THE ASTROPHYSICAL JOURNAL, 318:451–458, 1987 July 1

IS THE SOLAR OBLATENESS VARIABLE? MEASUREMENTS OF 1985

R. H. DICKE and J. R. KUHN
Joseph Henry Laboratories, Physics Department, Princeton University
AND
K. G. LIBBRECHT
Solar Astronomy, California Institute of Technology
Received 1986 November 10; accepted 1986 December 17

ABSTRACT

The solar oblateness measured in 1985 is $\Delta r = r_{eq} - r_p = 14.6 \pm 2.2$ arc ms, where the error is only a formal standard deviation assuming normally distributed and uncorrelated errors. The above result is significantly greater than the 1984 value which, in turn, is significantly less than the 1983 and 1966 values. The differences may be physically significant and are consistent with the hypothesis that the oblateness oscillates with the 11.14 yr period of the solar cycle. The data at present only weakly support this hypothesis.
Subject heading: Sun: general

I. INTRODUCTION

The Princeton solar distortion telescope was used during the summers of 1966, 1983, 1984, and 1985 to measure the ellipticity of the Sun. The 1966 measurements (Dicke and Goldenburg 1967, 1974; Dicke 1981) were made at Princeton, New Jersey, and the remainder were made on Mount Wilson, California (Dicke, Kuhn, and Libbrecht 1985, 1986). Prior to the 1983 measurements the instrument was modified and improved (Libbrecht 1984; Dicke, Kuhn, and Libbrecht 1985). The instrument was further slightly modified after the 1983 season to eliminate the need for a full-time observer.

It was found that the 1966 solar distortion could not be characterized as a simple solar oblateness, a P_2-shaped solar surface. The 1966 solar distortion was not axisymmetric and was rigidly rotating over the surface of the Sun, as a wave, with a sidereal period of 12.4 ± 0.1 days (Dicke 1977, 1981). It was suggested that this distortion of the solar surface might be due to a solar core distorted by a strong magnetic field, primarily toroidal, trapped in the core and rigidly rotating with this 12.4 day period. It was also noted that a torsional oscillation of this core with the 22.28 ± 0.03 yr period previously found for the solar cycle (Dicke 1978, 1979) might be responsible for the solar cycle and could generate a second harmonic distortion, an 11.14 yr oscillating distortion of the Sun.

Recent studies of the solar "5-minute oscillations" set limits to permissible rotation rates for the solar core and to permissible distortions of the Sun (if observed in the past few years when the 5 minute observations were made) (Duvall and Harvey 1984; Brown 1985; Libbrecht 1986). Rotation rates of the core as great as $1/12.4$ d^{-1} probably cannot be excluded at present, and solar oblatenesses as small as those observed in 1983, 1984 and 1985 are also permissible.

The publication of the 1966 results for the solar oblateness quickly led to controversy, for these results raised questions about the validity of Einstein's general relativity. Also, Hill and Stebbins (1975) made an oblateness measurement in 1973 and obtained a value only one-fifth the 1966 value. But the two measurements were not equivalent; different techniques were used and the measurements were made at different times.

The magnitude of a simple solar oblateness is defined as $\Delta r = r_{eq} - r_p$ and is expressed in arc ms. The observations of 1966, 1983, and 1984 give support to the conjucture that the solar oblateness might be time dependent. The oblatenesses obtained were, respectively, $\Delta r = r_{eq} - r_p = 41.9 \pm 3.3$, 19.2 ± 1.4, and 5.6 ± 1.3 arc ms. The conjecture that the oblateness is time dependent is further supported by the 1985 observations to be discussed below. The 1985 result is $\Delta r = 14.6 \pm 2.2$ arc ms.

The above errors are standard estimates of error computed with the assumption that the residuals of the least-square fits are normally distributed and are uncorrelated. There is some correlation of residuals of daily means differing in limb exposure. The resulting correction to the estimated error of Δr is small and is estimated in the last section.

The surface rotation of the Sun induces a 7.8 arc ms solar oblateness, and this part of the oblateness carries no gravitational quadrupole moment, J_2. The quadrupole moment obtained from the solar oblateness is $J_2 = (2/3)(\Delta r - 7.8)/r$, where $r = 9.6 \times 10^5$ arc ms is the solar radius. Thus for the 1984 results, $J_2 \approx 0$.

A small-amplitude torsional oscillation of a magnetic core could generate an oscillating oblateness, sinusoidal in form. Such a curve with a 11.14 yr period, resulting from a 22.28 yr (solar oscillation) period can be fitted to the four points (Fig. 1), but this functional form is not capable of a good fit. (It has $\chi^2 = 18.8$ with one degree of freedom.) In any case, the data points are too few to provide any significant support for the idea that the oblateness is oscillating with the solar cycle. The Hill-Stebbins point for 1973 has also been plotted in Fig 1.

For large-amplitude oscillations of the core, additional even harmonics of the basic frequency $1/22.3$ yr^{-1} can be obtained. As an example, consider a P_2 distortion of the core with its symmetry axis tilted β radians relative to the solar rotation axis. Assume that the tilt angle, β, oscillates sinusoidally with the frequency $1/22.28$ yr^{-1} and with an amplitude of π about the perpendicular position, $\beta = \pi/2$. The resulting oblateness oscillates with an 11.14 yr period and has a complex waveform. This oblateness is fitted to the four data points by adjusting three parameters, a magnitude, phase, and an additive constant. This fit has $\chi^2 = 7.2$ with one degree of freedom, and again the Hill-Stebbins point is compatible with this fit. Needless to say, there probably is no physical significance in this fit.

