, 1975) is interpreted by a small degree of "large-scale" homogeneity of the magnetic field, where "large-scale" means that the field line directions are nearly randomly distributed on scales small compared to the size of the cosmic source (typically $\sim 10^{17}$ cm for active galactic nuclei) but large compared to the Larmor radii of the radiating electrons ($r_L \simeq 10^6$ cm $(\gamma/10^3)(\mathrm{B}/1\mathrm{G})^{-1}$, γ: Lorentz factor of the electron), so that the synchrotron formulae are still applicable.

On the other hand, the magnetic fields are frozen in the highly conducting thermal plasma in these sources. The detailed analysis of ultraviolet and optical spectra from quasars (for 3C273, see Ulrich et al., 1980) has revealed the presence of dense ($n_e \simeq 10^7 - 10^9\ \mathrm{cm}^{-3}$) warm ($T \simeq 10^4 - 10^{4.7}$ K) partly ionized gas (clouds), probably in pressure equilibrium with hotter ($T \simeq 10^7$ K, $n_e \simeq 10^4 - 10^6\ \mathrm{cm}^{-3}$) completely ionized gas responsible for the X-ray emission of these objects (intercloud region). For the understanding of the underlying physical processes operating in these sources, it is highly desirable to investigate in detail the influence of the thermal plasma on the synchrotron emission of relativistic electrons. In the present paper, we study the extreme case of synchrotron radiation in a thermal plasma with completely random magnetic fields, since in this case the spectral synchrotron power of a single electron can be given in closed form (Crusius and Schlickeiser, 1986), allowing exact analytical expressions for the emissivity, absorption coefficient, intensity, and total energy loss of particles due to synchrotron radiation.

Our work generalizes earlier discussions by McCray (1966) and Zheleznyakov (1966) of the possibility of maser action in cosmic radio sources, who have used asymptotic approximations of the synchrotron absorption coefficient. It supplements the recent numerical study of Cawthorne (1985), who considered synchrotron masering in a tangled web of magnetic fields confined to the plane of the sky. Our analytical results allow a detailed discussion of the influence of various parameters and the behaviour of synchrotron emissivity, absorption coefficient, intensity and energy loss of particles.

The organization of the paper is as follows: In Sect. 2, we recall the theory of synchrotron emission in a plasma. The spectral synchrotron power in a thermal plasma with large-scale random magnetic fields is presented in Sect. 3. In Sects. 4 and 5 we calculate the emission and absorption coefficient for the special case of monoenergetic relativistic electrons, respectively, and discuss the influence of the various physical parameters. Section 6 contains a discussion of the necessary physical conditions for synchrotron masering in a thermal plasma with large scale random magnetic fields and monoenergetic relativistic electrons. In Sect. 7 we study the resulting synchtrotron intensity in a homogeneous slab geometry in this case. The mean energy loss of a single electron due to spontaneously emitted synchrotron radiation in a thermal plasma with large-scale random magnetic fields is calculated in Sect. 8. A summary and discussion of the results completes the paper.

Send offprint requests to: A. Crusius

2. Synchrotron emission in a plasma

Throughout this work we consider the synchrotron emission at frequencies ν which are large compared to the plasma frequency ν_p and the non-relativistic gyrofrequency $\nu_0 = eB/(2\pi mc)$ of the thermal electrons in the magnetic field of magnitude B, i.e.

$$\nu \gg \nu_p = 9 \cdot 10^3 \, (n_e/1 \text{ cm}^{-3})^{1/2} \text{ Hz} \tag{1}$$

and

$$\nu \gg \nu_0 = 2.8 \cdot 10^6 \, (B/\text{Gauss}) \text{ Hz} \tag{2}$$

Restriction (2) is always fulfilled since the typical frequency emitted by the relativistic electrons with Lorentz factor γ is given by $\nu_c = 3/2 \, \nu_e \gamma^2$, with $\nu_e = \nu_0 \sin\theta$, where θ denotes the angle between the velocity $\vec{v}$ of the radiating electron and the magnetic field $\vec{B}$. Under these conditions the refractive index n at frequency ν reads

$$n = \left(1 - \frac{\nu_p^2}{\nu^2}\right)^{1/2} \tag{3}$$

The influence of the thermal plasma on the synchrotron emission enters via this refractive index.

The total (i.e. summed over both polarization modes) spontaneously emitted spectral synchrotron power of a single electron in a homogeneous magnetic field is then given by (see Zheleznyakov, 1966; Yukon, 1968; Melrose, 1980)

$$p_h(\nu, \theta, \gamma) = \frac{q_0 \nu}{\gamma^2} \left[1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right] \int_{x/\sin\theta}^{\infty} dy \, K_{5/3}(y) \tag{4}$$

with

$$q_0 = \frac{4e^2\pi}{\sqrt{3}c}, \quad x = \frac{2\nu}{3\nu_0 \gamma^2} \left[1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right]^{3/2} \tag{5}$$

3. Synchrotron power in a thermal plasma with large-scale random magnetic fields

In a completely large-scale random magnetic field, the total spontaneously emitted spectral synchrotron power from a single electron is calculated by averaging $p_h(\nu, \theta, \gamma)$ over all possible values of the polar (θ) and azimuthal (ϕ) angles:

$$p_r(\nu, \gamma) = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \, p_h(\nu, \theta, \gamma) = \frac{1}{2} \int_0^{\pi} d\theta \sin\theta \, p_h(\nu, \theta, \gamma)$$

$$= \frac{q_0 \nu}{2\gamma^2} \left[1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right] \int_0^{\pi} d\theta \sin\theta \int_{x/\sin\theta}^{\infty} dy \, K_{5/3}(y). \tag{6}$$

Recently we have proven the identity (Crusius and Schlickeiser, 1986)

$$\int_0^{\pi} d\theta \sin\theta \int_{x/\sin\theta}^{\infty} dy \, K_{5/3}(y) = \pi CS(x) \tag{7}$$

with

$$CS(x) \equiv W_{0,\frac{4}{3}}(x) \, W_{0,\frac{1}{3}}(x) - W_{\frac{1}{2},\frac{5}{6}}(x) \, W_{-\frac{1}{2},\frac{5}{6}}(x), \tag{8}$$

where $W_{\lambda,\mu}(x)$ denotes Whittaker's function.

The properties of $CS(x)$ and its derivative $dCS(x)/dx$ together with their asymptotic expansions for small and large x are summarized in the Appendix. With Eq. (7) used in Eq. (6) we obtain

$$p_r(\nu, \gamma) = \frac{q_0 \pi \nu}{2\gamma^2} \left[1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right] CS\left[\frac{2\nu}{3\nu_0 \gamma^2} \left(1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right)^{3/2}\right] \text{erg s}^{-1} \text{ Hz}^{-1}. \tag{9}$$

For a given electron energy distribution function $N(E)$, the synchrotron emission coefficient $\varepsilon_r(\nu)$ and the synchrotron absorption coefficient $\mu_r(\nu)$ in large-scale random magnetic fields can be calculated as (e.g., Bekefi, 1966)

$$\varepsilon_r(\nu) = \frac{1}{4\pi} \int_0^{\infty} dE \, N(E) \, P_r(\nu, \gamma) \text{ erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1} \text{ ster}^{-1}, \tag{10}$$

$$\mu_r(\nu) = -\frac{c^2}{8\pi\nu^2} \int_0^{\infty} dE \left(\frac{d}{dE} \frac{N(E)}{E^2}\right) E^2 \, P_r(\nu, \gamma) \text{ cm}^{-1}, \tag{11}$$

respectively.

In case of a homogeneous source slab of thickness L, the emitted intensity is

$$I_r(\nu) = \frac{\varepsilon_r(\nu)}{\mu_r(\nu)} [1 - \exp(-\mu_r(\nu) L)]$$

$$\simeq \begin{cases} \varepsilon_r(\nu) L & \text{for } \mu_r(\nu) L \ll 1 \\ \dfrac{\varepsilon_r(\nu)}{\mu_r(\nu)} & \text{for } \mu_r(\nu) L \gg 1 \text{ and } \mu_r(\nu) > 0 \\ \dfrac{\varepsilon_r(\nu)}{|\mu_r(\nu)|} \exp(|\mu_r(\nu)| \, L) & \text{for } |\mu_r(\nu)| L \gg 1 \text{ and } \mu_r(\nu) < 0 \end{cases} \tag{12}$$

and can be determined straightforwardly from the respective expressions for $\varepsilon_r(\nu)$ and $\mu_r(\nu)$. In writing (12) as the solution of the radiation transport equation, it is assumed that the local intensity in the source is isotropic.

