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Spiral shocks and accretion in discs

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Summary. Recent numerical and analytical results on disc-like accretion with shock waves as the only dissipation mechanism are compared. The global properties of the process are similar to those of the viscous (α) disc model, but precise values of the effective α value as a function of the accretion rate can be calculated. At low values of the ratio of specific heats ($\gamma<1.45$) accretion is possible without radiative losses. Such adiabatic accretion can occur in practice at high accretion rates on to low mass objects and may be important in the formation of planets. Following Donner, and Lynden-Bell, it is pointed out that non-axisymmetric perturbations in the outer parts of a disc increase in amplitude as they propagate in and cause spiral shocks more easily in a disc than perturbations originating in the inner parts. It is suggested for this reason that the cause of spiral structure in normal spiral galaxies lies in moderate non-axisymmetries in their gaseous outer discs.

1 Introduction

Recent results (Sawada, Matsuda & Hachisu 1986a, b; Sawada *et al.* 1987; Spruit 1987) show that accretion discs around compact objects can support stationary spiral-shaped shock waves. The shocks strip the fluid of some of its angular momentum, so that it can accrete on to the central object. Though the shocks have to be set up by some disturbance in the outer parts of the disc, they extend all the way to the surface of the central object without further external forcing. Thus they provide a mechanism for accretion in the inner parts of the disc that is independent of the details of the perturbation in the outer parts. The numerical simulations by Sawada *et al.* show that shocks are easily set up by the tidal force due to the companion, in discs formed by Roche lobe overflow in a binary. Analytical and nearly-analytical solutions for discs accreting through such shocks, but without forcing by tidal effects of a companion, were found by Spruit (1987). The latter results confirm a conjecture of Lynden-Bell (1974) concerning the existence of self-sustained spiral shocks (as opposed to shocks induced by non-axisymmetries in the gravitational potential) and substantiate earlier work by Donner (1979). The possibility of accretion by shock waves has also been raised by Shu (1976) in the context of accretion in close binaries.

In the calculations of Sawada *et al.* (1986a, b, 1987) radiative losses from the accreting flow were not included. In Spruit (1987) solutions without radiative losses are given that can be compared with the results of Sawada *et al.*, as well as radiating solutions that can be compared with conventional thin (α-) disc theory. These comparisons are made in the present paper. The cooler solutions with radiative loss have more tightly wound and weaker shocks, and accrete less effectively than the adiabatic solutions.

An important aspect of the work mentioned above is that it demonstrates the possibility, under certain conditions, of accretion *without radiative losses* from the disc. The condition is that the ratio γ of specific heats is sufficiently small (below about 1.45). If this is satisfied the temperature increase due to dissipation in the flow is sufficiently low, even without radiative losses, that the gas remains bound to the central object. We call this mode of accretion *adiabatic*, in the thermodynamic sense (no exchange of heat within the fluid or with the surroundings) though it is not *isentropic* because irreversible processes (shocks) take place in the flow. Whereas for γ around 5/3 the accretion rate is limited by the rate of energy loss from the disc surface (see e.g. Pringle 1981), the accretion rate can be arbitrarily high if γ is below the critical value. This adiabatic accretion mode becomes possible when a significant part of an accretion disc has a low value of γ, for example due to molecular dissociation or ionization, and when the accretion rate is sufficiently high.

In the following, we first compare the results of Sawada *et al.* (1986a, 1987) with those of Spruit (1987). We shall find that the results agree where a meaningful comparison is possible. This supports the view that self-similar shocks form naturally in binary accretion on to a compact object, and that close to the accreting object the angular momentum is transported mostly by the shock rather than by the gravitational field of the companion. In Section 3 we describe the physics

