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# Is the standard accretion disc model invulnerable?

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**Summary.** Two-dimensional hydrodynamic calculations of a gas flow in a semi-detached close binary system with mass ratio unity are carried out again, using a different coordinate system from our previous work (Sawada, Matsuda & Hachisu). The Euler equation is solved using the second-order Osher scheme in a multi-box type of grid, which gives a high resolution about a mass-accreting compact object.

Spiral-shaped shock waves in the accretion disc are found to extend down to $r=0.01A$ , where $r$ and $A$ are the radial distance from the compact star and the separation of two stars respectively. It means that the tidal effect by the mass-losing star is important even so close to the compact object. It is also confirmed that the gas particles lose their angular momentum at the shocks and can spiral in without the help of a turbulent viscosity.

The fundamental assumptions of the standard accretion disc model, i.e. an axisymmetric thin disc, the important role of the turbulent viscosity etc., are questioned.

## 1 Introduction

The standard accretion disc model was proposed by Shakura (1972), Pringle & Rees (1972) and Shakura & Sunyaev (1973) and has been developed extensively by many workers (see reviews by Pringle 1981, Petterson 1983, Frank, King & Raine 1985 and Hoshi 1985).

The basic assumptions underlying the standard accretion disc model are as follows (Petterson 1983).

(1) The disc is axisymmetric. This implies that the gravitational influence of the mass-losing star is neglected.

(2) The disc is thin. Its half-thickness $h$ satisfies the inequality $h/r \ll 1$ at every radius $r$. Assuming that the disc is in a hydrostatic balance in the $z$ direction, we require that the local Kepler velocity should be highly supersonic for the disc to be thin (Frank *et al.* 1985).

(3) Molecular viscosity influences the flow only slightly because of the high Reynolds number, and therefore (3a) the disc is highly turbulent and (3b) radial inflow of the gas in the accretion disc is caused by the turbulent stress, whose component is given by

$$\sigma_{r\phi} = \eta r \frac{\partial \Omega}{\partial r}, \tag{1.1}$$

where $\eta$ and $\Omega$ are the turbulent viscosity coefficient and the angular speed of the gas around the compact object respectively. We can add the $\alpha$ model of Shakura & Sunyaev (1973) to the basic assumptions.

(4) The disc is stationary.

In our previous papers (Sawada, Matsuda & Hachisu 1986a, b) we showed the following.

(a) There exist spiral-shaped shocks on an accretion disc formed about a compact object in a semi-detached close binary system. The shocks seem to penetrate down to $r=0.03A$, where $r$ is the radial distance from the compact object and $A$ is the separation of two stars.

(b) The gas particles lose their angular momentum at these shocks and a fraction of gas can spiral in towards the compact object without the help of a conventional turbulent/magnetic viscosity.

If we accept result (a), the accretion disc is no longer axisymmetric and assumption (1) does not hold. It has been believed that the tidal effect of the mass-losing star is effective only at the outer part of the accretion disc. However, our result shows that it is important even at $r=0.03A$, which is the radius of our inner boundary.

The reason why earlier workers (Prendergast & Taam 1974; Flannery 1975; Sorensen, Matsuda & Sakurai 1975; Lin & Pringle 1976; Sorensen 1976; Hensler 1982) failed to find such shocks was that their numerical methods were too dissipative because of an excessive artificial viscosity. For example, Sorensen *et al.* (1975) used the FLIC code, which is a simple donor-cell method and has only a first-order accuracy in space. It has been realized that first-order schemes often need impossibly high resolution to give reliable results, and therefore higher-order schemes are more economical (van Albada 1985; Mulder 1985; Rozyczka 1985). The method used in our previous paper was the second-order Osher scheme.

If the existence of these spiral shocks is accepted it follows that subsonic or low Mach number pockets may exist just behind them. If this is the case, assumption (2) may no longer be valid and therefore the disc may be thick in the pocket.

Assumption (3a) is also questionable since there is no proof that the accretion disc is fully turbulent despite the high Reynolds number. These discs are textbook examples of flows which satisfy Rayleigh's stability criterion (Safronov 1969; Pringle 1981; Petterson 1983). Since the Rayleigh criterion deals with only axisymmetric modes in incompressible inviscid fluids, it is hoped that the higher modes for compressible fluids will become unstable, although it is not possible to derive a simple local dispersion relation for these modes.

It is worth noting here that one theory of the origin of the Solar System assumes that the solar nebula surrounding the proto-Sun is not turbulent (Safronov 1969; Hayashi 1981). The dynamics of the solar nebula and the accretion disc are essentially the same, except for the fact that the latter is affected by the tidal force due to a companion star.

Even if we assume that the disc is turbulent, there is no proof that the turbulent stress is given by equation (1.1). It tries to reduce an angular velocity gradient in the flow. In doing so, the angular momentum is transferred outwards in a Keplerian disc. Equation (1.1) would be correct if the turbulence were isotropic, but this is questionable in a rotating shearing disc.

Prandtl's semi-empirical mixing-length approach to turbulence gives a formula which reduces the angular momentum difference in the flow by particle exchange between different orbits. This formula is obtained because it would be more natural to assume that the fluid element conserves its angular momentum rather than its angular velocity during the exchange. If this is the case, the angular momentum of the gas is transferred inward rather than outward. These problems have been fully discussed by Safronov (1969).

There is a mechanism called 'negative viscosity' by which the angular velocity difference is enhanced rather than reduced (Starr 1968). This phenomenon is caused by coupling between shear flows and turbulent eddies. Ando (1985) discussed a coupling between a rotation and waves and concluded that the angular velocity difference is enhanced rather than diminished. This is also an example of negative viscosity phenomena. Negative viscosity phenomena seem to be occurring on stellar surfaces. If the negative viscosity is operative in the accretion disc, then angular momentum is also transferred inward. Even if we use the $\alpha$ prescription of the turbulent viscosity, we should ask whether $\alpha$ is positive at all.

If angular momentum is transferred inward, it would drive near-Keplerian flows towards a uniform angular momentum distribution. We would have a narrow ring rather than a spreading disc. Such a mechanism would not provide an accretion disc, whose existence is confirmed rather well observationally. The assumption that turbulence is the main viscous mechanism in discs is still problematic (Petterson 1983).

If we accept our result (b), we have an alternative mechanism for the angular momentum loss of the spiralling gas. The gas element hitting the shock gives its angular momentum directly to the orbital angular momentum through a gravitational interaction; the spiral-shaped density enhancements behind the shocks produce a torque on the two stars. If this is the case, assumption (3a) and (3b) may be unnecessary.

In our paper we neglect the magnetic effect. However, it would be useful to touch upon the effects of magnetic fields. If the compact object has a strong magnetic field, the accretion disc would not be formed (at least inside the Alfvén radius). Efforts have been made to derive the $\alpha$ model in a turbulent magnetic disc. The existence of turbulence is also essential in this mechanism.

Our numerical solutions show an oscillatory nature in one case, which therefore contradicts assumption (4). However, it is not clear whether this is significant with respect to the structure of the accretion disc. This point has yet to be clarified.