4. Synchrotron emission coefficient in a thermal plasma with large-scale random magnetic fields for monoenergetic electrons

In the following we discuss in detail the synchrotron emission coefficient for a monoenergetic distribution of the relativistic electrons,

$$N(E) \, dE = n(\gamma) \, d\gamma = N_0 \, \delta(\gamma - \gamma_0) \, d\gamma. \tag{13}$$

We choose this particular distribution function mainly for two reasons: (i) results for more complex distribution functions can be obtained by simple superposition of the expressions derived for the monoenergetic distribution presented here; (ii) it has been pointed out by one of us (Schlickeiser, 1984) that in active galactic nuclei a natural mechanism exists to accelerate relativistic electrons to almost monoenergetic energies, namely the combined influence of diffusive shock wave acceleration and radiation losses of particles in cases where the confinement time of particles is much longer than the acceleration time.

With Eq. (13) used in Eq. (10) we find

$$\varepsilon_r(\nu) = \frac{q_0 N_0 \nu}{8\gamma_0^2} \left[1 + \left(\frac{\gamma_0 \gamma_p}{\nu}\right)^2\right] CS\left[\frac{2\nu}{3\nu_0 \gamma_0^2} \left\{1 + \left(\frac{\gamma_0 \nu_p}{\nu}\right)^2\right\}^{3/2}\right]. \tag{14}$$

It is convenient to introduce the dimensionless frequency

$$f = \nu/(\gamma_0 \nu_p) = \frac{\nu}{9 \cdot 10^3 \text{ Hz} \, \gamma_0 \, (n_e/1\text{cm}^{-3})^{1/2}} \tag{15}$$

and the parameter

$$g_0 = \frac{3}{2}(\nu_0/\nu_p)\,\gamma_0 = 468\,\gamma_0\,(B/1\,\text{Gauss})\,(n_e/1\,\text{cm}^{-3})^{-1/2}. \quad (16)$$

In terms of these two quantities, Eq. (14) reads

$$\varepsilon_r(f) = b_1 f(1+f^{-2})\,CS\left[\frac{f}{g_0}(1+f^{-2})^{3/2}\right], \quad (17)$$

with

$$b_1 = \frac{q_0 N_0 \nu_p}{8\gamma_0}. \quad (18)$$

We are interested in the behaviour of $\varepsilon_r(f)$ for small frequencies ($f \ll 1$) and large frequencies ($f \gg 1$), as well as the influence of the parameter g_0.

4.1. *Large frequencies, $f \gg 1$*

For large frequencies, $f \gg 1$, (17) reduces to

$$\varepsilon_r(f) \simeq b_1 f\, CS(f/g_0). \quad (19)$$

With the asymptotic expansion (A3) of $CS(x)$,

$$CS(x) \simeq \begin{cases} a_0 x^{-2/3} & \text{for } x \ll 1 \quad (20a) \\ x^{-1} e^{-x} & \text{for } x \gg 1, \quad (20b) \end{cases}$$

$$a_0 = 2^{4/3}\,\Gamma^2(1/3)/(5\pi) = 1.15128, \quad (20c)$$

we may derive the behaviour of (19) for values of f small and large compared to g_0.

In case $g_0 \gg 1$ we obtain

$$\varepsilon_r(f) \simeq b_1 \begin{cases} a_0 g_0 (f/g_0)^{1/3} & \text{for } 1 \ll f \ll g_0 \quad (21a) \\ g_0 e^{-f/g_0} & \text{for } 1 \ll g_0 \ll f, \quad (21b) \end{cases}$$

which agrees with the behaviour in vacuum. In fact, if we introduce the definitions (15) and (16) we find $f/g_0 = \nu/\nu_c$ with $\nu_c = 3/2\,\nu_0\,\gamma^2$. This means that the influence of the plasma is negligible for frequencies $\nu \gg \gamma_0 \nu_p$. For values $g_0 \ll 1$, we find for all $f > 1$ that

$$\varepsilon_r(f) \simeq b_1\, g_0 \exp(-f/g_0). \quad (22)$$

4.2. *Small frequencies, $f \ll 1$*

For small frequencies, $f \ll 1$, (17) reduces to

$$\varepsilon_r(f) \simeq \frac{b_1}{f}\, CS\left(\frac{1}{g_0 f^2}\right) \quad (23)$$

In case $g_0 \gg 1$ we obtain

$$\varepsilon_r(f) \simeq b_1 \begin{cases} g_0 f \exp\left(\frac{1}{g f^2}\right) & \text{for } f \ll g_0^{-1/2} \ll 1 \quad (24a) \\ a_0 g_0 (f/g_0)^{1/3} & \text{for } g_0^{-1/2} \ll f \ll 1. \quad (24b) \end{cases}$$

So the vacuum behaviour (21) continues down to frequencies $f_v \simeq g_0^{-1/2}$, corresponding to (compare with Melrose, 1980, 4.129b)

$$\nu_v = \left(\frac{2\gamma_0 \nu_p^3}{3\nu_0}\right)^{1/2} = 420\,\gamma_0^{1/2}\, n_e^{3/4}\, B^{-1/2}\ \text{Hz} \quad (25)$$

For values $g_0 \ll 1$, we find for all $f < 1$,

$$\varepsilon_r(f) \simeq b_1\, g_0 f \exp\left(\frac{-1}{g_0 f^2}\right), \quad (26)$$

i.e. in case of $g_0 \ll 1$, from (22) and (26) find the emission to be suppressed in the whole frequency-range.

Figure 1 summarizes the behaviour of the emission coefficient $\varepsilon_r(f)$ for various values of the parameter g_0. In case of $g_0 \gg 1$ it can be seen that the vacuum behaviour extends down to the frequency $f_v \approx g_0^{-1/2}$, with $\varepsilon_r(f) \propto f^{1/3}$ for $f \lesssim g_0$ and $\varepsilon_r(f) \propto \exp(-f/g_0)$ beyond g_0, whereas below f_v the emission is exponentially reduced due to the presence of the thermal medium, $\varepsilon_r(f) \propto f \exp(-1/g_0 f^2)$.

The logarithmic bandwidth of the emission of monoenergetic electrons is therefore

$$\Delta\log(f) \simeq \log(g_0) - \log(g_0^{-1/2}) = 1.5\,\log(g_0). \quad (27)$$

The larger g_0, the larger the logarithmic bandwidth $\Delta\log(f)$, with its characteristic anisotropy around $f = 1$.

In case of $g_0 < 1$ Fig. 1 shows the exponential suppression of emission for all frequencies when compared to the $(g_0 \gg 1)$-case. The emission is very narrow-banded around frequency $f \simeq 1$. This can also be estimated from (27), which yields $\Delta\log(f) = 0$ for $g_0 = 1$.

4.3. *Influence of the relativistic electron Lorentz factor γ_0*

The parameter g_0 in Eq. (16) may be written as

$$g_0 = \frac{\gamma_0}{\gamma_R}, \quad (28)$$

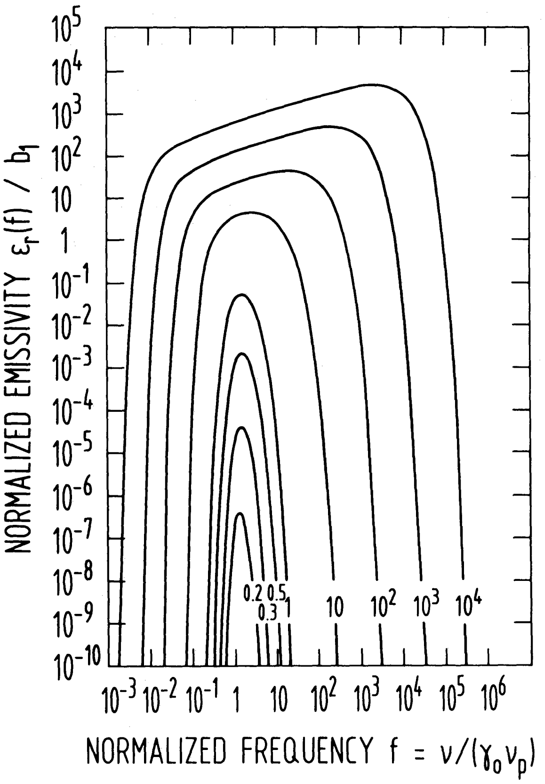

Fig. 1. Emission coefficient ε_r as a function of normalized frequency f for $g_0 = 0.2$, 0.3, 0.5, 1, 10, 10^2, 10^3, 10^4

where

$$\gamma_R = \frac{2\nu_p}{3\nu_0} = 2.1\ 10^{-3} (n_e/1\mathrm{cm}^{-3})^{1/2} (B/1\mathrm{Gauss})^{-1} \tag{29}$$

is called the Razin-Lorentz factor in a random magnetic field.