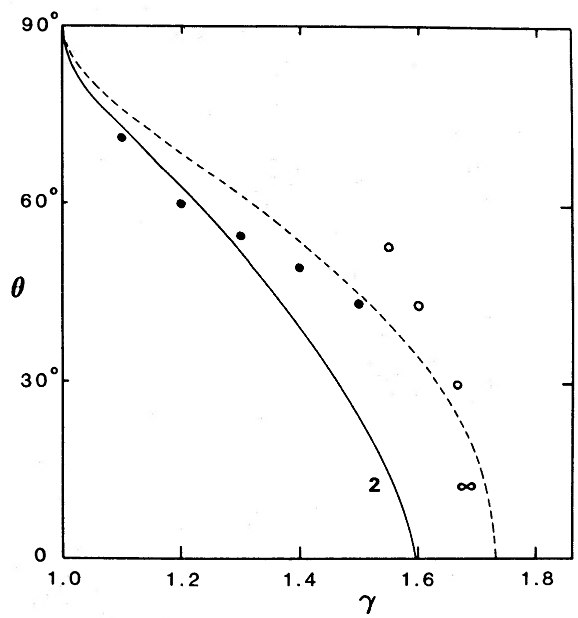

Figure 1. Dependence of the opening angle θ of the shock spiral on the ratio of specific heats γ, for adiabatic accretion. Solid line: self-similar stationary flows with two shocks. Broken line: same in the limit of many shocks. Circles: measured from time-dependent numerical simulations; filled circles: cases where the flow was observed to be stationary, open circles: approximate values for non-stationary cases.

of accretion by shock waves. In particular we discuss how angular momentum is transported by the shocks in the absence of turbulent viscosity or external forces, and how the existence of adiabatic solutions can be understood. In Sections 4, 5 and 6 the shock accretion process is compared with standard viscous disc theory and with the case of spherically symmetric accretion. In Section 7 the possible importance of shock accretion for various astrophysical objects is explored. We shall argue that most of the spiral structure in normal spiral galaxies may well be caused by relatively weak perturbations in the outermost gaseous parts.

In all of the calculations reported below, radiation pressure is explicitly left out. This restricts the applicability to accretion rates well below the Eddington value.

2 Comparison of time dependent and stationary calculations

Since the numerical simulations in Sawada *et al.* were done without radiative losses, we restrict the discussion in this section to the adiabatic case. In Spruit (1987) the calculations were simplified by assuming stationarity from the outset (guided by the observation that the numerical simulations by Sawada *et al.* showed the existence of stationary patterns in several cases) and by ignoring the presence of a companion. Further simplification was possible by restricting attention to *self-similar* solutions. An exact solution was found for the case where the number of shocks is large. For adiabatic shocks, this theory predicts that the opening angle of the spiral-shaped shocks (or equivalently the ratio of the temperature to the virial temperature) is a function *only* of the ratio of specific heats γ. For the case of two shocks, which is the pattern usually excited in the binary simulations, the opening angles are shown in Fig. 1. In the simulations the opening angles vary with distance to the central object. The values in Fig. 1 were measured close to the central object. The variation of the shock Mach number with distance is illustrated in Fig. 2.

For small γ the opening angles found in the simulations agree well with the self-similar results and the simulations show the decrease of θ with increasing γ expected from the self-similar results. They show time dependent behaviour for γ's of 1.55 and larger, while the self-similar calculations predict that no stationary solutions with two shocks exist for $\gamma>1.595$. For $1.595<\gamma<\sqrt{3}$ stationary solutions with more than two shocks are possible; indeed the simulations sometimes show three shocks in this parameter range (see fig. 5 in Sawada *et al.*). Such patterns are not stationary however but alternate with a two-shock pattern. This is presumably because of

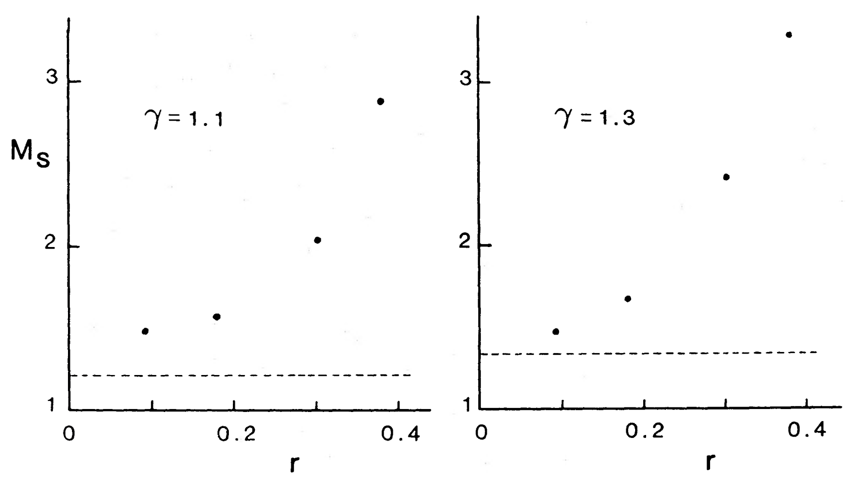

Figure 2. Dependence of shock strength on distance (r) to the accreting object. Broken line: self-similar shocks. Dots: time-dependent numerical simulations. The radius of the critical Roche surface (measured in the direction of the outer Lagrange point) is at r=0.5.

the strong twofold structure of the tidal field of the secondary. The accretion is also less effective above $\psi=1.55$, with a significant amount of mass being lost through the outer Roche lobe.