In view of the above discussions, it is important to confirm our previous result. The following criticisms of our results could be made.

(1) The numerical grid used in our previous work is too coarse to exclude the possibility that the numerical viscosity is the main cause of the angular momentum transfer.

(2) The radius of our 'compact object' is not really small enough to represent real compact objects.

(3) The cooling effect is not taken into account.

(4) The three-dimensional effect is not taken into account.

With regard to criticism (1), we performed a calculation on the grid using a half-mesh spacing and concluded that the essential features were not altered (Sawada *et al.* 1986a). Nevertheless, it is true that our numerical domain is too large to give a fine resolution near the compact object. Our O-type grid enclosing both the mass-losing star and the mass-accreting star might not be suitable for giving a fine resolution about the compact object. One purpose of the present paper is to remove this defect and to give a fine resolution, particularly close to the compact object, by using a multi-box type of grid. This will be discussed in Section 2.

An important question relating to criticism (2) is how deep the spiral shocks penetrate toward the mass-accreting compact object. Non-axisymmetry of the Roche potential about the mass-accreting object, which is the cause of the appearance of these shocks, diminishes towards the compact object. However, the velocity of gas flows increases. If the spiral shocks vanish before they reach the surface of the compact object or the Alfvén surface, we still require a conventional mechanism such as viscous interaction for the accretion to occur. If they reach either of these surfaces, the possibility of a new accretion mechanism arises. The radius of the compact star, which was assumed to be $0.03A$ in the previous work, is reduced to $0.01A$ in the present work.

The separation of two stars in close binary systems is given by the formula (Frank *et al.* 1985)

$$A=2.9\times10^{11}M_1^{1/3}(1+q)^{1/3}P_{\rm day}^{2/3}\,{\rm cm}$$

$$A=3.5\times10^{10}M_1^{1/3}(1+q)^{1/3}P_{\rm hr}^{2/3}\,{\rm cm}, \tag{1.2}$$

where $M_1$, $q$ and $P$ are the mass of the primary star in solar masses, the mass ratio and the rotational period respectively. This gives a typical separation of $10^{10}$–$10^{11}$ cm for close binaries with a period of hours or days.

Since we set the radius of the inner circular numerical boundary (our compact object) to be $0.01A$, it corresponds to $10^8$–$10^9$ cm. A typical radius of a white dwarf is about $10^9$ cm, and our 'compact star' corresponds to a white dwarf.

If the compact star has a magnetic field, its effect is dominant up to the radius $r_{\rm M}$ given by (Frank *et al.* 1985):

$$r_{\rm M}=5.1\times10^8\dot{M}_{16}^{-2/7}M_1^{-1/7}\mu_{30}^{4/7}\,{\rm cm}, \tag{1.3}$$

where all parameters on the right-hand side are quantities of the order of unity. If the compact star is a neutron star with a magnetic field of $10^{12}$ G, the Keplerian disc can extend only down to $5\times10^8$ cm, which corresponds to the radius of our compact star.

With regard to criticism (3), a cooling effect was simulated by using a lower value of the specific heat ratio, i.e. $\gamma=1.2$ (Sawada *et al.* 1986a). We found that the fraction of the gas accreted on to the compact object was rather sensitive to the choice of $\gamma$. The lower the value of $\gamma$ is, the larger is the accretion rate, because the shocks become stronger. Some 60–90 per cent of the gas ejected from the mass-losing star was accreted on to the compact object in the case $\gamma=1.2$. In the present paper we compute two cases: $\gamma=1.2$ and $\gamma=5/3$.

Full three-dimensional calculations are yet to be performed. In the present paper we compute a half-thickness $h$ of the disc by assuming a hydrostatic balance in the $z$ direction based on our two-dimensional calculation, although such a calculation may be inconsistent with our basic assumption of two-dimensionality.

## 2 Method of calculation

### 2.1 ASSUMPTIONS AND PARAMETERS

We consider a semi-detached binary system with a mass ratio of unity. Viscous effects, magnetic effects and radiative cooling and heating are all neglected. We consider only flows in the equatorial plane so that the two-dimensional Euler equation is solved.

The system separation is taken as a typical length scale, and the reciprocal of the angular velocity is taken as a time-scale. Density is normalized by the value of the gas ejected from the mass-losing component.

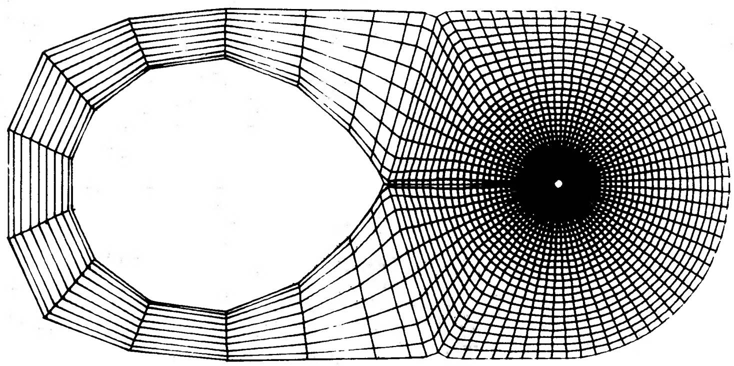

**Figure 1.** The multi-box type of grid used in the present work. It has 73 circumferential and 38 radial grids.

The parameters characterizing the flow are the sound velocity of the gas ejected from the mass-losing component, which is chosen to be $c_0$=0.1, and the velocity of gas inside the star surface, which is $u_0$=0. The density of the circumferential gas which fills up interstellar space at the initial moment is taken to be as low as $10^{-5}$ so that it does not affect the subsequent evolution. The temperature of this gas is assumed to be 10 times hotter than the value of the ejected gas in order to maintain stability at the interface of the two gases.

### 2.2 MULTI-BOX GRID

The coordinate system adopted in the present calculation is the so-called multi-box or multi-zone type shown in Fig. 1 (Rai 1985). The system has 73 circumferential and 38 radial grids. The

