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AN ANALYSIS OF THE LIGHT CURVES OF SHORT-PERIOD RS CANUM VENATICORUM STARS: STARSPOTS AND FUNDAMENTAL PROPERTIES

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ABSTRACT

We perform an analysis of selected light curves for the short-period RS CVn group: UV Psc, XY UMa, RT And, SV Cam, BH Vir, ER Vul, WY Cnc, and Cg Cyg. We optimize the photometric fitting parameters for the "distorted" light curves in order to derive the maculation wave for each system. We then fit a dark circular spot model to the maculation wave to infer the longitudes and sizes of one or two spot groups presumed to account for these effects. Using these spot properties, we "clean" the original light curves of their distortion waves and find new, optimal solutions, which give the adopted geometrical, orbital, and physical parameters for the stars of these systems, which fall on the lower end of the main sequence. We conclude that the secondary stars in these systems are all main sequence rather than subgiants typical for RS CVn binaries.

Subject heading: stars: binaries

I. INTRODUCTION

The short-period group of the RS CVn stars (Table 1), as defined by Hall (1976) and reviewed by Milano (1981), form a particular subgroup of binary systems that exhibit observation signs of magnetic activity. With orbital periods less than a day, the stars in the short-period group are relatively close to filling their Roche lobes. It has been proposed (e.g., Popper and Ulrich 1977) that the evolution of RS CVn stars is characterized by at least one of the component stars evolving off the main sequence (but *not* to the point of Roche overflow). Hence, these stars would appear to be optimal candidates for examining the properties of cool stars prior to some dramatic change in their evolutionary status.

To help in the determination of the physical properties of such lower main sequence stars (for which little reliable information exists, as noted by Popper 1980), an analysis of their photometric light curves offers itself as a well-known procedure. However, as has been long recognized (see Wood 1946), the "distortion waves" that are the photometric hallmark of the RS CVn systems complicate a reliable estimation of the system parameters that can be directly inferred from the optical light curves. These distortion waves do, however, convey information about the active ("spotted") regions of the photospheres.

The basic problem in dealing with the photometry of these stars is to extract appropriately all the available information—ideally to separate the effects of eclipses and normal proximity interactions from those associated with the "distortions." In this paper we attribute these latter effects to phenomena having the methodologically desirable feature of formal representability, which we shall call "starspots" or "maculation" effects. We also presume that the two classes of photometric variability are additive and separable.

The shortcomings of starspot modeling for the photometric maculation waves have been clearly pointed out by Rodonò (1981, 1986), Hall (1981), Oláh (1986), and others. The problem may be put, basically, as having more parameters to adjust than available information. There is, of course, an ambiguity concerning the latitude sign, as with orbital inclination; but even the value of the modulus of the latitude is difficult to pin down—particularly in the presence of its known strong correlation with the inclination and spot radius. However, the short-period group, as identified so far, consists only of eclipsing systems, and, for these, we can establish the orbital inclinations reliably. Also, eclipsing systems present the advantage of a well-defined reference—the primary eclipse—for finding longitudes; though the simultaneous presence of the "distortion" waves and eclipses does also make for some inherent potential complications (Kron 1952; Budding 1986).

As noted by Budding, Kadouri, and Gimenez (1982), for example, if the magnetic activity of RS CVn systems is connected to a stellar dynamo driven by convection and rotation, then the short-period systems have a proportionally larger supply of rotational energy to convert to magnetic energy. In a relative sense, we might expect activity as measured by ultraviolet lines (such as the *h* and *k* Mg II doublet) to correlate with rotation. Such a correlation does appear to be present with short-period systems (Budding and Zeilik 1986). If starspots represent a manifestation of the magnetic activity, then they should correlate also with orbital periods. Long-term changes in the starspots' properties—areas, temperatures, and positions—should then give an indication of activity cycles when tracked over a sufficiently long period of time (10 yr or more; see, e.g., Baliunas and Vaughan 1985). Such relative changes can be reliably inferred *even if* the photometric starspot solutions suffer from some ambiguities.

We would also like to ascertain to what extent, or in what ways, the short-period systems may differ from the "main group" of the RS CVn stars. Milano (1981) has suggested that the nature of the primary eclipse may distinguish the short-period systems. These eclipses may be transits, rather than the occultations we would expect for main group RS CVn's. Related to this point, we should consider the physical properties of the stars to see if they are main-sequence or evolved objects, and compare their properties as a group to those of the rest of the RS CVn class. Rao and Sarma (1984) have tentatively suggested that both components of the short-period systems are main-sequence stars.

We have as our main goals in this paper to establish an effective procedure and baseline for analyzing the starspot phenomena on the short-period RS CVn stars to reveal the nature of the activity cycles, and, as well, the physical properties of the stars themselves.

II. PROCEDURE

We expand and detail here the preliminary work of Zeilik and Budding (1986*a*). The general method is as follows. First, we collect high-quality ($S/N = 100$ or better) data on the short-period systems. As observations shows that these systems have transient changes in some tens of orbital cycles (Zeilik *et al.* 1982*d*), these "best" observations are those made to fill in the light curve as quickly as possible. If such data are not available, we then use normal points from observations completed over one observing season. If the large starspot groups are stable over months (such as seen for the Sun), such observations should give the general properties of these regions averaged over the span of time of the observations.