The critical values of γ depend on the way in which the variation of disc thickness with radius is included in the continuity equation. The results of this section were obtained by assuming a constant disc thickness. Probably, a more realistic assumption is to take the disc thickness proportional to the scale height derived from the temperature of the disc. This was done for the results reported in Section 4.

The largest differences between the calculations are found in the strength of the shocks (Fig. 2). Especially in the outer parts of the disc the shocks are significantly stronger in the simulations than expected for self-similar shocks. Towards the central object the strengths agree better. These differences should perhaps be expected, given that the self-similar calculations do not include the tidal field of the secondary, which forces the flow strongly in the outer parts of the disc. As discussed in Section 3, we expect that any non-axisymmetric perturbation in the outer parts of a disc will converge to a self-similar shock as it propagates in towards the accreting object provided the object is sufficiently small compared with the disc size. The results in Fig. 2 are consistent with this. We conclude that the agreement between the two sets of results is as good as may be expected, given the differences in the physics included.

3 Physics of accretion by shock waves

Accretion by shock waves obviously has strong connections with theories of spiral structure in galaxies. Numerical simulations of shocked gas flows in galaxies were done by several authors. For recent results and references see Mulder (1986), van Albada (1985), Matsuda *et al.* (1987). The self-similar shock calculations by Spruit (1987) are related most directly with the work of Donner (1979). The reader is referred to Donner's work for more extensive discussions of various points raised in this section.

Consider a disc with a (possibly weak) source of non-axisymmetric perturbations in its outer parts. This source could be a companion, an instability in the disc, or turbulence due to convection, for example. As such a perturbation propagates inward, it is sheared by differential rotation into a trailing spiral pattern. When the disc is sufficiently massive that self-gravity is important, these trailing waves can transport angular momentum outward directly by gravitational interaction, allowing part of the mass to move closer to the centre. Larson (1984) has proposed that this mechanism is important in the outer parts of star-forming discs.

Waves in a disc in general carry angular momentum (Toomre 1969; Shu 1970; for a recent discussion of wave angular momentum and the related wave action, various ways of defining these quantities in discs and their relation to instabilities see Narayan, Goldreich & Goodman 1987). As a result, waves can still transport angular momentum in the absence of self-gravity provided that a dissipative mechanism exists that allows them to transfer some of their angular momentum to the fluid. This can be made more quantitative by considering the wave's angular momentum. In the WKB (high-frequency) limit, the quantity

$$\tfrac{1}{2}\varrho v^2 m/\sigma \quad (1$$

can be identified with the angular momentum density of the wave. Here ϱ is the mass density, v the wave's velocity amplitude, m its azimuthal order [$v\sim\exp(im\phi)$], and σ the wave frequency *measured in a frame comoving with the fluid.* The integral of this quantity over a wave packet (confined to a sufficiently narrow range in radius) can be shown to be a constant during the packet's propagation, in the absence of dissipation (for a general discussion see Lighthill 1978). Near corotation (where $\sigma=0$) the identification of expression (1) with the wave angular momentum breaks down. Consider an inward propagating wave (in the sense that its group speed i directed inward), inside corotation ($m/\sigma>0$). Then in the frame of the fluid the azimuthal propagation of the wave is opposite to that of the fluid. Inward travelling perturbations set up at the outer boundary of the disc by a slowly rotating external agent (like a companion) or by corotating irregularities in the disc itself (as in the outer parts of spiral galaxies or protostellar discs) correspond to this case. Since m/σ is negative, the angular momentum of the wave is also *negative*. Though this may seem counterintuitive (since the pattern speed of the wave in an inertial frame may still be in the same sense as the fluid motion), it is easily understood when (weak) interaction between the wave angular momentum and the angular momentum of the fluid is considered (*cf.* Toomre 1969). Since the wave travels opposite in azimuthal direction to the fluid (in a corotating frame), any dissipation in the wave will slow the fluid down. Overall angular momentum conservation then implies that the wave's angular momentum has increased. But since its amplitude has decreased due to the dissipation, its angular momentum must have been negative. Thus inward propagating, trailing waves (i.e. inside corotation) can take up angular momentum by dissipating, and thereby allow mass to accrete through the disc.