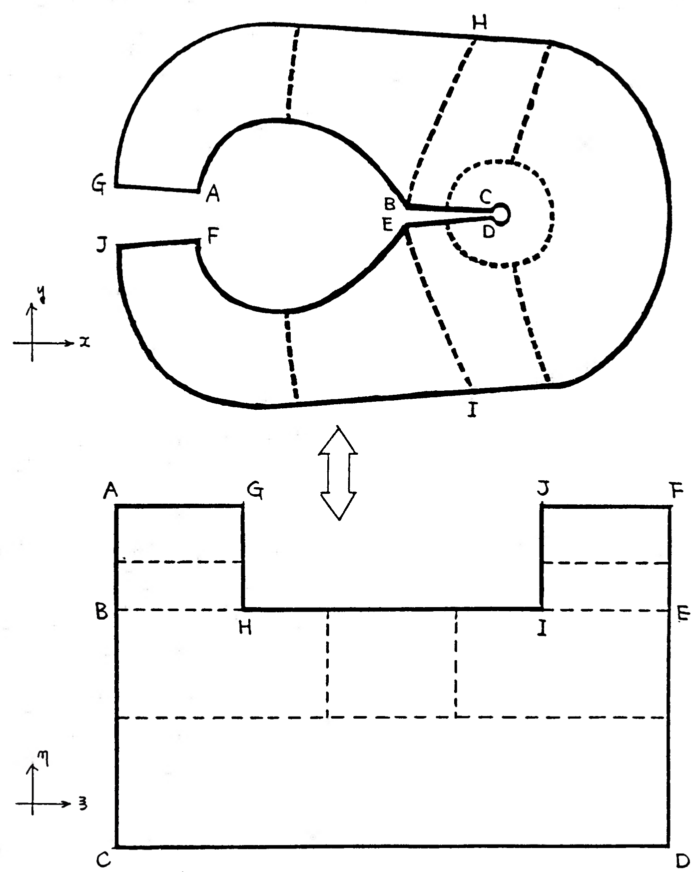


**Figure 2.** The correspondence between the physical space and the numerical space.

correspondence between the physical space and the numerical space is shown in Fig. 2. Rather a complicated mapping is adopted to preserve the grid qualities about the mass-accreting component where hypersonic circular flows are expected. It appears rather coarse about the mass-losing star. However, since our attention is concentrated on the mass-accreting component, it should give a satisfactory result.

The shortest mesh spacing is about $10^{-3}A$ in this grid system. Assuming the typical length to be $A$ and the typical speed to be $A\Omega$, we have a mesh Reynolds number of the order of $10^6$ in a second-order scheme. Although this seems to be an overestimate, we still have a value of $10^4$. Therefore we can conclude that the effect of numerical viscosity is not significant.

### 2.3 NUMERICAL METHOD

The numerical method employed is the second-order Osher scheme described in the previous work (see also Sawada *et al.* 1986 for details).

We assumed a vacuum state in the mass-accreting star. The boundary condition on the surface of the mass-accreting object is computed by solving a Riemann problem between this vacuum state and the state just outside the compact object. It is possible to adopt boundary conditions using a simple extrapolation procedure. However, this procedure sometimes causes a mass outflow from the mass-accreting star. In order to avoid this difficulty, we adopt the mass-sucking condition on the compact object. We also assume the mass-sucking condition at the outer numerical boundary.

Starting from an initial state in which the space is filled by the thin circumstellar gas, we follow the evolution up to about 4–5 periods of revolution. In the previous work we found that it took about 1 period of revolution for the accretion disc to reach an almost steady state.

Since we use an explicit time integration scheme and the minimum mesh spacing is very narrow, this procedure requires an enormous number of steps and a long CPU time. About $2.5\times10^5$ steps and 3 CPU hr were required using the Fujitsu VP200 vector processor whose maximum speed is 520 Mflops. The CPU time per step per grid is about $1.7\times10^{-5}$ s.

## 3 Results

### 3.1 THE CASE OF $\gamma=1.2$

Fig. 3 shows the time evolution of the density contours with the velocity vectors. These patterns are essentially the same as those obtained in the previous calculations. The initial transient stage, in which the elephant trunk develops, is very dynamic and is difficult to follow with the numerical calculation. After about 1 period of revolution, i.e. $t\approx2\pi$, the density pattern reaches an almost steady state. Even after this stage the flow field does not settle into a completely steady state; the density in the disc increases gradually, although the basic density pattern is unchanged.

It is difficult to see the density distribution and the structure of the shocks in the central part of the disc in Fig. 3. A perspective of the density distribution of the accretion disc for $r<0.3A$ at $t=35$ is shown in Fig. 4. The $\xi$ and $\eta$ axes show the circumferential and radial mesh numbers respectively. The lower left-hand side corresponds to the inner boundary, i.e. the surface of the compact star. The gas flow is from the lower right to the upper left.

It can be seen clearly that the cliff of shocks extends towards the compact star and forms spiral shocks. The shock strength appears to become weak very close to the compact object. This is because of the sucking boundary condition on the inner boundary and is artificial.

As was stated in Section 1, pockets of low Mach number appear just behind the shocks. Fig. 5 shows the Mach number contours at $t=35$. The maximum Mach number is $M=8$ which occurs in the elephant trunk. The Mach number near the compact object is $M=4$, which is not excessively large because of the high temperature of the gas.

In order to have some idea of the thickness of the disc, let us assume hydrostatic balance in the $z$ direction. Then we have

$$\frac{\partial P}{\partial z}=-\frac{\varrho Gmz}{r^3}, \tag{3.1}$$

where $\varrho$, $P$ and $m$ are the density and pressure of the gas and the mass of the compact object respectively. If the typical scale height of the disc in the $z$ direction is $h$, we can set $\partial P/\partial z \sim P/h$ and $z \approx h$. With $P \approx \varrho c^2$, where $c$ is the velocity of sound, we have

$$h \approx cr\left(\frac{r}{Gm}\right)^{1/2}. \tag{3.2}$$

Fig. 6(a) and (b) show the distribution of the scale height $h$ based on equation (3.2). Examination of these figures suggests that the accretion disc is thin except at the shocks and the peripheral region. This is because the Mach number is greater than unity everywhere in the disc. The scale height increase behind the shocks is moderate. If $\gamma$ is raised or a cooling effect is neglected, we have a slightly thicker disc. Therefore it can be concluded that the basic assumption (2) is acceptable.

However, if we consider X-ray emission from the central region of the accretion disc, the temperature at the ridges rises because of the illumination and the cooling effect may be compensated. It is difficult to decide which effect is dominant at this stage of the investigation.

Fig. 7 shows the time history of the mass-loss rate from the mass-losing star, the mass-accretion rate on to the compact star and the mass-loss rate from the computational domain. The initial peak of the mass-loss rate from the mass-losing star is due to an initial violent out-gassing due to the extremely low density in the initial atmosphere. It can be seen that the mass-accretion rate does not reach a completely steady state even at $t=35$, although its increase is very low.

The fraction of the gas accreted on to the compact object is about two-thirds of the ejected gas in the present model. This value is consistent with our previous result (Sawada *et al.* 1986a). We can conclude that the mechanism of angular momentum loss at the shocks is really working.

### 3.2 THE CASE OF $\gamma=5/3$

In this section we give a result for the case of $\gamma=5/3$, i.e. an adiabatic gas. We computed this case up to $t=26$. Fig. 8 shows the density contours with velocity vectors at $t=18$. Fig. 9 shows a perspective view of the density distribution of the central part of the accretion disc. We can observe spiral shocks. The pitch angle of the spirals is larger than in the case of $\gamma=1.2$.

Fig. 10 shows the Mach number contours, which indicate the presence of subsonic pockets behind the shocks. Fig. 11(a) and (b) show the scale height distributions. The disc is thicker than in the case of $\gamma=1.2$ because the Mach number is lower.