Second, we use a 16 parameter optimizing curve fitter (Budding 1973; Budding and Najim 1980) to develop theoretical models to produce light curves which, in some sense, will "best" represent the initial data set (Fig. 1). The models include "reflection" and "ellipticity" effects (Budding 1974; Budding and Ardabili 1978). The difference between such a theoretical curve and the observed light curve defines, for our present purpose, the distortion wave that we associate with the effects of starspots.

Third, we apply another curve fitter, corresponding to the case of one or two dark, circular spots (Budding 1977), to construct representations of the distortion waves (Fig. 2). The essential difference between the two utilized optimization programs (for close eclipsing binaries and starspot effects) is that different fitting functions are used in the calculation of the goodness of fit estimator (χ^2). The spot fitter provides the parameters of the starspots (sizes, relative fluxes, and positions) on an active star.

Fourth, we "clean" the original observational data by correcting for the presence of the distortion waves by adding the theoretical distortion wave effects back to the data with the opposite sign. We than apply the fitter program to find new optimized parameters of the system (Fig. 3). These "cleaned" parameters should then provide us with information about the physical properties of the stars, which we argue to be more reliable than the original solution for the distorted light curve. A basic test for this hypothesis will come when we repeat this process for observations at different epochs, which should reveal any evolution in the starspots. The "cleaned" system parameters, should remain essentially constant with time—and, as far as the main geometric elements are concerned in fact, should be the same for all observations at all wavelengths from different photometers.

Some more details on the underlying methodology and programs have been provided elsewhere (Zeilik and Budding 1986*b*). The approach has developed from that presented by Budding and Najim (1980) and reflects some points discussed in the mathematical appendix to that paper. Thus, a significant feature of our procedure is the calculation of the curvature Hessian (second-order derivatives of χ^2 with respect to the various parameters whose values are to be optimized), as well as its inverse (the error matrix), which are numerically evaluated in the vicinity of the adopted optimum. The character of this curvature Hessian gives valuable insight into the nature of the determinacy of the fitting. Genuine determinacy is signaled by positive definiteness of the Hessian, i.e., all of its eigenvalues are positive. A failure of this condition points up, in a more objective way, the nonuniqueness problem. The appendix to Budding and Najim's (1980) paper shows that increasing the number of parameters to be determined to match a given data set increases the likelihood of a breakdown of determinacy; i.e., at some stage a negative eigenvalue will appear. This condition was observed when we allowed spot evaluation with the other

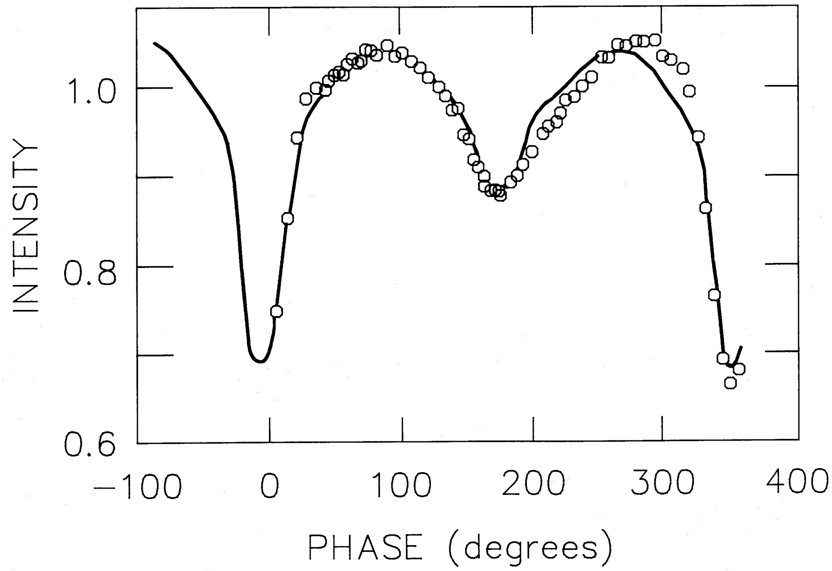

FIG. 1.—Model fit (*solid line*) to *V* band observations (*open symbols*) made at Capilla Peak Observatory. The solution is the one whose parameters are given in Table 3 for XY UMa.

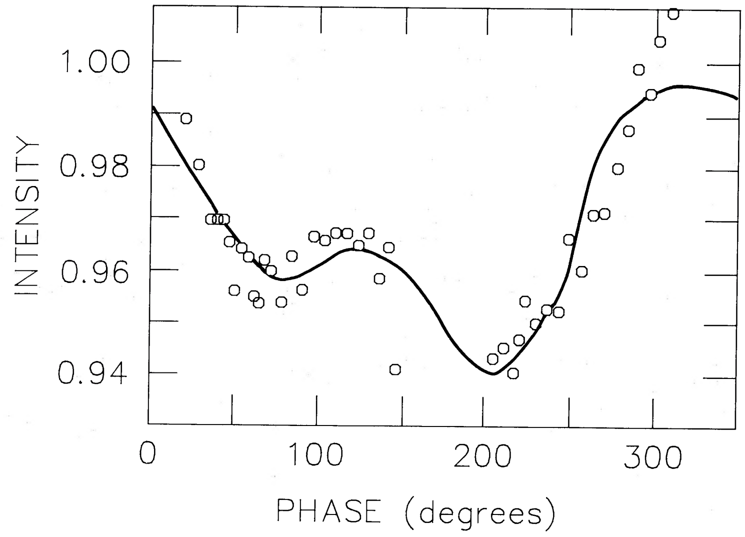

FIG. 2.—Two-spot model fit (*solid line*) to the distortion waves (*open symbols*) calculation from the *difference* between the observational points and model in Fig. 1

parameters (longitudes and radii). It is in this sense, therefore, that we speak of indeterminacy of latitudes in the particular data sets we have examined in this way.