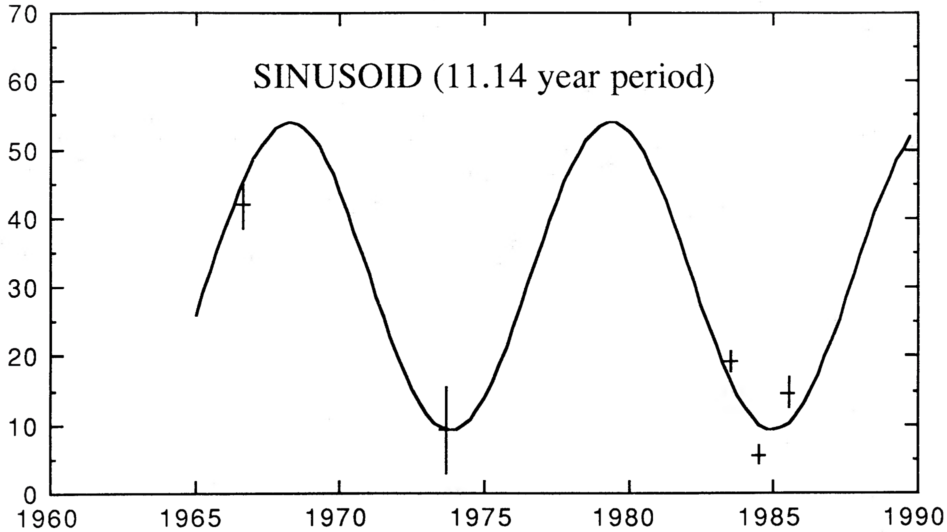

FIG. 1.—Weighted least-squares fit of sinusoidal curve of solar cycle period to the four plotted solar oblateness points of 1966, 1983, 1984, and 1985. The Hill-Stebbins point of 1973 is also plotted but not fitted. The ordinate is the solar oblateness in arc milliseconds.

II. THE 1985 OBSERVATIONS

During the interval 1985 June 1–September 30 observations of the solar ellipticity were made on Mount Wilson for a total of 72 days of which 4 days were rejected as incomplete, with one or more limb exposures missing, and 3 days were rejected as defective with 2 or more 3σ residuals per day. A full day's observations required 8 hr. An observer was needed only to start and stop the day's observations. Owing to poor weather, fewer data were obtained in 1985 than in either 1983 or 1984 for which data from 116 and 111 days, respectively, were analyzed.

The design and operation of the Princeton solar distortion telescope has been previously described (Dicke and Goldenberg 1974; Libbrecht 1984; Dicke, Kuhn, and Libbrecht 1986). The telescope is vertically mounted and projects a centered image of the Sun on an occulting disk slightly smaller than the Sun. The angular distribution of the light flux past the occulting disk is measured using a rotating disk perforated by two diametrically opposed radial slots, 2° wide. The combined light flux passed by these two slots is summed in 128 angular bins, 1°41 wide. The flux measurement is made simultaneously in two color bands, "red" and "green." Observations are made for three different amounts of the solar limb projecting beyond the occulting disk, 19″7, 14″1, and 9″1 in the "red" and designated $d = 1, 2$, and 3, respectively.

To eliminate telescope and occulting disk errors for even harmonics up to the 16th, measurements are combined for eight angular positions of the instrument, about the optic axis. In similar fashion the astigmatic errors due to the distortions of the two mirrors are removed by making the observations with four different combinations of mirror positions, requiring $\pm 90°$ rotations about their normals.

Two parameters are required to describe the solar ellipticity, and they are chosen to be the vertical and diagonal components of the ellipticity, defined, respectively, as

$$\Delta r_v = -(2/\pi) \int_0^{2\pi} \delta r(\alpha) \cos(2\alpha) d\alpha , \quad (1)$$

and

$$\Delta r_d = (2/\pi) \int_0^{2\pi} \delta r(\alpha) \sin(2\alpha) d\alpha , \quad (2)$$

where $\delta r(\alpha)$ is the limb extension beyond some arbitrary centered reference circle, such as the edge of the occulting disk, and α is the clockwise position angle of a point on the limb relative to the terrestrial north point on the solar disk, *not the solar north pole*. The differential $\delta r(\alpha)$ is expressed in arc ms.

The (positive) vertical component of the ellipticity has its minor axis in the north-south orientation. For the (positive) diagonal component, the minor axis has a northeast-southwest orientation.

For an oblate Sun with $\Delta r = r_{eq} - r_p$, $\Delta r_v = \Delta r \cos(2P)$, and $\Delta r_d = \Delta r \sin(2P)$, where $P(t)$ is the counterclockwise position angle of solar north pole relative to the north point on the solar disk. During the 1985 observational season $P(t)$ varied from $-15°6$ to $25°9$.

The mirror error measurement technique mentioned above does not determine the off-axis astigmatic error of a spherical mirror, but, averaged over the day, such an error occurs only in the vertical component of the ellipticity. To avoid this error, only the diagonal component is used to calculate the solar oblateness.

The limb extension, $\delta r(\alpha)$, which appears in equations (1) and (2), is obtained from the light flux past the occulting disk as a function of the angle α. As noted above, because of the two scanning holes, this flux is averaged across diameters. It is summed in 128 angular bins 1°41 apart.