Fig. 12 shows the time history of the mass-loss and mass-accretion rates. It can be seen that the mass-loss rates from the mass-losing star and the computational domain show very violent oscillations at about $t=14$ and $t=21$. In our previous paper, we obtained indications of this phenomenon, but we did not follow the evolution for such a long period. The accretion disc is substantially deformed at about $t=14$ by the infalling gas from the outer numerical boundary, and

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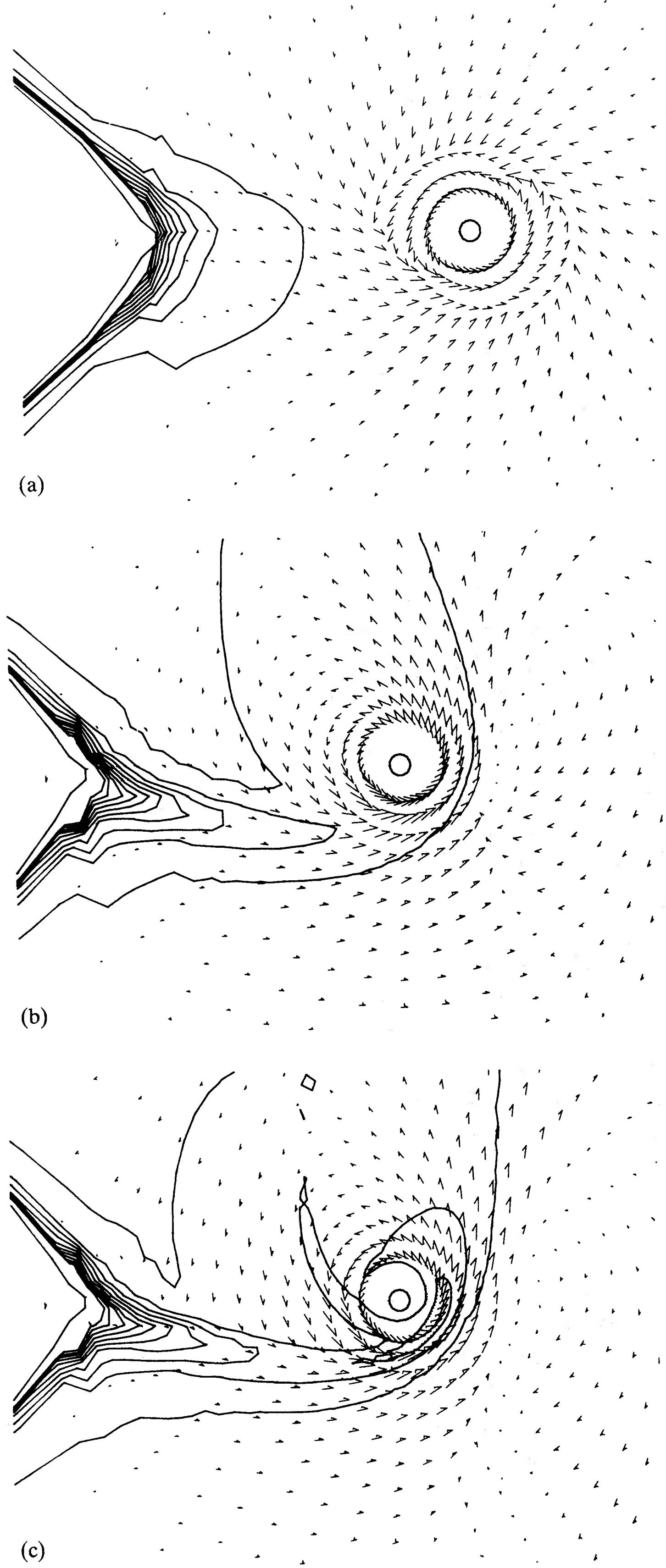
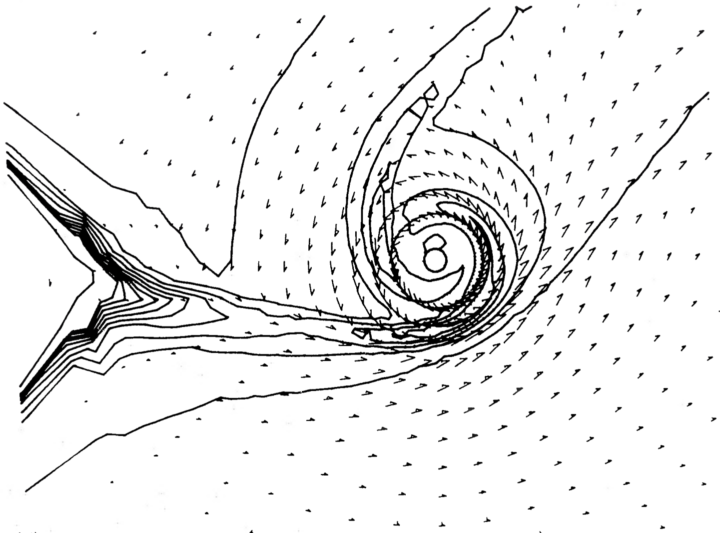


(a)

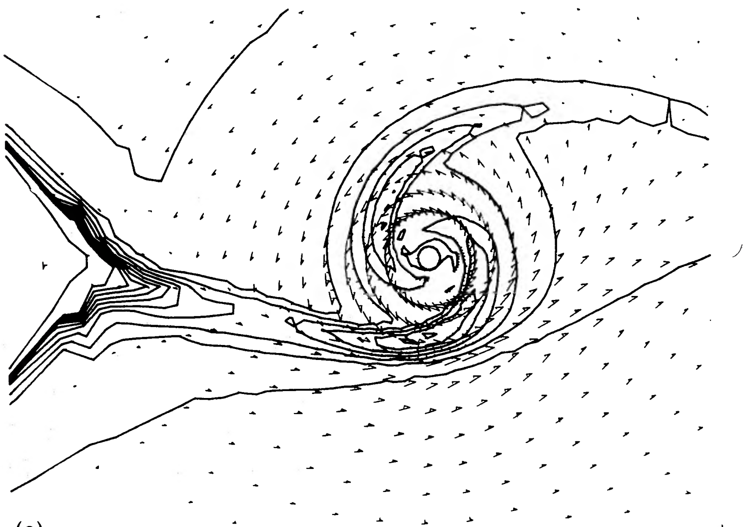
(b)

(c)

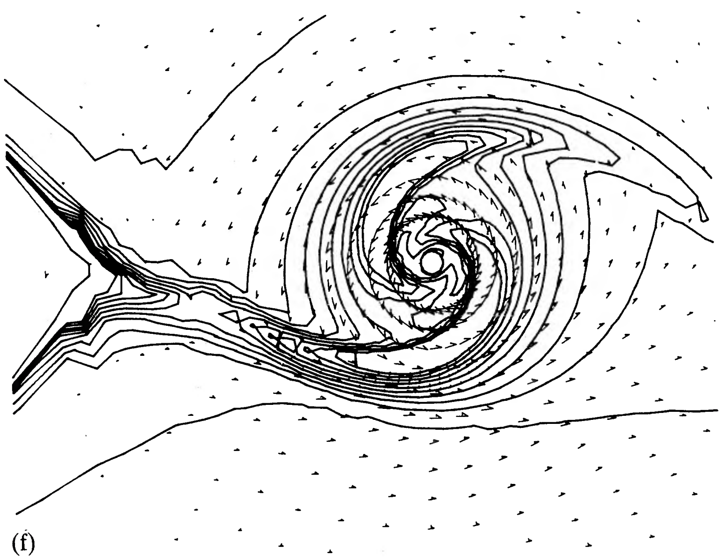
**Figure 3.** Time evolution of the density contours for the case of $\gamma=1.2$: (a) $t=0.5$; (b) $t=1.5$; (c) $T=2.5$; (d) $t=4.0$; (e) $t=6.0$; (f) $T=35$. The portion near the compact object is enlarged. The density range is 0.001–0.2, which is divided by 10 contour lines with an equal spacing.