We apply the foregoing strategy to each of the short-period systems discussed in what follows. Note that we are constrained by inherent limitations and assumptions. Theoretically, we have confined ourselves to dark, circular spot groups, which probably can represent the physical situation only very approximately. We confine our attention really only to differential effects on the brighter (primary) star. The problem of detectability affecting an array of spots distributed more or less uniformly in longitude was noted already by Evans (1971) and has been discussed, in some more particular contexts, by others since (e.g., Vogt 1981; Poe and Eaton 1985). Also, in the systems we shall consider, the primary usually contributes a large proportion of the light (~80%). It may seem reasonable to suppose that spots of roughly comparable size may well exist on both components, and that the observed variation would therefore be dominated by effects from the primary, than posit *very* large spots on the secondaries, with no contribution from the primaries, often only a few spectral subclasses up the main sequence from the cooler components.

However, it should be noted that this point of view, which is the one we have followed, has not been properly supported by previous evidence, and may even introduce some inaccuracies. Thus, Barden (1985), for instance, demonstrates an accord between a chromospheric (Hα) diagnostic of activity and a Rossby parameter for either component of several short-period RS CVn stars, including two systems discussed also in the present study. Apart from a reasonably good empirical correlation existing between spectroscopic (chromospheric) and photometric (spots) activity tracers on this group of stars (Budding and Zeilik 1986), theoretical reasons have also been advanced (e.g., Mullan 1973) why spot radii might correlate broadly with the convective turnover time scale, which forms the denominator of Barden's Rossby parameter (see also

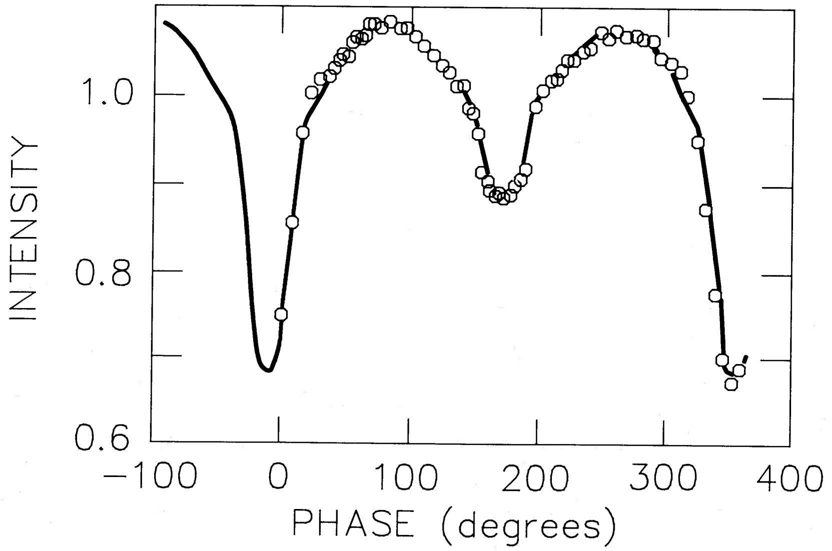

FIG. 3.—Model fit (*solid line*) to the "CLEAN" light curve (*open symbols*), derived by subtracting the model distortion wave in Fig. 2 from the observed data points (Fig. 1).

Vilhu 1984). This point merits more detailed investigation with a sufficiently extensive database. The role of eclipses too, about which we have used a quasi-empirical, interpolative approach in accounting for the interaction of maculation and eclipse effects (as in Budding 1975), could well help determine more clearly the configuration of the spotting with regard to each component, if analyzed more properly. Then, as noted before, we usually cannot resolve the latitude ambiguity; however, we do find that in a few cases the data do indicate whether the spot group is at a higher or lower latitude. The photometric models are much better constrained when accompanied by high-quality spectroscopic data, so that the spectral types are known and the effective temperatures can be inferred. Independent sources of information are especially important to inferences about the absolute physical parameters of the stars.

Observationally, the information content of the data is limited by the signal-to-noise ratio of the observations. We have found that the modeling would usually be considered informative only if the $S/N > 100$. Such precision allows us to pull out maculation effects with amplitudes ~ 0.1 mag or greater. We also note that the modeling procedure is sensitive to all parts of the light curve, so that the coverage must be complete to this degree of precision. (We weight each observational point equally.) Future photometry of RS CVn systems for the purpose of photometric light curve analysis needs to conform to these demands derived from the modeling procedure.

III. INDIVIDUAL SYSTEMS

We discuss our analysis applied to each of the short-period systems (Tables 1 and 2) and summarize the results in Tables 3, 4, and 5.