For electron densities of $n_e \simeq 10^8\ \mathrm{cm}^{-3}$ and magnetic field strengths of $B \simeq 10^{-2}$ Gauss in quasars, γ_R is of the order of 10^3 (see also Table 1). Only the electrons with Lorentz factors $\gamma_0 > \gamma_R$ give rise to substantial unsuppressed synchrotron emission, whose logarithmic bandwidth increases as $\Delta\log(g_0) \propto \log(\gamma_0)$ with increasing γ_0 around $\nu = \gamma_0 \nu_p$. This source would become visible in the optically thin case only at frequencies above

$$\nu_R \equiv \nu_V(\gamma_0 = \gamma_R) = \gamma_R \nu_p = \frac{2\nu_p^2}{3\nu_0} = 20\, (n_e/1\mathrm{cm}^{-3}) (B/1\mathrm{Gauss})^{-1}\ \mathrm{Hz}, \tag{30}$$

which is the Razin-Tsytovich frequency. With the above given values of n_e and B we find that ν_R is of the order 10^{11} Hz in active galactic nuclei (AGN), making it plausible that most (90 percent) of the quasars are radio-quiet if the radio emission can be associated with the broad line region. However, according to standard knowledge the filling factors of the broad line clouds are very small ($\simeq 10^{-4}$), and it is not easy to imagine how the radio source could be compressed into such tiny clumps. Suppression could also be associated with the intercloud gas below $\simeq 20$ GHz if the magnetic field strength in this region is as low as 10^{-3} G. The radio-loud quasars we detect may either be those AGN whose Razin-Tsytovich frequency is low enough ($\lesssim 10^{10}$ Hz), or whose radio emission comes from another population in these objects not associated with the nucleus.

Table 1 summarizes values of the Razin-Lorentz factor γ_R and the Razin-Tsytovich frequency ν_R in different astrophysical objects, indicating that objects like molecular clouds, the interstellar medium, and quasars are visible in the light of synchrotron radiation only above frequencies of order $10^8 - 10^{11}$ Hz.

5. Synchrotron absorption coefficient in a thermal plasma with large-scale random magnetic fields for monoenergetic electrons

Using Eqs. (9) and (13) in Eq. (11) and partially integrating the RHS of (11) yields

$$\mu_r(\nu) = \frac{N_0 q_0}{16 m \gamma_0^2 \nu} \left[\frac{d}{dy} \left\{ \left[1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2 \right] CS\left(\frac{2\nu}{3\nu_0 \gamma^2} \left[1 + \left(\frac{\gamma\nu_p}{\nu}\right)^2 \right]^{3/2} \right) \right\} \right]_{y=y_0}$$

$$= \frac{N_0 q_0}{8 m \gamma_0^2 \nu} \left[\frac{\nu_p^2 \gamma_0}{\nu^2} CS(y_0) + \frac{\nu_p^2}{3\nu_0\gamma_0\nu} \left(1 - \frac{2\nu^2}{\nu_p^2\gamma_0^2}\right) \left(1 + \frac{\nu_p^2\gamma_0^2}{\nu^2}\right)^{3/2} \cdot \left[\frac{dCS(y)}{dy}\right]_{y=y_0} \right], \tag{31}$$

where

$$y_0 = \frac{2\nu}{3\nu_0\gamma_0} \left[1 + \left(\frac{\gamma_0\nu_p}{\nu}\right)^2 \right]^{3/2}.$$

In terms of the normalized frequency f (see (15)) and the parameter g_0 (see (16)), Eq. (31) can be written as

$$\mu_r(f) = \frac{b_2}{f^3} \left[CS\left[\frac{f}{g_0} (1+f^{-2})^{3/2} \right] + \frac{1}{2}(1-2f^2) \left[y \frac{dCS(y)}{dy} \right]_{y=\frac{f}{g_0}(1+f^{-2})^{3/2}} \right], \tag{32}$$

with

$$b_2 = \frac{N_0 q_0}{8 m \nu_p \gamma_0^4}. \tag{33}$$

As shown in the Appendix, the derivative of $CS(y)$ is given by

$$\frac{dCS(y)}{dy} = -\frac{1}{y} W_{\frac{1}{2}, \frac{5}{6}}(y) W_{-\frac{1}{2}, \frac{5}{6}}(y), \tag{34}$$

which is negative for all positive values of y. Combining Eqs. (34) and (8) we find

$$y \frac{dCS(y)}{dy} = CS(y) - W_{0,\frac{4}{3}}(y)\, W_{0,\frac{1}{3}}(y), \tag{35}$$

which yields for Eq. (32)

$$\mu_r(f) = \frac{b_2}{f^3} \left[\left(\frac{3}{2} - f^2\right) CS\left[\frac{f}{g_0}(1+f^{-2})^{3/2} \right] + \left(f^2 - \frac{1}{2}\right) W_{0,\frac{4}{3}}\left[\frac{f}{g_0}(1+f^{-2})^{3/2} \right] W_{0,\frac{1}{3}}\left[\frac{f}{g_0}(1+f^{-2})^{3/2} \right] \right]. \tag{36}$$

Table 1. Razin-Lorentz factor γ_R and Razin-Tsytovich frequency ν_R in different astrophysical objects

Object	Electron density n_e(cm^{-3})	Magnetic field strength B(Gauss)	γ_R	ν_R(Hz)
Solar flare	10^9	10^2	0.68	$2\,10^8$
Supernova remnant	10^{-2}	10^{-5}	20	$2\,10^4$
Molecular cloud	10^2	10^{-5}	$2\,10^3$	$2\,10^8$
Interstellar medium	1	$3\,10^{-6}$	680	$7\,10^7$
Quasar: broad line region	10^8	10^{-2}	$2\,10^3$	$2\,10^{11}$
Quasar: intercloud region	10^6	10^{-3}	$2\,10^3$	$2\,10^{10}$

For the following discussion it is useful to note the asymptotic behaviour of (see Appendix)

$$W_{0,\frac{4}{3}}(y)W_{0,\frac{1}{3}}(y)\simeq\begin{cases}5/3\,a_0\,y^{-2/3} & \text{for } y\gg1 \quad (37a)\\ e^{-y} & \text{for } y\gg1 \quad (37b)\end{cases}$$

with a_0 given in Eq. (20c). If we compare (37b) with (20b) we see that for large arguments y the second term in (36) dominates the first term involving $CS(y)$ by a factor $O(y)$. Because of the expression $(1-2f^2)$ in front of the second term of Eq. (32) the absorption coefficient can become negative for frequencies $f<1/\sqrt{2}$, allowing for induced synchrotron emission to occur (case (12c)). We will come back to this point in Sect. 6.

We now consider the behaviour of $\mu_r(f)$ for small ($f\ll1$) and large ($f\gg1$) frequencies, as well as the influence of the parameter g_0.