As has been frequently noted, it is necessary to eliminate the effect of "brightness signals," principally excess light flux due to faculae, before a trustworthy measure of the limb extension can be obtained from the light flux. One of the modifications of the distortion telescope permits the observation and the rejection of faculae signals directly from the data. The occurrence of a facular signal or sunspot signal in an angular bin at the 3σ level is used to flag this and neighboring bins. These flagged bins are then ignored for the whole day. Instead of obtaining the components of the ellipticity from the integrals (1) and (2), a least-squares fit is used with the flagged bins omitted from the fit. This fit is

$$\delta r(\alpha_n) \leftarrow -0.5\Delta r_v \cos(2\alpha_n) + 0.5\Delta r_d \sin(2\alpha_n) + C , \quad (3)$$

where $n = 1, 2, \ldots, 128$ and $\alpha_{128} = \pi$. The quantities Δr_v and Δr_d are obtained as regression coefficients from the fit.

The above technique appears to eliminate the principal brightness signals satisfactorily, but other minor flux contaminants might still be present. Thus Δr_v and Δr_d cannot be interpreted directly as solar ellipticity components.

III. ATMOSPHERIC DISTORTION

More important than the effect of possible weak additional brightness signals is the effect of the atmosphere in distorting the solar image. This distortion is conveniently divided into two parts, the effect of an idealized plane laminar atmosphere and the effect of the departure from the idealized state. The ellipticity induced by a plane laminar atmosphere is easily computed from the index of refraction of the atmosphere at the observatory. It is independent of the distribution of the index with height. The necessary observatory values of the temperature, barometric pressure, and relative humidity are continually monitored. Several other atmospheric parameters are also monitored.

The laminar atmospheric distortion is large when the Sun has a zenith angle greater than 45°. Consequently, the laminar distortion is computed and removed from the vertical and diagonal components of the ellipticity for each ellipticity measurement.

The remainder of the distortion is designated as "local." It is mainly due to temperature inhomogeneity, primarily in the vicinity of the observatory and much of it at the observatory itself. The primary reason for choosing the terrestrial north point on the solar disk as the reference point, rather than the solar north pole, is that the observatory and terrain are stationary in this coordinate frame. The seeing distortion depends on the position of the Sun in the sky relative to the terrestrial coordinates.

The individual measurements of the components of the ellipticity are averaged, day by day, in six solar hour angle bins 20° wide with mid angles of $-50°, -30°, \ldots, 50°$. These six bins are designated $h = 1, 2, \ldots, 6$. The solar declination is stationary ~June 21 and is only slowly varying for most of the observational period. Furthermore, an expansion of the declination angle in cos $(2P)$ and sin $(2P)$ components yields only a small amplitude for sin $(2P)$.

The atmospheric line-of-sight atmospheric transmission averaged over the day shows very little sin $(2P)$ dependence. The north and east components of the wind velocity show no systematic variation with the season. The temperature difference between the inside and outside of the observatory averages only 1.2 °F over the season.

A substantial variation of the local atmospheric distortion with solar hour angle is expected and seen, but for each hour angle bin, h, and limb exposure, d, the contribution of the local atmospheric distortion to the diagonal component of the ellipticity is expected to be approximately constant over most of the season when the solar declination is slowly varying. By contrast, the contribution from the solar oblateness should vary through the season as sin $(2P)$, and it should be independent of d and h. But a contribution of atmospheric distortion to the sin $(2P)$ term, if appreciable, would be expected to depend upon both d and h. These expectations can be tested through the least-square fits:

$$M(d, h, P) \leftarrow \Delta r(d, h) \sin(2P) + C_A(d, h) , \quad (4)$$

and

$$D(d, h, P) \leftarrow \Delta r'(d, h) \sin(2P) + C'_A(d, h) , \quad (5)$$

where $\Delta r(d, h)$, $C_A(d, h)$, $\Delta r'(d, h)$, and $C_A'(d, h)$ are regression coefficients computed by least squares.

Here

$$M = 0.5[\Delta r_d(\text{green}) + \Delta r_d(\text{red})] \quad (6)$$

is the mean of the red and green diagonal components. Also

$$D = 0.5[\Delta r_d(\text{green}) - \Delta r_d(\text{red})] \quad (7)$$

is the color semidifference. The green and red measures of the solar oblateness should be equal and should have the same values in M. The solar oblateness contribution to sin $(2P)$ should be zero in D.

The results from the 18 least-square fits (eq. [4]) are given in Table 1. The last two columns of Table 1 are the standard estimated error in $M(d, h)$ and the number of data points, respectively. The 18 values of $\Delta r(d, h)$ do not differ significantly from their mean $\langle \Delta r \rangle = 18.1 \pm 2.4$ arc ms, and the mean has $\chi^2 = 17.4$ with 17 degrees of freedom. The oblateness, $\Delta r(d, h)$ and the seasonal mean of the local atmospheric distortion, $C_A(d, h)$, are not significantly correlated. As a function of the limb extension, $d = 1$, 2, and 3, the mean values of $\Delta r(d, h)$ are $\Delta r(d) = 17.6$, 18.4, and 18.4. These results are all compatible with the assumption that the local atmospheric distortion does not contribute appreciably to $\Delta r(d, h)$.

This conclusion is further supported by examining the 18 least-square fits (eq. [5]). The measure of the solar distortion should be color independent. Consequently the solar distortion should be absent from D, and $\Delta r'(d, h)$ should be either due to a seasonal variation of color-dependent atmospheric distortion or be due to a color-dependent solar brightness signal.