We now show that such dissipation is also a natural consequence, for *inward* propagating, trailing waves (*cf.* Donner 1979). The volume occupied by the wave packet is $4\pi rH\Delta$, where H is the disc half-thickness and Δ the radial extent of the wave packet. The leading and trailing edges of the packet move with the local sound speed c, so that Δ is proportional to c. With expression (1) the conservation of wave angular momentum then yields

$$rHc\varrho v^2 m/\sigma=\text{const.}$$

Well inside corotation, σ is of the order $m\Omega$, where Ω is the orbital frequency, and hence varies as $r^{-3/2}$. The variation of v^2 with distance r determined by this equation depends on the disc structure ϱ, c, $H(r)$, but in general the velocity amplitude must increase inward. For example, for a self-similar disc structure (*cf.* Section 5) we have

$$\varrho\sim r^{-3/2}; \qquad c\sim r^{-1/2}, \qquad H\sim r,$$

so that

$$v\sim r^{-3/4}.$$

Hence the amplitude increases inward as long as the wave does not dissipate. Given enough room (in the sense that the inner radius of the disc is sufficiently small compared with the outer radius) the wave will eventually form a shock, which increases in amplitude until further steepening is balanced by dissipation in the shock. A balance is then reached at some radius, and the strength of the shock inside this radius is *independent* of the initial amplitude of the perturbation. It is this behaviour of inward propagating waves that makes it possible to construct consistent disc models with shocks in which the disc structure and the accretion process are determined entirely by the shocks. In practice, the inner radius of the disc is finite, so that a finite amplitude perturbation is required at the outer edge for a shock to form before the wave reaches this inner radius.

Note that the *opposite* happens to a wave generated by a slowly rotating perturbation in the *inner* parts of the disc [see Michel (1984) for such a process in the context of accreting neutron stars]. Here, the wave frequency σ decreases outward and conservation of wave angular momentum makes the amplitude decay as the wave travels outward towards the corotation radius. For waves generated near the disc centre by a *rapidly* rotating perturbation the situation is a little more complicated. Suppose the wave is generated by a perturbation that rotates with the local orbital frequency Ω_0. As the wave propagates outward, the wave's frequency σ increases and asymptotically approaches $m\Omega$. Initially, the amplitude will then increase outward, but at larger distances the wave may either amplify or decrease depending on the details of the density variation in the disc. The spirals generated in galaxies by a central bar correspond to this case.

Though numerical simulations produce satisfactory spiral structure in this case (for recent result: see van Albada 1985; Matsuda *et al.* 1987), spiral structure may be explained more readily ii many cases by inward propagating perturbations because they amplify under a wider range o circumstances and do not require a bar to be present.

Consider again a perturbation generated in the outer parts of the disc. Let r_0 be the radia distance where the wave becomes a shock. If the disc is sufficiently large compared with the size *r* of the accreting object, and the initial amplitude of the perturbation large enough, there is a rang of radial distances r such that $r_i \ll r \ll r_0$. Suppose that the pattern speed of the wave (measured ii an inertial frame) is small compared with the local rotation rate (consistent with the wave's origi in the slowly rotating outer parts). The equations governing such nearly stationary flow aroun the central object contain no explicit length or time-scales, and allow for the existence of *self similar* solutions. In the range $r_i \ll r \ll r_0$ the wave does not depend on the conditions at r_i and r_0 hence it must in fact be one of these self-similar solutions. Thus we have made plausible tha waves generated in the outer parts of large discs lead, in the inner parts, to accretion by self similar, spiral-shaped shock waves. The generation of stable spiral shocks in the time dependen calculations by Sawada *et al.* substantiates this picture.