5.1. Large frequencies, $f\gg1$

For large frequencies, $f\gg1$, Eq. (36) reduces to

$$\mu_r(f)\simeq\frac{b_2}{f}\left[W_{0,\frac{4}{3}}\left(\frac{f}{g_0}\right)W_{0,\frac{1}{3}}\left(\frac{f}{g_0}\right)-CS\left(\frac{f}{g_0}\right)\right]$$
$$=\frac{b_2}{f}W_{\frac{1}{2},\frac{5}{6}}\left(\frac{f}{g_0}\right)W_{-\frac{1}{2},\frac{5}{6}}\left(\frac{f}{g_0}\right), \quad (38)$$

which is positive for all values of (f/g_0). Equation (38) implies as a special case the impossibility of a synchrotron maser in vacuum, since for $v_p\to0$ it reduces to the vacuum behaviour

$$\mu_r(\nu)=\frac{N_0q_0}{8m\gamma_0^3}\frac{1}{\nu}W_{\frac{1}{2},\frac{5}{6}}\left(\frac{\nu}{\nu_C}\right)W_{-\frac{1}{2},\frac{5}{6}}\left(\frac{\nu}{\nu_C}\right), \quad (39)$$

with $\nu_C=3/2\,\nu_0\gamma_0^2$, for all frequencies $\nu>0$.

With Eqs. (20) and (37) it is straightforward to derive the asymptotic behaviour of (38) for values of f small and large compared to g_0.

In case $g_0\gg1$ we find

$$\mu_r(f)\simeq b_2\cdot\begin{cases}2/3\,a_0g_0^{2/3}f^{-5/3} & \text{for } 1\ll f\ll g_0 \quad (40a)\\ f^{-1}\,e^{-f/g_0} & \text{for } 1\ll g_0\ll f \quad (40b)\end{cases}$$

which agrees with the vacuum behaviour.

For values $g_0\ll1$ we find for all $f\gg1$ that

$$\mu_r(f)\simeq b_2f^{-1}\,e^{-f/g_0}. \quad (41)$$

5.2. Small frequencies, $f\ll1$

For small frequencies, $f\ll1$, Eq. (36) reduces to

$$\mu_r(f)\simeq\frac{b_2}{2}f^{-3}\left[3CS\left(\frac{1}{g_0f^2}\right)-W_{0,\frac{4}{3}}\left(\frac{1}{g_0f^2}\right)W_{0,\frac{1}{3}}\left(\frac{1}{g_0f^2}\right)\right]. \quad (42)$$

In case of $g_0\gg1$ we obtain

$$\mu_r(f)\simeq b_2\cdot\begin{cases}\frac{2}{3}a_0\,g_0^{2/3}f^{-5/3} & \text{for } g_0^{-1/2}\ll f\ll1 \quad (43a)\\ -\frac{1}{2}f^{-3}\exp\left(\frac{-1}{g_0f^2}\right) & \text{for } f\ll g_0^{-1/2}\ll1 \quad (43b)\end{cases}$$

It is interesting to note, that (43a) agrees exactly with (40a), indicating that the vacuum behaviour continues down to the frequency $f_v\simeq g_0^{-1/2}$. Also it can be seen that at frequencies below f_v the absorption coefficient is negative.

For values $g_0\ll1$ we find

$$\mu_r(f)\simeq-\frac{b_2}{2}f^{-3}\exp\left(\frac{-1}{g_0f^2}\right), \quad (44)$$

a negative but exponentially small absorption coefficient for all frequencies $f<1$.

5.3. $g_0\gg1$

Figure 2 summarizes the behaviour of the synchrotron absorption co-efficient $\mu_r(f)$ in the case $g_0\gg1$. Exact numerical calculations of (36) are compared with the asymptotic results (40) and (43) for a value of $g_0=10^4$. For frequencies $f>f_v=g_0^{-1/2}=10^{-2}$ we obtain the vacuum case behaviour, with $\mu_r(f)\propto f^{-5/3}$ for $f\lesssim g_0$, whereas below f_v the absorption coefficient becomes negative, $\mu_r(f)\propto-f^{-3}\exp(-1/g_0f^2)$. As can be seen, apart from the vicinity of f_v, the asymptotic expressions agree well with the exact curve.

In Fig. 3 $Y=\mu_r/(b_2\,g_0^{3/2})$ is plotted against $X=f/g_0^{-1/2}$ for $g_0=1,10,10^2,10^6$. For values $g_0>10^2$ the form of the curves does not change. This becomes plausible from (40) and (43), since we can write

$$Y(X)\simeq b_2\begin{cases}2/3\,a_0\,X^{-5/3} & \text{for } 1\ll X\ll g_0^{3/2} \quad (45a)\\ -1/2X^{-3}\exp(-1/X^2) & \text{for } X\gg1 \quad (45b)\end{cases},$$

which is independent of g_0. This means that the minimal and maximal values of $\mu_r(f)$ are proportional to $b_2\,g_0^{3/2}$, whereas the corresponding frequencies are proportional to $g_0^{-1/2}$.

5.4. $g_0\ll1$

Figure 4 summarizes the behaviour of the absorption coefficient $\mu_r(f)$ in the case $g_0\ll1$. Exact numerical calculations of (36) are compared with the asymptotic results (41) and (44) in the case $g_0=0.3$. At small frequencies, $f\ll1$, the absorption coefficient is negative, $\mu_r(f)\propto-f^{-3}\exp(-1/g_0f^2)$, but exponentially small, whereas at large frequencies, $f\gg1$, it is positive $\mu_r(f)\propto f^{-1}\exp(-f/g_0)$.

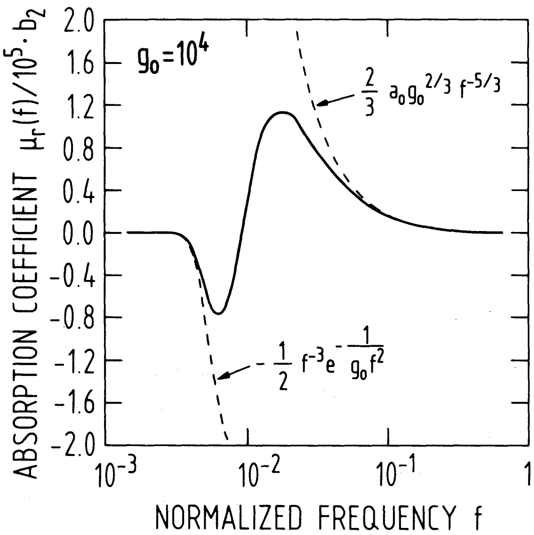

Fig. 2. Absorption coefficient μ_r as a function of normalized frequency $f=\nu/(\gamma_0\,\nu_p)$ in the case $g_0=10^4$ together with the asymptotic expansions

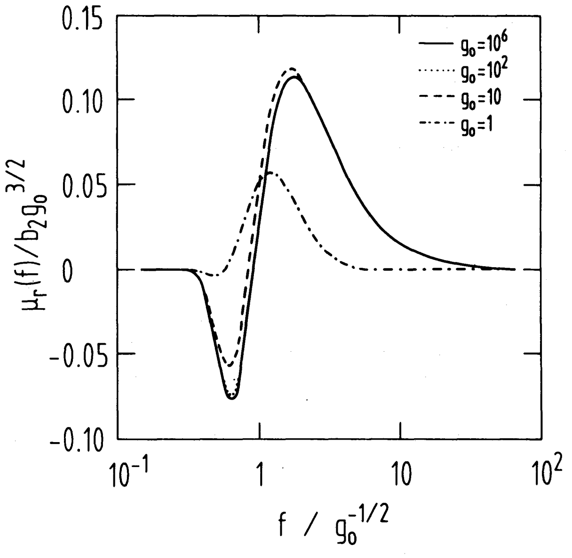

Fig. 3. Normalized absorption coefficient $\mu_r/b_2 g_0^{3/2}$ plotted against normalized frequency $f/g_0^{-1/2}$ for $g_0=1$, 10, 10^2, 10^6

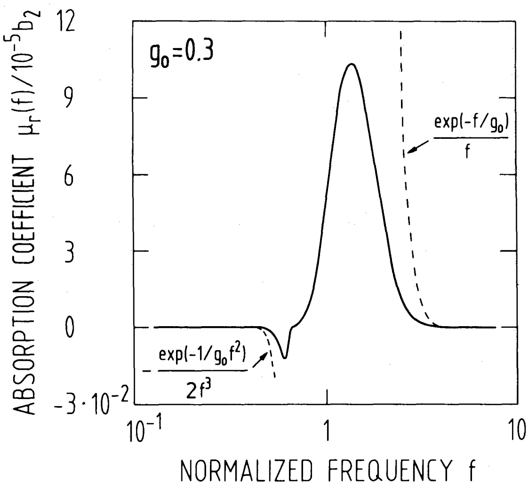

Fig. 4. Absorption coefficient $\mu_r(f)$ together with its asymptotic expansions in the case $g_0=0.3$ (negative part of μ_r expanded by a factor 100)

The scale of the negative part is extended by a factor of 100 over the positive part.