(d)



(e)



(f)

**Figure 3**–*continued*

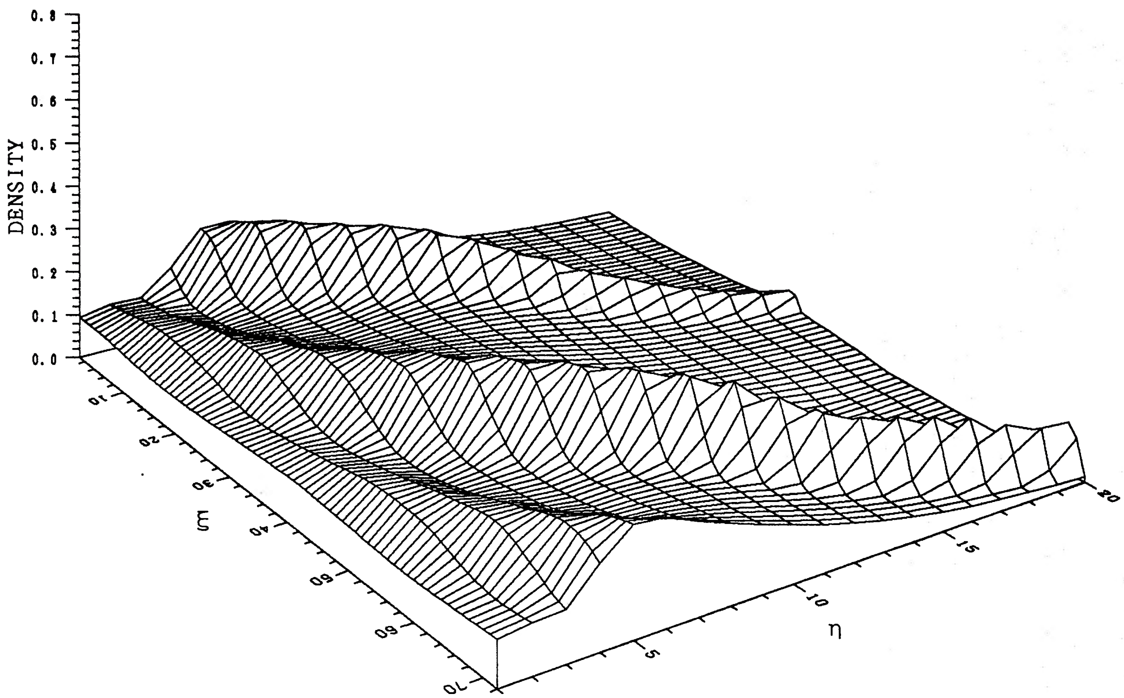



**Figure 4.** A perspective view of the density distribution of the inner accretion disc ($r<0.3A$ and $t=35$). The $\xi$ and $\eta$ axes represent the mesh numbers in the circumferential and radial directions respectively. The lower left-hand side corresponds to the surface of the compact object.

the spiral shocks almost disappear. However, the disc and the shocks appear again as is shown in the above figures. The mean accretion rate on to the compact object is about 1/3.

We do not consider that this oscillation is due to the instability in the Osher scheme, since it shows marked stability in other problems (Shima *et al.* 1985; Sawada *et al.* 1986a, b). It is possible that the oscillation is caused by an improper choice of numerical boundary conditions. In order to

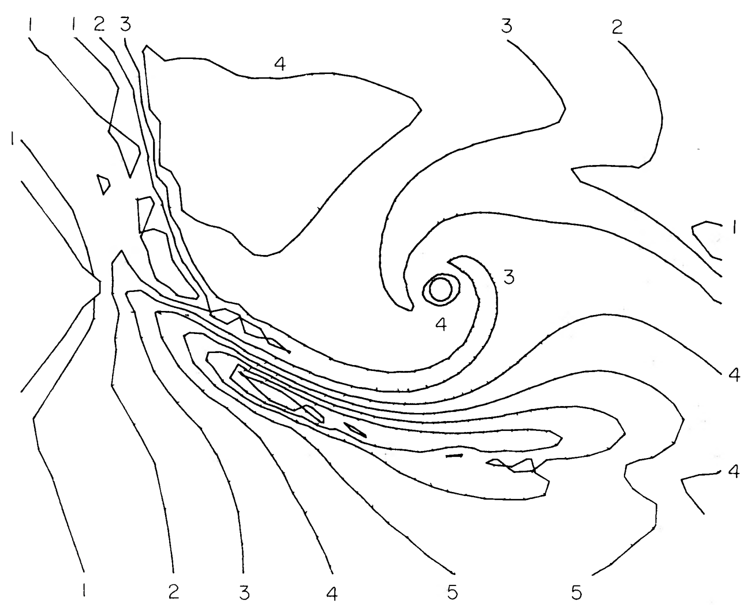
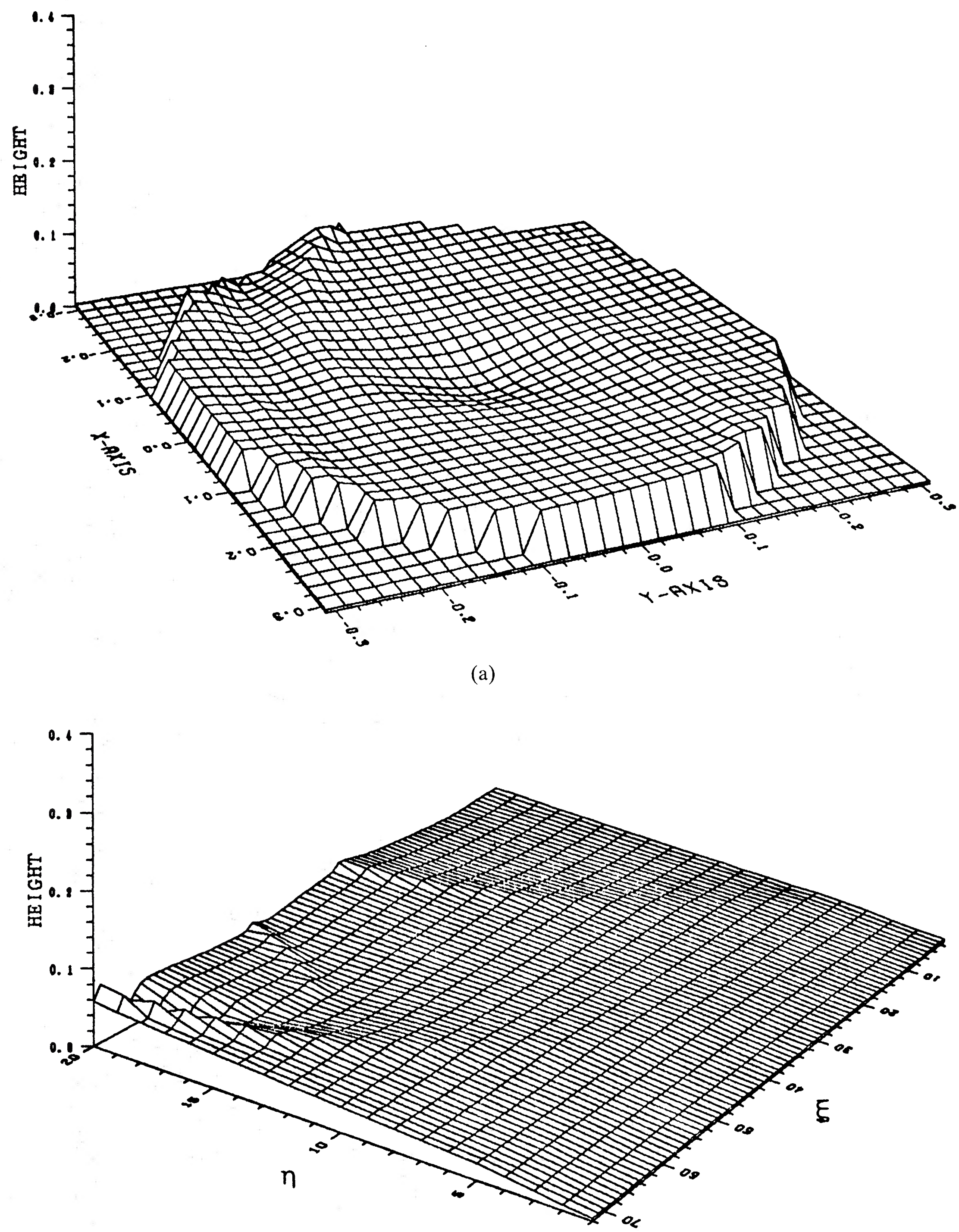