Table 3A lists the geometric r_1, r_2, and i, for which the notation is standard (see, e.g., Budding 1974) together with

TABLE 1

ACCEPTED PRELIMINARY DATA ON SHORT-PERIOD GROUP STARS

Parameters	XY UMa (star 50)	SV Cam (star 9)	RT And (star 1)	CG Cyg (star 22)	ER Vul (star 54)	BH Vir (star 105)	WY Cnc (star 14)	UV Psc (star 44)
Period (days)............	0.479	0.593	0.629	0.631	0.698	0.817	0.829	0.861
Spectral types(s)	G3 V	G3 V	F8 V	G9 V	G0 V	G0 V	G5 V	G2 V
	(K5 V)	(K4 V)	(K0 V)	(K0 V)	(G5 V)	(G2 V)	(M0 V)	(K0 IV)
Color ($B-V$)	0.93	0.72	0.50	0.87	0.68	0.64	0.61	0.81
Temperature(s) (K)	5700	5750	6250	5000	6000	6000	5500	5860
	4100	4300	4900	4800	5500	5850	3500	4800
Mass ratio (M_2/M_1)	0.80	0.71	0.65	1.0	0.90	1.02	0.60	0.80

TABLE 2

SOURCES OF UTILIZED LIGHT CURVES, WAVELENGTHS, AND CORRECTIONS FOR ZERO PHASE

Information	XY UMa (star 50)	SV Cam (star 9)	RT And (star 1)	CG Cyg (star 22)	ER Vul (star 54)	BH Vir (star 105)	BH Vir (star 105)	WY Cnc (star 14)	UV Psc (star 44)	UV Psc (star 44)
Data base	Capilla	van Woerden	Capilla	Capilla	Capilla	Sadik	Scaltriti *et al.*	Chamblis	Sadik	Capilla
	1982.17	(J) 1951.8	1981.8	1981.75	1982.83	1978.58	1984.6	1964.5	1977.9	1981.84
Wavelength	V	493.0 nm	V	V	V	V	V	"yellow"	V	V
Determined zero phase correction ($\Delta\theta_0$) (degrees)	8.4	0.0	5.9	−5.0	−15.9	−0.5	−0.6	0.1	−6.8	0.0

TABLE 3A

CURVE FITTING PARAMETERS—GEOMETRIC ELEMENTS

Parameter	XY UMa	SV Cam	RT And	CG Cyg	ER Vul	BH Vir (Sadik)	BH Vir (Scaltriti)	Wy Cnc (Chambliss)	UV Psc (Sadik)	UV Psc (Capilla)
Distorted Light Curves										
r_1 ($=r_h$)...........	0.323	0.316	0.306	0.208	0.192	0.248	0.252	0.246	0.246	0.238
	±0.003	±0.003	±0.004	±0.03	±0.004	±0.001	±0.001	±0.003	±0.004	±0.003
r_2 ($=r_c$)...........	0.158	0.204	0.231	0.245	0.237	0.240	0.226	0.158	0.182	0.181
	±0.003	±0.003	±0.004	±0.03	±0.008	±0.001	±0.001	±0.006	±0.004	±0.007
i (°)................	87.7	90.0	84.8	82.5	72.8	86.5	87.1	86.2	90.0	85.9
	±0.4	±0.5	±0.4	±0.3	±0.3	±0.2	±0.2	±0.2	±0.2	±0.2
"CLEANed" Light Curves										
r_1 ($=r_h$)...........	0.327	0.321	0.311	0.241	0.254	0.254	0.255	0.250	0.247	0.246
	±0.002	±0.003	±0.003	±0.006	±0.001	±0.001	±0.003	±0.003	±0.003	±0.006
r_2 ($=r_c$)...........	0.168	0.213	0.225	0.226	0.204	0.256	0.231	0.612	0.185	0.186
	±0.002	±0.003	±0.004	±0.009	±0.002	±0.002	±0.007	±0.003	±0.003	±0.007
i (°)................	88.2	90.0	88.9	82.8	71.6	90.0	86.8	86.0	90.0	86.0
	±0.4	±0.5	±0.2	±0.1	±0.1	±0.2	±0.3	±0.2	±0.2	±0.2

TABLE 3B

CURVE FITTING PARAMETERS—λ-DEPENDENT AND T-DEPENDENT QUANTITIES

Parameter	XY UMa	SV Cam	RT And	CG Cyg	ER Vul	BH Vir (Sadik)	BH Vir (Scaltriti)	WY Cnc (Chambliss)	UV Psc (Sadik)	UV Psc (Capilla)
					Distorted Light Curves					
L_1	0.955	0.893	0.857	0.581	0.354	0.595	0.625	0.975	0.803	0.866
	±0.004	±0.01	±0.01	±0.01	±0.01	±0.002	±0.003	±0.002	±0.008	±0.02
L_2	0.045	0.107	0.143	0.419	0.646	0.405	0.375	0.025	0.197	0.134
					"CLEANed" Light Curves					
L_1	0.930	0.901	0.866	0.745	0.652	0.573	0.634	0.976	0.811	0.878
	±0.003	±0.008	±0.008	±0.01	±0.01	±0.003	±0.01	±0.004	±0.008	±0.015
L_2	0.070	0.099	0.134	0.253	0.348	0.427	0.366	0.024	0.189	0.122
					Adopted Auxiliary Parameters					
u_1	0.70	0.64	0.65	0.65	0.7	0.83	0.83	0.65	0.65	0.70
u_2	0.70	0.88	0.75	0.70	0.7	0.80	0.80	0.88	0.80	0.80
τ_1	1.20	1.27	1.07	1.31	1.14	1.19	1.12	1.20	1.15	1.14
τ_2	1.27	1.70	1.27	1.40	1.20	1.14	1.14	1.77	1.38	1.32
E_1	0.96	2.23	0.53	1.43	0.92	1.02	1.02	2.50	0.54	0.63
E_2	1.58	0.97	2.54	1.27	1.48	1.26	1.26	0.85	2.93	2.37