The noise in M must be largely due to color-independent image fluctuation, for the standard error in M is found to be 5.7 times as great as that of D. The values of $\Delta r'(d, h)$ obtained from equation (5) have a mean value of only -0.9 ± 0.8 arc ms. The three means $\Delta r'(d) = -2.6$, -0.9, and 0.8 are small. Contrary to the fits obtained from equation (4), $\Delta r'(d, h)$ and $C'_A(d, h)$ are now correlated with a correlation coefficient $r = -0.47$, but this is only 2 σ significant. The small mean

TABLE 1

RESULTS OF LEAST-SQUARE FITS: EQUATION (4)

d	h	Δr (arc ms)	$\sigma(\Delta r)$ (arc ms)	C_A (arc ms)	$\sigma(C_A)$ (arc ms)	$\sigma(M)$ (arc ms)	N
1......	1	25.5	12.2	35.3	5.7	34.6	51
1......	2	16.8	6.7	4.4	3.0	20.2	59
1......	3	12.7	6.1	0.1	2.8	18.2	58
1......	4	20.2	8.4	2.0	3.9	24.4	55
1......	5	13.5	7.2	8.5	3.4	20.2	50
1......	6	16.8	10.8	35.5	4.9	28.6	46
2......	1	34.7	9.8	19.5	4.5	27.9	53
2......	2	14.4	8.2	11.1	3.7	24.8	59
2......	3	19.3	7.4	−1.9	3.4	22.0	57
2......	4	15.1	8.2	−1.6	3.9	23.7	54
2......	5	17.2	8.4	10.2	3.9	23.6	50
2......	6	9.7	9.9	43.1	4.4	25.6	45
3......	1	43.4	11.5	31.2	5.3	33.1	54
3......	2	25.4	8.0	8.3	3.6	24.6	60
3......	3	23.2	7.5	−1.5	3.4	22.4	58
3......	4	12.1	8.9	3.3	4.2	26.1	55
3......	5	8.2	8.0	9.3	3.7	22.3	50
3......	6	−1.8	10.5	32.8	4.6	27.2	46

$\Delta r' = \langle \Delta r'(d, h) \rangle = -0.9 \pm 0.8$ is not statistically significant. Apparently the sin $(2P)$ dependence of the local atmospheric distortion is unimportant, and this distortion is adequately described by the seasonal mean $C_A(d, h)$.

The contribution of local atmospheric distortion to the oblateness term, sin $(2P)$, can be determined in another way, if an assumption can be made. Assuming that the functional form of the seasonal variation of the local atmospheric distortion is the same for all of the solar hour angle bins, the sin $(2P)$ variation of the atmospheric distortion is proportional to the seasonal mean $C_A(d, h)$. Then the least-squares fit $\Delta r(d, h) \leftarrow \Delta r'' + \lambda^* C_A(d, h)$ to the 18 values of $\Delta r(d, h)$ from Table 1 gives the regression coefficients $\lambda = 0.11 \pm 0.18$ and $\Delta r'' = 17.3 \pm 3.6$ arc ms, where $\Delta r''$ is the mean corrected oblateness, with the atmospheric contribution to $\Delta r(d, h)$ eliminated. This value of $\Delta r''$ can be compared with the average over d and h of $\Delta r(d, h)$, $\langle \Delta r \rangle = 18.8$. The correction of Δr required to eliminate the contribution from atmospheric distortion is $\Delta r'' - \langle \Delta r \rangle = -1.5$. It should be noted that $\lambda = 0.11 \pm 0.18$ and that this correction is not statistically significant.

If this computation is made using the 1983 data, $\lambda = 0.12 \pm 0.08$ and the correction $\Delta r'' - \langle \Delta r \rangle = -1.0$ arc ms is again insignificant. If, nonetheless, this correction is made, the result for the solar oblateness in 1983 is 18.2 ± 1.4 arc ms, where the error is now 1 standard deviation. This correction is *not* made.

For the 1984 data the corresponding results are $\lambda = 0.17 \pm 0.08$, with the correction $\Delta r' - \langle \Delta r \rangle = -2.2$ arc ms. This correction was made to the published value of Δr for 1984.

In computing the daily means of the diagonal component of the solar ellipticity it is necessary first to correct the individual binned values, $\Delta r(d, h)$, $d = 1, 2, 3$, and $h = 1, 2, \ldots, 6$, for the seasonal means of the local atmospheric distortion. If this is not done the noise level is increased by the fluctuation in the length of the observational days.

The correction for the local atmospheric distortion is made separately for each color band. Instead of using the fits (4) and (5) applied to the red and green diagonal components, an alternative improved fit based on a physical model to the atmospheric distortion is used. In an expansion of the atmospheric distortion in moments of the seeing disk the two dominant terms are the cylindrical lens distortion $C(h)$ and the anisotropic seeing distortion $S(d)A(h)$, where $S(d) = 1$, 1.3, and 1.8 (Dicke and Goldenberg 1974). These two terms are included in the least-square fits

$$\Delta r_d(d, h, P) \leftarrow \Delta r(d) \sin(2P) + C(h) + S(d)A(h) . \quad (8)$$

Here $\Delta r(d)$, $C(h)$, and $A(h)$ are regression coefficients computed by least squares.

These fits are made separately for the red and green values of $\Delta r_d(d, h, P)$. Owing to the possibility that there is a residual limb exposure dependent brightness signal, the oblateness term $\Delta r(d)$ is assumed to be a function of d for each color. The results of the fits (eq. [8]) are given in Table 2.

After correcting the diagonal components $\Delta r_d(d, h, P)$ for the atmospheric distortion by subtracting $C(h) + S(d)A(h)$, daily means of the residuals are obtained by forming weighted averages over h to give $\Delta r_d(d, P)$. For each day (parameterized by P) there are six daily means of Δr_d; i.e. there are two colors and three limb exposures.

The Sun is low in the sky for $h = 1$ and 6, and these data are inferior. A second set of daily means is obtained with $h = 1$ and 6 omitted. Owing to the loss of three more days of data through incompleteness, these means represent 62 days of data. These four-bin means are believed to be more reliable than the 65 days of six-bin means. Except where noted, these four-bin means are used below.