6. Necessary conditions for synchrotron masering

For a synchrotron maser, of the type described above, to occur in nature, three conditions have to be satisfied:

(1) The absolute value of the (negative) synchrotron absorption coefficient has to be larger than the sum of all other (positive) absorption coefficients, in particular the free-free absorption coefficient μ_{FF}:

$$|\mu_r(f_M)| > \mu_{FF}(f_M) \qquad (46)$$

with (Melrose, 1980, p. 182)

$$\mu_{FF}(\nu) \simeq \zeta n_e^2\, \nu^{-2}\, T_e^{-3/2}\ \mathrm{cm}^{-1} \qquad (47)$$

for $\nu \gg \nu_p$, ζ a number in the range 0.1 to 0.2, T_e in Kelvins, and n_e in particles per cm^3; f_M is the frequency of the maser-line, i.e. where $\mu_r(f)$ reaches its minimum.

(2) When making use of the simple expression (3) for the refraction index n it has been assumed that the relativistic electrons generate the synchrotron photons but that they do not affect the propagation of the electromagnetic waves through the plasma. We can neglect the role of the relativistic particles when evaluating n provided (Ginzburg, 1979)

$$\frac{\nu_p^2\, \nu_0}{\nu_M^3} \gg \frac{c\, \mu_r(\nu_M)}{4\pi\, \nu_M}. \qquad (48)$$

(3) The absolute value of the optical depth τ has to be greater than unity for a significant enhancement of the radiation to occur:

$$|\tau| = |\mu_r(f_M)| L > 1, \qquad (49)$$

where L is the thickness of the emitting slab.

In Sect. 5.3 we found for the frequency f_M and for the minimum value of the absorption coefficient $\mu_r(f_M)$:

$$f_M \simeq 0.63\, g_0^{-1/2} \qquad (50)$$

and

$$\mu_r(f_M) = -0.08\, b_2\, g_0^{3/2}\ \mathrm{cm}^{-1}, \qquad (51)$$

which holds well for $g_0 > 10^2$. Even for $g_0=10$ the deviations are small ($f_M = 0.6\, g_0^{-1/2}$, $\mu_r(f_M) = -0.06\, b_2\, g_0^{3/2}$). For $g_0=1$ the deviations are somewhat bigger ($f_M=0.48$, $\mu_r(f_M) = -0.003\, b_2$).

Using Eqs. (50) and (51) in Eq. (46) we obtain the condition

$$\frac{N_0}{n_e} > 0.06\, n_e^{1/2}\, g_0^{-5/4}\, \gamma_0^2\, T_e^{-3/2} = 3.10^{-5}\, n_e^{9/8}\, \gamma_0^{3/4}\, B^{-5/4}\, T_e^{-3/2}, \qquad (52)$$

and the maser-frequency is given by

$$\nu_M = 0.63\, g_0^{-1/2}\, \nu_p \gamma_0 = 2.5\cdot 10^2\, n_e^{3/4}\, \gamma_0^{1/2}\, B^{-1/2}\ \mathrm{Hz}, \qquad (53)$$

where in the second step in (52) and (53) equation (16) has been used. Condition (48) can be rewritten as

$$N_0 \ll 3.7\cdot 10^5\, n_e^{1/2}\, g_0^{-1/2}\, \gamma_0^2\, B = 1.7\cdot 10^4\, n_e^{3/4}\, \gamma_0^{3/2}\, B^{1/2}\ \mathrm{cm}^{-3} \qquad (54)$$

and restricts the number of relativistic particles.

In writing $\mu_r(f_M)$ in the form (51) we have to introduce the condition $g_0 > 1$, since for $g_0 < 1$ the absorption coefficient becomes exponentially small, which is not contained in the formula (51). $g_0 > 1$ is equivalent to

$$n_e < 2.2\cdot 10^5\, \gamma_0^2\, B^2\ \mathrm{cm}^{-3}. \qquad (55)$$

From (49) we get the condition

$$L > 2.10^7\, n_e^{1/2}\, g_0^{-3/2}\, \gamma_0^4\, N_0^{-1}\ \mathrm{cm} = 2.10^3\, n_e^{5/4}\, \gamma_0^{5/2}\, B^{-3/2}\, N_0^{-1}\ \mathrm{cm}, \qquad (56)$$

which should be satisfied by the thickness L of the slab.

In Fig. 5 the condition (54) does not affect the diagram in the shown range of parameters. The condition $g_0 > 1$ in the form of Eq. (55) is shown as a vertical line. For maser-action to be possible the points $(N_0/n_e, n_e)$ must be to the left of it. The condition that the free-free absorption is small enough in the

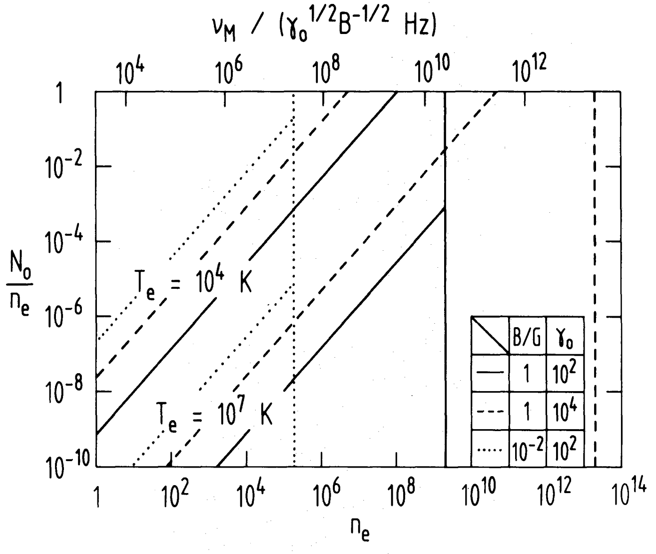

Fig. 5. Physical restrictions of a synchrotron maser. Maser action is possible above the lines with slope 9/8 (free-free absorption negligible) and left to the vertical lines ($g_0 \geq 1$).

form of Eq. (52) is a straight line with slope 9/8. A synchrotron-maser can only work on the upper-left side of it. These restrictions are shown for three different combinations of (B/Gauss, γ_0) $= [(1,10^2); (1,10^4); (10^{-2}, 10^2)]$ each at two temperature ($T_e = 10^4$, 10^7 K). Condition (56) has to be treated separately.

If the density of the thermal plasma is given, the magnetic field strength and/or the Lorentz factor must be large enough, so that the vertical boundary is to the right of the actually present value.

For the length L not to become too large (L should be smaller than the dimension of the source) the Lorentz factor has to be small and the field strength large enough.

High temperatures are necessary in order that free-free absorption becomes negligible at high densities n_e. The maser-frequency ν_M depends on the Lorentz factor and the modulus of the field as $\nu_M \propto (\gamma_0/B)^{1/2}$.

We now want to emphasize that another necessary condition has been used implicitly: namely the assumption of a mono-energetic distribution of the relativistic electrons. As can be seen from Eq. (11) the energy distribution must have a slope >2 in some energy interval, otherwise the absorption coefficient is positive for all frequencies.

A mechanism which may produce nearly monoenergetic electrons is the interplay of first and second order Fermi acceleration (energy gain processes) with synchrotron and inverse Compton losses (Schlickeiser, 1984). The particles accumulate at an energy where gains and losses exactly balance each other.

A further constraint is associated with the radiation field. Cawthorne (1985) has pointed out that inverse Compton scattering and stimulated Compton scattering severely restrict the feasibility of synchrotron masers. For example, to show that stimulated Compton scattering is likely to be important, consider the parameters used in Eq. (62), $n_e \sigma_T L \simeq 2$. For $n_e \sigma_T L (K T_b/mc^2) < \tau_M \simeq 40$ requires $K T_b/mc^2 < 20$, i.e. $T_b < 10^{11}$ K. A source of 10^{11} K and size 1 pc at a redshift $z = 1$ has a flux density of only 0.1 mJy at 1 GHz. So for high-redshift powerful radio sources the problem with stimulated Compton scattering becomes well apparent.