Motivated by arguments of Lynden-Bell (1974, last paragraph), which are akin to the above self-similar waves were first considered by Donner (1979), who also calculated numerically som shock solutions. His calculations were intended especially for spiral galaxies. They include viscosity, and do not address explicitly the case of accretion by shock waves on to a compac object. They also contain an equation of state which is somewhat artificial for an accretion disc Spruit's (1987) calculations use more realistic equations of state and energy and give som analytical shock wave solutions, but use the gravitational field of a central point mass and do no include viscosity.

Donner also discusses the problem of how to generate waves in the outer parts of a disc galaxy It seems to us that irregularities in the mass distribution of the gas seen extending beyond th luminous disc in spiral galaxies (Bosma 1981) could be quite adequate as a source of suc perturbations.

4 Accretion with and without radiative losses

A striking aspect of the results of Sawada *et al.* is the apparent ability of the flow to accret adiabatically, i.e. without radiative loss. This may seem to be at variance with conventional thi disc theory, where one finds that all the energy dissipated during the accretion is radiated away a the disc surface. The self-similar results of Spruit however show that the global properties of flow accreting by shocks are very much like those of α-discs. In Section 5 we show how these parts o accretion theory fit together but before doing so we illustrate the case with some results for self similar shocks with radiative loss.

In Fig. 3 the disc temperature is shown as a function of the accretion rate, for radiating self similar shock accretion, in which the disc thickness is taken proportional to the vertical pressur scaleheight. (This is different from the calculations in Sawada *et al.*, where the disc thickness wa effectively taken constant.) The dimensionless accretion rate shown ($\dot{m}$) is related to the physica accretion rate $\dot{M}$ through

$$\dot{M} = K\dot{m}, \tag{2a}$$

$$K = 4\pi^{1/2}\tilde{\sigma}^{1/2}(\gamma-1)^{1/2}(\mu/R_g)^2(GM)^{7/4}\varkappa_0^{-1/2}r_0^{-1/4}, \tag{2b}$$

where M is the mass of the central object, $\tilde{\sigma}$ is the radiation constant, μ the mean molecula weight, R_g the gas constant, r_0 the radial distance at a representative point of the disc and $\varkappa_0$ th opacity there. The relation between disc temperature and accretion rate is not very sensitive t

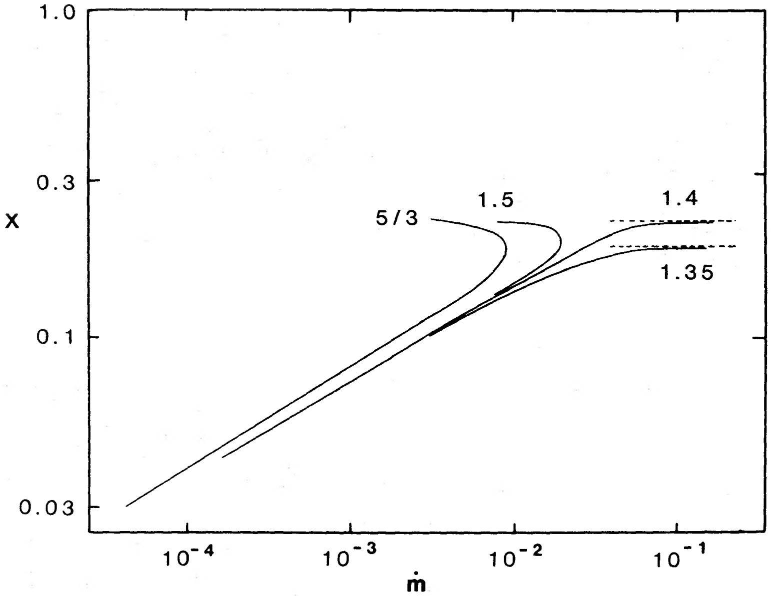

Figure 3. Disc temperature [in units of the virial temperature $\mu GM/(rR_g)$] as a function of the dimensionless accretion rate $\dot{M}/K$, for self-similar shock accretion. Parameter is the ratio of specific heats γ. Solid lines: including radiative loss. Broken line: adiabatic accretion.

the value of γ at low rates, but the behaviour at large rates depends strongly on γ. The figure shows that for γ above the critical value, which is $\sqrt{6}-1=1.45$ for this case [see Spruit (1987) for details] there is a maximum above which the accretion cannot be increased. This maximum depends on γ. Below the critical γ, the disc temperature reaches a finite value as the accretion rate tends to infinity. At these high accretion rates the flow is optically so thick that radiative losses play no role, and the solution asymptotically approaches an adiabatic solution (horizontal lines in Fig. 3). As shown below, this behaviour is also present in α-discs.