TABLE 2

OBLATENESS AND ATMOSPHERIC DISTORTION: EQUATION (8)

	d	h	Red (arc ms)	$\sigma(R)$ (arc ms)	Green (arc ms)	$\sigma(G)$ (arc ms)
$\Delta r(d)$	1	...	20.4	3.4	15.6	3.7
	2	...	18.9	3.2	16.7	3.5
	3	...	19.0	3.5	20.3	3.7
$C(h)$	...	1	6.9	8.4	52.0	9.7
	...	2	−14.0	7.9	14.8	9.1
	...	3	−6.3	8.0	2.2	9.3
	...	4	9.9	8.2	−7.9	9.6
	...	5	25.0	8.6	−5.2	10.0
	...	6	65.0	8.9	33.1	10.3
$A(h)$	...	1	11.6	5.9	−7.3	7.1
	...	2	11.4	5.6	−0.5	6.6
	...	3	2.1	5.6	−0.8	6.7
	...	4	−6.1	5.8	5.7	6.9
	...	5	−9.4	6.1	6.8	7.3
	...	6	−18.0	6.3	−2.8	7.5

The four parameters describing the mirror errors are found to be functions of both the color and the limb exposure d. The most likely explanation of this is that geometrical optics is not adequate to describe the mirror distortion and that the small-scale mirror irregularities diffract the reflected light through angles of the order of $\sim 10''$. But such diffraction effects should depend upon the angle of incidence of light on the mirror, and, for the primary mirror, this angle varies with the solar declination.

Thus a correction to the daily means of $\Delta r_d(d, P)$ is required by the seasonal variation of the angle of incidence of sunlight on the primary mirror. This correction is obtained from the six least-squares fits:

$$\Delta r_d(d, P) \leftarrow \Delta r(d) \sin(2P) + M_p(P) \times [A_0(d) + A_1(d)I(P) + A_2(d)I(P)^2] + C(d) , \quad (9)$$

where the five coefficients $\Delta r(d)$, $A_0(d)$, $A_1(d)$, $A_2(d)$, and $C(d)$ are obtained by least squares. Here $M_p(P) = \pm 1$ is the position variable of the primary mirror and $I(P)$ is the angle of incidence of the sunlight on the mirror, both expressed as a function of P. After determining the three coefficients $A_0(d)$, $A_1(d)$, and $A_2(d)$, the bracketed term in equation (9) is subtracted from the daily means, $\Delta r_d(d, P)$, to correct for this contribution to the mirror astigmatism. The $\Delta r_d(d, P)$ appearing below has been corrected in this way.

IV. BRIGHTNESS SIGNALS

As discussed above, a sufficiently large variation of brightness of the Sun about the solar limb could invalidate the measure of the solar ellipticity. The most important brightness signals are due to faculae and the significant facular signals have already been eliminated, day by day, by dropping angular bins (and adjacent bins) from all the least-squares fits for the ellipticity for 1 day if the bins contain a facular signal at the 3 σ level at least once for that day.

This technique is reliable, but there may be other brightness signals such as facular signals below the 3 σ level, slight photospheric darkening (or brightening) associated with facular patches, and/or a slight latitude-dependent variation of the effective temperature of the photosphere. For all three of these possibilities and for any signal due to a small change in the photospheric temperature the green brightness signal is expected to be $\sim 50\%$ greater than the red. This permits a least-squares determination of this (color-dependent) brightness signal by using the color semidifference

$$D(d, P) = 0.5[\Delta r_d(\text{green}) - \Delta r_d(\text{red})] \quad (10)$$

as a measure of the brightness signal. The color mean

$$M(d, P) = 0.5[\Delta r_d(\text{green}) + \Delta r_d(\text{red})] \quad (11)$$

is also defined. Equations (10) and (11) differ from equations (7) and (6) in that Δr_d in equations (10) and (11) are daily averages corrected for local atmospheric distortion. The correction of the primary mirror astigmatism due to variation of the angle of incidence of light on the mirror is also removed.

With the assumption that these corrections are adequate, the daily means Δr_d contain only the solar ellipticity and the brightness signals. But then equation (10) contains only the color-dependent brightness signals.

In comparison with $M(d, P)$, $D(d, P)$ has an unusually low noise level. This was also true of the 1984 and 1983 data. As with the 1984 and 1983 data, the noise content of $D(d, P)$ for $d = 1$ and 2 can be neglected. There are then two independent brightness signals contained in D. Owing to the substantially larger noise content of $D(3, P)$, the third brightness signal is not sufficiently free of noise to be used. The first brightness signals is defined as

$$D_1(P) = D(1, P) , \quad (12)$$

and the second (orthogonal) signal is

$$D_2(P) = D(2, P) - 0.73D(1, P) . \quad (13)$$

The statistical properties of the D's for all 3 yr are similar. For both the 1983 and the 1984 data the coefficient of $D(1, P)$ is -0.70, essentially the same as the above. For the past 2 yr the 3×3 covariance matrices of D, computed from the residuals of D after the removal of $\Delta r \sin (2P)$, are very similar. For the 1985 data it has the elements $C_{11}, C_{22}, C_{33}, C_{12}, C_{23}, C_{31} =$ 7.3, 7.7, 26.3, 5.3, 12.6, 8.4 arc ms^2 and for 1984 the corresponding elements are 8.8, 6.1, 26.3, 6.1, 9.1, and 7.6. For 1983 the elements are 6.2, 4.5, 12.1, 4.3, 4.7, and 3.5.