**Figure 5.** Mach number contours at $t=35$.

determine whether this phenomenon has a numerical origin, we tested other boundary conditions at the outer numerical boundary: a simple extrapolation, a tenuous gas at rest beyond the boundary and a similar gas moving to balance gravity. We also tested the case of an enlarged numerical domain. A lower temperature of the initial atmospheric gas was also tested. All cases




(a)

(b)

**Figure 6.** (a) A bird's eye view of the vertical scale-height distribution of the inner accretion disc at $t$=35; (b) a topologically deformed representation of (a). The $\xi$ and $\eta$ coordinates shown in the figure are the mesh numbers of the circumferential and radial directions respectively. The surface of the compact object is at the lower right.

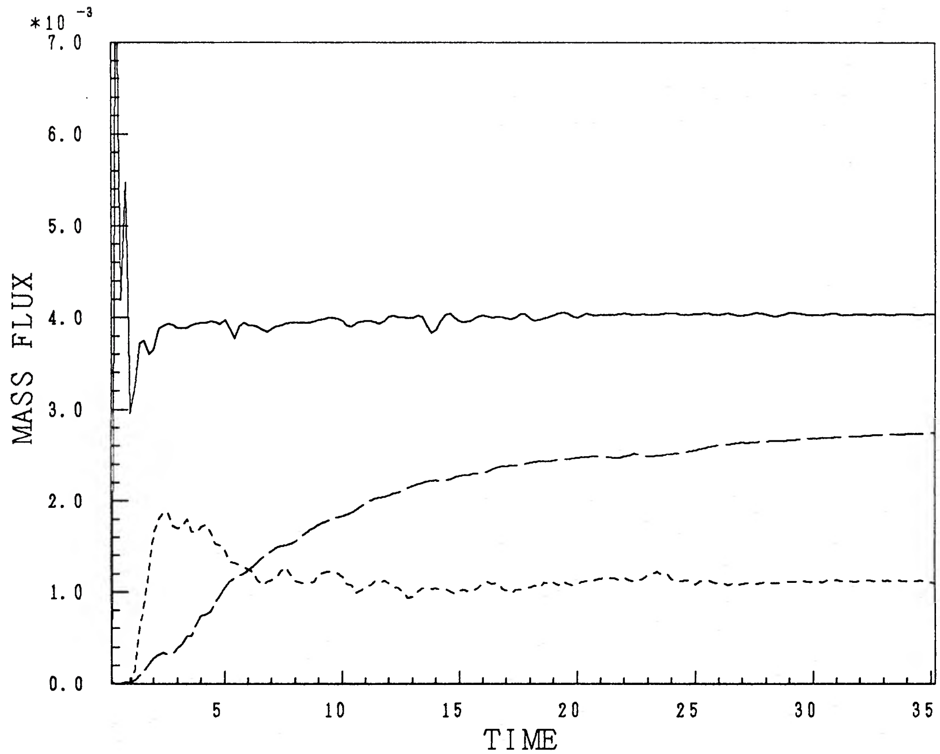



**Figure 7.** Time history of the mass-loss rate from the mass-losing star (full curve) the mass-accretion rate (broken curve) and the mass-loss rate from the numerical domain (dotted curve).

showed a similar phenomenon differing only in the exact time history. We cannot decide yet whether this oscillation is a real phenomenon or is simply numerical.

An encouraging result is that the rate of mass accretion on to the compact object does not show marked oscillation despite the violent oscillation of other quantities. The accretion disc works as a