5 Relation to viscous disc theory

The α-disc model has properties quite similar to those shown in Fig. 3. In this model a viscosity ν is assumed, given by

$$\nu=\alpha\Omega H^2, \tag{3}$$

where Ω is the local Keplerian rotation rate, H the pressure scaleheight and α a dimensionless function of the accretion rate. The scaleheight is related to the isothermal sound speed c by

$$c^2=P/\varrho=\tfrac{1}{2}\Omega^2H^2. \tag{4}$$

The energy equation is

$$\varrho T\frac{dS}{dt}=-\mathrm{div}\,F_{\mathrm{rad}}+\varrho Q, \tag{5}$$

where ϱ is the density, S the specific entropy, F_{rad} the radiative flux, and Q the viscous dissipation per unit mass, which for a Keplerian disc is given by (e.g. Pringle 1981)

$$Q=-\tfrac{9}{8}\nu\Omega^2. \tag{6}$$

Assuming stationarity, and denoting the radial flow speed by u, the equations of motion yield (e.g. Pringle 1981)

$$u=-\tfrac{3}{2}\nu/r, \tag{7}$$

and the energy equation becomes

$$\varrho T u \frac{\partial S}{\partial r} = -\text{div } F_{\text{rad}} + \tfrac{9}{8}\varrho \nu \Omega^2. \tag{8}$$

For thin discs ($H/r \ll 1$) the left-hand side is small compared with the last term on the right, and is usually ignored. The terms on the right-hand side must then balance, i.e. all the energy dissipated is radiated away at the disc surface. We keep the term on the left side here. To solve equation (8) we need an expression for the radiative flux. We assume radiative energy transport. Since most of the energy is dissipated near the midplane, the radiative energy flux is roughly constant with height. Assuming $\varkappa$ to be roughly constant with height in the disc as well, this yields a relation between the surface temperature and the temperature at the midplane:

$$H \text{ div } F_{\text{rad}} = \tilde{\sigma} T_s^4 = \tfrac{4}{3}\tilde{\sigma} T^4/(\varkappa\Sigma). \tag{9}$$

where Σ is half the mass column density, measured perpendicular to the disc plane. By integrating the density over the disc thickness, its value is found to be of the order

$$\Sigma = H\varrho. \tag{10}$$

It is related to the mass accretion rate by

$$\dot{M} = 6\pi\nu\Sigma. \tag{11}$$

Using (11) to eliminate Σ, the energy equation can be written as

$$\dot{M}^2\left(Tr\frac{dS}{dr} + \frac{3}{4}\Omega^2 r^2\right) = 32\pi^2 \frac{\tilde{\sigma} T^4}{\varkappa} \nu r^2. \tag{12}$$

This equation relates the disc temperature to the accretion rate. The radial dependence of T is particularly simple if we assume that the opacity varies like

$$\varkappa \sim r^{-1/2}. \tag{13}$$

The disc is then self-similar, and the dependences on r are

$$H \sim r, \qquad P \sim r^{-5/2}, \qquad \varrho \sim r^{-3/2}. \tag{14}$$

With

$$S = c_v \ln(P/\varrho^\gamma). \qquad c_v = \frac{R_g}{\mu(\gamma-1)}, \tag{15}$$

$\dot{M}$ can be written as:

$$\dot{M}^2\left(x\frac{\gamma-5/3}{\gamma-1} + \frac{1}{2}\right) = \frac{2}{3}(8\pi)^2 \tilde{\sigma}(\mu/R_g)^4 (GM)^{7/2} \varkappa^{-1} r^{-1/2} \alpha x^5, \tag{16}$$

where

$$x = \frac{c^2 r}{GM} \tag{17}$$

is the (constant) ratio of disc temperature to the virial temperature. With equation (2) this can also be written as

$$\dot{M} = (8\pi/3)^{1/2} K \alpha^{1/2} x^{5/2} [x(\gamma-5/3) + (\gamma-1)/2]^{-1/2}. \tag{18}$$

In this relation, as well as in all of the above, α may be a function of x, to be determined by the