Assuming that the color mean (eq. [11]) consists of two parts, the solar oblateness and a color-dependent brightness signal, these two parts can be separated using the least-squares fit

$$M(d, P) \leftarrow \Delta r(d) \sin (2P) + C_1(d)D_1(P) + C_2(d)D_2(P) + C_0(d) . \quad (14)$$

Here the oblateness, $\Delta r(d)$, is assumed to be a function of d to allow for the possibility of a color-independent brightness signal which increases with increasing limb exposure ($d = 3$, 2, and 1). The results of this fit are given in the first three lines of Table 3. The last column is the postfit estimated error of the residuals.

The distribution of the combined residuals of these three fits is plotted in Figure 2 together with a least-squares fitted normal distribution curve (three-parameter fit). A χ^2 test of the goodness of this fit yields a $\chi^2 = 7.5$ with 9 degrees of freedom.

The 3×3 covariance matrix C_{ij} obtained from these residuals has the components $C_{11} = 119$, $C_{22} = 124$, $C_{33} = 89$, $C_{12} = 9.1$, $C_{23} = 29.3$, and $C_{31} = 1.83$ arc ms^2.

The last three lines of Table 3 are obtained from equation (14) with $C_1(d)$ and $C_2(d)$ set equal to zero. None of the six values of $\Delta r(d)$ differs significantly from their means over d. Thus there is no indication of an exposure-dependent brightness signal. The means for the first and second groups of three lines are, respectively, $\langle \Delta r \rangle = 13.3$ and 9.5 arc ms.

Comparing the above two mean values of $\Delta r(d)$, 13.3 ± 2.0 and 9.5 ± 2.4 arc ms, shows that this brightness signal contribution to $\Delta r(j)$ is small and negative, $-3.8 = (9.5 - 13.3)$ arc ms. Whatever its cause, if this color-dependent brightness signal is meaningful, it is greatest in the polar regions. For neither the 1983 or 1984 data is the contribution of a color-dependent brightness signal to $\Delta r(d)$ appreciable. An effective temperature excess of ~ 0.4 K in the polar regions would develop a brightness signal this great (-3.8 arc ms) for $d = 1$. A small polar excess of the effective temperature has been previously seen in the 1983 data in the fourth harmonic (Kuhn, Libbrecht, and Dicke 1985). Preliminary results from the 1984 and 1985 data show an excess effective temperature at the pole consistent with the 1985 result (Kuhn 1986).

The possibility of slight photospheric darkening or very weak faculae associated with facular patches suggests the inclusion of the term $C_F \Delta r_F(P)$ in the above least-squares fit. Here $\Delta r_F(P)$ is the diagonal component of the "oblateness" computed from the angular bins which contained 3σ faculae and which were dropped in fitting for the ellipticity.

With no indication of an exposure dependence of the oblateness or of C_1 and C_2 on d, the d dependence of these terms is omitted and the least-squares fit is written:

$$M(d, P) \leftarrow \Delta r \sin (2P) + C_1 D_1(P) + C_2 D_2(P) + C_F \Delta r_F(P) + C_0(d) . \quad (15)$$

The results obtained from this fit are

$$\Delta r = 14.6 \pm 2.2 \text{ arc ms}, \quad C_1 = 2.1 \pm 0.3 , \quad C_2 = 1.7 \pm 0.4 ,$$
$$C_F = -0.3 \pm 0.15 , \quad \text{and} \quad C_0(d) = 0.2 \pm 1.5 , -0.5 \pm 1.5, \quad \text{and} \quad -2.4 \pm 1.5 \text{ arc ms} ,$$

TABLE 3

LEAST-SQUARES FIT: EQUATION (14)

d	$\Delta r(d)$ (arc ms)	$\sigma(\Delta r)$ (arc ms)	$C_1(d)$	$\sigma(C_1)$	$C_2(d)$	$\sigma(C_2)$	$C_0(d)$ (arc ms)	$\sigma(C_0)$ (arc ms)	$\sigma(M)$ (arc ms)
1......	14.5	3.7	2.69	0.53	0.57	0.74	−0.3	1.7	11.3
2......	14.0	3.8	1.60	0.54	2.61	0.75	−0.5	1.7	11.5
3......	11.9	3.2	2.37	0.46	2.24	0.64	−1.9	1.4	9.8
1......	9.7	4.3	0.00	0.00	0.00	0.00	0.5	1.9	13.4
2......	11.2	4.2	0.00	0.00	0.00	0.00	−0.6	1.9	13.2
3......	7.8	4.0	0.00	0.00	0.00	0.00	−1.7	1.8	12.4

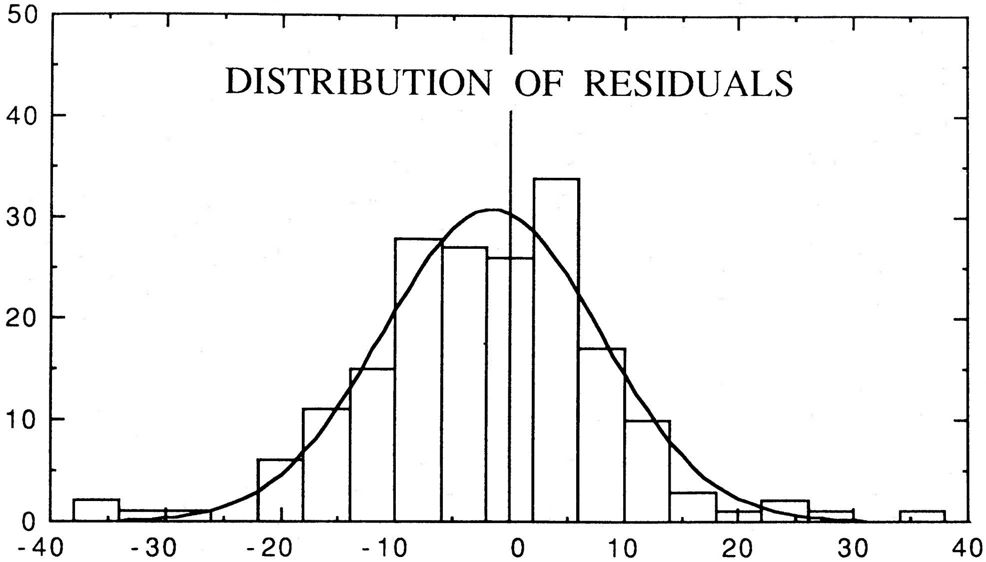

FIG. 2.—Distribution of residuals from least-squares fit (eq. [14]). Abscissa is residual in arc milliseconds.