7. Emitted synchrotron intensity from a homogeneous slab

In case of a homogeneous slab of thickness L the emitted synchrotron intensity from monoenergetic relativistic electrons in a thermal plasma with large-scale random magnetic fields is given by Eq. (12) after inserting the derived emission coefficient (17) and the absorption coefficient (36).

7.1. Optically thin case

In the optically thin case, $|\mu_r(f)| L \ll 1$, the intensity follows directly from the emission coefficent $\varepsilon_r(f)$,

$$I_r(f) = \varepsilon_r(f) L, \tag{57}$$

the behaviour of which was thoroughly discussed in Sect. 4.

7.2. Optically thick case

In the optically thick case, $|\mu_r(f)| L \gg 1$, we find according to (12b) and (12c) that

$$I_r(f) \simeq \begin{cases} \dfrac{\varepsilon_r(f)}{\mu_r(f)} & \text{for } \mu_r(f) > 0 \\[2ex] \dfrac{\varepsilon_r(f)}{|\mu_r(f)|} \exp(|\mu_r(f)| L) & \text{for } \mu_r(f) < 0. \end{cases} \tag{58}$$

Here the emitted intensity depends very much on the sign of the absorption coefficient: drastic amplification of emission is possible in ranges where $\mu_r(f) < 0$.

Figure 6 shows the optical depth $\tau(f) = \mu_r(f) L$ in the case where $g_0 = 100$ and $L = 0.05/b_2$ with b_2 according to (33). For $f \to 0$ and $f \to \infty$ we have $\tau(f) \to \pm 0$, which means that the emission becomes optically thin (region 1 and 5). Since the absorption coefficient goes through zero, there is a small frequency interval where the emission is also optically thin (region 3). Regions 2 and 4 are the maser and self-absorbed regions, respectively.

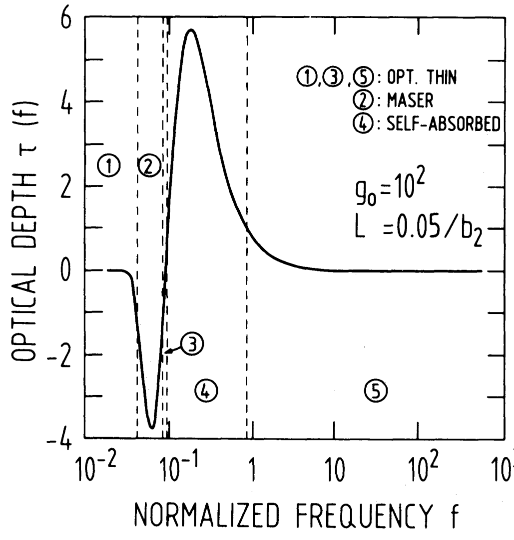

Fig. 6. Optical depth $\tau(f)$ in the case $g_0 = 10^2$ and length $L = 0.05/b_2$.

In Fig. 7 the intensity spectrum of the radiation is plotted for the optical depths τ and 2τ with τ according to Fig. 6. For $f\to 0$ we get from (24a)

$$I_r(f)\simeq b_2\, g_0\, L f \exp\left(\frac{-1}{g_0 f^2}\right) \quad \text{for } f \ll g_0^{-1/2}=10^{-1}. \tag{59}$$

Since $g_0 f^2 \ll 1$, in this case, the intensity grows very rapidly. It follows a continuous transition to the "maser-line". The next part of the spectrum can be approximated by (see (21a), (24b) and (40a), (43a)):

$$I_r(f)\simeq \frac{3b_1}{2b_2} f^2 \quad \text{for } g_0^{-1/2} \ll f < f_\tau. \tag{60}$$

This region of the spectrum resembles the classical optically thick Rayleigh-Jeans spectrum. f_τ is that frequency where τ again becomes smaller than unity, i.e. the source is optically thin for $f>f_\tau$ and the spectrum is described by the vacuum behaviour of the emission coefficient (21a, b):

$$I_r(f)\simeq b_1 L \begin{cases} a_0\, g_0^{2/3} f^{1/3} & \text{for } f_\tau < f \ll g_0 \\ g_0 \exp(-f/g_0) & \text{for } f \gg g_0 = 10^2. \end{cases} \tag{61}$$

In the case where $g_0=1$, i.e. $\gamma_0=\gamma_R$, we find for the optical depth at the maser frequency ν_M

$$|\tau(\nu_M)|_{g_0=1} = 38 \left(\frac{n_e}{10^6\ \text{cm}^{-3}}\right)^{-5/2} \left(\frac{B}{10^{-2}\ \text{G}}\right)^4 \left(\frac{N_0}{10^4\ \text{cm}^{-3}}\right)\left(\frac{L}{1\text{pc}}\right) \tag{62}$$

with

$$\nu_M(g_0=1)=0.9\left(\frac{n_e}{10^6\ \text{cm}^{-3}}\right)\left(\frac{B}{10^{-2}\ \text{G}}\right)^{-1}\ \text{GHz} \tag{63}$$

where we have used typical parameter values for compact non-thermal sources (Jones, O'Dell, Stein 1974; Burbidge, Jones, O'Dell 1974). Thus the occurrence of a synchrotron-maser might be possible in these sources.

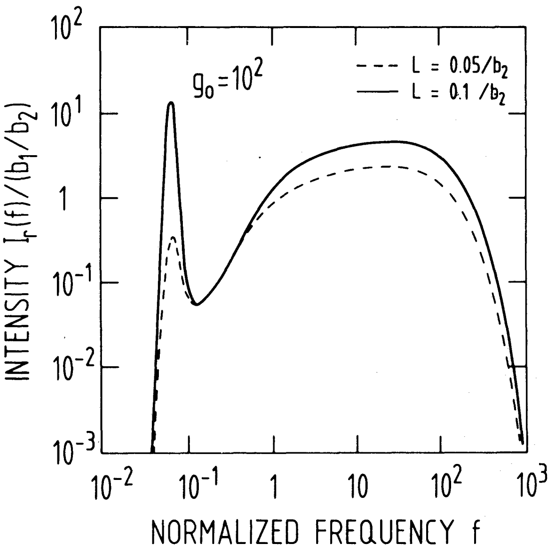

Fig. 7. Synchrotron-spectrum for optical depth τ (dashed line) and 2τ (continuous line) with τ from Fig. 6.

8. Mean energy loss of a single electron due to spontaneously emitted synchrotron radiation in a thermal plasma with large-scale random magnetic fields

The analytical expression (9) for the spontaneously emitted synchrotron power of a single electron allows a simple calculation of the total energy loss:

$$\frac{dE}{dt}=mc^2\frac{d\gamma}{dt}=-\int_0^\infty d\nu\, p_r(\nu,\gamma)=$$

$$=-\frac{q_0\pi}{2\gamma^2}\int_0^\infty d\nu\,\nu\left[1+\frac{\gamma^2\nu_p^2}{\nu^2}\right] CS\left[\frac{2\nu}{3\nu_0\gamma^2}\left(1+\frac{\gamma^2\nu_p^2}{\nu^2}\right)^{3/2}\right]. \tag{64}$$

Introducing as before

$$f=\nu/(\gamma\nu_p), \tag{65}$$

$$g=\frac{3}{2}(\nu_0/\nu_p)\,\gamma=\frac{\gamma}{\gamma_R}, \tag{66}$$

and

$$y=\frac{f}{g}(1+f^{-2})^{3/2}\simeq\begin{cases}\dfrac{1}{gf^2} & \text{for } f\ll 1\\ \dfrac{f}{g} & \text{for } f\gg 1\end{cases} \tag{67}$$

yields

$$\frac{d\gamma}{dt}=-\frac{\pi q_0\nu_p^2}{2mc^2}\left[\int_0^\infty df\,f\,CS(y)+\int_0^\infty df\,f^{-1}\,CS(y)\right]. \tag{68}$$

Using the asymptotic behaviour of $y(f)$ for f small and large compared to unity, we may approximate (68) as

$$\frac{d\gamma}{dt}\simeq-\frac{\pi q_0\nu_p^2}{2mc^2}\left[\int_0^1 df\,f\,CS\left(\frac{1}{gf^2}\right)+\int_1^\infty df\,f\,CS\left(\frac{f}{g}\right)\right.$$

$$\left.+\int_0^1 df\,f^{-1}\,CS\left(\frac{1}{gf^2}\right)+\int_1^\infty df\,f^{-1}\,CS\left(\frac{f}{g}\right)\right]$$

$$=-\frac{\pi q_0\nu_p^2}{2mc^2}\left[\frac{3}{2}\int_{1/g}^\infty du\,u^{-1}\,CS(u)\right.$$

$$+\frac{1}{2}g^{-1}\int_{1/g}^\infty du\,u^{-2}\,CS(u)$$

$$\left.+g^2\int_{1/g}^\infty du\,u\,CS(u)\right] \tag{69}$$

after obvious changes of the integration variable. We consider the two limits $g\gg 1$ and $g\ll 1$, respectively.