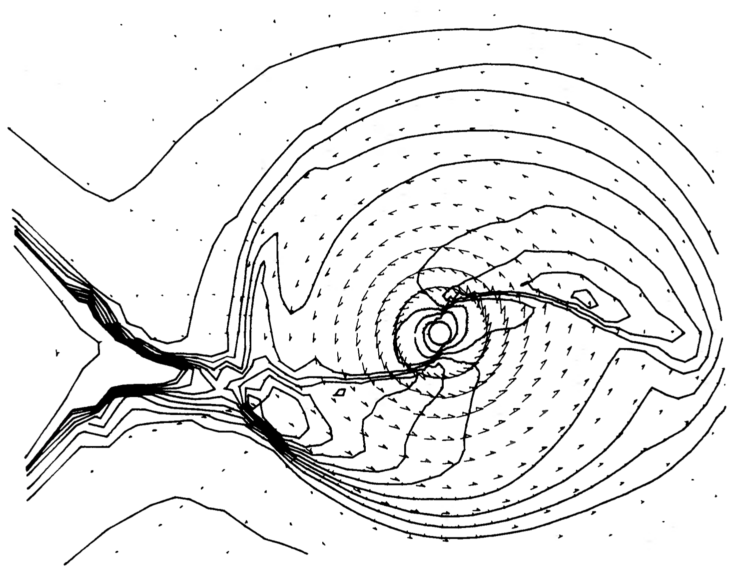


**Figure 8.** Density contours with velocity vectors at $t=18$ for the case of $\gamma=5/3$.

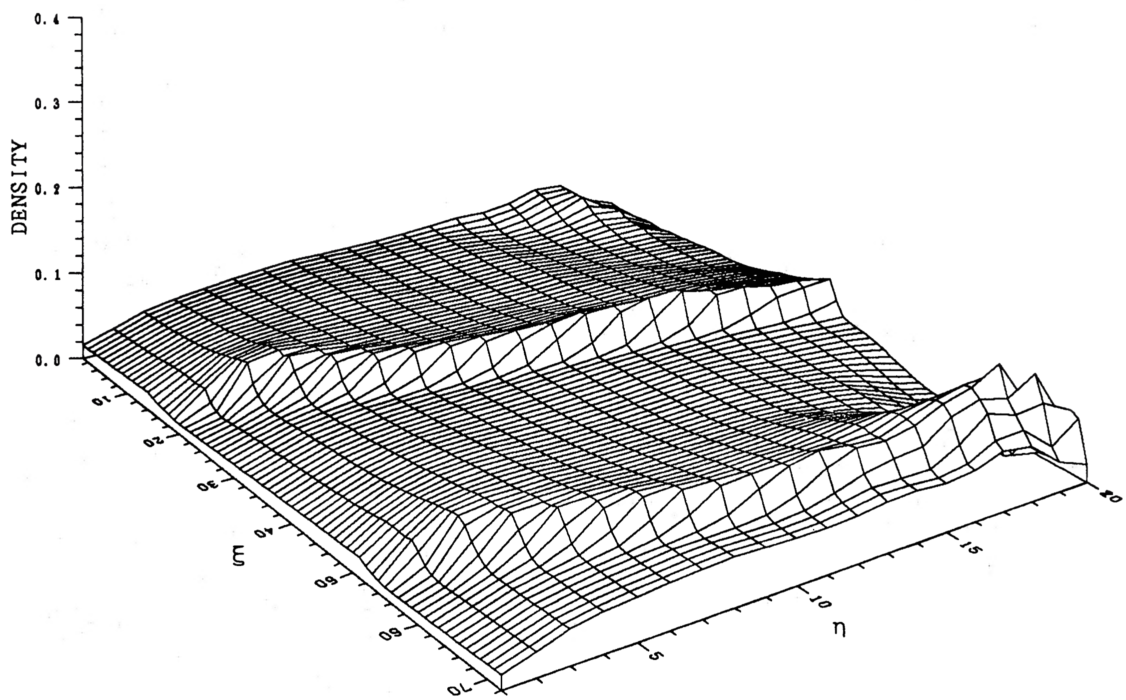



**Figure 9.** As Fig. 4 but with $\gamma=5/3$ and $t=18$.

reservoir for the accreting gas. The basic density pattern and the accretion rate are the same for all cases tested.

Recently, Spruit (1986) sought a self-similar solution in an accretion disc. He showed that there are steady self-similar solutions exhibiting logarithmic spiral shocks. However, he could not find a steady solution with two spiral shocks in a disc of constant thickness for $\gamma>1.6$. This fact may be consistent with our result on the oscillatory nature of the flow for $\gamma=5/3$.

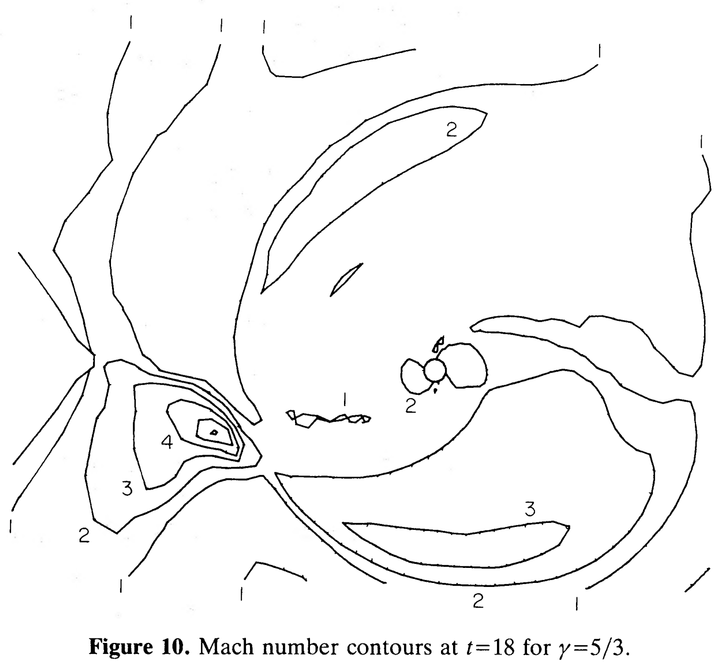
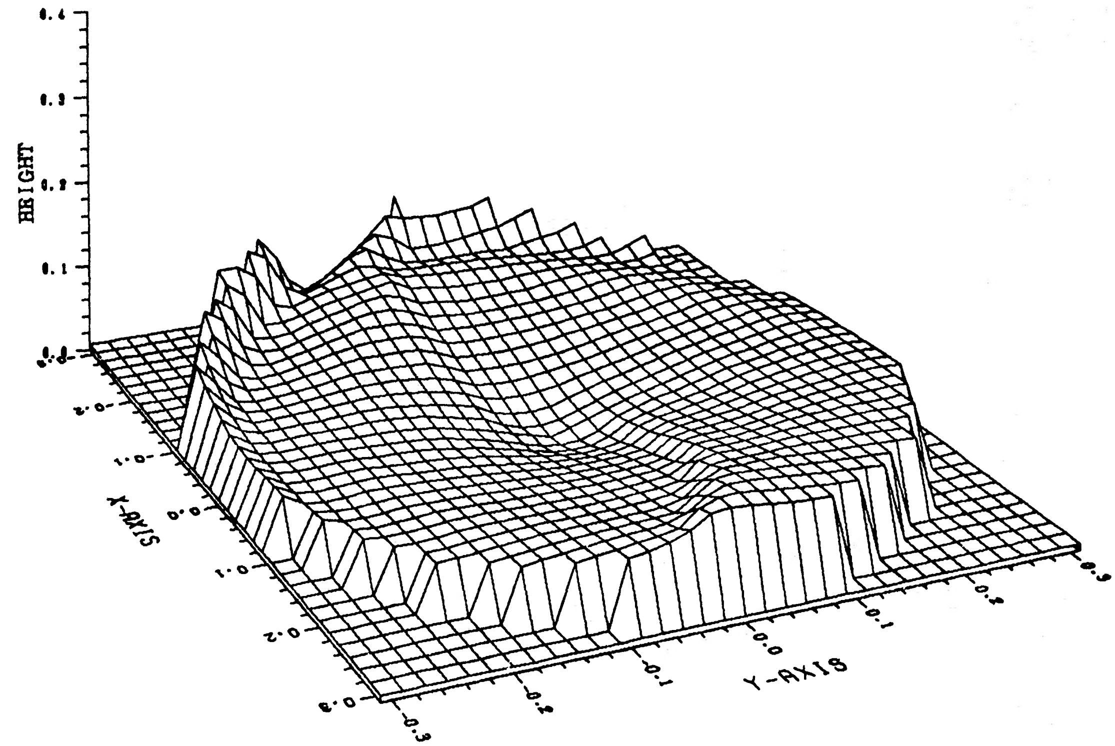
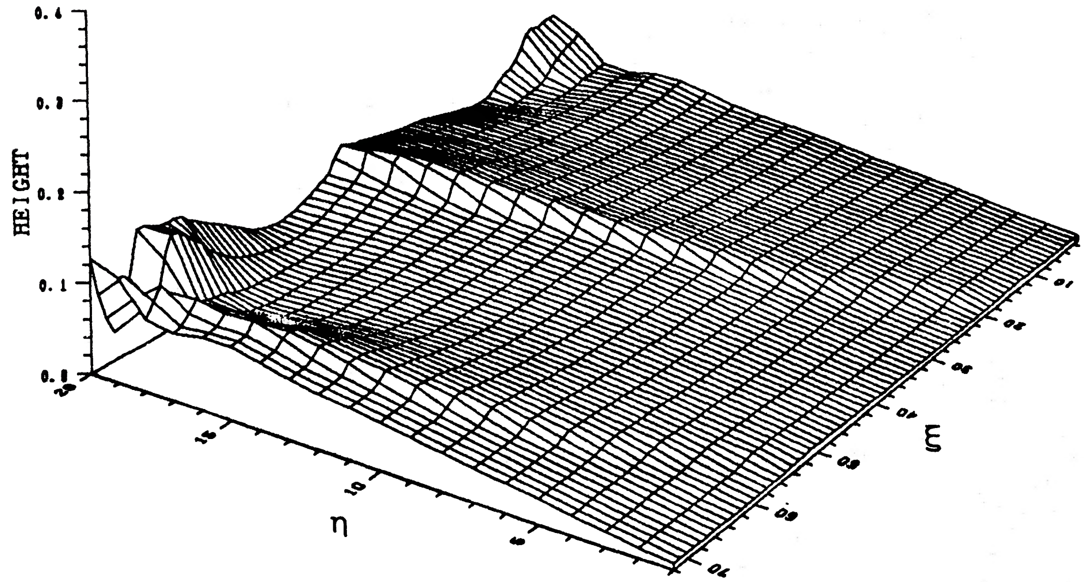