for $d = 1$, 2, and 3. The negative sign of C_F would be obtained from a slight decrease of the photospheric brightness in the vicinity of a facular patch. But it should be noted that C_F is only 2σ significant. This term was not significant for the data obtained in 1983 and 1984. If the above least-squares fit is obtained from color means and semidifferences based on the six-bin daily means, the solar oblateness $\Delta r = 19.1 \pm 2.2$ arc ms is obtained. But the four-binned daily means are believed to be more reliable, and the value $\Delta r = 14.6 \pm 2.2$ arc ms is "adopted" for the solar oblateness in 1985. Again, the standard error is only formal and assumes normally distributed and uncorrelated errors. As noted below, the estimated standard errors should be increased by a factor of ~ 1.2 by the correlation between residuals of the same day but different disk number.

V. POWER SPECTRUM

It is of interest to examine the 1985 solar ellipticity data to see if there is any indication of the distortion seen in the 1966 data rotating rigidly over the surface with a sidereal period of 12.4 ± 0.1 days. To test for the presence of the corresponding synodic frequency, 0.078 d^{-1} and its harmonics, the residuals of $M(d, P)$ derived from the least-squares fit (eq. [14]) are averaged over d and the power spectrum of this mean is calculated. The power spectrum of these residuals is plotted in arbitrary units in Figure 3 together with vertical lines showing the locations of the frequency 0.078 d^{-1} and its harmonics. The second, third, fourth, and sixth harmonics of 0.078 d^{-1} may be present in the spectrum. There is also an indication of some of these frequencies in the 1983 data, but not in the 1984 data.

While the occurrence of four out of six frequency peaks might seem persuasive, it should be noted that the power peaks in question are not the most prominent ones, and there are five other power peaks in the spectrum as large as or larger than the four peaks in question. Also the distribution of the power peaks is similar to that expected from Gaussian noise. It is concluded that the 12.4 day period may be present in the 1985 data but that the statistical support for this is not persuasive.

VI. SUMMARY AND CONCLUSION

The observations of 1983, 1984, and 1985 have much in common, and for all 3 years:

1. The facular signals greater than 3σ are rejected.
2. Other color-dependent brightness signals are significant but do not contribute importantly to the oblateness term, $\Delta r \sin 2P$.
3. The color-independent brightness signal varying with the amount of exposed limb is statistically insignificant.
4. The local atmospheric distortions are statistically similar.
5. The three standard errors, $\sigma(\Delta r) = 1.4$, 1.3, 2.2, are substantially equal after making allowances for the number of observational days in each of the years.

The most significant differences in the results are in the solar oblateness, $\Delta r = 19.2 \pm 1.4$, 5.6 ± 1.3, and 14.6 ± 2.2 arc ms obtained, respectively, for the years 1983, 1984, and 1985.

The comparison of the result of 1966, $\Delta r = 41.9 \pm 3.3$, with the above results is not as direct as the above intercomparisons. The instrument and observatory site were different in 1966, and the local atmospheric distortion was different. The facular signal rejection method was not used. Instead a correction for the facular signal was made by introducing a proxy facular function, based on the assumption that the signal is proportional to the area of the facular patch. This facular function was included in the least-squares fit for the oblateness (Dicke and Goldenberg 1974). It was found that the facular signals were relatively unimportant and were well corrected by the least-squares procedure. The dependence of the facular signal on limb exposure obtained using this technique is consistent with the recent direct determinations of the facular signals (Libbrecht and Kuhn 1984, 1985; Dicke, Libbrecht, and Kuhn 1983).

The 1966 observations were made in a single color band. Hence there was no independent test for a color-dependent brightness signal. But after the correction for the facular signal there was no indication of an additional brightness signal varying with limb exposure. The conclusions about brightness