8.1. Vacuum case $g\gg 1$ ($\gamma\gg\gamma_R$)

From an order of magnitude analysis, we see that the three terms in (69) are of respective order

$$1:1:g^2. \tag{70}$$

so that in the range $g\gg 1$ we may neglect the first two terms, to obtain

$$\left.\frac{d\gamma}{dt}\right|_{g\gg 1}\simeq-\frac{q_0\pi\nu_p^2 g^2}{2mc^2}\left[\int_0^\infty du\,u\,CS(u)-\int_0^{1/g} du\,u\,CS(u)\right]. \tag{71}$$

The first integral in (71) solves exactly (see Appendix, A17) as

$$\int_0^\infty du\, u\, CS(u) = \frac{32\sqrt{3}}{81}, \tag{72}$$

whereas in the second one we may approximate (26a)

$$CS(u) \simeq a_0\, u^{-2/3}, \tag{73}$$

since $u \leqslant g^{-1} \ll 1$, yielding

$$\int_0^{1/g} du\, u\, CS(u) \simeq \frac{3}{4} a_0\, g^{-4/3}. \tag{74}$$

Inserting (72) and (74) in (71) we find

$$\frac{d\gamma}{dt}(\gamma \gg \gamma_R) \simeq -\frac{16\sqrt{3}\,\pi\, q_0\, v_p^2\, g^2}{18\, mc^2}\left[1 - \frac{18\sqrt{3}}{128}\, a_0\, g^{-4/3}\right]$$

$$= -\frac{4\pi\, q_0\, v_0^2\, \gamma^2}{3\sqrt{3}\, mc^2}\left[1 - \frac{81\sqrt{3}}{128}\, a_0 \left(\frac{\gamma}{\gamma_R}\right)^{-4/3}\right], \tag{75}$$

The first term corresponds exactly to the behaviour in vacuum (see, e.g., Blumenthal and Gould, 1970). The second term is the lowest order plasma-correction.

8.2. *Plasma case $g \ll 1$ ($\gamma \ll \gamma_R$)*

In this case the argument of the function CS in (69) is very large compared to unity, $u \geqslant g^{-1} \gg 1$, so that we may approximate according to (20b)

$$CS(u) \simeq u^{-1}\, e^{-u}. \tag{76}$$

We obtain for Eq. (69)

$$\left.\frac{d\gamma}{dt}\right|_{g \ll 1} \simeq -\frac{\pi\, q_0\, v_p^2}{2mc^2}\left[\frac{3}{2}\int_{1/g}^\infty du\, u^{-2}\, e^{-u} + \frac{1}{2} g^{-1}\int_{1/g}^\infty du\, u^{-3}\, e^{-u} + g^2\, e^{-1/g}\right]. \tag{77}$$

Partially integrating the two integrals gives

$$\left.\frac{d\gamma}{dt}\right|_{g \ll 1} \simeq -\frac{\pi\, q_0\, v_p^2}{2mc^2}\left[\frac{7}{4} g e^{-1/g} - \frac{1}{4} e^{-1/g} + g^2\, e^{-1/g} + \left(\frac{1}{4g} - \frac{3}{2}\right) E_1(1/g)\right] \tag{78}$$

where $E_1(x)$ denotes the exponential integral of order one. Using the asymptotic expansions of $E_1(x)$ for large arguments (Abramowitz and Stegun, 1970, p. 231)

$$E_1(x) \approx x^{-1}\, e^{-x}\left(1 - \frac{1}{x} + \frac{2}{x^2}\right) \tag{79}$$

gives

$$\frac{d\gamma}{dt}(\gamma \ll \gamma_R) \simeq -\frac{3\pi\, q_0\, v_p^2}{2mc^2}\, g^2\, e^{-1/g} = \frac{-27\pi\, q_0\, v_0^2}{8mc^2}\, \gamma^2\, e^{-\gamma_R/\gamma}. \tag{80}$$

This result has the following implications. If the Lorentz factor γ of the relativistic electrons is smaller than the Razin-Lorentz factor γ_R, then the electrons lose an exponentially small amount of energy by synchrotron radiation compared to the vacuum-case.

9. Summary and discussion

In this paper we discuss the effects of a thermal plasma on the synchrotron emission process. We assume a large-scale random magnetic field, which is suggested by many polarization observations of radio sources. This gives us the opportunity to use a closed expression for the synchrotron power of a single particle. The investigation of the emission-coefficient, the absorption-coefficient, and the resulting intensity spectrum in a plasma with random magnetic fields is therefore more transparent, compared to previous discussions by other authors who either used aproximations of the relevant formulas or made numerical calculations. We study synchrotron radiation of a monoenergetic distribution function (all relativistic electrons have the Lorentz factor γ_0) since in this case the influence of the plasma shows up in its purest form. The emission and absorption coefficients for any other distribution can be obtained by a superposition of our results for different energies (one integration over energy).

The influence of the thermal ionized medium is described by the parameter g_0, which in turn depends on three source parameters ($g_0 = (3v_0/2v_p)\gamma_0 = 468\ \gamma_0\ B\ n_e^{-1/2}$ in cgs units). The emission coefficient and the optically thin spectrum is exponentially cut down for frequencies $v < v_v = g_0^{-1/2}\, \gamma_0\, v_p$. At the same frequency the absorption coefficient becomes negative allowing a maser-type emission. This effect produces a "line" in the spectrum whose height and width depends on the optical depth. At high frequencies ($v > v_v$) we rediscover the vacuum case behaviour.

For large electron densities n_e of the plasma, free-free absorption prevents the formation of the maser-line, restricting synchrotron masering to objects with low n_e. This restriction is temperature-dependent, due to the dependence of the free-free absorption coefficient upon $T_e(\mu_{FF} \propto T_e^{-3/2})$. In the last part of the paper we calculate the total energy loss of a single relativistic electron moving in a thermal plasma. In case of $g \gg 1$ the influence of the medium is negligible, $d\gamma/dt \propto \gamma^2$, as in the vacuum case. But for $g \lesssim 1$ the medium heavily suppresses emission in the whole frequency range. The energy loss is exponentially reduced, $d\gamma/dt \propto \gamma^2\, \exp(-\gamma_R/\gamma)$, with $\gamma_R = 2v_p/3v_0$. Consequently particles with $\gamma > \gamma_R$ lose energy through the synchrotron mechanism until they reach the energy $E_R = \gamma_R mc^2$. Then the plasma suppresses the emission drastically, which leads to negligible synchrotron energy loss.

Acknowledgements. We thank U. Runkel for her careful typing of the manuscript. R. S. acknowledges the hospitality of Prof. Heinz Völk at the MPI Kernphysik, Heidelberg, where part of this work was performed.