TABLE 3C

CURVE FITTING STATISTICS

Parameter	XY UMa	SV Cam	RT And	CG Cyg	ER Vul	BH Vir (Sadik)	BH Vir (Scaltriti)	WY Cnc (Chambliss)	UV Psc (Sadik)	UV Psc (Capilla)
					Distorted Light Curves					
Δl	0.01	0.007	0.01	0.01	0.02	0.01	0.015	0.01	0.02	0.03
$\Delta l'$	0.014	0.021	0.02	0.03	0.01	0.007	0.015	0.01	0.02	0.02
N	67	98	109	120	133	265	153	56	115	65
χ^2	71.7	96.1	86.6	99.6	118.8	193.0	177.9	60.5	162.1	56.2
ν	61	93	103	114	127	260	147	50	110	60
χ^2/ν	1.18	1.04	0.84	0.87	0.94	0.74	1.21	1.21	1.47	0.94
					"CLEANed" Light Curves					
$\Delta l'$	0.01	0.014	0.014	0.02	0.007	0.007	0.01	0.01	0.02	0.02
N	67	98	106	120	133	265	153	56	115	65
χ^2	51.2	88.35	115.3	108.0	139.2	203.9	173.8	35.8	69.4	47.0
ν	61	93	110	115	126	260	147	50	110	60
χ^2/ν	0.84	0.95	1.15	0.94	1.10	0.78	1.18	0.72	0.63	0.78

TABLE 4

STARSPOT PARAMETERS

Parameter	XY UMa	SV Cam	RT And	CG Cyg	ER Vul	BH Vir (Sadik)	BH Vir (Scaltriti)	WY Cnc (Chambliss)	UV Psc (Sadik)	UV Psc (Capilla)
L_1	0.95	0.89	0.86	0.59	0.5	0.58	0.58	0.96	0.82	0.90
$i(°)$	88.2	90.0	88.9	82.8	71.6	90.0	86.8	...	90.0	86.0
u_1	0.70	0.64	0.70	0.70	0.7	0.83	0.83	0.65	0.65	0.70
κ_λ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$\lambda_1°$	81.4	212.0	127.8	120.8	100.8	94.0	97.4	256.1	224.6	187.9
	±3.2	±1.6	±8.0	±2.7	±5.1	±2.9	±5.7	...	±4.0	±2.0
$\beta_1°$	45	60	45	45	45	45	45	45	45	...
$\gamma_1°$	13.0	29.9	8.3	20.8	9.2	16.8	10.7	9.7	18.3	15.9
	±0.3	±0.3	±0.1	±0.5	±0.3	±0.4	±0.8	...	±0.4	±0.3
$\lambda_2°$	202.8	...	...	219.4	234.9	259.5	234.7	...	...	288.8
	±2.5	...	...	±6.8	±3.1	±2.9	±2.2	...	...	±1.6
$\beta_2°$	45	...	...	45	45	45	45	...	...	45
$\gamma_2°$	15.9	...	...	12.5	10.9	20.7	16.7	...	...	17.4
	±0.4	...	...	±0.6	±0.3	±0.5	±0.4	...	...	...
$\Delta l'$	0.007	0.01	0.01	0.014	0.0	0.015	0.02	0.005	0.01	0.007
N	44	66	88	84	105	174	111	42	72	52
χ^2	32.0	50.8	86.4	74.4	88.5	212.2	103.6	44.0	61.6	57.0
ν	39	63	85	79	100	166	106	39	70	47
χ^2/ν	0.82	0.81	1.02	0.94	0.89	1.28	0.98	1.13	0.88	1.21

TABLE 5

MASSES, RADII, AND TEMPERATURES

Parameter	XY UMa	SV Cam	RT And	CG Cyg	ER Vul	BH Vir (Sadik)	BH Vir (Scaltriti)	WY Cnc (Chambliss)	UV Psc (Sadik)	UV Psc (Capilla)
M_1 (0)	1.0[a]	0.93	1.04	0.72	0.96	1.00	1.01	0.85	0.95	0.95
M_2 (0)	0.73[a]	0.67	0.68	0.72	0.89	0.98	0.99	0.50	0.72	0.71
R_1 (0)	1.01	1.11	1.15	0.84	1.03	1.18	1.18	1.03	1.12	1.11
R_2 (0)	0.52	0.74	0.83	0.79	0.83	1.18	1.07	0.67	0.84	0.84
T_1 (K)............	5770	5750	6250	5200	6100	6000	6000	5500	5860	5860
T_2 (K)............	4580	4500	4800	4400	5800	5600	5500	<4000	4900	4900
q (spectr.)	...	0.71	0.65	1.0	0.93	0.98	0.98	0.59	0.75	0.75

[a] Assumed to agree with MS spectral types.

their formal standard deviation error assessments. Generally speaking, we should note a decrease in the value of the latter quantities as we move from the distorted to the "cleaned" light curves in response to the improved closeness of the fit.