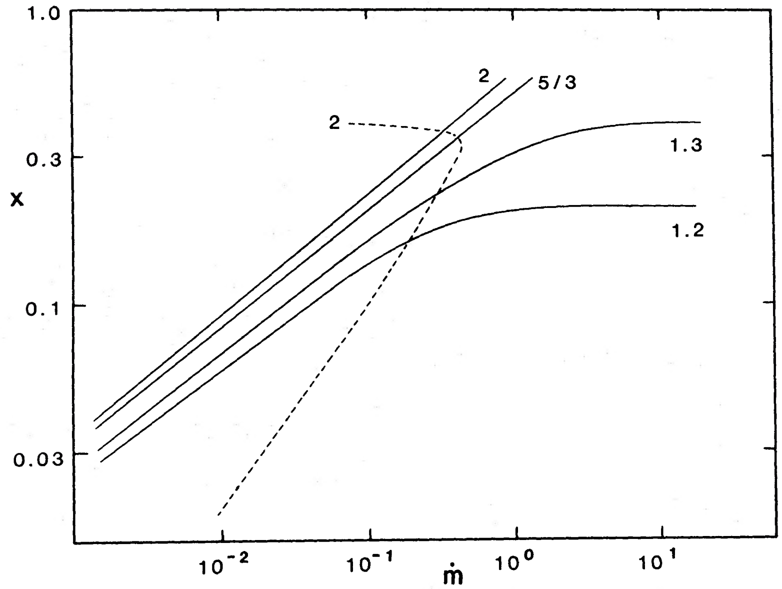

Figure 4. Same as Fig. 3, but for the α-disc model, with $\alpha=1$. Dotted line gives the corresponding line for spherical accretion with $\gamma=2$.

theory which specifies the dissipation mechanism. For a fixed $\alpha=1$ the relation is shown in Fig. 4 in the form of a dimensionless accretion rate $\dot{m}=\dot{M}/K$. The relation is quite similar to that for accretion by shocks (equation 2, Fig. 3), with the important difference that $\alpha(x)$ is a known function in that case. Its dependence on x accounts for the difference in slope between Figs 3 and 4. The behaviour at high accretion rates is similar in both cases: below a critical value of γ the disc temperature reaches an asymptotic value below the virial temperature as the rate is increased. Though the thin disc assumptions break down at high temperatures (both for the α-disc and the shock model), we show in the following that the behaviour shown in Figs 3 and 4 is generic and not an artefact of the thin disc assumption.

Consider first under which conditions relation (16) allows adiabatic accretion. This amounts to setting the Stefan–Boltzmann constant $\tilde{\sigma}$ equal to zero. For $\dot{M}\neq 0$ this yields

$$x=T/T_{\rm virial}=\frac{1}{2}\,\frac{\gamma-1}{5/3-\gamma}. \tag{19}$$

Adiabatic accretion is therefore possible only for $\gamma<5/3$. For γ sufficiently close to unity, adiabatic accretion is possible with low temperatures ($x\ll 1$). Then, with radiative loss, the disc temperature approaches expression (19) asymptotically from below as $\dot{M}$ is increased. Thus for low γ, the disc is thin, and the above analysis valid, for *all* accretion rates. For $\gamma>5/3$ this is not the case and instead the temperature reaches the virial temperature at a finite accretion rate.

Together this shows that for accretion with angular momentum there exists a maximum γ below which accretion is possible at arbitrary rates, while above this value of γ there exists a maximum to the accretion rate. To determine the precise value of this critical value of γ, a specific model for the dissipation is needed, and discs of finite thickness must be considered.

The reason for the existence of a critical γ is the same as in the case of spherical accretion, even though the details of the accretion process are quite different.

6 Relation to the case of spherical accretion

This problem has already been studied by Bondi (1952), who showed existence of a critical value $\gamma=5/3$. In the spirit of the calculations above, assume that the opacity varies as $r^{-1/2}$ and assume stationary accretion. For the heat flux we use the diffusion approximation:

$$F_{\rm rad}=-\frac{4\sigma T^3}{3\varkappa\varrho}\frac{dT}{dr}. \tag{20}$$

For $\gamma>5/3$ there exist again self-similar exact solutions of the equations of motion, continuity and energy (equation 8 with $\nu=0$), which combine to yield a relation between the temperature and the accretion rate:

$$\dot{M}=2/3(2\pi)^{1/2}(\gamma-5/3)^{-1/2}Kx^{3/2}(2-5x)^{1/4}, \qquad (\gamma>5/3) \tag{21}$$

where x is again the ratio of temperature to the virial temperature, and K is given by equation (2), as before. This relation is shown in Fig. 5. There is a maximum accretion rate:

$$\dot{M}_{\rm max}=0.19(\gamma-5/3)^{-1/2}K, \tag{22}$$

which is reached for $x=12/35$. This is evidently the same limit as applies to the shock and viscous accretion models. For $\gamma<5/3$ no stationary self-similar solutions exist in the spherical case, and as a result the physics of spherical accretion (Bondi 1952; Begelman 1979; Freihoffer 1981) is somewhat different from that of accretion with angular momentum, for these values of γ.

7 Role of the Eddington limit – applications

In the foregoing sections we have found a characteristic accretion rate, K, which plays the following roles. For γ *below* a critical value (which is near 5/3), it is the rate *above* which the accretion proceeds essentially adiabatically. For γ above the critical value it is the maximum possible accretion rate with angular momentum.

Since γ is at most equal to 5/3 in most astrophysical accretion problems, the above would imply that in practice no limits would exist on the rate at which a gravitating object can accrete. At high rates the process would just become adiabatic. In the analysis we have neglected radiation pressure, however, so that the Eddington limit does not appear anywhere in the formulation. It turns out that for most astrophysical situations the Eddingtoin limit is far smaller than the characteristic rate K, which limits the applicability of the above discussion somewhat. Radiation pressure can probably be included easily in numerical calculations of shock accretion, but general analytic results are difficult to get because there are no self-similar solutions any more.

By comparing the characteristic rate K with the Eddington rate

$$\dot{M}_{\rm E}=4\pi Rc/\varkappa, \tag{23}$$

where R is the size of the accreting object and c the speed of light, we find that radiation pressure has to be included before K is reached, for accretion on to relativistic objects, white dwarfs, and main-sequence stars. From the form of K it is clear that one must look for low mass objects. Indeed, for *planets* the Eddington limit is larger than K. For example, with $M=10^{-5}M_\odot$, $r=2\times10^9$, $\varkappa\approx1$, $\mu=2$, we have

$$K\approx1.7\times10^{-8}\,M_\odot\,{\rm yr}^{-1}, \tag{24}$$

whereas the Eddington rate is

$$\dot{M}_{\rm E}\approx1.1\times10^{-5}\,M_\odot\,{\rm yr}^{-1}. \tag{25}$$

Since the molecular gas accreted from the protoplanetary cloud also has a low γ, it seems that adiabatic accretion, for example by shock waves, may be an important process in the formation of planets.

The process of accretion by shocks (including radiative loss) is significant in general for the theory of accretion discs by providing a reliable *lower* limit to the effective value of α, at least for 'live' accretion discs (those for which there is a mechanism adding mass at the outer edge). In those cases, non-axisymmetric perturbations will be set up at the outer edge unless the mechanism providing the mass to the disc is perfectly symmetric. The intrinsic increase of these perturbations as they travel in (due to wave action conservation) means that the shocks discussed in this paper will eventually form if the size of the central object is small enough (*cf.* Section 2). For the accretion rates found in Cataclysmic Variables for example, self-similar shock accretion would have α ranging from 10^{-4} to 10^{-3} (Spruit 1987). In the outer parts of these discs, the value would be higher, because the tidal field of the companion maintains higher shock strengths there (Fig. 2).

An interesting application of these ideas would be to normal (unbarred) spiral galaxies (*cf.* Lynden-Bell 1974). In these galaxies the case for driving of the spiral pattern by a non-axisymmetric gravitational potential does not seem very convincing. Whereas a source outside the visible disc such as that produced in M51 by its companion would almost certainly work, it is important to realize that perturbations of much smaller magnitude may work almost as well, because inward propagating trailing waves amplify due to conservation of wave action. Random motions in gas outside the visible discs of galaxies may well be sufficient. This possibility is especially relevant because of abundant evidence (e.g. Bosma 1981) that the gaseous discs of spiral galaxies extend far beyond the luminous discs. In this interpretation the gas in the outer parts of the disc contains no stars just because in this region the inward propagating perturbations have not yet steepened into the shocks that produce young stars.

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