Appendix: properties of the function $CS(x)$

The function $CS(x)$ is defined as the integral

$$CS(x) = \frac{1}{\pi}\int_0^\pi d\theta\, \sin\theta \int_{x/\sin\theta}^\infty dt\, K_{5/3}(t), \tag{A1}$$

where $K_{5/3}(t)$ is the modified Bessel function of the second kind and of order 5/3. In a recent paper (Crusius and Schlickeiser, 1986) we have proven that

$$CS(x) = W_{0,\frac{4}{3}}(x)\, W_{0,\frac{1}{3}}(x) - W_{\frac{1}{2},\frac{5}{6}}(x)\, W_{-\frac{1}{2},\frac{5}{6}}(x), \tag{A2}$$

where $W_{\lambda,\mu}(x)$ denote Whittaker functions. We also gave the asymptotic expansions of $R(x) = (\pi x/2)\ CS(x)$ from which we immediately obtain

$$CS(x) \simeq \begin{cases} a_0 x^{-2/3} & \text{for } x \ll 1 \\ x^{-1} e^{-x} & \text{for } x \gg 1 \end{cases} \tag{A3}$$

with

$$a_0 = \frac{2^{4/3}}{5\pi} \Gamma^2(1/3) = 1.151275. \tag{A4}$$

To calculate the first derivative $CS(x)$ we differentiate (A1) with respect to x and obtain

$$\frac{dCS(x)}{dx} = -\frac{2}{\pi} \int_0^{\pi/2} d\theta \, K_{5/3}\left(\frac{x}{\sin\theta}\right) \tag{A5}$$

Substituting $\theta = \pi/2 - \beta$ yields

$$\frac{dCS(x)}{dx} = -\frac{2}{\pi} \int_0^{\pi/2} d\beta \, K_{5/3}\left(\frac{x}{\cos\beta}\right) \tag{A6}$$

which is a special case of the formula (Gradshteyn and Ryzhik, 1965, p. 741)

$$\int_0^{\pi/2} d\beta \frac{\cos(2\lambda\beta)}{\cos\beta} K_{2\mu}\left(\frac{x}{\cos\beta}\right) = \frac{\pi}{2x} W_{\lambda,\mu}(x) \, W_{-\lambda,\mu}(x), \tag{A7}$$

so that (A6) becomes ($\lambda = 1/2$, $\mu = 5/6$)

$$\frac{dCS(x)}{dx} = -\frac{1}{x} W_{\frac{1}{2},\frac{5}{6}}(x) \, W_{-\frac{1}{2},\frac{5}{6}}(x) = \frac{1}{x}[CS(x) - W_{0,\frac{4}{3}}(x) \, W_{0,\frac{1}{3}}(x)]. \tag{A8}$$

With the asymptotic expansions of the Whittaker functions for small and large arguments (Abramowitz and Stegun, 1970; Buchholz, 1953) we derive

$$\frac{dCS(x)}{dx} \simeq \begin{cases} -\frac{2}{3} a_0 x^{-5/3} & \text{for } x \ll 1 \\ -\frac{1}{x} e^{-x} & \text{for } x \gg 1 \end{cases}. \tag{A9}$$

From (A8), (A3) and (A9) we deduce

$$W_{0,\frac{4}{3}}(x) \, W_{0,\frac{1}{3}}(x) \simeq \begin{cases} 5/3 \, a_0 x^{-2/3} & \text{for } x \ll 1 \\ e^{-x} & \text{for } x \gg 1 \end{cases}. \tag{A10}$$

Finally we calculate the integral

$$j = \int_0^\infty du \, u \, CS(u) = \int_0^\infty du \, u^2 K_{4/3}\left(\frac{u}{2}\right) K_{1/3}\left(\frac{u}{2}\right) - \int_0^\infty du \, u \, W_{\frac{1}{2},\frac{5}{6}}(u) \, W_{-\frac{1}{2},\frac{5}{6}}(u) \tag{A11}$$

where we used the relation

$$K_\lambda(z) = \left(\frac{\pi}{2z}\right)^{1/2} W_{0,\lambda}(2z). \tag{A12}$$

Using (Gradshteyn and Ryzhik, 1965, p. 858)

$$\int_0^\infty dx \, x^{\rho-1} W_{\lambda,\mu}(x) \, W_{-\lambda,\mu}(x) = \frac{\Gamma(\rho+1)\,\Gamma(\rho/2+1/2+\mu)\,\Gamma(\rho/2+1/2-\mu)}{2\Gamma(1+\rho/2+\lambda)\,\Gamma(1+\rho/2-\lambda)} \tag{A13}$$

($\mathrm{Re}\rho > 2|\mathrm{Re}\mu| - 1$)

and (Erdélyi et al., 1954, p. 334)

$$\int_0^\infty dx \, x^{\rho-1} K_\mu(ax) \, K_\nu(ax) = 2^{\rho-3} a^{-\rho} [\Gamma(\rho)]^{-1} \cdot \Gamma\left[\frac{1}{2}(\rho+\mu+\nu)\right] \Gamma\left[\frac{1}{2}(\rho-\mu+\nu)\right] \Gamma\left[\frac{1}{2}(\rho+\mu-\nu)\right] \Gamma\left[\frac{1}{2}(\rho-\mu-\nu)\right] \tag{A14}$$

($\mathrm{Re}\rho > |\mathrm{Re}\mu| + |\mathrm{Re}\nu|$)

gives

$$j = \frac{16}{9\pi} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right) - \frac{32}{27\pi} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right). \tag{A15}$$

With the reflection formula

$$\Gamma(z)\,\Gamma(1-z) = \pi/\sin\pi z \tag{A16}$$

we obtain

$$j = \frac{16}{27\pi} \cdot \frac{\pi}{\sin(\pi/3)} = \frac{32\sqrt{3}}{81}. \tag{A17}$$

References

Abramowitz, M., Stegun, I.A.: 1970, Handbook of Mathematical Functions, National Bureau of Standards (Washington)

Angel, J.R.P., Stockman, H.S.: 1980, *Ann. Rev. Astron. Astrophys.* **18**, 321

Bekefi, G.: 1966, Radiation Processes in Plasmas, Wiley (New York)

Blumenthal, G. R., Gould, R.J.: 1970, *Rev. Mod. Phys.* **42**, 237

Buchholz, H.: 1953, Die konfluente hypergeometrische Funktion, Springer-Verlag (Berlin)

Burbidge, G.R., Jones, T.W., O'Dell, S.L.: 1974, *Astrophys. J.* **193**, 43

Cawthorne, T.V.: 1985, *Monthly Notices Roy. Astron. Soc.* **216**, 795

Crusius, A., Schlickeiser, R.: 1986, *Astron. Astrophys.* **164**, L16

Erdélyi, A., Magnus, W., Oberhettinger, F., Tricomi, F.G.: 1954, Tables of Integral Transforms, Vol. 1, McGraw-Hill Book Co., Inc. (New York)

Ginzburg, V.L.: 1979, Theoretical Physics and Astrophysics, Pergamon Press (Oxford)

Gradshteyn, I.S., Ryzhik, I.M.: 1965, Table of Integral Transforms, Series, and Products, Academic Press (New York)

Jones, T.W., O'Dell, S.L., Stein, W.A.: 1974, *Astrophys. J.* **192**, 261

Kellermann, K.I., Pauliny-Toth, I.I.K.: 1981, *Ann. Rev. Astron. Astrophys.* **19**, 373

McCray, R.: 1966, *Science* **154**, 1320

Melrose, D.B.: 1980, Plasma Astrophysics, Vol. 1, Gordon and Breach Science Publishers (New York)

Miley, G.: 1980, *Ann. Rev. Astron. Astrophys.* **18**, 165

Moffet, A.T.: 1965, in G.P. Kuiper and B.M. Middlehurst (eds.),

Stars and Stellar Systems, Chicago University Press, Vol. 9, p. 211
Schlickeiser, R.: 1984, *Astron. Astrophys.* **136**, 227
Ulrich, M.H., Boksenberg, A., Bromage, G., Carswell, R., Elvius, A., Gabriel, A., Gondhalekar, P.M., Lind, J., Lindegren, L., Longair, M.S., Penston, M.V., Perryman, M.A.C., Pettini, M., Perola, G.C., Rees, M., Sciama, D., Snijders, M.A.J., Tanzi, E., Tarenghi, M., Wilson, R.: 1980, *Monthly Notices Roy. Astron. Soc.* **192**, 561
Yukon, S.P.: 1968, *Astrophys. Letters* **2**, 181
Zheleznyakov, V.V.: 1966, *Zh. Eksp. Teo. Fiz.* **51**, 570; *Soviet Phys.* JETP **24**, 381