The fractional luminosities L_1 and L_2 given in Table 3B are constrained to sum to unity. The other six quantities which influence the shape of the light curves are the limb darkening (u), gravitational (τ), and radiative (E) interaction parameters. The u-values have been taken, for the temperatures and wavelengths given in Table 1 from the data of Al-Naimiy (1978). The τ's and E's are prescribed by the simple approximation formulae given in Budding and Najim (1980). The simplicity of the underlying formulae (blackbody, Lambert's law representations, and so on) is not significant for the scale of proximity effect (~ 0.1 mag) under consideration.

Table 3C gives quantities relevant for interpreting the goodness of fit of the theoretical curves to the observed points (in sets of number N: number of "degrees of freedom" ν). The quantity Δl gives an observational estimate of the expected standard deviation error of a photometric point for the data in question. Of course, for a reasonably acceptable model χ^2/ν should be of order unity, hence in order to give a more realistic appraisal to the quoted formal errors of the determined parameters (which depend on the adopted value of observational accuracy), the observational accuracy estimate Δl has been rescaled to $\Delta l'$, with a consequent implication concerning what may be regarded as "accidental" errors from the curve-fitting point of view. That is, we are obviously prepared to disregard the systematic errors of the "distorted" light curves in order to determine a first-order solution for the parameters. Generally speaking, what should be expected, if the originally quoted value of Δl was indeed appropriate for the data in question, is that $\Delta l' > \Delta l$ for the initial fits, but then either $\Delta l' \rightarrow \Delta l$ and the parameter formal errors decrease, or in any case χ^2/ν decreases as we go from distorted to "cleaned" light curves with the same value of $\Delta l'$.

The main unknowns for the curve fitting in Table 4 are the longitudes (λ) and radii (γ) of spots. They are expressed in degrees. In some cases (e.g., SV Cam), a single spot only has been used. In all cases, except SV Cam the latitudes (β) have been set at the "intermediate" latitude 45°. For SV Cam, a distinct improvement in χ^2 was observed by increasing the latitude of the spot. The formal errors are standard deviations with the assigned datum accuracies ($\Delta l'$). L_1, i, and u_1 were set from the eclipsing binary curve fits. The ratio of spot to photospheric flux κ_λ has been set to zero ("black" spots) in all cases for the optical wavelengths of observation.

Note that the spots are assumed to exist on the brighter star. In most cases, this component dominates the combined light at optical wavelengths. A spot on the secondary would have to be larger by $\sim L_1/L_2$ in area and displaced 180° in nominal stellar longitude, to produce the same effect.

At the present time, we cannot distinguish formally between these alternatives; though we consider it unlikely that enormous spots would exist on one star, while a companion just a little bit larger and a few hundred degrees hotter (i.e., the primary) would go unspotted. On the other hand, if spot parameters, similar to those given in Table 4, were to apply also to the secondaries, they would produce a fairly negligible additional photometric variation, such as may well occur in practice.

In Table 5, a useful determinant of masses in eclipsing binary systems is provided by the combination of Kepler's Third Law with the mass luminosity relation, usually applied to the primary star, and taking the form

$$\log M_1 = 0.22[\log (1 + q) + 2 \log P + 3 \log r_1 + 6 \log T_1 - 20.99] ,$$

where M_1 is in solar units and P is the orbital period in days, r_1 is the quantity listed in Table 3A, T_1 is derived from the adopted spectral type and the mass ratio q may be known from spectroscopy (or perhaps some other procedure). A knowledge of the masses allows the absolute separation to be derived, and hence the relative radii r_1, r_2 can be converted to absolute values R_1 and R_2. Generally, it has been found that the masses and radii conform to a main-sequence–type relationship, and this may be checked further by using the mean surface flux ratio, coming from the quantities listed in Table 3B as $(L_1 r_2^2)/(L_2 r_1^2)$, and model atmosphere data to compare the appropriate pair of temperatures both with those initially adopted in Table 1 and those corresponding to the main sequence. There appears, in Table 4, to be a good overall consistency with the main-sequence picture apart, perhaps, from XY UMa, where a lack of spectroscopic data, combined with the rather low derived ratio of radii, makes for some uncertainty.

We will not review the literature on observations comprehensively for each; for more information see Milano (1981). Moreover, the present review is not exhaustive on existing data: it is recognized that there is much scope for further similar analyses on material from various sources and wavelengths. Most of the modeling is based on V band observations made at the Capilla Peak Observatory with a microcomputer-controlled photometer (Elston and Zeilik 1982) equipped with a cooled EMI 461A phototube and a Kitt Peak $UBVR$ filter set. These observations were analysed in real time so that $S/N > 100$ for each datum; if this standard was not met, a new observation was made immediately. For the stars that were not observed from Capilla (SV Cam, WY Cnc, and BH Vir), we have relied on observations from other sources, as noted in Table 2. We present the systems in order of increasing orbital period and identify them by variable star name and Hall catalog number (Hall *et al.* 1986).

a) XY UMa (Star 50)

By measure of Mg II 280 nm emission, XY UMa is the most active of all the short-period group systems (Budding, Kadouri, and Giminez 1982; Budding and Zeilik 1986). Along with ER Vul and UV Psc, XY UMa is one of the short-period RS CVn stars of our review which have been recently observed to be sources of 6 cm radio emission (Drake, Simon, and Linsky 1986) confirming Geyer's (1977) previous provisional identification. Long-term observations by Geyer (1980) indicate large-scale starspot activity on the primary (hotter) star (G2–G5 V). XY UMa is one of two short-period systems (the other is SV Cam) to have white-light flares observed from it (Zeilik, Elston, and Henson 1983). The observations we model here were made during the same time as the flaring episode (Zeilik *et al.* 1983).