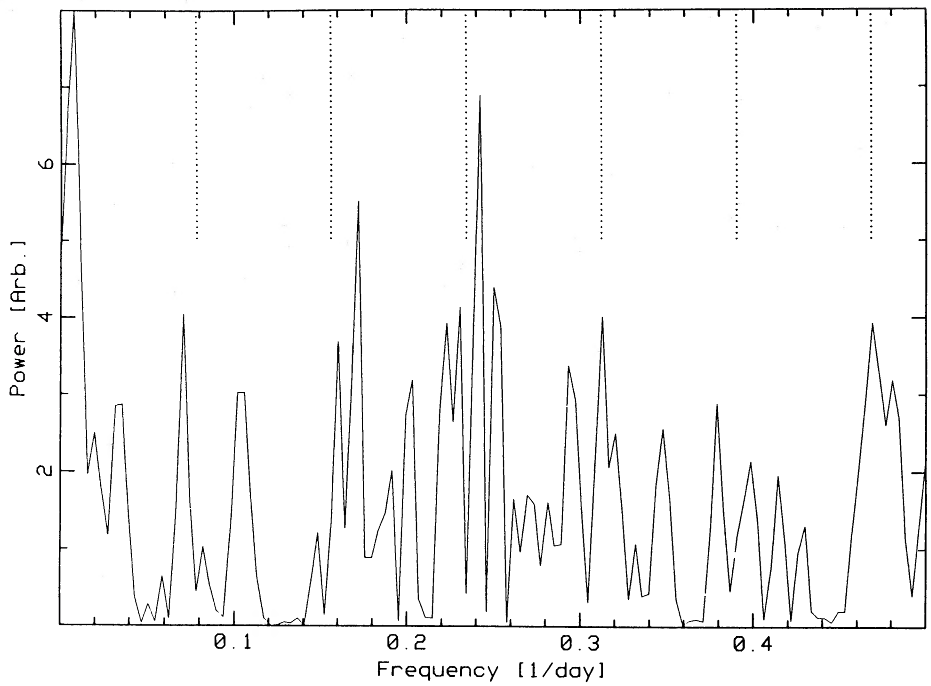

FIG. 3.—Power spectrum of the residuals from the least-squares fit (eq. [14]), averaged over *d*. Vertical lines are harmonics of the 0.078 d^{-1} synodic frequency found in the 1966 data. This frequency was interpreted as the rotation rate of the (distorted) solar core (Dicke 1981).

signals drawn in recent years are apparently consistent with the 1966 conclusions.

The larger error, 3.3 arc ms, found for the 1966 oblateness was mainly due to the relatively few observational days available in 1966. Taking this into account gives standard errors only slightly larger than those obtained in the past 3 yr.

From the above values for Δr it is evident that, if the standard errors can be accepted and if the results are free of systematic bias, the results obtained in the four years 1966, 1983, 1984, and 1985 are incompatible with the assumption that the oblateness is constant. While the main discrepancy comes from the 1966 result, the elimination of this result does not remove the difficulty. A weighted fit of a constant to the remaining three values of Δr yields a mean of 12.3 arc ms with a χ^2 of 51.9 with two degrees of freedom, a hopelessly inadequate fit.

It is possible that the above errors of the four values of the oblateness are much too small. But if they were larger by up to a factor of 3.3, the hypothesis that the oblateness was constant could still be rejected with a 99% confidence limit. Also if there were reasons to believe that the 1966 result was defective, the errors of the remaining three points could be larger by up to a factor of 2.4 with rejection of the hypothesis still at the 99% confidence limit.

There is no reason known to us why the above errors should be much too small. The residuals are normally distributed, and the errors are determined from the distribution of residuals. There is one small correction of the errors needed. The above errors assume that the residuals of the least-squares fit (eq. [15]) are independent, whereas the covariance matrix of these residuals shows that residuals of the same day but different *d* are somewhat correlated. To include the effect of this correlation the above error for the 1985 results should be increased by a factor of ~ 1.2 with similar corrections 1983 and 1984. This correction was already made to the 1966 results.

The above correction factor is obtained from the 3×3 covariance matrix obtained, above, from the residuals of the fit (eq. [4]).

One conceivable source of an error might be a year-to-year variation of some presently unknown hypothetical physical phenomenon which mimics the sin (2*P*) seasonal variation in the diagonal component of the ellipticity associated with the solar oblateness.

There is no reason known to us why the 1966 result should be defective. The symmetry of local atmospheric distortion at the Princeton site permitted its elimination from daily means. There was no significant dependence of Δr on the limb exposure, *d*. Hence, consistent with the observations of 1983–1985, there is no indication of a disk-dependent brightness signal in the oblateness after correction for the effect of faculae. Also the correction for facular signals appears to be reliable.

The quantity Δr may vary with the 11.14 yr period of the solar cycle, e.g. as in Figure 1, but perhaps with a more complex waveform. It is apparent that more observations are needed before a convincing interpretation of the apparent variation of the solar oblateness is feasible.

This research was supported in part by the NSF and by the AFGL under contract no. F19628-84-K-0024.

REFERENCES

Brown, T. M. 1985, *Nature*, **317**, 591.
Dicke, R. H. 1977, *Ap. J.*, **218**, 547.
———. 1978, *Nature*, **276**, 676.
———. 1979, *New Sci.*, **83**, 12.
———. 1981, *Proc. Nat. Acad. Sci.*, **78**, 1309.
Dicke, R. H., and Goldenberg, H. M. 1967, *Phys. Rev. Letters*, **18**, 313.
———. 1974, *Ap. J. Suppl.*, **27**, 131.
Dicke, R. H., Kuhn, J. R., and Libbrecht, K. G. 1983, *Nature*, **304**, 326.
———. 1985, *Nature*, **316**, 687.
———. 1986, *Ap. J.*, **311**, 1025.

Duvall, T. L., Jr., and Harvey, J. W. 1986, *Nature*, **310**, 19.
Hill, H. A., and Stebbins, R. T. 1975, *Ap. J.*, **200**, 471.
Kuhn, J. R. 1986, *Proc. Sac. Peak Conf.*, in press.
Kuhn, J. R., Libbrecht, K. G., and Dicke, R. H. 1985, *Ap. J.*, **290**, 758.
Libbrecht, K. G. 1984, Ph.D. thesis, Princeton University.
———. 1986, *Nature*, **319**, 753.
Libbrecht, K. G., and Kuhn, J. R. 1984, *Ap. J.*, **277**, 889.
———. 1985, *Ap. J.*, **299**, 1047.

R. H. Dicke: Joseph Henry Laboratories, Physics Department, Princeton University, Princeton, NJ 08544

J. R. Kuhn: Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824

K. G. Libbrecht: Solar Astronomy 264–33, California Institute of Technology, Pasadena, CA 91125