**Figure 10.** Mach number contours at $t=18$ for $\gamma=5/3$.




(a)




(b)

**Figure 11.** (a) Scale-height distribution at $t=18$ for $\gamma=5/3$; (b) topologically deformed graph of (a).

## 4 Discussion

In the present work we neglect the viscosity, but we cannot claim that the gas is dissipationless. It is really operating in the narrow shock layers and our accretion disc is dissipative in this sense. If we construct a one-dimensional accretion disc model by taking a mean value in the circumferential direction, we would have a model similar to the standard accretion disc. An important difference is that the angular momentum is not transferred in the gas but is just lost through gravitational interaction.

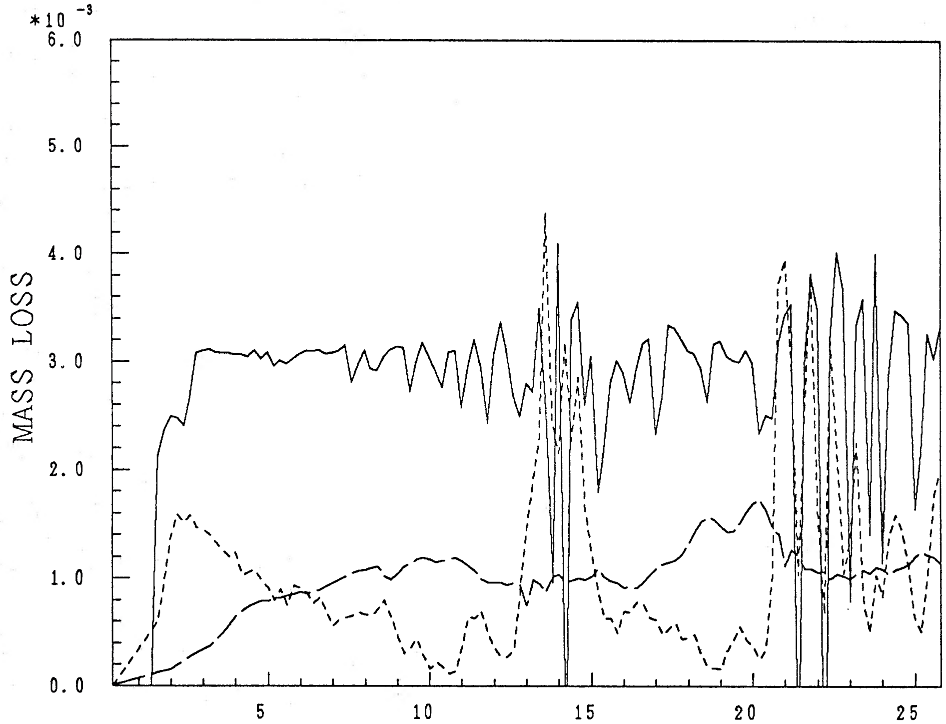



**Figure 12.** As Fig. 7 but with $\gamma=5/3$.

Spruit (1986) computed an effective $\alpha$ parameter based on his self-similar flow and obtained a value of $10^{-2}$. This value may be too small to explain the observations, and it is inconsistent with our result. This is because the maximum Mach number in Spruit's flow is only 1.360, while our Mach number is much larger. Is this discrepancy due to the assumption of self-similarity of the flow, which does not hold in a real accretion flow? Alternatively, is our numerical viscosity still too large? We need to make further investigations to clarify this discrepancy.

The size of our 'compact star' corresponds to that of a white dwarf. A neutron star is typically a factor of $10^{-3}$ of the size of a white dwarf. If a neutron star has no magnetic field or only a weak magnetic field, the accretion disc is on a much smaller scale than that investigated in the present work. Therefore, one of the most important questions relating to the present work is: How deep can spiral shocks penetrate?

Relating to this question, we can ask why tidal effects can operate so close to the accreting object where they should be weak. With regard to this point it is useful to note that a shock is an envelope of compression waves originating from a flow compression. Therefore it is possible to form shocks very close to the compact object if compression waves formed at the outer region, where tidal effects are prominent, can propagate deep inside and form envelopes. In fact Spruit's (1986) logarithmic spiral shocks show this phenomenon.

Another interesting possibility was proposed by Michel (1984), who considered spiral shocks (he called them hydraulic jumps) originating from the non-axisymmetric magnetosphere of a neutron star. Michel's waves propagate outward, while ours propagate inward. The sign of the pitch is opposite (Spruit 1986).

In the present model we fixed the value of $\gamma$. If we take account of a phase change in the gas, e.g. ionization of the gas etc., the effective value of $\gamma$ may vary from place to place and from time to time. Such a model may explain the time variability of the accretion rate and hence the variation of the luminosity of the disc. It is hoped to investigate further the structure of the accretion disc using the present model.

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