The modeling (Figures 1 and 2) shows two minima (phases 0.23 and 0.56), the second at the same phase as the flare (peak at phase 0.57), which is also when the larger spot group of the two is facing Earth. The "cleaned" solution (Fig. 3) is a transit one (as has also been noted by Geyer 1980) and indicates that the secondary is, in fact, a mid-K star. However, our first transit solution indicates that the secondary is an unusually small star relative to the primary ($k = r_2/r_1 = 0.5$). We do not have the spectroscopic data for additional information on this point. We did attempt to find a solution in which the ratio of the radii was fixed at 0.7—a value that would represent a mid-K star. The optimization procedure found a solution with a lower inclination angle and a similar value for χ^2. In the absence of corroborative data, our picture of the secondary remains somewhat uncertain.

b) SV Cam (Star 9)

SV Cam was not observed from Capilla Peak; we have instead used one light curve from the massive study of this system by van Woerden (1957). These observations were made without a filter; the effective wavelength for the SV Cam observations taken to be 493.0 nm (van Woerden 1957). We chose light curve "J" from the collection, for it has the most data points and the most complete coverage of phases. Our analysis shows a single maculation wave with a minimum at phase 0.60—the same phase as that of the onset of white-light flare 1 observed by Patkós (1981). Our spot model showed a distinctly better fit for a high-latitude (~70°) rather than low-latitude spot (less than 45°).

c) RT And (Star 1)

The Capilla observations (Zeilik *et al.* 1982*c*) spanned 2 months to fill in the light curve. They indicate a single, small maculation wave with a minimum at phase =0.35; however, the solution is not well defined in this case.

Mancuso *et al.* (1981) have examined in detail the issue of whether a transit or occultation solution best fits the photometric data and the inferred physical properties of the stars. They conclude that the transit solution is the better choice, even though an optimized occultation solution is possible. In terms of the fitting procedure, any transit solution has a corresponding occultation one (though the reverse is not always the case), and only an appeal to the physical parameters of the stars (via the mass-luminosity relation and Kepler's Third Law) can resolve the question. Our solution for the "cleaned" light curve is also a transit one ($k = 0.72$), confirming the arguments of Mancuso *et al.* (1981).

d) CG Cyg (Star 22)

CG Cyg has a clearly migrating maculation wave in its light curve (Milone *et al.* 1979), as well as large apparent period variations (Milone and Ziebarth 1974). Our 1981 Capilla observations (Zeilik *et al.* 1982*c*) fit into this historic trend. The modeling analysis indicated two distortion wave minima (phases 0.34 and 0.61), which, when combined, dramatically changed the "clean" light of the system. Our fittings of the "clean" curve allowed either an occultation or a transit solution, but with a somewhat better χ^2 fit for the transit case. Jassur (1980) has analyzed his 1978 light curves and decided that the occultation solution was the more promising of the two. We note, however, that for his solution, he averaged all points outside of the eclipses to one point on each branch. We would argue that significant information is lost by this oversimplification, which can have bearing on the alternative model representations. With appropriate methods to deal with proximity and maculation effects now computable such a procedure should be bypassed.

We find that the transit solution not only gives a smaller value of χ^2 than the occultation one; it also gives a more consistent picture of a system containing two main-sequence stars, one G9 and the other K0, in agreement with the findings of Milone *et al.* (1979).

e) ER Vul (Star 54)

Our 1981 light curves (Zeilik *et al.* 1981*a*) indicated that ER Vul undergoes rapid (~1 week) light variations, with a higher dispersion in observations outside of eclipse (Zeilik *et al.* 1982*d*; Arevalo and Fuensalida 1985). Our 1982 observations, which we analyze here, were completed within 1 week (Zeilik *et al.* 1982*a*). They contain a very subtle maculation wave, with two minima (at phases 0.28 and 0.65). Our solution appears, superficially, to be similar to that of Al-Naimiy (1981), who also derived a transit-primary picture with a similar orbital inclination value. The derived stellar radii of Al-Naimiy, however, though of similar relative proportions to those presented here, are significantly larger. The photometry of Al-Naimiy, on which several maculation contributions appear to be superposed, do suggest wider eclipse minima than on our light curves, possibly as a result of these distorting effects. However, Al-Naimiy did not clarify any allowance he may have made for the effects of distortions on his photometfic solutions.

f) BH Vir (Star 105)

BH Vir is a recent addition to the RS CVn class, though Budding, Kadouri, and Gimenez (1982) have noted its similarities to the stars in the short-period list and so included BH Vir in the *IUE* observations. For the modeling, we used the observations of Sadik (1978), which were carried out in 1978. They indicate a double-minima maculation wave, but the scatter in the observations was rather large (at least 0.02 mag), so that we were not very confident of the validity of the spot fit. As a check on the results, we also ran an analysis on the light curve of Scaltriti, Cellino, and Busso (1985) for data obtained in 1984. We found a maculation wave of the same shape with two minima at about the same phases (0.26 and 0.72 from Sadik's 1978 data, and 0.27 and 0.65 from Scaltriti *et al.*'s 1984 data). Hence, we find a long-term persistence in spot groups for this system.

The optimized elements for the "clean" solutions are very similar for both light curves and also similar to the elements found by Sadik (1978). Hence, we gain confidence from this one test that our cleaning procedure is basically suitable to the situation and will result in firmer physical parameters for the stars in these systems.

g) *WY Cnc* (*Star 14*)

Awadalla (1981) and Awadalla and Budding (1979) have comprehensively reviewed the physical properties of the stars in the WY Cnc system. They concluded that, on the basis of a transit photometric solution to Awadalla's 1978 data, the secondary has a low mass, corresponding to spectral type of around M2. To have a comparison to these results, we fitted the 1964–65 yellow-band data of Chambliss (1965) and found a modest maculation wave with one minimum (at phase 0.71). The spot fitting indicated that the spot group lies below a lattitude of 45°, rather than at high latitudes. Our fit to the "clean" curve gives results generally similar to the transit solution presented by Awadalla and Budding (1979). However, we have a rather hotter and therefore more massive secondary star, probably no later than spectral type M0.

h) *UV Psc* (*Star 44*)

We used the Capilla 1981 data (Zeilik *et al.* 1982*b*) rather than our 1980 data (Zeilik *et al.* 1981*b*), which did not completely cover the light curve. The curve fit resulted in a distortion wave with two minima (phases 0.52 and 0.80). For comparison, we also fitted 1977 data from Sadik (1978), which gave us a maculation wave with one minimum (at phase 0.62). Hence, we tentatively conclude that the spot groups have evolved on the star over a four-year interval. Sadik (1978) also modeled his data with one spot and found a best fit for an equatorial spot at longitude 250° with a fractional radial size of 0.23. So the two separate procedures for the same data set give reasonably similar results, increasing our confidence in the inferred spot parameters. Finally, the "clean" solution is clearly a primary transit one and corresponds reasonably well to the optimized results of Sadik (1978).

Rao and Sarma (1984) have observed UV Psc in the 1976–1979 seasons. They rectified the normal points of the composite light curve to eliminate the maculation wave, which exhibited two minima. Their solution of their "cleaned" curve implied a transit primary with $k = 0.75$, in close accord with our results.

Busso, Scaltriti, and Cellino (1986) have examined the history of the maculation wave of UV Psc to study the spot motions and variability cycle (including the Capilla data). They note that in 1976 and 1977, the maculation wave showed two minima that they took to indicate a complex organization of spot groups. Our data show that a two-group configuration also appears at a later time, and that, in fact, the second minimum was buried in our original light curve and hard to discern.

IV. DISCUSSION AND CONCLUSIONS

Our goal has been to establish the suitability of our method for an analysis of starspots and the fundamental stellar properties of the short-period RS CVn group. We emphasize that the photometric fitting techniques are subject to severe limitations (especially when not accompanied by spectroscopic information). We cannot generally establish with certainty the latitude of spot groups, though we can say in some cases if the data are better fit with a high-latitude or low-latitude solution. We have greatly simplified the nature of the spot groups by assuming that they are circular and black (relative to the photosphere). We also have confined our attention to one star (the primary) and neglected any uniform background to the maculation effect. However, such limitations should not unduly hinder the recognition of spot cycles (if they exist). The solar analogy is the sunspot number, *R*, which severely simplifies the complexity of the Sun's spot structure, morphology, and characteristics. Yet, over time, *R* does reveal the 11 yr sunspot number cycle. In the same way, we now have established a procedure for finding *R*-like parameters for the RS CVn systems. With a homogeneous, time-serial database, we can then make consistent inferences about the gross properties of the spot groups and the nature of any spot cycles. Observations with different telescopes may be used as long as they are of high precision ($S/N > 100$) and refer to a standard photometric system.

We note that our procedures and intentions differ somewhat from those of Eaton and Hall (1979) and Poe and Eaton (1985). First, these workers have aimed at finding solutions to variations in color in order to infer the spot temperatures, as well as their geometrical properties. For only *V* band photometry, it is sufficient to assume that the spots are dark, without specifying a temperature. Second, Eaton and Hall (1979) did *not* use a numerical optimization technique (as was done by Poe and Eaton and us). Third, Poe and Eaton analyzed mostly noneclipsing systems, with the one exception of LX Per, for which the light curves had limited coverage for the distortion waves. We do agree with Poe and Eaton's conclusion that one or two circular spot groups can accurately reproduce the photometric data. However, we find that we *can* extract information from the photometry as to whether the spots are equatorial or polar from our χ^2 analysis. The situation is helped, as Poe and Eaton suggest, by the additional information coming from the circumstances of eclipses. By concentrating on eclipsing systems, and using the strategy discussed in § II, our work complements and extends that of Poe and Eaton.

As an intended consequence of this starspot modeling project, we are able to infer from the "cleaned" light curves the absolute physical parameters of the stars (Table 5). We note that the short-period group contains unevolved, main-sequence stars, one typically of spectral class (early) G, the other of spectral class (late) G or K. This configuration is associated with the transit nature of the primary eclipses (rather than occultations). This result strongly supports the suggestion of Rao and Sarma (1984) that the short-period systems contain luminosity class V stars. What drives the hotter star, which probably does not have relatively deep convective zones, to such enhanced activity? It may be an interaction with the cooler star, which should have deeper convective zones and magnetic flux tubes that may connect from the cooler to the hotter star (Uchida and Sakurai 1983). Such stellar magnetodynamics need detailed examination and a direct tie to observations of activity.

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